IISER Kolkata | Class Test - I

MA 1201 : Mathematics II

February 10, 2025 | 7 :15 - 8 :45 PM

INSTRUCTIONS

- Start each problem on a new page.
- The maximum marks you can score is **20 points.**
- Answers without justification will receive **NO** credit.
- You may use any result/theorem proved in class (unless explicitly asked to prove it), BUT you must clearly state it while applying it.

Problem 1.

(a) Find an injective map from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ and prove that it is an injection.

(b) State Schroeder–Bernstein theorem and use it to show $\mathbb{N} \times \mathbb{N}$ countable.

[4 + (2 + 3) = 9 points]

Problem 2.
Prove that $\{1, 2\}^{\mathbb{N}}$ is uncountable. [4 points]

Problem 3.
Let A be a 2×2 real matrix such that $AB = BA$ for all 2×2 real matrices B. Show that $A = aI_2$ for some real number a, where I_2 denotes the 2×2 identity matrix.

[4 points]

Problem 4.
Find the inverse of the matrix A by the Gauss-Jordan method starting with $[A \,|\, I_3]$, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix},$$

where I_3 denotes the 3×3 identity matrix. [5 points]

Vihan Ghiya

IISER Kolkata Class Test - II

MA 1201 : Mathematics II

April 07, 2025 7:15 - 8:45 PM

INSTRUCTIONS

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Problem 1.
If A is a 54 by 27 matrix of rank 13, how many independent vectors satisfy $Ax = 0$? How many independent vectors satisfy $A^T y = 0$? Justify your answer. [3 points]

Problem 2.
Find a basis for the row space, column space and null space of the matrix A and rank (A), where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}.$$

[5 points]

Problem 3.
Construct a matrix with the required property, or explain why you can't.

(a) Column space has basis $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, nullspace has basis $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

(b) Row space and nullspace both contain $(0, 0, 1)$. [3 + 4 = 7 points]

Problem 4.
Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T(1,0) = (1,4)$, and $T(1,1) = (2,3)$. What is $T(x,y)$ for $(x,y) \in \mathbb{R}^2$? Is T one-to-one? [3 + 2 = 5 points]

Problem 5.
Let $T : \mathcal{P}_1(\mathbb{R}) \to \mathcal{P}_1(\mathbb{R})$ be the linear operator defined by $T(p) = p'$, the derivative of p. Let $\beta = \{1, x\}$ and $\beta' = \{1+x, 1-x\}$ Use the change of basis matrix from β to β' to find $[T]_{\beta'}^{\beta'}$ and verify by computing $[T]_{\beta'}^{\beta'}$ independently. [5 points]

MA 1201 : Mathematics II

April 19, 2025 — 7:00 - 8:30 PM

INSTRUCTIONS

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Problem 1.
Find the determinant of the following 4×4 matrix using Gaussian elimination :

$$\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}.$$

[5 points]

Problem 2.
Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

without computing the determinant. Prove or disprove that there exists a matrix P such that $P^{-1}AP$ is diagonal. [6 points]

Problem 3.
Solve the IVP : $y' = -y^2$, with $y(2) = -\frac{1}{3}$. Find the largest interval of validity of the solution. [4 points]

Problem 4.
Solve the IVP : $xy\frac{dy}{dx} + y^2 = 2x^2$, with $y(1) = \sqrt{2}$. Find the largest interval of validity of the solution. [6 points]