LAGRANGE'S METHOD OF UNDETERMINED MULTIPLIERS

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Lagrange Multipliers: A General Definition

- Mathematical tool for constrained optimization of differentiable functions.
- Provides a strategy for finding the maximum/minimum of a function subject to constraints.

Key Terms

- **Gradient** - a normal vector to a curve (in two dimensions) or a surface (in higher dimensions)
- **Lagrange Multiplier** - a constant that is required in the lagrange function because although both gradient vectors are parallel, the directions and magnitudes of the gradient vectors are generally not equal

How to Use the Method of Lagrange Multipliers

To find the maximum and minimum values of $f(x,y,z)$ where x,y,z are subject to a constraint equation $g(x,y,z)=0$

We define a function

$$F(x,y,z,\lambda)=f(x,y,z)+\lambda g(x,y,z)$$ where λ is called Lagrange multiplier which is independent of x,y,z.

The necessary conditions for a maximum or minimum are

$$\partial F/\partial x=0,\ \partial F/\partial y=0 \text{ and } \partial F/\partial z=0$$

EXAMPLE

QUESTION The temperature $u(x,y,z)$ at any point in space is $u=400xyz^2$.Find the highest temperature on surface of the sphere $x^2+y^2+z^2=1$

SOLUTION

GIVEN:- $u=f=400xyz^2$

$g=x^2+y^2+z^2-1$

$$F(x,y,z,\lambda)=400xyz^2+\lambda(x^2+y^2+z^2-1) \dashrightarrow 1$$

Fx=0

$400yz^2+\lambda(2x)=0$

$400yz^2=-2\lambda x$

$400yz^2/2x=-\lambda$

$200yz^2=-\lambda$ ---> 2

Fy=0

$400xz^2+\lambda(2y)=0$

$400xz^2=-\lambda 2y$

$400xz^2/2y=-y$

$200xz^2/y=-\lambda$ ---> 3

$F_z=0$

$800xyz+\lambda(2z)=0$

$800xyz=-\lambda 2z$

$800xyz/2z=-\lambda$

$400xy=-\lambda$ ---> 4

From 2 and 3

$200yz^2/x=200xz^2/y$

$y/x=x/y$

$y^2=x^2$ ---> 5

From 3 and 4

$200xz^2/y=400xy$

$z^2/y=2y$

$2y^2=z^2$ ---> 6

From 5 and 6

$x^2=y^2=(½)z^2$

We have $x^2+y^2+z^2=1$

$(½)z^2+(½)z^2+z^2=1$

$(z^2+z^2+2z^2)/2=1$

$(4z^2)/2=1 \Rightarrow 2z^2=1 \Rightarrow z^2=½ \Rightarrow z=\pm 1/\sqrt{2}$

Therfore $x^2=½(½)=¼ \Rightarrow x^2=¼ \Rightarrow x=\pm 1/2$

$y^=½(½)=¼ \Rightarrow y^2=¼ \Rightarrow y=\pm 1/2$

Temperature $u=400xyz^2$

$=400(½)(½)(1/\sqrt{2})^2$

$=400*(¼)*(½)=50$

Maximum temperature is 50

THANK YOU ...