~~AI~~ x,y First Order Logic :-

First order logic is superset of propositional logic

Constants → objects

in wumpus world – pit, wumpus & objects

VNIT, Ram, 2, 3.14.

Predicates → Relations

→ unary – Smelly (Wumpus)

Tall (Ram)

→ Binary – Brother (Ram, shyam)

(relate two objects) Adjacent ([1,1], [1,2])

Functions ::

f(const) $\xrightarrow{\text{maps}}$ another constant → only one value for the given input

↓ refers object ↓ object

Home (Wumpus) → [3,1] → given object must be related to exactly one object in this way

BestFriend (Ram) $\xrightarrow{\text{maps to}}$ Laxman

Ground Term:

Is a constant or function applied on some constant

const, Function (constant)

Ram, Father (Ram)

Variable: x, y

Connectives: $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$

Equality: = [Father (Ram), Dashrath]

Quantifiers :

$\forall x$: Universal quantifier

$\forall x$ Greedy (x)

$\exists x$: Existential quantifier

$\exists x$ Greedy (x)

Interpretation :

Constant - All objects map to particular constant

It gives meaning to every object in the domain.

Ex: Everyone / Anyone at VNIT is Smart.

Nouns - constants
adjectives - predicates

natural connective

$\forall x \; At(x, VNIT) \rightarrow Smart(x)$

⇒ Person only at VNIT are smart.

$\forall x \; At(x, VNIT) \leftrightarrow Smart(x)$

⇒ $\forall x \; At(x, VNIT) \wedge Smart(x)$

All persons are at VNIT and all persons are smart.

⇒ Someone at VNIT is smart

natural connective

$\exists x \; At(x, VNIT) \wedge Smart(x)$

$\exists x \; At(x, VNIT) \rightarrow Smart(x)$

It becomes true even for persons who is not at VNIT

$F \rightarrow T$ T

$F \rightarrow F$ T.

It is true for any person not at VNIT
Is true even if there is no one at VNIT

$\Rightarrow$ $\forall x$ Friend (John, x)
John is friends with everybody.

$\forall x$ Friend (x, John)
Everybody is friend of John

$\forall x y$ Friend (x, y) $\rightarrow$ symmetric
Everybody is friend of everybody.

$\forall x, y$ Friend (x, y) $\rightarrow$ Friend (y, x)
$\Downarrow$
Symmetric

Relation b/w quantifiers:

$\forall x \; P(x) \equiv \neg \exists x \; \neg P(x)$

$\exists x \; P(x) \equiv \neg \forall x \; \neg P(x)$

$\Rightarrow$ A person's is the person's female parent

$\forall x, y$ Mother (x, y) $\leftrightarrow$ Female (y) $\wedge$ parent (x, y)

John has atleast one brother.

$\exists x$ Brother (John, x)

John has atleast two brothers.

$\exists x, y$ Brother (John, x) $\wedge$ Brother (John, y) $\wedge \neg(x = y)$

John has exactly one brother

Brother (John, x) $\wedge$ $\forall y$ Brother (John, y) $\rightarrow$ ($x = y$)

John has exactly two brothers

$\exists x, y$ $\neg(x = y)$ $\wedge$ Brother (John, x) $\wedge$
Brother (John, y) $\wedge$
$\forall z$. Brother (John, z) $\rightarrow$ ($x = z$) $\vee$ ($y = z$)

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John has exactly one brother

$\exists x, y$ Brother (John, x) $\wedge$ Brother (John, y) $\wedge$ (x≠y)

1. King (John)
2. Person (Richard) $\Big\}$ KB
3. $\forall x$ King (x) $\rightarrow$ Person (x)

?: King (z)

Substitution list : z / John

?: $\exists p$ Perspon(p)

If no sublist return NULL

Sub list : P / Richard
P / John

1. King (John)
2. Greedy (John)
3. Person (Richard)
4. $\forall x$ King (x) $\rightarrow$ Person (x)
5. $\forall x$ King (x) $\wedge$ Greedy (x) $\rightarrow$ Evil (x)
6. $\forall x$ Greedy (x)

Q: $\exists z$ Evil (z)

z / John

Inference in FOL:

<u>Propositionalization</u>:

Universal Instantiation

Remove universal quantifier by replacing ~~removing~~ with ground term

$\forall x \quad P(x) \equiv P(G_1)$

$P(G_2)$

$\vdots$

$P(G_n)$

with rules 1, 2, 5

4: ~~Key~~ King(John) $\wedge$ Greedy(John) $\rightarrow$ Evil(John)

with rules 1, 2, 3, 5

5: King(Richard) $\wedge$ Greedy(Richard) $\rightarrow$ Evil(Richard)

Now in KB we have 1, 2, 3, 4, 5

Q: $\exists x$ Evil(x)

α : $\exists z$ Evil(z)

$\neg\alpha \equiv \neg \exists z$ Evil(z)

$\forall z \; \neg$Evil(z)

Existential ~~Quanti~~ Instantation / skolemization

Remove existential quantifier

$\exists x \; P(x) \rightarrow$ replace with constant P(K) such that in the domain

existential quantifier

K is not there

elements of the KB

skolem constant

Modus Ponen : $\dfrac{A \rightarrow B, A}{B}$ then B is true

$A \rightarrow B$ is True & A is True

modus ponen on 1, 2, 5 we get Evil(Richard).

if we have another two stmt in KB:

7. Father(John)

Father(Richard)

then we get another pL

6. King(Father(John)) $\wedge$ Greedy(Father(John)) $\rightarrow$ Evil(Father(John))

Universal Instantiation never ends (infinite)

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Inference in FOL

Propositionalization

Universal Instantitation

Existensial Instantiation - skolemization

<u>Herbrand's</u> <u>Theorem</u>:

If a sentence α is entailed by a FOL KB, it is also entailed by a ~~first~~ finite sized propositionalized version of the KB

Natnum(0)

$\forall n$ Natnum(n) $\rightarrow$ Natnum (succ(n))

<u>Church</u> - <u>Turing</u> <u>Hypothesis</u>:

Entailment in FOL is semidecidable.

$\forall x$ king(x) $\wedge$ Greedy (x) $\rightarrow$ Evil(x)

$\forall y$ Greedy (y)

King (John)

Brother (John, Richard)

Friend (John, Jack)

Q: $\exists x$ Evil(x) ?

Generalised Modus Ponen (GMP) for FOL

$$\forall \quad \frac{P_1 \wedge P_2 \ldots\ldots \wedge P_n \rightarrow q \;,\; P_1', P_2' \ldots P_n'}{q|\theta}$$

s.t $P_1|_\theta = P_1'|\theta$

$P_2|\theta = P_2'|\theta$

$\vdots$

$P_n|\theta = P_n'|\theta$

1. only $\forall$ is used
2. All terms may have variables
3. $P_i|\theta$ means that the variables in P_i are substitution satisfied by the terms given in the list θ
4. Substitution list $\theta = \{x/GT_1, x/GT_2 \ldots\}$

$$\forall x \quad \frac{King(x) \wedge Greedy(x) \rightarrow Evil(x) \;,\; King(John), \forall y \; Greedy(y)}{Evil(x)\,/_{x/John}}$$

$P_1|_{x/John} = P_1'|$

$P_2| \qquad = P_2'|_{y/x}$

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if P_i and P_i' match under some substitution θ

~~Generali~~ then

$\forall x \quad king(x) \wedge Greedy(x) \rightarrow Evil(x)$

$\forall y \quad greedy(y)$

king(John)

Brother(John, Richard)

Unification:

Indexing of the terms using predicates

① $\forall x$ knows (John, x) knows (John, Jack) x / Jack

② $\forall x$ knows (John, x) knows (John, Jack) x / Richard

$\forall y$ knows (y, Richard) y / John

③ $\forall x$ knows (John, x) knows (John, Jack)

if x is john we don't get same meaning of sentences

$\forall y$ knows (y, Richard)

$\forall x$ knows (x, Richard) Failure

x is not unique

④ ~~All~~ the above ~~$\forall y,x$ knows(y, z)~~

$\forall x$ knows (John, x) $\forall y,z$ knows (y, z)

$\left.\begin{array}{l} y \mid John \\ x \mid z \end{array}\right\}$ MGU

x can be substituted with variable z

most generalised unified term

① It is a crime for an American to sell weapons to hostile nations.

② A country, Nono has some missiles an enemy

③ All of the missiles of Nono were sold to it by col. west, who is an american

Q: Is col west a criminal?

① $\forall_{xyz}$ American(x) $\wedge$ Weapon(y) $\wedge$ Hostile nation(z) $\wedge$ Sell(x,y,z) $\rightarrow$ Criminal(x)

(seller, object, purchaser)
x y z
sold

Hostilenation – it should be hostile to only america.

Hostilenation (America, z)

~~Fro~~ For now we assume all term in KB belongs to same country

Hostile nation $\xrightarrow{\text{used instead}}$ Hostile

② Hostile (Nono) – Fact.

$\forall x'$ Missile(x') $\rightarrow$ weapon(x')

$\exists m$ Missile(m) $\wedge$ Has$(nono, m)$

③ American(Col.west) ground term

$\forall m'$ Missile(m') $\wedge$ Has$(Nono, m')$ $\rightarrow$ Sell$(col.west, m', nono)$

~~$\forall_{x',y',z'}$ Sell(x',y',z') $\rightarrow$ Has(z',y')~~ already we assumed.

purchaser has what the seller sold.

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1. $\forall xyz$ American(x) ∧ Weapon(y) ∧ Sells(x, y, z)
∧ Hostile(z) → Criminal(x)

2. $\exists x_1$ Owns$($Nono$, x_1)$ ∧ Missile(x_1)

3. $\forall x_2$ Missile(x_2) ∧ Owns$($Nono$, x_2)$ →
sells$($west$, x_2$, Nono$)$

4. $\forall m$ Missile(m) → weapon(m)

5. $\forall c$ Enemy$(c$, America$)$ → Hostile(c)

6. American$($west$)$

7. Enemy$($Nono, America$)$

Query :
criminal$($west$)$?

Forward chaining:

2 becomes 2a, 2b

2a. owns$($Nono$, M_1)$ 2b. Missile(M_1)

3. Generalized modus ponen, we can find substitution for all the terms on left hand side so sells$($west$, x_2$, Nono$)$ becomes true

~~Forwar~~

8. sells$($west$, M_1$, Nono$)$

9. weapon(M_1)

10. Hostile$($Nono$)$

2nd iteration: x/west y/M z/Nono

11. Criminal$($west$)$

[if the new formed fact is on left side then we have to do again in the next iteration]

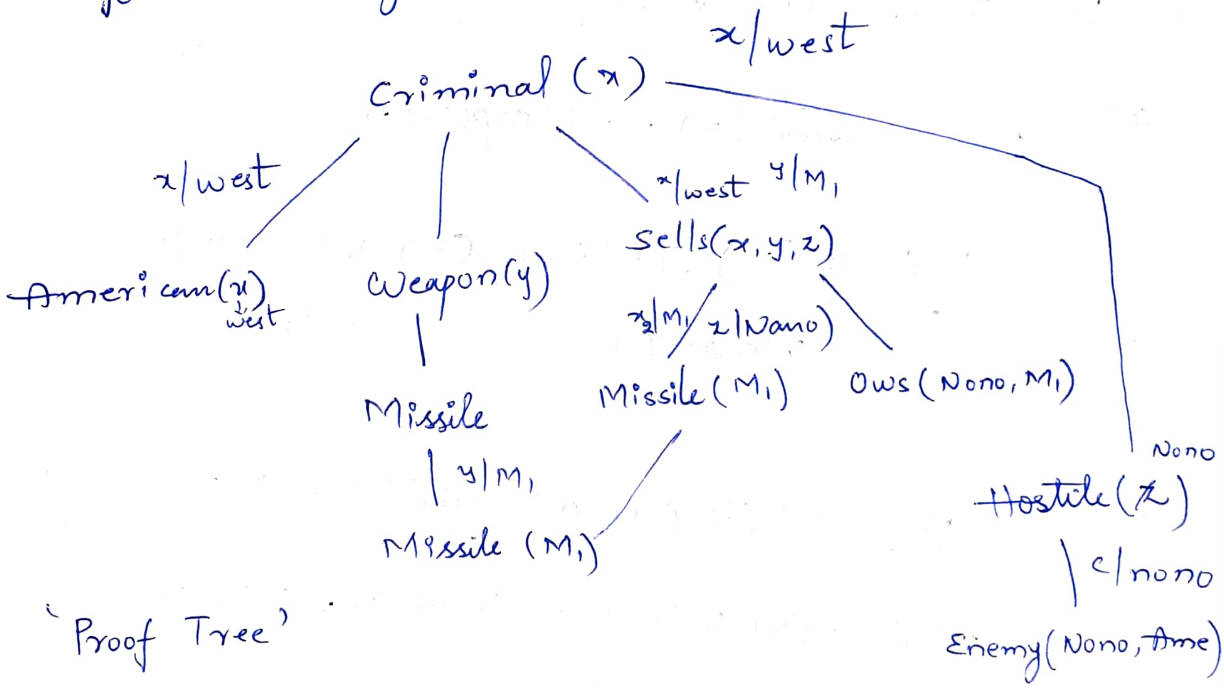
Conjunct ordering:

Heuristic: choose the one with minimum remaining values

in which order should we proceed like $(a \wedge b)$ substitute a first or b first

Backward chaining

↳ why not directly consider what is required for answering the question.



'Proof Tree'

is west criminal entailed.

Query: $\exists c_2$ Criminal (c_2)

who is criminal?

Forward & Backward chaining can be used only if KB contains horn clauses

Resolution for FOL: we can use one clause multiple times (resolute should be different)

$$\frac{l_1 \vee \ldots \vee l_n \ , \ P_1 \vee \ldots \vee P_n}{l_1 \vee l_{i-1} \vee l_{i+1} \ldots l_n \vee P_1 \ldots P_{j-1} \vee P_{j+1} \vee \ldots P_m / \theta}$$

Unify $(l_i, \neg P_j) = \theta$

Entire KB must be converted to CNF

1) Missile (M)

$\forall m \ \neg Missile(m) \vee Weapon(m)$

2) $\forall m \ Missile(m) \rightarrow Weapon(m)$

$$\frac{Missile(M), \neg Missile(m) \vee Weapon(m)}{Weapon(M) \qquad m/M}$$

3) $\forall x \ Missile(x) \wedge Owns(Nono, x) \rightarrow Sells(West, x, Nono)$

$\forall x \ \neg Missile(x) \vee \neg Owns(Nono, x) \vee Sells(West, x, Nono)$

$$\frac{① , ③ \qquad x/M}{\neg Owns(Nono, M) \vee Sells(West, M, Nono)}$$

① Everyone who loves all animals is loved by someone.

$\forall x [[\forall y \ Animal(y) \rightarrow Loves(x,y)] \rightarrow \exists z \ loves(z,x)]$

1. Eliminate $\rightarrow, \leftrightarrow$

~~$\forall x$~~ $\forall x \ [\neg[\forall y \ Animal(y) \rightarrow loves(x,y)] \vee \exists x \ loves(z,x)$

$\forall x\ [\neg[\forall y\ \neg Animal(y) \vee Loves(x,y)] \vee \exists z\ loves(z,x)]$

2. Push $\neg$ inside, $\neg \forall_x P \equiv \exists_x \neg P$

$\neg \exists_x q \equiv \forall_x \neg q$

$\forall x [(\exists y\ \neg(\neg Animal(y) \vee loves(x,y)) \vee \exists z\ loves(z,x)]$

$\forall x [\exists y\ (Animal(y) \wedge \neg loves(x,y)) \vee \exists z\ loves(z,x)]$

3. skolemize - wherever we have $\exists$ symbol replace that symbol with constant

$\cancel{\forall x [(Animal(A_1) \wedge \neg loves(x, A_1)) \vee loves(A_2, x)]}$

y should be replaced but we can't just replace it with constant

we should replace it with function that takes argument as the universal quantifier

$\forall x\ (Animal(F(x)) \wedge \neg loves(x, F(x))) \vee loves(G(x), x)]$

$F(x)$ - denotes animal that is particular to x

$G(x)$ - denotes person

if $\exists$ occurs at outermost level then we can replace $\exists$ with constant.

if it occurs inside then replace it with function

4. Distribute $\vee$ over $\wedge$

$\forall x\ [(Animal(F(x)) \vee loves(G(x), x)) \wedge$

$(\neg loves(x, F(x)) \vee loves(G(x), x))]$

Drop $\forall$ to get the equivalent CNF clauses

A_1: $[Animal(F(x)) \lor loves(G(x), x)) \land$

A_2: $\neg loves(x, F(x)) \lor loves(G(x), x))]$

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② Anyone who kills an animal is loved by no one.

~~$\forall x_1$ Animal~~

$\forall x_1 [[\exists y_1\ Animal(y_1) \land Kills(x_1, y_1)] \rightarrow \neg \exists z_1\ loves(z_1, x_1)$

① remove $\rightarrow$

$\forall x_1 [\neg [\exists y_1\ Animal(y_1) \land kills(x_1, y_1)] \lor \neg \exists z_1\ loves(z_1, x_1)$

② Push $\neg$ inside & apply De-morgans

$\forall x_1 [\forall y_1\ \neg Animal(y_1) \lor \neg kills(x_1, y_1)] \lor$
$\forall z_1\ \neg loves(z_1, x_1)]$

Drop $\forall$.

$\Rightarrow$ B: $\neg Animal(y_1) \lor \neg kills(x_1, y_1) \lor \neg loves(z_1, x_1)$

③ Jack loves all animals

$\forall y_2\ Animal(y_2) \rightarrow loves(Jack, y_2)$

C: $\neg Animal(y_2) \lor loves(Jack, y_2)$

④ Either Jack or Curiosity killed the cat Tuna.

D: $Kills(Jack, Tuna) \lor Kills(Curiosity, Tuna)$

⑤ All cats are animals

$\forall x_2 \; cat(x_2) \rightarrow Animal(x_2)$

E: $\neg cat(x_2) \vee Animal(x_2)$

⑥ F: $cat(Tuna)$

Question: Did curiosity kill Tuna?

α: $kills(Curiosity, Tuna)$

E F

$x_2 | Tuna$

H: $Animal(Tuna)$ C

$y_2 | Tuna$

$loves(Jack, Tuna)$

G D

J: $kills(Jack, Tuna)$ B

$x_1 | Jack$

$y_1 | Tuna$

K: $\neg Animal(Tuna) \vee \neg loves(z_1, Jack)$ H

L: $\neg loves(z_1, Jack)$

C A2

$x | Jack$

$y_2 | F(Jack)$

M: $\neg Animal(F(Jack)) \vee loves(G(Jack), Jack)$ A1

$x | Jack$

N: $Loves(G(Jack), Jack)$

N L

$z_1 / G(Jack)$ $\square$ Null clause

Q: who killed Tuna?

α: $\exists z_2$ kills(z_2, Tuna)

G: $\neg$ Kills(z_2, Tuna)

substitute z_2 with Jack & curiosity
with jack we won't get null clause
with curiosity we get null clause
so curiosity killed tuna.

Uncertainity:

Sources of uncertainity.

1. Action outcomes
2. Lack of entire evidence

Fever → Malaria
(Diagnostic rule)

Fever → Malaria ∨ Flu . . . . .

(Causal Rule)

Malaria → Fever

Malaria ∧ Bloodcount > value ∧ . . . . → Fever

Fever - Boolean

P(Fever = T) = 0.01

Weather - Discrete

[hot, humid, rainy, cold]

$P(\text{weather} = \text{hot}) = 0.4$

Continuous random var

Axioms:

$0 \le P(e) \le 1$

$\sum_{e_i \in \text{atomic events}} P(e_i) = 1$

$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

Conditional Probabilities:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

b: evidence

$$P(a \wedge b) = P(a/b) \times P(b) \quad \text{— (1)}$$

$$P(b|a) = \frac{P(a \wedge b)}{P(a)}$$

$$P(a \wedge b) = P(b|a) \times P(a) \quad \text{— (2)}$$

Baye's Rule: [equate (1) & (2)]

$$P(a|b) = \frac{P(b|a)\, P(a)}{P(b)}$$

Baye's rule is important bcoz sometimes finding probability from one side is easy than the other

i.e) calculating $P(b|a)$ easier than $P(a|b)$.

Cavity

Toothache

Joint Conditional Probability Distribution Table (CPT).

Cavity	Toothache	Catch	P
T	T	T	0.108
T	T	F	0.012
T	F	T	0.072
T	F	F	0.08
F	T	T	0.016
F	T	F	0.064
F	F	T	0.144
F	F	F	0.576

$P(\text{Cavity} = T) = 0.108 + 0.012 + 0.072 + 0.008$

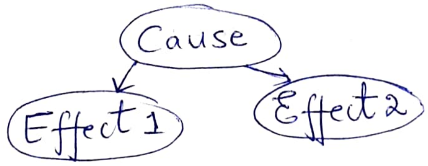
$P(\text{Cavity} = T \wedge \text{Toothache} = F) = 0.072 + 0.008 = 0.08$

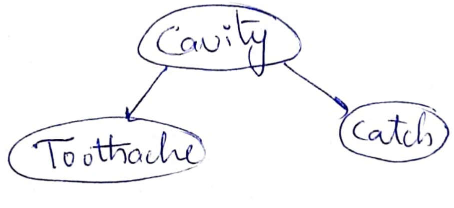
$$P(\text{cavity} = T \mid \text{Toothache} = T) = \frac{P(\text{cavity} = T, \text{Toothache} = T)}{P(\text{Toothache} = T)}$$

$$= \frac{0.108 + 0.012}{0.108 + 0.012 + 0.016 + 0.064}$$

$$= 0.6$$

Conditional Independence:





Cavity	P(Toothache | cavity)
T	
F	

For all effects 2 experiments and for cause one experiment.

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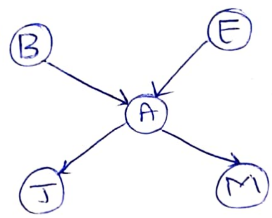
Bayesian Network:

$P(E=T) = 0.002$

$P(B=T) = 0.001$

from Past police records



A	P(J=T|A)
T	0.9
F	0.05

B	E	P(A=T/B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(M=T|A)
T	0.7
F	0.01

$$P(B=T \mid J=T, M=T) = \frac{P(B=T, J=T, M=T)}{P(J=T, M=T)}$$

Similarly, $P(B=F \mid J=T, M=T) = \dfrac{P(B=F, J=T, M=T)}{P(J=T, M=T)}$

Treat denominator as a normalizing constant α

Compute $P(B \mid J=T, M=T) = \alpha P(B, J=T, M=T)$

$$\alpha P(B, J=T, M=T) = \alpha \sum_{e\in E}\sum_{a\in A} P(B, e, a, J=T, M=T)$$

E & A are hidden variables

$$= \alpha \sum_{e\in E}\sum_{a\in A} P(B)\, P(e)\, P(a \mid B, e)\, P(J=T \mid a)\, P(M=T \mid a)$$

$$= \alpha P(B) \sum_{e\in E} P(e) \sum_{a\in A} P(a \mid B, e) \cdot P(J=T \mid a)\, P(M=T \mid a)$$

For B=T

$$0.001 \times [\underbrace{0.002}_{e=T} \times \{\underbrace{0.95}_{a=T} \times 0.9 \times 0.7 + \underbrace{(1-0.95)}_{a=F} \times 0.05 \times 0.01\} +$$

$$(1-0.002) \times \{\underbrace{0.94}_{a=T} \times 0.9 \times 0.7 + (1-0.94) \times 0.05 \times 0.01\}]$$

$= 0.000592$

For $B = F$:

$$(1-0.001) \times [\underset{e=T}{0.002} \times \{\underset{a=T}{0.29} \times 0.9 \times 0.7 + (1-0.09) \times 0.05 \times 0.01\}$$

$$+ \underset{e=F}{(1-0.002)} \times \{\underset{a=T}{0.001} \times 0.9 \times 0.7 + (1-0.001) \times 0.05 \times 0.01$$

$= 0.001492$

$\langle 0.000592, 0.001492 \rangle \rightarrow$ Normalize - summation should be 1.

$B = T \quad B = F$

$\langle 0.284, 0.716 \rangle$ $\quad \alpha$ - normalizing constant

Time complexity is exponential (dfs)

space complexity is polynomial

How to build network?

start from root causes and proceed.

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Approximate Inference:

T - True

F - False

Sampling a Baysian N/W

B E A J M (Topological sort of the N/w)

1. Sample B conditioned on its parents

   suppose F

2. Sample E conditioned on its parents

   suppose T

3. Sample $A | B, E$ $\quad P(A | B=F, E=T) = 0.29$

   suppose T

4. Sample $J | A$ $P(J | A=T) = 0.9$

Suppose T

5. Sample $M | A$ $P(M | A=T) = 0.7$

Suppose T

$\langle F, T, T, T, T \rangle$

<u>Rejection Sampling</u> : Repeat & store all samples

Once given a query, find probability of the query given samples we created

$P(B=T | J=T, M=T)$

1. Reject all samples which do not match with the evidence vars.
2. Out of the remaining samples

$count_{B=T}$, $count_{B=F}$

Normalize the counts.

Likelihood weighting :

$P(B=T | J=T, M=T)$

$w = 1.0$

B E A J M (Topsort order)

1. Sample B [B has no parents so take its prior prob.]

suppose T

2. Sample E [E has no parents]

suppose ~~E~~ F

3. Sample A $P(A \mid B=T, E=F) = 0.94$ is not evidence so we sample it

Suppose $A=T$

4. J is evidence var, so we don't sample it
we will update the weight.

$w = w * P(J=T \mid A=T) = 0.9$

5. M is evidence var

$w = w \times P(M=T \mid A=T) = 0.63$

$$\left.\begin{array}{l} \text{sum } B=T \\ \text{sum } B=F \end{array}\right\} \text{Normalize two sums}$$

normalize $\langle \text{sum}_{B=T}, \text{sum}_{B=F} \rangle$

Markov chain Monte Carlo (MCMC)

Markov blanket of a var

[Prev we sampled var based on parents now we did it based on markov blanket]

$P(x \mid mb(x))$

$= P(x \mid \text{parents}(x)) * \prod_{y \in \text{children}(x)} P(y \mid \text{parents}(y))$

$P(A \mid mb(A)) = P(A \mid B, E) * P(J \mid A)^{\to T} * P(M \mid A)^{\to T}$

Compute for both $A=T$ & $A=F$ & normalize

$P(B=T \mid J=T, M=T)$

Initial sample

$$\langle \underbrace{\begin{matrix} B & E & A \\ F & T & T \end{matrix}}_{\text{random}} \begin{matrix} J & M \\ T & T \end{matrix} \rangle$$

set $J, M = T$
remaining random values

1. Sample B

$$P(B \mid mb(B)) = \overset{\text{no parents}}{1} \times P(A=T \mid B, E=T)$$

For $B=T$, 0.95

For $B=F$, 0.29

Normalize $\langle 0.95, 0.29 \rangle$

Suppose T

2. Sample E

$$P(E \mid mb(E)) = 1 \times P(A=T \mid B=T, E)$$

For $E=T$, 0.95

For $E=F$, 0.94

Normalize $\langle 0.95, 0.94 \rangle$

suppose F

$$\Rightarrow P(A \mid B=T, E=F) \times P(J=T \mid A) \times P(M=T \mid A)$$

For $A=T$ $\quad 0.95 \times 0.9 \times 0.7 = V_1$

For $A=F$ $\quad (1-0.95) \times (0.05) \times (0.01) = V_2$

Normalize $\langle V_1, V_2 \rangle$

Suppose T

$\langle T, F, T, T, T \rangle$ $\quad \langle \text{count}_{B=T}, \text{count}_{B=F} \rangle$

Normalize

$\neg B_{11}$, $\neg P_{11}$, $\neg P_{12}$, $\neg P_{21}$, B_{12}
(5) (6) (7) (8) (9)

$B_{11} \leftrightarrow P_{12} \vee P_{21}$

convert to CNF (using DPLL)
enumeration

$\left.\begin{array}{l} \neg B_{11} \vee P_{12} \vee P_{21} \\ \neg P_{12} \vee B_{11} \\ \neg P_{21} \vee B_{11} \end{array}\right\}$ CNF

$B_{12} \leftrightarrow P_{11} \vee P_{13} \vee P_{22}$ → Required.

CNF

α_1 : P_{13}

α_2 : P_{22}

(1) $\neg B_{12} \vee P_{11} \vee P_{13} \vee P_{22}$

(2) $\neg P_{11} \vee B_{12}$

(3) $\neg P_{13} \vee B_{12}$

(4) $\neg P_{22} \vee B_{12}$

10: $\neg \alpha_1$: $\neg P_{13}$

→ Assign True to B_{12} (9)

omit (2) and (3) and (4)

(1) becomes $\cancel{\neg B_{12}} \vee P_{11} \vee P_{13} \vee P_{22}$

→ By (10)

(1) becomes $P_{11} \vee P_{22}$.

By (6)

(1) becomes P_{22}

Model exists

Resolution - Theorem proving

Enumeration - Truth Table, DPLL