28/01/19

MODULE - 1

DISCRETE PROBABILITY DISTRIBUTION

The term probability refers to chance of happening or not happening of an event. It also gives the variability in the outcome of any experiment whose exact outcome cannot be predicted with certainity. Theory of probability provides a numerical measure of the element of uncertainty.

Numerical value of probability P(a)

$$= \frac{\text{No. of favourable cases}}{\text{Total no. of cases}}.$$

Random Experiment

It is an experiment which can be repeated n number of times under the same conditions, but doesnot give unique results. Random exprmt is also called a trial and the outcomes are called events. Eg:- Tossing a coin is a trial, getting head is an event.

Sample Space

A set of all possible outcomes of random experiment is called a sample space.

Eg:- When a coin tossed sample space S

$S = \{H, T\}$

When a coin tossed twice

$S = \{HH, HT, TH, TT\}$

when a coin tossed thrice

$S = \{HHH, HHT, HTT, TTT, TTH, THH, THT, HTH\}$

Q Consider tossing a coin thrice, find the probability of getting head

Ans. Let X be a random variable which is defined as getting head

Here the sample space is

$S = \{HHH, HHT, HTT, TTT, TTH, THH, THT, HTH\}$

Let x be 0, 1, 2, 3

$\therefore f(0) = P(X=0) = \frac{1}{8}$ $\quad f(2) = P(X=2) = \frac{3}{8}$

$f(1) = P(X=1) = \frac{3}{8}$ $\quad f(3) = P(X=3) = \frac{1}{8}$

Total probability $= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1$

we can represent this in the table form

X = getting head.

x	0	1	2	3
$P(X=x)$	1/8	3/8	3/8	1/8

Note:-

1) $\sum_x f(x) = 1$

2) $f(x) \geq 0$

Probability Distribution Function / Probability Density Function (pdf) of a discrete Random Variable

If a random variable takes only a finite number of values, it is called discrete random variable

Eg:- Marks obtained in a test.

Q Check whether the following can serves has a probability distribution. i) $f(x) = \frac{x-2}{2}$, $x = 1, 2, 3, 4$

Ans. $f(1) = -\frac{1}{2}$ $\quad f(3) = \frac{1}{2}$ $\quad$ Here sum

$f(2) = 0$ $\quad f(4) = 1$ $\quad \sum_x f(x) = 0 + \frac{1}{2} - \frac{1}{2} + 1$

$= 1$

But $f(1)$ is negative so, this is not a probability distribution function.

Q $f(x) = \frac{x^2}{25}$, $x = 0, 1, 2, 3, 4$

Ans- $f(0) = 0$ $\quad f(2) = \frac{4}{25}$ $\quad f(4) = \frac{16}{25}$

$f(1) = \frac{1}{25}$ $\quad f(3) = \frac{9}{25}$

$$\sum_x f(x) = 0 + \frac{1}{25} + \frac{4}{25} + \frac{9}{25} + \frac{16}{25} = \frac{30}{25} \neq 1$$

$\therefore$ It is not a probability distribution function

Q Find the value of k, if $f(x) = \frac{k}{2^x}$ is a probability distribution for $x = 0, 1, 2, 3, 4$.

Ans. $f(0) = \frac{k}{1} = k$ $\quad f(2) = \frac{k}{4}$ $\quad f(4) = \frac{k}{16}$.

$f(1) = \frac{k}{2}$ $\quad f(3) = \frac{k}{8}$

We know that $\sum_x f(x) = 1$

$\therefore K + \frac{K}{2} + \frac{K}{4} + \frac{K}{8} + \frac{K}{16} = 1$

$K\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right] = 1 \quad \therefore K = \frac{1}{\frac{31}{16}}$

$\therefore K = \frac{16}{31}$

29/1/19 **Cumilative distribution Functions**

The function $F(x) = P(X \leq x) = \sum_{x_i \leq x} P(X = x_i)$, $\forall -\infty < x < \infty$ is called cumilative distribution or simply distribution function of the function or random variable.

Q Find the cdf of following table.

x	-2	-1	0	1	2	3
$F(x)$	0.1	$\frac{1}{15}$	0.2	$\frac{2}{15}$	0.3	$\frac{3}{15}$

Ans. $F(x) = \begin{cases} 0, & x < -2 \\ 0 + 0.1, & -2 \leq x < -1 \\ 0 + 0.1 + \frac{1}{15}, & -1 \leq x < 0 \\ 0 + 0.1 + \frac{1}{15} + 0.2, & 0 \leq x < 1 \\ 0 + 0.1 + \frac{1}{15} + 0.2 + \frac{2}{15}, & 1 \leq x < 2 \\ 0 + 0.1 + \frac{1}{15} + 0.2 + \frac{2}{15} + 0.3, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$

$$F(x) = \begin{cases} 0, & x < -2 \\ 0.1, & -2 \le x < -1 \\ \frac{1}{6}, & -1 \le x < 0 \\ 0.367, & 0 \le x < 1 \\ 0.5, & 1 \le x < 2 \\ 0.8, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$$

Q Find the cumilative Distributive function of the following table

x	1	2	3	4	5	6
$F(x)$	1/6	1/6	1/6	1/6	1/6	1/6

→ Equally likely

Ans. $$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{6}, & 1 \le x < 2 \\ \frac{1}{3}, & 2 \le x < 3 \\ \frac{1}{2}, & 3 \le x < 4 \\ \frac{2}{3}, & 4 \le x < 5 \\ \frac{5}{6}, & 5 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$

Q A random variable has the following distribution function

x	-2	-1	0	1	2	3
$F(x)$	0.1	K	0.2	$2K$	0.3	$3K$

i) Find the value of K.
ii) Evaluate $P(X \leq 2)$
iii) Evaluate $P(-2 < x < 2)$

Ans. i) Total probability = 1

$$\therefore 0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$$
$$0.6 + 6K = 1$$
$$6K = 0.4$$
$$K = \frac{1}{15}.$$

$\therefore$

x	-2	-1	0	1	2	3
$F(x)$	0.1	$\frac{1}{15}$	0.2	$\frac{2}{15}$	0.3	$\frac{3}{15}$

ii) $P(X \leq 2) = P(X=-2) + P(X=-1) + P(X=0) + P(X=1) + P(X=2)$

$$= 0.1 + \frac{1}{15} + 0.2 + \frac{2}{15} + 0.3$$
$$= \frac{12}{15} = \frac{4}{5} = \underline{0.8}$$

iii) $P(-2 < x < 2) = P(X=-1) + P(X=0) + P(X=1)$

$$= \frac{1}{15} + 0.2 + \frac{2}{15} = \frac{2}{5}$$

Note :-

$P(X \leq a) = 1 - P(x > a)$

$P(X \geq a) = 1 - P(x < a)$

From the above example $P(X \leq 2) = 1 - P(x > 2)$

$= 1 - P(x = 3) = 1 - \frac{3}{15} = \frac{4}{5}$

Q Determine whether the following can be a pdf. of a random variable which takes the values 1, 2, 3, 4 with $f(1) = 0.24$, $f(2) = 0.24$, $f(3) = 0.24$, $f(4) = 0.24$.

Ans. $F(1) + f(2) + f(3) + f(4) = 0.24 + 0.24 + 0.24 + 0.24$

$= 0.96$.

Total probability $\neq 1$

$\therefore$ above is not a pdf.

Q Find distribution function (cdf) $F(x)$ for

x	0	1	2	3	4
$F(x)$	16/31	16/62	16/124	16/248	16/496

$\rightarrow$ pdf

Ans. $F(x) = \begin{cases} 0, & x < 0. \\ \frac{16}{31}, & 0 \leq x < 1. \\ \frac{24}{31}, & 1 \leq x < 2. \\ \frac{28}{31}, & 2 \leq x < 3 \\ \frac{30}{31}, & 3 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$

$\rightarrow$ cdf.

Q) Find the value of k so that the following is a pdf

x	0	1	2	3
$f(x)$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k+1}{3}$	$\frac{2k-1}{6}$

Ans. Total probability is 1

$$\therefore \frac{k}{2} + \frac{k}{3} + \frac{k+1}{3} + \frac{2k-1}{6} = 1$$

$$\frac{3k}{6} + \frac{2k}{6} + \frac{2k+2}{6} + \frac{2k-1}{6} = 1$$

$$3k + 2k + 2k + 2 + 2k - 1 = 6$$

$$9k + 1 = 6$$

$$9k = 5$$

$$k = \frac{5}{9}$$

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Mean & Variance of a Probability distribution

$$\text{Mean}(\mu) = E(x) = \Sigma x f(x)$$

$$\text{Variance } (\sigma^2) = E(x^2) - E(x)^2$$

$$= \Sigma x^2 f(x) - \mu^2$$

standard deviation SD $(\sigma) = \sqrt{\text{variance}}$.

E - expectation.

Note :-

$E(aX+b) = aE(X)+b.$

$V(aX+b) = a^2 V(X)$

Q. Find the mean of p.d. of no. of heads obtained in 3 flips of a coin.

Ans. $S = \{HHH, HHT, HTT, HTH, THT, THH, TTH, TTT\}$

X = getting heads.

$x = 0 \quad 1 \quad 2 \quad 3$

x	0	1	2	3
$f(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$\text{Mean}(\mu) = \Sigma x f(x)$$

$$= 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8}$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

Q. Find Mean and Variance of

x	0	1	2	3	4	5
$f(x)$	0.2	0.3	0.2	0.1	0.1	0.1

Ans. $\text{Mean}(\mu) = \Sigma x f(x).$

$= 0\times0.2 + 1\times0.3 + 2\times0.2 + 3\times0.1 +$

$4\times0.1 + 5\times0.1$

$= 1.9$

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Variance $(\sigma^2) = \Sigma x^2 f(x) - \mu^2$.

$= (0^2 \times 0.2) + (1^2 \times 0.3) + (2^2 \times 0.2) + (3^2 \times 0.1)$
$+ (4^2 \times 0.1) + (5^2 \times 0.1) - (1.9)^2$

$= 6.1 - (1.9)^2$

$= \underline{2.49}$

Q Find mean and variance of

x	0	1	2	3	4
$f(x)$	0.05	0.2	0.45	0.2	0.1

Ans Mean $(\mu) = \Sigma x f(x)$

$= (0 \times 0.05) + (1 \times 0.2) + (2 \times 0.45) +$
$(3 \times 0.2) + (4 \times 0.1)$

$= \underline{2.1}$

Variance $(\sigma^2) = E(x^2) - E(x)^2$

$= \Sigma x^2 f(x) - \mu^2$

$= (0^2 \times 0.05) + (1^2 \times 0.2) + (2^2 \times 0.45) +$
$(3^2 \times 0.2) + (4^2 \times 0.1) - (2.1)^2$

$= 5.4 - (2.1)^2$

$= \underline{0.99}$

Q Find mean and variance of p.d.

x	1	2	3	4
~~$f(x)$~~ $P(X=x)$	$1/6$	$2/6$	$2/6$	$1/6$

. Also calculate Mean & and variance of

(i) $y = 2\cancel{x}+3$

(ii) $z = \sin X$ (sin values should calculate in radian).

Ans. Mean $(\mu) = \Sigma x f(x)$

$$= \left(1\times\frac{1}{6}\right) + \left(2\times\frac{2}{6}\right) + \left(3\times\frac{2}{6}\right) + \left(4\times\frac{1}{6}\right)$$

$$= \frac{1}{6} + \frac{4}{6} + \frac{6}{6} + \frac{4}{6}$$

$$= \frac{5}{2}$$

Variance$(\sigma^2) = E(x^2) - E(x)^2$

$$E(x^2) = \left(1^2\times\frac{1}{6}\right) + \left(2^2\times\frac{2}{6}\right) + \left(3^2\times\frac{2}{6}\right) + \left(4^2\times\frac{1}{6}\right)$$

$$= \frac{1}{6} + \frac{8}{6} + \frac{18}{6} + \frac{16}{6}$$

$$= \frac{43}{6}$$

$$\sigma^2 = \frac{43}{6} - \left(\frac{5}{2}\right)^2$$

$$= \frac{11}{12}$$

(i) $Y = 2X+3$

Mean $= E(Y) = E(2X+3) = 2E(X)+3$

$= 2 \times \frac{5}{2} + 3 = \underline{\underline{8}}$

Variance $(Y) = V(2X+3) = 2^2 \times \cancel{E(x)} V(x)$

$= 4 \times \frac{11}{12} = \underline{\underline{\frac{11}{3}}}$

Mean $= E(Z) = E(\sin x)$ $\quad E(x) = \Sigma x f(x)$

$= \Sigma \sin x \, f(x)$

$= \sin 1 \times f(1) + \sin 2 \, f(2) + \sin 3 \, f(3) +$

$\sin 4 \times f(4)$

$= \left(0.8415 \times \frac{1}{6}\right) + \left(0.9093 \times \frac{2}{6}\right) + \left(0.1411 \times \frac{2}{6}\right) +$

$\left(-0.7568 \times \frac{1}{6}\right)$

$= \underline{\underline{0.36425}}$

Variance $= V(Z) = V(\sin x)$

$E(z^2) = \Sigma \sin^2 x \, f(x)$

$= \left(\sin^2(1) \times \frac{1}{6}\right) + \left(\sin^2(2) \times \frac{2}{6}\right) +$

$\left(\sin^2(3) \times \frac{2}{6}\right) + \left(\sin^2(4) \times \frac{1}{6}\right)$

$= \left(0.7081 \times \frac{1}{6}\right) + \left(0.8268 \times \frac{2}{6}\right) +$

$\left(0.0199 \times \frac{2}{6}\right) + \left(0.5728 \times \frac{1}{6}\right)$

$= 0.4957$

$\therefore$ Variance (z) $= 0.4957 - (0.36425)^2$

$= \underline{0.3630}$

Q) Find expectation of $\overline{Y}$, $(E(Y))$, where $Y = X^2$, Let X be the random variable with probability distribution $P(X=1) = 0.1$, $P(X=2) = 0.2$, $P(X=3) = 0.4$, $P(X=4) = 0.3$.

Ans.

x	1	2	3	4
$f(x)$	0.1	0.2	0.4	0.3

$E(Y) = E(X^2) = \Sigma x^2 f(x)$

$= (1^2 \times 0.1) + (2^2 \times 0.2) + (3^2 \times 0.4) + (4^2 \times 0.3)$

$= \underline{\underline{9.3}}$

Q) The possible values of a discrete random variable are 1, 2, 3, 4, 5. If all the values are equally likely. Find probability mass function (pdf) and cdf of X. Find also the probabilities

(i) $P(2 \leq X < 5)$

(ii) $P(x > 3)$

(iii) $P(x$ is an even integer$)$

Ans.

x	1	2	3	4	5
$f(x)$	K	K	K	K	K

Here we know total probability $\sum f(x) = 1$

$5K = 1 \therefore K = \frac{1}{5}$.

$\therefore K = \frac{1}{5}$.

$\therefore$ pdf is

x	1	2	3	4	5
$f(x)$	$\frac{1}{5}$	$1/5$	$1/5$	$1/5$	$1/5$.

$f(x) = \frac{1}{5}$, $x = 1, 2, 3, 4, 5$.

0, otherwise.

cdf.

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{5}, & 1 \le x < 2 \\ \frac{2}{5}, & 2 \le x < 3 \\ \frac{3}{5}, & 3 \le x < 4 \\ \frac{4}{5}, & 4 \le x < 5 \\ 1, & x \ge 5 \end{cases}$$

$$P(2 \le X < 5) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

$$= \frac{3}{5}$$

$$P(x > 3) = 1 - P(X \le 3)$$

$P(X=4) + P(X=5) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

iii) $P(X=2) + P(X=4) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

Q. The cdf of random variable X is given by ~~F(x) =~~

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 2 \\ 0.75, & 2 \le x < 3 \\ 0.90, & 3 \le x < 5 \\ 1, & x \ge 5 \end{cases}$$

Find probability mass function, also find $P(X \le 3)$ and $P(2 \le x < 5)$.

Ans. pdf is given by,

x	0	2	3	5
$f(x)$	0.5	0.25	0.15	0.10

$P(X \le 3) = P(X=0) + P(X=2) + P(X=3)$

$= 0.5 + 0.25 + 0.15 = \underline{0.9}$

$P(2 \le x < 5) = P(X=2) + P(X=3)$

$= 0.25 + 0.15 = \underline{0.4}$

Q. The values of a random variable X are integers from 0 to 6. The table below gives the cumilative distribution at each of this value. Find pmf, mean and variance of X.

X	0	1	2	3	4	5	6
F(x)	0.05	0.20	0.42	0.64	0.81	0.91	1

Ans. From the table, pmf is

x	0	1	2	3	4	5	6
f(x)	0.05	0.15	0.22	0.22	0.17	0.1	0.09

Mean $(\mu) = E(x)$

$= \sum x f(x)$

$= (0\times 0.05) + (1\times 0.15) + (2\times 0.22) + (3\times 0.22) + (4\times 0.17) + (5\times 0.1) + (6\times 0.09)$

$= \underline{2.97}$

Variance $(\sigma^2) = E(x^2) - E(x)^2$

$E(x^2) = (0^2\times 0.05) + (1^2\times 0.15) + (2^2\times 0.22) + (3^2\times 0.22) + (4^2\times 0.17) + (5^2\times 0.1) + (6^2\times 0.09)$

$E(x^2) = \underline{11.47}$

$\therefore \sigma^2 = 11.47 - \mu^2$

$= 11.47 - (2.97)^2$

$= \underline{2.6491}$

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Binomial distribution

Consider an experiment with 2 possible outcomes called success or failure. Let the probability of success is same for each trial. And outcomes of different trials are independant. These trials are called Bernoulli trial. Let X be the random variable that equals no. of successes in n trials and let probability of success be p. Therefore, probability of failure is $q = 1-p$. Therefore probability of success and failure $= p^x \times q^{n-x}$.

No. of ways in selecting x successes from n trials is ${}^nC_x$. Therefore pdf of binomial distribution is given by

$$\boxed{b(x,n,p) = {}^nC_x\, p^x q^{n-x}}$$

where n - no. of trials

x - no. of success

p - probability of success

q - probability of failure.

Mean and variance of binomial distribution

$$\text{mean } (\mu) = E(x) = \sum_{x=0}^{\infty} x\, f(x)$$

$$= \sum_{x=0}^{\infty} x\, {}^nC_x\, p^x q^{n-x}$$

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$$= \sum_{x=0}^{\infty} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} \cancel{x} \frac{n(n-1)!}{\cancel{x}(x-1)!(n-x)!} p^x q^{n-x}$$

If $x=0$, $(0-1)!$, $(-1)!$ undefin

$$= np \sum_{x=1}^{\infty} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{\infty} {}^{n-1}C_{x-1} \, p^{x-1} q^{n-x}$$

$$= np \sum \left[{}^{n-1}C_0 \, p^0 q^{n-1} + {}^{n-1}C_1 \, p^1 q^{n-2} + \dots \right.$$

$$\left. \dots \cancel{+n-1} \right]$$

$$= np \sum [q+p]^{n-1}, \quad q+p=1$$

$$= np \, 1^{n-1}$$

$$= \underline{np}.$$

Variance of binomial Distribution

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - n^2p^2 \quad \text{—(1)}$$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 f(x)$$

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$$= \sum_{x=0}^{\infty} [x(x-1)+x] f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) f(x) + \sum_{x=0}^{\infty} x f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)\, {}^{n}C_{x}\, p^{x} q^{n-x} + np.$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{n!}{x!\,(n-x)!} p^{x} q^{n-x} + np.$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \frac{n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!\,(n-x)!} p^{x} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{\infty} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{\infty} {}^{n-2}C_{x-2}\, p^{x-2} q^{n-x} + np.$$

$$= n(n-1)p^2 \left[{}^{n-2}C_0\, p^0 q^{n-2} + {}^{n-2}C_1\, p^1 q^{n-3} + {}^{n-2}C_2\, p^2 q^{n-4} + \cdots \right] + np.$$

$$= n(n-1)p^2 [p+q]^{n-2} + np.$$

Substitute in (1)

$$\therefore \sigma^2 = n(n-1)p^2 [p+q]^{n-2} + np - n^2p^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$= n^2p^2 - np^2 + np - n^2p^2.$$

$= np[1-p]$

$= npq$

Q Find the probability of getting ~~the~~ 4 head. in 6 tosses of a coin

Ans. Let X be the random variable of getting heads.

$n = 6.$

$p = P(\text{getting head})$

$p = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X=4) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= {}^{6}C_{4}\, p^{4}\, q^{6-4} = {}^{6}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{2}$

$= \frac{15}{64}$

Q 10 coins are tossed simultaneously. Find the probability of getting atleast 7 heads.

Ans. Let X be the random variable of getting head.

$n = 10$

$p = P(\text{getting head})$

$= \frac{1}{2}.$

$q = \frac{1}{2}$

$P(\text{atleast 7 heads}) = P(X=7) + P(X=8) + P(X=9) + P(X=10).$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} +$
${}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}.$

$= \frac{15}{128} + \frac{45}{1024} + \frac{5}{512} + \frac{1}{1024}$

$= \frac{11}{64} = \underline{0.1719}$

10/2/19

Q. It has be claimed that in 60% of all solar heat installations. The utility bill is reduced by atleast $\frac{1}{3}$rd. What are the probabilities that, the utility bill will be reduced by atleast $\frac{1}{3}$rd,

a) 4 of 5 installations

b) atleast 4 of 5 insts

Ans. Let x be the random variable of installing solar heates so that the utility bill is reduced by atleast $\frac{1}{3}$rd.

i.e, $P = P(\text{installing solar heat})$

$= \frac{60}{100} = \underline{0.6}$

$q = 1 - 0.6 = 0.40.$

$n = 5$

(a) $P(X=4) = {}^nC_x \, p^x q^{n-x}$

$= {}^5C_4 \, p^4 q^{5-4}$

$= 5 \times (0.6)^4 \times (0.4)^1$

$= \underline{0.2592}$

b) $P(\text{atleast } 4) = P(X=4) + P(X=5)$

$= {}^5C_4 (0.6)^4 \times (0.4)^{5-4} + {}^5C_5 (0.6)^5 \times (0.4)^{5-5}$

$= 0.2592 + 0.0778$

$= \underline{0.3370}$

Q A machine manufacturing screws is known to produce 5% defective in a random variable of 15 screws. What is the probability that

a) Exactly 3 defectives

b) Not more than 3 defectives.

Ans. Let X be a random variable of defective screw.

$P(\text{defective screw}) = p = \frac{5}{100} = 0.05$

$q = 1 - 0.05$

$= 0.95$

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$n = 15$.

a) $P(X=3) = {}^{15}C_3 (0.05)^3 (0.95)^{15-3}$

$= 0.0307$

b) P(Not more than 3 defectives) =

$P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{15}C_0 (0.05)^0 (0.95)^{15} + {}^{15}C_1 (0.05)^1 (0.95)^{14} +$

${}^{15}C_2 (0.05)^2 (0.95)^{13} + {}^{15}C_3 (0.05)^3 \times (0.95)^{12}$

$= \underline{0.9945}$

Q In a large consignment of electric bulb, 90% are good bulbs. A random sample of 20 is taken for inspection. Find the probability that

a) All are good ones

b) Atmost 3 are defective.

c) Exactly 3 are defective.

Ans. Let x be the random variable of defective bulb

P(defective bulb) = $p = 10\% = 0.10$

$\therefore q = 1 - 0.10 = 0.90$

& $n = 20$.

a). P(all are good ones) = $P(X=0)$

$$= {}^{20}C_0 (0.1)^0 (0.9)^{20}$$

$$= 0.1216$$

b) P(atmost 3 are defective) = $P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= {}^{20}C_0 (0.1)^0 (0.9)^{20} + {}^{20}C_1 (0.1)^1 (0.9)^{19} + {}^{20}C_2 (0.1)^2 (0.9)^{18} + {}^{20}C_3 (0.1)^3 (0.9)^{17}$$

$$= \underline{\underline{0.8671}}$$

c) (Exactly 3 are defective) = $P(X=3)$

$$= {}^{20}C_3 (0.1)^3 \times (0.9)^{17}$$

$$= \underline{\underline{0.1901}}$$

Note :-

1) P(atleast 3 defectives) = $P(x \geq 3)$

$$= 1 - P(x < 3)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - 0.677 = \underline{\underline{0.323}}$$

2) P(atmost 19 defective) = $P(X \leq 19)$

$$= 1 - P(X > 19) = 1 - P(X=20)$$

$$= 1 - {}^{20}C_{20} (0.1)^{20} (0.9)^0$$

$$= \underline{\underline{1}}$$

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2/2/19

Q 6 dice are thrown 729 times. How many times do you ~~exeepect~~ expect atleast 3 dice ~~6~~ to show 5 or 6. ~~Let~~

Ans Let x be random variable that shows a dice 5 or 6.

$P(\text{getting } 5 \text{ or } 6) = p(x=5) + p(x=6)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}$

$n = 6 \quad N = 729$

$p(\text{atleast } 3 \text{ dices}) = P(x \geq 3)$

$= P(X=3) + P(X=4) + p(X=5) + P(X=6)$

$= {}^6C_3 \left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^3 + {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1 +$

${}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0$

$= 0.2195 + 0.0823 + 0.0165 + 0.0014$

$= 0.3197$

Number of times we expect atleast 3 dice to show 5 or 6 $= N \times 0.3197$

$= 729 \times 0.3197$

$= 233.0613$

$\approx \underline{233}$

Q In 256 sets of 12 tosses of a coin in

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how many cases one may expect 8 head and 4 tails.

Ans. Let x be the random variable of getting head.

$p = P(\text{getting heads}) = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 12$

$N = 256$

$P(\text{getting 8 heads}) = P(X = 8)$

$= {}^{12}C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^4$

$= 0.1208$

Number of cases one may expect 8 heads and 4 tails

$= N \times 0.1208$

$= 256 \times 0.1208$

$= 30.9248$

$\approx \underline{31}$

Q) Probability that a bomb droped from a plane with be strike the target is 0.2. If 6 bombs are dropped find the probability that

a) Exactly 2 will strike the target

b) Atleast 2 will strike the target

c) Atmost ~~Atleast~~ 2 will strike the target

Ans. Let X be the random variable of striking the target

$p = P(\text{striking the target}) = 0.2$

$q = 1 - p = 1 - 0.2 = 0.8$

$n = 6$

a) $P(X=2) = {}^6C_2 (0.2)^2 \times (0.8)^4$

$= 0.2458$

b) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}^6C_0 (0.2)^0 \times (0.8)^6 + {}^6C_1 (0.2)^1 \times (0.8)^5]$

$= 1 - [0.2621 + 0.3932]$

$= 1 - 0.6553$

$= 0.3447$

c) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.2621 + 0.3932 + 0.2458$

$= 0.9011$

Q Find $P(X=1)$ if mean $= 6$ and variance $= 4$

is binomial distribution.

Ans. Mean = $np = 6$

Variance = $nqp = 4$.

$$\frac{\cancel{n}\cancel{p}q}{\cancel{n}\cancel{p}} = \frac{4}{6} \therefore q = 0.6666$$

$$q = 0.6666 = \frac{2}{3}$$

$$p = 1 - \frac{2}{3} = \frac{1}{3}$$

$$np = 6$$

$$n = \frac{6}{p} = \frac{6}{\frac{1}{3}} = 6 \times 3 = 18$$

$$P(x=1) = {}^{18}C_1 \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{17}.$$

$$= \underline{0.0061}$$

Q If mean and variance of binomial distribution are ~~8~~ 30 and 25 respectively find $P(x=3)$.

Ans Mean $np = 30$

Variance = $npq = 25$

$$\frac{\cancel{n}\cancel{p}q}{\cancel{n}\cancel{p}} = \frac{25}{30} \therefore q = \frac{5}{6}$$

$$p = 1 - \frac{5}{6} = \frac{1}{6}$$

$$np = 30$$

$$n = \frac{30}{1/6} = 180.$$

$$P(x=3) = {}^{180}C_3 \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^{177}$$

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$= 4.2742 \times 10^{-11}$

Q If the binomial variable X satisfies the relation $P(X=4) = P(X=2)$, $n=6$. Find P.

Ans. $P(X=4) = {}^6C_4\, p^4 q^2$.

$P(X=2) = {}^6C_2\, p^2 q^4$

${}^6C_4 = {}^6C_2$ $\{ {}^nC_r = {}^nC_{n-r} \}$

Here $P(X=4) = P(X=2)$

$\Rightarrow \cancel{{}^6C_4}\, p^4 q^{6-4} = \cancel{{}^6C_4}\, p^2 q^{6-2}$

$\Rightarrow \cancel{p^4}\, q^2 = p^2\, \cancel{q^4}$

$p^2 = q^2$

$p^2 = (1-p)^2$

$p = 1-p$

$2p = 1$

$p = 1/2$

Q If the probability is 0.05 that a certain wide range column will fail under axial load. What are the probabilities that among 16 such column

a) Atmost 2 will fail

b) Atleast 4 will fail.

Ans. Let x be a random variable for getting wide range column will fail under a axil road

P(getting column will fail) = 0.05

$q = 1 - 0.05$

$= 0.95$

$n = 16$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{16}C_0 (0.05)^0 \times (0.95)^{16} + {}^{16}C_1 (0.05)^1 \times (0.95)^{15}$

$+ {}^{16}C_2 (0.05)^2 \times (0.95)^{14}$

$= 0.4401 + 0.3706 + 0.1463$

$= 0.9570$

b) $P(X \geq 4) = 1 - P(X < 4)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)$

$+ P(X=3)]$

$= 1 - [0.4401 + 0.3706 + 0.1463$

$+ 0.0359]$

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$= 0.0071$

Q An agriculture cooperative claims that 90% of watermelon shopped out are ripe and ready to eat. Find the probability that among 18 watermelon shopped out:

a) All 18 are ripe and ready to eat.

b) Atleast 16 are ripe and ready to eat

c) Atmost 14 are ripe and ready to eat

Ans. Let X be random variable that watermelon are ripe and ready to eat

$p = 90\% = 0.90 \quad q = 1 - 0.90 = 0.10$

$n = 18$

a) $P(X = \text{getting } 18 \text{ ripe watermelon}) = {}^{18}C_{18} (0.90)^{18} \times (0.10)^{0}$

$= 0.1501$

b) $P(\text{Atleast } 16 \text{ are ripe}) = P(X \geq 16) = \cancel{1 - [P(X < 16)]}$

$P(X=16) + P(X=17) + P(X=18)$

$= {}^{18}C_{16} (0.9)^{16} \times (0.1)^{2} + {}^{18}C_{17} (0.9)^{17} (0.1)^{1} +$

${}^{18}C_{18} (0.9)^{18} (0.1)^{0}$

$= 0.2835 + 0.30082 + 0.1501$

$= 0.7338$

c) $P(\text{Atmost } 14) = P(X \leq 14) = 1 - P(X > 14)$

$= P(X = 15) + P(X=16) + P(X=17) + P(X=18)$

$= 0.1680 + 0.7338 = 0.9018$

$1 - 0.9018 = 0.0982$

8/2/19

If 6 of 18 new buildings in a city. Violating the building ~~cour~~ codes. What is the probability that the building inspector select 4 of the new building will catch

a) None of the new building will violate the building code

b) One of the new building violate the building code

c) Atleast least 2 of new building.

Ans. Let X be a random variable that building violate the building code.

$p = \frac{6}{18} = \frac{1}{3} = P$(building that violate building code)

$\therefore q = \frac{2}{3}$

$n = 4$.

a) $P(X=0) = {}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4$

$= 0.1975$

b) $P(X=1) = {}^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3$

$= 0.3951$

c) $P(\text{Atleast two}) = P(X \geq 2) = \cancel{P(X \leq 2)} + P \quad 1 - P(X < 2)$

$= 1 - P(X=0) + P(X=1)$

$= 1 - [0.1975 + 0.3951] = 0.407494$

Q An Insurance company agent accept policies of 5 men all of identical age and good health probability that a man of this age will be alive 30 years ~~to buy~~ 2/3. Find the probability that in 30 years

(a) All 5 men are alive.

(b) Atleast one men will be alive

Ans. Let the random variable that the men will be alive 30 years.

$p = P(\text{men will be alive}) = \frac{2}{3}$

$q = 1 - P = 1 - \frac{2}{3} = \frac{1}{3}$

$n = 5$

$P(\text{All 5 men are alive}) = P(X=5)$

$= {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^0 = 0.1317$

$P(\text{Atleast one men}) = P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0) = 1 - {}^5C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4$

$= \cancel{0.04115}\ \cancel{0.9588}\ 0.9959$

Q The probability that noise level of a white band amplifier will exceed 2dB is 0.05. Find pro that among 12 such amplifiers the noise level of

a) One will exceed 2 dB.
b) Atmost 2 will exceed
c) 2 or more will exceed.

Ans. Let X be random variable for which amplifier will exceed 2dB

$p = P(\text{amplifier will exceed } 2\,dB)$

$= 0.05$

$q = 0.95$

$n = 12$

a) $P(X=1) = {}^{12}C_1 (0.05)^1 (0.95)^{11}$

$= \underline{0.3413}$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{12}C_0 (0.05)^0 (0.95)^{12} + {}^{12}C_1 (0.05)^1 (0.95)^{11}$

$+ {}^{12}C_2 (0.05)^2 (0.95)^{10}$

$= 0.5404 + 0.3413 + 0.0988$

$= \underline{0.9804}$

c) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X=0) + P(X=1)$

$= 1 - (0.5404 + 0.3413)$

$= \underline{0.1183}$

Q) During one stage the manufacture of integrated circuits chips. A coating must be applied If 70% of chips recieved thick enough coating. Find the probability that among 15 chips. At

a) Atleast 12 will have thick enough coating

b) Atmost 6 will have thick enough coating.

c) Exactly 10 will have " " " "

Let X be the random

$p = 0.70$

$q = 0.30$

$n = 15.$

a) $P(\text{Atleast } 12) = P(x \geq 12)$

$= P(X=12) + P(X=13) + P(X=14)$

$+ P(X=15$.

$= {}^{15}C_{12}(0.70)^{12} \times (0.30)^3 + {}^{15}C_{13}(0.70)^{13} \times (0.30)^2 +$

${}^{15}C_{14}(0.70)^{14}(0.30)^1 + {}^{15}C_{15}(0.70)^{15}(0.30)^0$

$= 0.1700 + 0.0916 + 0.0305 + 0.0047.$

$= 0.2968.$

b) $P(\text{Atmost } 6) = P(X \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$P(X=6)$

$= 0 + 0 + 0 + 0 + 0.0006 + 0.0030 + 0.0116$

$= 0.0152$

c) $P(\text{exactly } 10) = P(X=10) = {}^{15}C_{10} (0.7)^{10} \times (0.3)^5$

$= 0.2061$

5/2/19

Poisson Distribution

A discrete random variable X is said to follow poisson distribution, if it has pdf, $P(X=x) = f(x, \lambda)$

$$= \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, 3, \cdots$$

$\lambda > 0$, where λ is a parameter.

Mean of pd

$$\text{Mean } (\mu) = E(x) = \sum_{x=0}^{\infty} x f(x)$$

$$= \sum_{x=0}^{\infty} x \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=1} \cancel{x} \frac{\lambda^x e^{-\lambda}}{\cancel{x}(x-1)!}$$

$$= \frac{\lambda^1 e^{-\lambda}}{0!} + \frac{\lambda^2 e^{-\lambda}}{1!} + \frac{\lambda^3 e^{-\lambda}}{2!} + \cdots$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \right]$$

$$= \lambda e^{-\lambda} \times e^{\lambda}$$

$$= \underline{\underline{\lambda}}$$

Variance of pd

$$\text{Variance } \sigma^2 = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - \lambda^2 \longrightarrow ①$$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 f(x)$$

$$= \sum_{x=0}^{\infty} x^2 \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=0}^{\infty} x[x-1]+x \times \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{\lambda^x e^{-\lambda}}{x!} + \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_{x=2}^{\infty} \cancel{x}\cancel{(x-1)} \frac{\lambda^x e^{-\lambda}}{\cancel{x}\cancel{(x-1)}(x-2)!} + \lambda$$

$$= \left[\frac{\lambda^2 e^{-\lambda}}{0!} + \frac{\lambda^3 e^{-\lambda}}{1!} + \frac{\lambda^4 e^{-\lambda}}{2!} + \cdots\right] + \lambda$$

$$= \lambda^2 e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right] + \lambda$$

$$= \lambda^2 e^{-\lambda} \times e^{\lambda} + \lambda$$

$$= \lambda^2 + \lambda$$

(1) $\therefore$ Variance $= E(x^2) - \lambda^2$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

Q) At a checkout counter, customers arrived at an average of 1.5/min. Find the probabilities that.

a) Atmost 4 will arrive.

b) Atleast 3 will arrive

c) Atmost 5 will arrive.

Ans. Averag λ = mean = 1.5

a) $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$.

$$= \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!} + \frac{\lambda^4 e^{-\lambda}}{4!}$$

$$= e^{-1.5}\left[\frac{(1.5)^0}{0!} + \frac{1.5}{1!} + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!}\right]$$

$$= 0.9814$$

b) $P(X \geq 3) = 1 - P(X < 3)$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!}\right]$$

$$= 1 - e^{-1.5}\left[1 + 1.5 + 1.125\right]$$

$$= \underline{\underline{0.1912}}$$

c) $P(X \leq 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$$= \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!} + \frac{\lambda^4 e^{-\lambda}}{4!} + \frac{\lambda^5 e^{-\lambda}}{5!}$$

$$= \underline{\underline{0.9955}}$$

Q) In a given city 6 % of drivers get atleast 1 parking ticket per year, use poisson distribution to determine the probability that among 80 drivers.

a) 4 will get atleast 1 parking ticket

b) atleast 3 will get atleast one parking ticket

c) 3 to 6 inclusive. will get one parking ticket.

Ans. Let X be the random variable that the drivers get atleast 1 parking ticket.

$$P = \frac{6}{100} = 0.06$$

$n = 80$

$\lambda = np = 0.06 \times 80$

$= 4.8$

a) $P(X=4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(4.8)^4 e^{-4.8}}{4!}$

$= \cancel{0.0439}$ $\underline{0.1820}$

b) $P(X \geq 3) = \cancel{P(X=0)}$ $1 - \overline{P(X < 3)}$

$= 1 - P(X=0) + P(X=1) + P(X=2)$

$\cancel{= 1 -} \left[\frac{(4.8)^0 \times e^{-4.8}}{0!} \right.$

$= 1 - e^{-4.8} \left[\frac{(4.8)^0}{0!} + \frac{(4.8)^1}{1!} + \frac{(4.8)^2}{2!} \right]$

$= \underline{0.8575}$

c) $P(3 \leq x \leq 6) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= e^{-4.8} \left[\frac{(4.8)^3}{3!} + \frac{(4.8)^4}{4!} + \frac{(4.8)^5}{5!} + \frac{(4.8)^6}{6!} \right]$

$= \underline{0.6483}$

6/2/19

Q) ~~P~~ If 0.8 % of ~~juices~~ fuses delivered are defective. Use poision distribution to determine probability that 4 fuses will be defective in a random sample of 400.

Ans. Let X be the random variable that fuse will be defective.

$p = \frac{.8}{100} = 0.008$

$n = 400$

$\lambda = np = 400 \times 0.008$

$= \underline{\underline{3.2}}$

$P(X=4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(3.2)^4 e^{-3.2}}{4!}$

$= \underline{\underline{0.1781}}$

Q. ~~atleast~~ Find the probability that atmost 5 defective fuses will found in a box of 200 fuses, if the experience shows that 2% of such ~~featur~~ fuses are defective.

Ans. ~~p~~ Let X be the random variable that fuses will be defective

$p = \frac{2}{100} = 0.02$

$n = 200$. $\lambda = np = 200 \times 0.02 = 4$.

$P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \frac{(4)^0 \times e^{-4}}{0!} + \frac{(4)^1 \times e^{-4}}{1!} + \frac{4^2 \times e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!} + \frac{4^4 e^{-4}}{4!} +$

$$+ \frac{4^5 e^{-4}}{5!} = 0.7851$$

Q In a factor of razor blades, there is a small chance of $\frac{1}{500}$ for any blade to be defective, the blades ~~of~~ are in packets of 10. Use poisson distribution ~~to~~ to calculate approximate no. of packet containing

a) No defective

b) 1 defective

c) 3 defective blades in a consignment of 10,000 packet.

Ans. Let X be the random variable that the blade is defective.

$$P = \frac{1}{500} = 0.002$$

$$n = 10. \quad N = 10,000$$

$$\therefore \lambda = np = 0.002 \times 10 = 0.02$$

$$\therefore P(X=0) = \frac{(0.02)^0 e^{-0.02}}{0!} = 0.9802$$

$\therefore$ ~~In 10000 packet.~~

No. of packet containing no defective $= N \times P(X=0)$

$$= 10000 \times 0.9802$$

$$= 9802$$

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b) $P(X=1) = \frac{\lambda^x e^{-x}}{x!} = \frac{(0.02)^1 e^{-0.02}}{1!} = 0.0196$

$\therefore$ No of packet containing 1 defective.

$= N \times P(X=1)$

$= 10,000 \times 0.0196.$

$= 196.$

c) 3 defective blade.

$P(X=3) = \frac{(0.02)^3 \times e^{-0.02}}{3!} = \underline{\underline{0}}$

No. of packet containing 3 defective.

$= 10,000 \times \cancel{0.013}\ 1.3 \times 10^{-3}$

$= 0.013.$

$\underline{\underline{0}}$

Q It is known that 5% of the books bind at a certain bind shop have defective binding. Find the probability that 2 of 100 books bind by this bindary will have defective binding using a) binomial distribution.

b) Poisson distribution.

Ans. Let x be the random variable that book binding is defective.

$\underline{bd}$

$n = 100.$

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$p = \frac{5}{100} = 0.05$

$q = 1 - 0.05 = 0.95.$

a) $P(X=2) = {}^{100}C_2 \, (0.05)^2 \times (0.95)^{98}$

$= 0.0812$

b) By Poisson distribution. (Accurate)

~~Rel~~ $\lambda = np$

$= 100 \times 0.05 = 5$

$P(X=2) = \dfrac{5^2 e^{-5}}{2!} = 0.0842$

Q A heavy machinary manufacturer has 3840 generators in the field that are under warranty. If the probability is $\frac{1}{1200}$ that any one will fail during the given year. Find the probability that 2 generato will fail during the given year.

Ans. Let X be random variable that generator will fail during the given year.

$n = 3840$

$p = \frac{1}{1200} = 0.0008.$

$\lambda = np = 3.2.$

$\therefore P(X=2) = \frac{(3.2)^2 \times e^{-3.2}}{2!}$

$= 0.2087$

Q A consulting engineer an average of 0.7 (λ) request permit follow poisson process. Find the probability that
a) atleast 1 request
b) atmost 3 request.

Ans. $\lambda = 0.7$.

a) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - \frac{(0.7)^0 \times e^{-0.7}}{0!}$

$= 0.5034$

b) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.9942$

1/2/19

Q No. of gamma rays emitted by a certain radio active substance is a random variable having poisson distribution with $\lambda = 5.8$. Find probability that, this instrument becomes inoperative, if there are more than 6 rays per second.

Ans. $\lambda = 5.8$.

$P(X>6) = 1 - P(X \le 6)$

$= 1 - P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= 1 - e^{-5.8}\left[\frac{(5.8)^0}{0!} + \frac{(5.8)^1}{1!} + \frac{(5.8)^2}{2!} + \frac{(5.8)^3}{3!} + \frac{(5.8)^4}{4!} + \frac{(5.8)^5}{5!} + \frac{(5.8)^6}{6!} \right]$

$= \underline{\underline{0.3616}}$

Poisson Distribution as a limiting case of binomial distribution.

Pdf of binomial distribution is given by

$P(x, n, p) = {}^nC_x\, p^x\, q^{n-x}$.

where n and p are parameters

taking $n \to \infty$, very large, $p \to 0$, very small, $np = \lambda$ is finite.

$P(x, n, p) = {}^nC_x\, p^x\, q^{n-x}$.

$= \frac{n!}{x!\,(n-x)!}\, p^x (1-p)^{n-x}$.

$= \frac{n(n-1)(n-2)\cdots(n-x-1)\cancel{(n-x)!}}{x!\,\cancel{(n-x)!}} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right.$

$$= \frac{\cancel{n^x} \, 1 \times \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \cdots \left(1 - \frac{x-1}{n}\right)}{x!} \cdot \frac{\lambda^x}{\cancel{n^x}} \times \frac{\left(1 - \frac{\lambda}{n}\right)^n}{\left(1 - \frac{\lambda}{n}\right)^x}$$

Taking $n \to \infty$

$$= \frac{1 \times 1 \times \cdots \times 1}{x!} \lambda^x \times \frac{\lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^n}{1}$$

$$= \frac{\lambda^x}{x!} \underset{n \to \infty}{Lt} \left[\left[1 + \frac{1}{-n/\lambda}\right]^{-n/\lambda}\right]^{-\lambda}, \quad \boxed{\underset{n \to \infty}{Lt} \left(1 + 1/n\right)^n = e}$$

$$= \frac{\lambda^x}{x!} \times e^{-\lambda}$$

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