

LeetCode Online Coding Interview Questions – 错题集

deepwaterooo

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Chapter 1

Weekly Plan

1.1 总结题型

- 等 dynamic programming 专题的几道题目写完后，会试图再注册一个账号，将这个专题的题目重新再写一遍。这将会是第一个着力重新写的专题，也是因为它有难度，值得挑战。
- 这个周第二次回去刷了一些 dynamic programming 的题目，真的是感觉似曾相识，这次再回去做的过程中，可以清楚地感觉到以前不会的、或是想得不透彻的地方现在能够想得相对深远一些了，说明好好刷一遍、重要的话说三遍，典型题型多刷几遍是有效用的，认识深刻，二刷的感觉完全不一样
- 等把这第二次的 dynamic programming 专题的题目刷完，我会边刷题，一边准备 android projects. 算法题目中关于数组的部分，我觉得不重要，可刷可不刷。(这两天，今天傍晚、明天和后天我需要搬房间到另一个空出来的，所以这两天周一和周二可能做题会少一点儿)
- I will keep his repository alive for quite a while. And from this week, I will try daily updates later this week after I have finished moving.
- 这周刷得跟上一周差不多。计划接下来几周，每天刷 10 个题左右，一周 60-70 个题目，边刷边记错题进小本，到接下来的周六刷到 1250 左右，多作总结，希望比赛的成绩能更稳定一些；
- 这个周主要复习了基础的部分，hashing, hashtable string two pointers, 以前很讨厌字符串，这个周写字符串看来，感觉基础的只要自己能够想得清楚的，都能很快实现出来，写得还比较得心应手。TreeMap, TreeSet 也用得比较顺一点儿了
- 写树的题目，dfs, bfs graph 各种 node, 现在也是写得很顺心了，只是通过不断地测试加强巩固
- 完成打基础的部分：hashing, hashmap, string, two pointer, sliding window, 这些基础部分的题目，希望扫完
- 如果某天头脑比较清醒、精力比较好的时候，会试图去慢速解决自己平时困难的地方：动态规划/ hashmap/hashing 中数数组的个数，不常用的算法等
- 数组相关的 segment tree, binary index tree 等的基础，希望能够理解得再彻底一些，到能活学活用的程度
- Deque 双端队列 O(n) 解法的概念在建立，还需要很多的练习和熟悉
- 最讨厌扫描线，几个双数怎么也数不清楚 heap 等。。。。。这个周扫几个出去
- 如果说以前是迷迷糊糊刷题求 AC，现在基本的概念在建立，希望从以前代码和题目的算法效率向代码优化中等偏优，寻求高效、最优解法的提升
- bit manipulation, bitmasks 基础知识基本掌握，还剩几道难题 take my time 慢慢解（感觉现在对 bit 操作，相对自信得心应手得多了！）
- union find 的几个题，基本算是基本扫完吧，剩下的几下慢慢写。。。。。
- 很喜欢现在自己搭建出来的 window 刷题环境：WSL system, Zsh power shell, emacs configurations, locally everything, except Leetcode server is too slow, have to tolerant its latency.

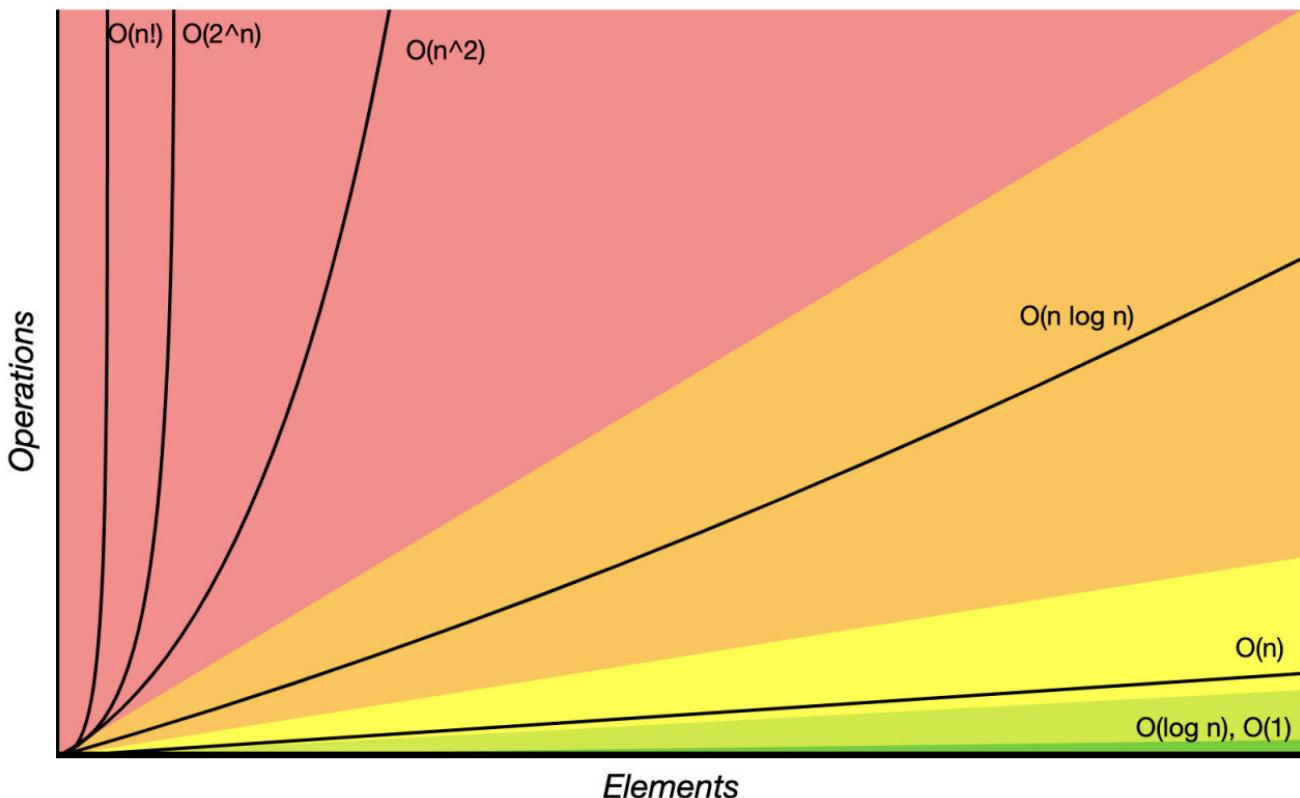
- 至昨天晚上我终于意识到确定右侧单耳耳鸣，搬到现居住处后发病的（感觉现居住处到处都是电磁波干扰、洗衣烘衣的车床，厨房的冰箱，曾整小时整小时开过的洗手间风扇等），已经有几个周了。对于自 2013 年秋天野鸡大学的住宿环境以来，备受各种居住环境的困扰，尤其是 2019 年 9 月 10 月以来，我自小的听力受损，现在单侧耳鸣，可能的原因有家族遗传性高血压、遗传性脑血管肿瘤（外公舅舅和妈妈都受此脑溢血困扰过，大我五岁的亲姐姐前几个月也刚发此重病一次）等等。今天在网上稍搜索了一下相关信息，回想这几年的居住环境噪音和人为打扰与睡眠干扰、心里戚戚很不是滋味，希望我不至于会失去听力。
- But my suffering is still only my/a personal suffering. Unless I could find an appropriate job, nobody cares if you are sick or not. 因此，自信是本能，向往强大也是一种本能的向往。Anything happens, 我还是必须努力努力刷题，直到找到合适的工作. 所以会近几周把剩下的一点儿题目刷完 (hard and medium only, keep easy untouched dont care) 不喜欢数字，也数字无缘，不打算写数组题目，如果一定要写，可能也只会把难题写一写吧

1.2 比赛

- 会尽量多参加一些比赛，比赛时的效率还是相对好一点儿，所以 leetcode 上以后每周一次、每半月一次，以及以后 codeforces 上的比赛，希望都能够尽量地多参加一些。
- codeforces 的刷题界面还不是很熟悉，对于暴内存之类的问题尚没有思考，需要再熟悉一下，希望也能尽量多参加他们家的比赛，希望参加他们家的比赛能够自己写得出答案

1.3 数据规模与算法

Input Size	Complexity
50000	$O(n)$
20000	$O(n \log n)$
1000	$O(n^2)$
30	$O(n^4)$
16 (20)	$O(2^n)$



算法	时间复杂度			空间复杂度
	最佳	平均	最差	
快速排序	$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	$O(\log(n))$
归并排序	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(n)$
Timsort	$O(n)$	$O(n \log(n))$	$O(n \log(n))$	$O(n)$
堆排序	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(1)$
冒泡排序	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$
插入排序	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$
选择排序	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$
希尔排序	$O(n)$	$O((n \log(n))^2)$	$O((n \log(n))^2)$	$O(1)$
桶排序	$O(n + k)$	$O(n + k)$	$O(n^2)$	$O(n)$
基数排序	$O(nk)$	$O(nk)$	$O(nk)$	$O(n + k)$

节点 / 边界管理	存储	增加顶点	增加边界	移除顶点	移除边界	查询
邻接表	$O(V + E)$	$O(1)$	$O(1)$	$O(V + E)$	$O(E)$	$O(V)$
邻接矩阵	$O(V ^2)$	$O(V ^2)$	$O(1)$	$O(V ^2)$	$O(1)$	$O(1)$

类型	时间复杂度						
	建堆	查找最大值	分离最大值	提升键	插入	删除	合并
(排好序的) 链表	-	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$	$O(m + n)$
(未排序的) 链表	-	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$
二叉堆	$O(n)$	$O(1)$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(m + n)$
二项堆	-	$O(1)$	$O(\log(n))$	$O(\log(n))$	$O(1)$	$O(\log(n))$	$O(\log(n))$
斐波那契堆	-	$O(1)$	$O(\log(n))$	$O(1)$	$O(1)$	$O(\log(n))$	$O(1)$

Chapter 2

Dynamic Programming, 动态规划

2.1 总结

2.2 第二次仍不会的题号记这里

- 1872 (stone game 8), 647 329 494 131 518 1723(need summary)
- 根据 CLRS, 动态规划分为两种:
- top-down with memoization (递归记忆化搜索)

等价于带缓存的, 搜索树上的 DFS 比较贴近于新问题正常的思考习惯

- bottom-up (自底向上循环迭代)

以"reverse topological order" 处理每个子问题下面依赖的所有子问题都算完了才开始计算当前一般依赖于子问题之间天然的"size"

2.2.1 647. Palindromic Substrings - Medium

Given a string s, return the number of palindromic substrings in it.

A string is a palindrome when it reads the same backward as forward.

A substring is a contiguous sequence of characters within the string.

```
public int countSubstrings(String t) {
    int n = t.length(), ans = 0;
    char [] s = t.toCharArray();
    boolean [][] dp = new boolean [n][n];
    for (int i = n-1; i >= 0; i--)
        for (int j = i; j < n; j++) {
            dp[i][j] = s[i] == s[j] && (j-i <= 2 || dp[i+1][j-1]);
            if (dp[i][j]) ans++;
        }
    return ans;
}
```

2.2.2 516. Longest Palindromic Subsequence - Medium

Given a string s, find the longest palindromic subsequence's length in s.

A subsequence is a sequence that can be derived from another sequence by deleting some or no elements without changing the order of the remaining elements.

只要把原字符串反过来, 两个字符串找最长公共子序列, 就是最长回文了

```
public int longestPalindromeSubseq(String tt) {
    int n = tt.length();
    char [] s = tt.toCharArray(); // ori
    char [] t = (new StringBuilder(tt).reverse().toString()).toCharArray(); // reverse
    int [][] dp = new int [n+1][n+1];
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++) {
```

```

        if (s[i-1] == t[j-1]) dp[i][j] = dp[i-1][j-1] + 1;
    else dp[i][j] = Math.max(dp[i-1][j], dp[i][j-1]);
}
return dp[n][n];
}

```

2.2.3 1872. Stone Game VIII - hard 需要好好理解消化

Alice and Bob take turns playing a game, with Alice starting first. There are n stones arranged in a row. On each player's turn, while the number of stones is more than one, they will do the following: Choose an integer $x > 1$, and remove the leftmost x stones from the row. Add the sum of the removed stones' values to the player's score. Place a new stone, whose value is equal to that sum, on the left side of the row. The game stops when only one stone is left in the row. The score difference between Alice and Bob is (Alice's score - Bob's score). Alice's goal is to maximize the score difference, and Bob's goal is the minimize the score difference. Given an integer array stones of length n where $\text{stones}[i]$ represents the value of the i th stone from the left, return the score difference between Alice and Bob if they both play optimally.

1. 解题思路与分析

这里我原始的做法 `dfs + 记忆数组` 会超时，是因为数组发生了改变，其盘状态发生了改变，所以记忆无效!!!! 才会超时（感觉还理解得不透，这里）

所以采用反向遍历的方法，将 $O(N^2)$ 变为 $O(N)$

```

dp(i) := max relative score current player
can achieve by taking the first i stones.

dp(i) = sum(S[0:i])
      - max(dp(i+1), dp(i+2), ..., dp(n))

base case: dp(n) = sum(S)

ans = max(dp(2), ..., dp(n))

Naive implementation takes O(n2) time =>
TLE

```

```

# Author: Huahua 59/78 Passed
class Solution:
    def stoneGameVIII(self, stones: List[int]) -> int:
        n = len(stones)
        sums = [0] * (n + 1)
        for i in range(n):
            sums[i + 1] = sums[i] + stones[i]

        @cache
        def dp(i: int) -> int:
            if i == n: return sums[n]
            return sums[i] - max(dp(j) for j in range(i + 1, n + 1))

        return max(dp(i) for i in range(2, n + 1))

```

```

dp(i) = sum(S[0:i])
      - max(dp(i+1), dp(i+2), ..., dp(n))

Since dp(i) depends on dp(j) where j>i, we
can compute dp in reverse order e.g compute
dp(n) first and store the best dp so far.

• best = dp(n)
• dp(n-1) = sum(S[0:n-1]) - best
• best = max(best, dp(n-1))
• dp(n-2) = sum(S[0:n-2]) - best
• best = max(best, dp(n-2))
• ...
• dp(2) = sum(S[0:2]) - best

dp(i) = sum(S[0:i]) - best
ans = best

Time complexity: O(n), Space -> O(1)

```

```

best = [sums[n]] # dp(n)

@cache
def dp(i: int) -> int:
    if i == n: return sums[n]
    ans = sums[i] - best[0]
    best[0] = max(ans, best[0])
    return ans
    n to 2

return max(dp(i) for i in range(n, 1, -1))

class Solution {
public:
    int stoneGameVIII(vector<int>& stones) {
        const int n = stones.size();
        for (int i = 1; i < n; ++i)
            stones[i] += stones[i - 1];
        int ans = stones.back(); // dp[n]
        for (int i = n - 1; i >= 2; --i)
            ans = max(ans, stones[i - 1] - ans);
        return ans;
    }
};

```

```

// 使用 dp(i) 表示还剩下  $[i, n]$  要选择的情况下, Alice 所能得到的最大分数差。
// 对于某个玩家来说, 其对应决策可以分为两种:
// 选取当前数及之前的所有数 (等价于 pres[pos], 其中 pos 为上个玩家选完后的下个位置), 那么 dp[i] = pres[i] - dp[i+1]。
// 这是因为 bob 也会最大化发挥。
// 不选择当前数 (可能选下一个, 下下一个。。。 etc), 那么 dp[i] = dp[i + 1]
public int stoneGameVIII(int[] stones) {
    int n = stones.length;

```

```

int [] dp = new int [n];
Arrays.fill(dp, Integer.MIN_VALUE);
int [] pre = new int [n+1];
for (int i = 1; i <= n; i++)
    pre[i] = pre[i-1] + stones[i-1];
dp[n-1] = pre[n];
for (int i = n-2; i >= 0; i--)
    dp[i] = Math.max(dp[i+1], pre[i+1]-dp[i+1]);
return dp[0];
}

```

- 更精简的代码如下

```

public int stoneGameVIII(int[] stones) {
    int n = stones.length;
    for (int i = 1; i < n; i++)
        stones[i] += stones[i-1]; // 原位求前缀和
    int ans = stones[n-1];
    for (int i = n-1; i >= 2; i--)
        ans = Math.max(ans, stones[i-1] - ans); // 一遍反向遍历求最优解
    return ans;
}

```

2.2.4 464. Can I Win 这个题：为什么顺序无关了？

In the "100 game" two players take turns adding, to a running total, any integer from 1 to 10. The player who first causes the running total to reach or exceed 100 wins. What if we change the game so that players cannot re-use integers? For example, two players might take turns drawing from a common pool of numbers from 1 to 15 without replacement until they reach a total ≥ 100 . Given two integers maxChoosableInteger and desiredTotal, return true if the first player to move can force a win, otherwise, return false. Assume both players play optimally.

```

// state 是前走的人走完之后的局面, sum 是当前数字总和, 返回的是当前走的人是否能赢
private boolean dfs(int max, int target, int state, int val) {
    if (dp[state] != -1) return dp[state] > 0;
    if (val >= target) { // 如果对方取数的时候总和达到 target 了, 则当前走的人输了, 做记忆并返回 false
        dp[state] = 0;
        return false;
    }
    for (int i = 1; i <= max; i++) { // 枚举当前人取哪个数
        if ((state >> i-1 & 1) == 0 && !dfs(max, target, state | (1 << i-1), val + i)) {
            dp[state] = 1;
            return true;
        }
    }
    dp[state] = 0;
    return false;
}
int [] dp;
public boolean canIWin(int maxChoosableInteger, int desiredTotal) {
    if (desiredTotal <= maxChoosableInteger) return true;
    if (desiredTotal > (maxChoosableInteger + 1)*maxChoosableInteger / 2) return false;
    dp = new int[1 << maxChoosableInteger]; // 时空复杂度 O(2^m) O(2^m)O(2^m)
    Arrays.fill(dp, -1);
    return dfs(maxChoosableInteger, desiredTotal, 0, 0);
}

```

- 另外这第二次又看见的解法

```

public boolean canIWin(int maxChoosableInteger, int desiredTotal) { // 这个题与其它类假题相比, 为什么顺序无关?
    if (desiredTotal == 0) return true; // 如果 1 到最大能选的值所有都不能满足目标值, 那么肯定失败
    if ((maxChoosableInteger+1) * maxChoosableInteger / 2 < desiredTotal) return false;
    char [] state = new char [maxChoosableInteger];
    for (int i = 0; i < maxChoosableInteger; i++) state[i] = '0';
    return dfs(desiredTotal, state, new HashMap<>());
}
private boolean dfs(int sum, char [] st, Map<String, Boolean> map) {
    String key = new String(st);
    if (map.containsKey(key)) return map.get(key);
    for (int i = 0; i < st.length; i++) {
        if (st[i] != '0') continue;
        st[i] = '1';
        if (sum <= i+1 || !dfs(sum - (i+1), st, map)) {
            map.put(key, true);
            st[i] = '0';
            return true;
        }
    }
}

```

```

        }
        st[i] = '0';
    }
    map.put(key, false);
    return false;
}

```

- // 下面这个效率更高

```

public boolean canIWin(int maxChoosableInteger, int desiredTotal) {
    if (desiredTotal <= 0) return true;
    int sum = (maxChoosableInteger + 1) * maxChoosableInteger / 2;
    if (sum < desiredTotal) return false;
    boolean[] vis = new boolean[maxChoosableInteger+1];
    return helper(desiredTotal, vis);
}

Map<Integer, Boolean> map = new HashMap<>();
public boolean helper(int desiredTotal, boolean[] vis) {
    if (desiredTotal <= 0) return false;
    int symbol = format(vis);
    if (map.containsKey(symbol)) return map.get(symbol);
    for (int i = 1 ; i < vis.length ; i++) {
        if (!vis[i]) {
            vis[i] = true;
            if (!helper(desiredTotal-i, vis)) {
                vis[i] = false; // 这里不回复状态会影响其它结果
                map.put(symbol, true);
                return true;
            }
            vis[i] = false;
        }
    }
    map.put(symbol, false);
    return false;
}

public int format(boolean[] vis) {
    int symbol = 0;
    for (boolean select : vis) {
        symbol <= 1;
        if (select) symbol |= 1;
    }
    return symbol;
}

```

2.2.5 494. Target Sum - Medium

You are given an integer array `nums` and an integer `target`.

You want to build an expression out of `nums` by adding one of the symbols '+' and '-' before each integer in `nums` and then concatenate all the integers.

For example, if `nums` = [2, 1], you can add a '+' before 2 and a '-' before 1 and concatenate them to build the expression "+2-1". Return the number of different expressions that you can build, which evaluates to target.

- 该题是一道非常经典的题目，在面试中很可能会考到。该题有多种解法。
- 第一种解法：DFS, brute force。我们对 `nums` 数组中的每个数字，都尝试在其前面添加正号和负号，最后暴力求解，统计数组中各数字组合值为 `target` 的情况。（该理解是错误的，我们可以使用带备忘录机制的自顶向下的 DP 方法，代码见下）

1. 回溯 O (2^N)

```

private int getAllSums(int [] a, int target, int idx, int sum, int cnt) { // (2^20) 可否一试呢？理论上是可以过的
    if (idx == a.length) {
        if (sum == target) cnt++;
        // n < 17 比较好 这个 2^N 的复杂度，真要命呀。。。。。。
        return cnt; // 有 return int 代码更简洁，但是全局变量 cnt 效率更高
    }
    // for (int i = idx; i < a.length; i++) { // 为什么要画蛇添足，加个多余的 for loop 呢？
        // getAllSums(a, target, idx+1, sum + a[idx]);
        // getAllSums(a, target, idx+1, sum - a[idx]);
    }
    return getAllSums(a, target, idx+1, sum + a[idx], cnt)
        + getAllSums(a, target, idx+1, sum - a[idx], cnt);
}
public int findTargetSumWays(int[] a, int target) {
    int n = a.length;
    return getAllSums(a, target, 0, 0, 0);
}

```

2. 解题思路与分析: dfs 记忆化搜索

```

private int dfs(int[] a, int target, int idx, int sum) {
    String key = idx + "_" + sum;
    if (dp.containsKey(key)) return dp.get(key);
    if (idx == n) {
        if (sum == target) return 1;
        else return 0;
    }
    int add = dfs(a, target, idx+1, sum + a[idx]);
    int sub = dfs(a, target, idx+1, sum - a[idx]);
    dp.put(key, add+sub);
    return add + sub;
}
Map<String, Integer> dp = new HashMap<>();
int n;
public int findTargetSumWays(int[] a, int target) {
    n = a.length;
    return dfs(a, target, 0, 0);
}

```

- 上面的方法比较慢，下面这个效率更好一点儿

```

private int dfs(int[] a, int sum, int idx) {
    if (idx == a.length) {
        if (sum == 0) return 1;
        else return 0;
    }
    Map<Integer, Integer> tmp = dp.get(idx);
    if (tmp != null) {
        if (tmp.containsKey(sum))
            return tmp.get(sum);
    } else {
        tmp = new HashMap<>();
        dp.put(idx, tmp);
    }
    int cnt = dfs(a, sum - a[idx], idx+1) + dfs(a, sum + a[idx], idx+1);
    tmp.put(sum, cnt);
    return cnt;
}
Map<Integer, Map<Integer, Integer>> dp = new HashMap<>();
public int findTargetSumWays(int[] nums, int target) {
    return dfs(nums, target, 0);
}

```

3. DP

```

// sum[p] + sum[n] = sum[nums];
// sum[p] - sum[n] = S;
// 2sum[p] = sum[nums] + S
// sum[p] = (sum[nums] + S) / 2
public int findTargetSumWays(int[] a, int S) {
    int sum = Arrays.stream(a).sum(), target = (sum + S) / 2; // 根据推导公式，计算出 target
    if (S > 0 && sum < S || S < 0 && -sum > S) return 0; // 如果和小于 S，说明无法得到解，返回 false。(注意 S 有可能为负)
    if ((sum + S) % 2 != 0) return 0; // 如果计算出的 target 不是整数，返回 false。
    int[] dp = new int[target + 1]; // dp[i] 表示在原数组中找出一些数字，并且他们的和为下标 i 的可能有多少种。
    dp[0] = 1; // 初始化 dp[0] 为 1
    for (Integer v : a) {
        // for (int i = target-v; i >= 0; i--) { // 从 0 循环到 target - n，注意逆序
        //     if (dp[i] > 0) // dp[i] 大于 0 说明，存在 dp[i] 种组合，其和为 i 的可能性
        //         dp[i+v] += dp[i]; // 既然存在和为 i 的可能，那么 i 加上当前数字的和也是存在的
        // }
        for (int i = target; i >= v; i--) // 从 0 循环到 target - n，注意逆序
            dp[i] += dp[i-v]; // 两种写法都对
    }
    return dp[target];
}

```

4. dp todo 我们使用 V_i 来表示数组中的前 i 个数所能求得的和的集合。初始化时

```

V0 = {0}      // 表示前 0 个数的和为 0
Vi = {V(i-1) + ai} U {V(i-1) - ai}

```

V_n 就是 nums 数组所有数字的组合值之和的集合

根据上面的思路，我们知道数组中数字若全为正号其和为 sum ，全为负号其和为 $-\text{sum}$ 。若不选数组中任何一个数，则和为 0。因此，我们设立一个长度为 $2*\text{sum}+1$ 的数组 ways ， $\text{ways}[i]$ 表示我们选择前 m 个数，其和可能为 i 的情况数， $m = 0, 1, \dots, \text{nums.length}$ 。可参考下图

Why DP works?

Let's look at a simpler problem: Can the following equation can be true?

$$\pm a_1 \pm a_2 \pm a_3 \dots \pm a_n = \text{target}$$

$O(2^n)$ combinations, but $S = 2 * \text{sum}(a) + 1$ total possible sums, $S \leq 2000 + 1$

We use V_i to denote the possible sums by using first i elements

$$V_0 = \{0\}$$

$$V_i = \{V_{i-1} + a_i\} \cup \{V_{i-1} - a_i\}$$

Check target in V_n

DP works because $|V_n| \leq S \ll O(2^n)$

Time complexity is $\text{Sum}\{2^{|V_i|}\} \leq n * S = O(n * S)$

input: [1,1,1,1,1], target = 3

$2^5 = 32$ combination, but total 6 distinct values, max is 11 ($2^5 + 1$)

i	a_i	V_i
0	-	{0}
1	1	{-1, 1}
2	1	{-2, 0, 2}
3	1	{-3, -1, 1, 3}
4	1	{-4, -2, 0, 2, 4}
5	1	{-5, -3, -1, 1, 3, 5}

input: [1,1,1,1,1], target = 3

sum = 5, range = -5 ~ 5

i	a_i	W[i][j]												
		-5	-4	-3	-2	-1	0	1	2	3	4	5		
0	-						1							
1	1					1		1						
2	1				1		2		1					
3	1			1		3		3		1				
4	1		1		4		6		4		1			
5	1	1		5		10		10		5		1		

Sum of (W[5][j]) is 32 = 2^5

<http://zxi.mytechroad.com/blog/>

<https://www.cnblogs.com/cnoodle/p/14869498.html> [https://leetcode.com/problems/target-sum/discuss/97334/Java-\(15-ms\)-C++-\(3-ms\)-O\(ns\)-iterative-DP-solution-using-subset-sum-with-explanation/239290](https://leetcode.com/problems/target-sum/discuss/97334/Java-(15-ms)-C++-(3-ms)-O(ns)-iterative-DP-solution-using-subset-sum-with-explanation/239290) <http://www.noteanddata.com/leetcode-494-Target-Sum-javascript-solution-note.html> <https://www.i4k.xyz/article/gqk289/54709004> <https://github.com/cherryljr/LeetCode/blob/master/Target%20Sum.java>

2.2.6 518. Coin Change 2

You are given an integer array coins representing coins of different denominations and an integer amount representing a total amount of money. Return the number of combinations that make up that amount. If that amount of money cannot be made up by any combination of the coins, return 0. You may assume that you have an infinite number of each kind of coin. The answer is guaranteed to fit into a signed 32-bit integer.

```
public int change(int target, int[] nums) {
    int[] dp = new int[target + 1];
    // 初始化 dp[0] 为 1
    dp[0] = 1;
    // 循环数组中所有数字
    for (int val : nums) {
        for (int i = 0; i <= target - val; i++) {
            // dp[i] 大于 0 说明，存在 dp[i] 种组合，其和为 i 的可能性
            if (dp[i] > 0) {
                // 既然存在和为 i 的可能，那么 i 加上当前数字的和也是存在的
                dp[i + val] += dp[i];
            }
        }
    }
}
```

```

        }
    }
    return dp[target];
}

```

2.2.7 1444. Number of Ways of Cutting a Pizza - Hard

Given a rectangular pizza represented as a rows x cols matrix containing the following characters: 'A' (an apple) and '.' (empty cell) and given the integer k. You have to cut the pizza into k pieces using k-1 cuts.

For each cut you choose the direction: vertical or horizontal, then you choose a cut position at the cell boundary and cut the pizza into two pieces. If you cut the pizza vertically, give the left part of the pizza to a person. If you cut the pizza horizontally, give the upper part of the pizza to a person. Give the last piece of pizza to the last person.

Return the number of ways of cutting the pizza such that each piece contains at least one apple. Since the answer can be a huge number, return this modulo $10^9 + 7$.

1. 解题思路与分析: 自底向上

常规的矩阵 DP 做法, 这里还需要通过前缀和的思想来快速获取指定范围矩阵的苹果数量。

首先是建立状态表示数组, 通过一个三维数组, 分别代表矩阵左上角顶点 xy 坐标和需要分配的人数, 数组值表示分该状态下的配方案数;

然后是进行状态转移, 从右下角开始枚举所有以该点为状态中左上角的状态, 再从低到高枚举需要分配的人数, 接着进行切的操作, 可以横着切和竖着切, 分别枚举所有可能的切除的长度, 当前状态的方案数需要从切除后剩下的矩阵状态中进行转移累加。

最后返回以原矩阵左上角为顶点的, 分配人数为 k 的方案数即可。

这里为什么需要将状态表示中的 xy 设定为矩阵的左上角, 还有为什么苹果数的前缀和也是求的右下角的前缀和呢?

因为题意中的切除操作后, 要将上半部分或者左半部分给分掉, 所以只有右下部分是剩余状态的, 我们需要从切除之前的状态获取剩余状态。

```

public int ways(String[] pizza, int p) {
    int mod = (int)1e9 + 7;
    int m = pizza.length, n = pizza[0].length;
    int [][] cnt = new int [m+1][n+1]; // 苹果数的前缀和, 用于快速获得在指定矩阵范围内的苹果数量, 两个维度也分别是左上角的 x、y
    for (int i = m-1; i >= 0; i--) {
        for (int j = n-1; j >= 0; j--) {
            cnt[i][j] = cnt[i+1][j] + cnt[i][j+1] - cnt[i+1][j+1] + (pizza[i].charAt(j) == 'A' ? 1 : 0);
        }
    }
    int [[[dp = new int [m+1][n+1][p+1]; // 状态数组, 三个维度分别表示以 x、y 为左上角的矩阵中, 分给 k 个人, 元素值表示方案数
    for (int i = m-1; i >= 0; i--) // 遍历矩阵, 获取指定左上角矩阵中范围内的苹果数量
        for (int j = n-1; j >= 0; j--) { // 从右下角开始, 向左上角开始枚举所有状态
            if (cnt[i][j] > 0) dp[i][j][1] = 1; // 如果这个矩阵内存在苹果, 那么这个矩阵肯定可以分给 1 个人, 且方案数为 1
            for (int k = 2; k <= p; k++) { // 枚举所有人数状态下的方案, 前面已经判断了人数为 1 的状态, 所以这里只需要从 2 开始枚举
                for (int x = m-1-i; x >= 0; x--) // 横着切, 枚举所有切法
                    if (cnt[i][j] - cnt[i+x][j] > 0) // 如果当前切掉的矩阵内存在苹果, 则可以进行状态转移
                        dp[i][j][k] = (dp[i][j][k] + dp[i+x][j][k-1]) % mod;
                for (int y = n-1-j; y >= 0; y--) // 竖着切
                    if (cnt[i][j] - cnt[i][j+y] > 0)
                        dp[i][j][k] = (dp[i][j][k] + dp[i][j+y][k-1]) % mod;
            }
        }
    }
    return (int)dp[0][0][p];
}

```

2. 解题思路与分析: 自顶向下

先用 dp 方法求出以 (i,j) 位置为右下角, 左上角为 (0,0) 的区域的苹果数量

建立 3 维数组, $dp[i][j][k]$ 表示切完 k 次后, 剩余蛋糕左上角在 i, j 位置时的方案数

初始化, $dp^{1,1,1} = 1$

样本维度为切的次数 k

状态维度, 这次切之前的状态 (蛋糕左上角位置 i, j)

状态转移, 这次切完后蛋糕左上角位置 (横向切, ni, nj; 竖向切, i, nj, 切的次数 +1)

转移条件: 切出去的蛋糕当中有苹果 (用上面求得的苹果数量, dp 公式求得)

最后求结果总和: 最后的一块蛋糕中有苹果, $sum += dp[i][j][k-1]$

¹DEFINITION NOT FOUND.

```

public int ways(String[] pizza, int p) { // 自顶向下: 与自底向上相比
    int mod = (int)1e9 + 7;
    int m = pizza.length, n = pizza[0].length();
    int [][] cnt = new int [m+1][n+1]; // 苹果数的前缀和, 用于快速获得在指定矩阵范围内的苹果数量, 两个维度也分别是左上角的 x、y
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            cnt[i][j] = cnt[i-1][j] + cnt[i][j-1] - cnt[i-1][j-1] + (pizza[i-1].charAt(j-1) == 'A' ? 1 : 0);
    int [[[dp = new int [m+1][n+1][p]; // dp[i][j][k] 表示切完 k 次后, 剩余蛋糕左上角 在 i,j 位置时的方案数
    dp[1][1][0] = 1; // 初始值是为了程序的运行,
    for (int k = 1; k < p; k++)
        for (int i = 1; i <= m; i++)
            for (int j = 1; j <= n; j++) {
                System.out.println("(dp[i][j][k-1] == 0) : " + (dp[i][j][k-1] == 0));
                if (dp[i][j][k-1] == 0) continue; // 上一次 cut 完后, 剩余蛋糕左上角在 i,j
                for (int x = i+1; x <= m; x++) // 横向切, 切完后的剩余左上角为 x, j
                    if (cnt[x-1][n] - cnt[i-1][n] - cnt[x-1][j-1] + cnt[i-1][j-1] > 0)
                        dp[x][j][k] = (dp[x][j][k] + dp[i][j][k-1]) % mod;
                for (int y = j+1; y <= n; y++) // 坚向切
                    if (cnt[m][y-1] - cnt[m][j-1] - cnt[i-1][y-1] + cnt[i-1][j-1] > 0)
                        dp[i][y][k] = (dp[i][y][k] + dp[i][j][k-1]) % mod;
            }
    long ans = 0;
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            if (cnt[m][n] - cnt[i-1][n] - cnt[m][j-1] + cnt[i-1][j-1] > 0) // 先前并没有确认切的结果有效, 即最后剩下的那块是否有苹果
                ans = (ans + dp[i][j][p-1]) % mod; // 统计结果的时候, 要先确保有效
    return (int)ans;
}

```

2.3 字符串、数组等双序列

一、题目分析

题意：实现支持'?'和'*'的通配符匹配的字符串匹配。

分析：类似 [最长公共子串](#), [最长公共子序列](#), [编辑距离](#)等求2个字符串(或数组)之间的某种关系的题目，一般来说都是有动态规划的解法的。

1、状态定义：

$dp[i][j]$ 表示 p 的前 i 个字符和 s 的前 j 个字符是否匹配。

2、状态转移：

如果 $p[i - 1] == s[j - 1]$ 或 $p[i - 1] == ?$ ，表示当前的字符串是匹配的， $dp[i][j]$ 可以从 $dp[i - 1][j - 1]$ 转移而来。

如果 $p[i - 1] == \ast$ ，这个位置可以匹配0到若干个字符。那么 $dp[i][j]$ 可以从 $dp[i - 1][j]$ 转移而来（表示当前星号没有匹配字符），也可以从 $dp[i][j - 1]$ 转移而来（表示当前星号匹配了当前位置的字符）。因为只要任意一种匹配即可，所以这里是逻辑或的关系。

状态转移方程如下：

$$dp[i][j] = \begin{cases} dp[i - 1][j - 1] & p[i - 1] == s[j - 1] \text{ || } p[i - 1] == ? \\ dp[i - 1][j] \mid dp[i][j - 1] & p[i - 1] == \ast \end{cases}$$

3、初始条件

$dp[0][0] = true$ 表示空串是匹配的。

处理一下匹配串 p 以若干个星号开头的情况。因为星号是可以匹配空串的：

```
for (int i = 1; i <= p.length(); i++) {
    if (p.charAt(i - 1) != '*') {
        break;
    }
    dp[i][0] = true;
}
```

2.3.1 题目拓展

1. 718. 最长重复子数组 (类似题目，只是由字符串变为数组)
2. 72. 编辑距离
3. 1143. 最长公共子序列
4. 10. 正则表达式匹配
5. 583. 两个字符串的删除操作
6. 727. 最小窗口子序列

你会发现这些都是求2个字符串(或数组)之间的某种关系的题目

2.3.2 10. Regular Expression Matching - Hard

Given an input string s and a pattern p , implement regular expression matching with support for '.' and '*' where:
'.': Matches any single character.
'*': Matches zero or more of the preceding element. The matching should cover the entire input string (not partial).

1. 解题思路与分析

也就是我们「浪费」了一个字符 + 星号的组合，没有匹配任何 s 中的字符。

在匹配 1, 2, 3, … 次的情况下，类似地我们有

$$\begin{aligned} f[i][j] &= f[i-1][j-2], && \text{if } s[i] = p[j-1] \\ f[i][j] &= f[i-2][j-2], && \text{if } s[i-1] = s[i] = p[j-1] \\ f[i][j] &= f[i-3][j-2], && \text{if } s[i-2] = s[i-1] = s[i] = p[j-1] \\ &\dots \end{aligned}$$

如果我们通过这种方法进行转移，那么我们就需要枚举这个组合到底匹配了 s 中的几个字符，会增导致时间复杂度增加，并且代码编写起来十分麻烦。我们不妨换个角度考虑这个问题：字母 + 星号的组合在匹配的过程中，本质上只会有两种情况：

- 匹配 s 末尾的一个字符，将该字符扔掉，而该组合还可以继续进行匹配；
- 不匹配字符，将该组合扔掉，不再进行匹配。

如果按照这个角度进行思考，我们可以写出很精巧的状态转移方程：

$$f[i][j] = \begin{cases} f[i-1][j] \text{ or } f[i][j-2], & s[i] = p[j-1] \\ f[i][j-2], & s[i] \neq p[j-1] \end{cases}$$

- 在任意情况下，只要 $p[j]$ 是 `.`，那么 $p[j]$ 一定成功匹配 s 中的任意一个小写字母。

最终的状态转移方程如下：

$$f[i][j] = \begin{cases} \text{if } (p[j] \neq '*') = \begin{cases} f[i-1][j-1], & \text{matches}(s[i], p[j]) \\ \text{false}, & \text{otherwise} \end{cases} \\ \text{otherwise} = \begin{cases} f[i-1][j] \text{ or } f[i][j-2], & \text{matches}(s[i], p[j-1]) \\ f[i][j-2], & \text{otherwise} \end{cases} \end{cases}$$

其中 $\text{matches}(x, y)$ 判断两个字符是否匹配的辅助函数。只有当 y 是 `.` 或者 x 和 y 本身相同时，这两个字符才会匹配。

```
public boolean isMatch(String s, String p) {
    int m = s.length();
    int n = p.length();
    boolean[][] f = new boolean[m + 1][n + 1];
    f[0][0] = true;
    for (int i = 0; i <= m; ++i)
        for (int j = 1; j <= n; ++j)
            if (p.charAt(j - 1) == '*') {
                f[i][j] = f[i][j - 2];
                if (matches(s, p, i, j - 1))
                    f[i][j] = f[i][j] || f[i - 1][j];
            } else {
                if (matches(s, p, i, j))
                    f[i][j] = f[i - 1][j - 1];
            }
    return f[m][n];
}
public boolean matches(String s, String p, int i, int j) {
    if (i == 0) return false;
    if (p.charAt(j - 1) == '.') return true;
    return s.charAt(i - 1) == p.charAt(j - 1);
}
```

2.3.3 将一个数组分为两个部分，分别求和 $S1$ 与 $S2$ ，使得 $|S1-S2|$ 最小

一、题目描述：

将一个数组分为两个部分，分别求和 $S1$ 与 $S2$ ，使得 $|S1 - S2|$ 最小。

解法：动态规划

解法描述：转化为求数组的一个子集，使得这个子集中的元素的和尽可能接近 $sum/2$ ，其中 sum 为数组中所有元素的和。这样转换之后这个问题就很类似 0-1 背包问题了：在 n 件物品中找到 m 件物品，他们的可以装入背包中，且总价值最大不过这里不考虑价值，就考虑使得这些元素的和尽量接近 $sum/2$ 。

状态转移方程：

$dp[i][j]$ 表示前 i 件物品中，总和最接近 j 的所有物品的总和，其中包括两种情况：

- 第 i 件物品没有包括在其中
如果第 i 件物品没有包括在其中，则 $dp[i][j] = dp[i-1][j]$
- 第 i 件物品包括在其中
如果第 i 件物品包括在其中，则 $dp[i][j] = dp[i-1][j - vec[i]]$

当然，这里要确保 $j - vec[i] \geq 0$ 。

所以状态转移方程为：

$$dp[i][j] = \max(dp[i-1][j], dp[i-1][j - vec[i]] + vec[i]);$$

```
public static int getMaxDiff(int[] array) {
    int sum = Arrays.stream(array).sum();
    int length = array.length;
    int [][] f = new int[length+1][sum/2+1];
    for (int i = 0; i < length; i++) {
        for (int j = 1; j <= sum/2; j++) {
            f[i+1][j] = f[i][j];
            if (array[i] <= j && f[i][j-array[i]] + array[i] > f[i][j])
                f[i+1][j] = f[i][j-array[i]] + array[i];
        }
    }
    return sum-2*f[length][sum/2];
}
```

2.3.4 给定一个序列，不保证有序，求这个序列的最长等差序列的长度。

二、题目描述

给定一个序列，不保证有序，求这个序列的最长等差序列的长度。

解法：动态规划

解法描述：令 $f[i][j]$ 表示以 i 为结尾的某子序列（该子序列的等差为 j ）的最大长度；

状态转移方程：

$$f[i][j] = f[i-1][Num[i] - Num[j]] + 1, i > 0$$

```
private static int lengthOfLongest(int[] set){
    Arrays.sort(set);
    int n = set.length;
    if (n <= 2) return n;
    int llap = 2;
    int [][] dp = new int[n][n];
    for (int i=0; i<n; i++) dp[i][n-1] = 2;
    for (int j=n-2; j>=1; j--) {
        int i=j-1, k=j+1;
        while (i>=0 && k<=n-1) {
            if (set[i] + set[k] < 2 * set[j])
                k++;
            else if (set[i] + set[k] > 2 * set[j]) {
                dp[i][j] = 2;
            } else
                dp[i][j] = 3;
        }
    }
}
```

```

        i--;
    } else {
        dp[i][j] = dp[j][k] + 1;
        llap = Math.max(llap, dp[i][j]);
        i--;
        k++;
    }
}
while (i >= 0) {
    dp[i][j] = 2;
    i--;
}
}
return llap;
}

```

2.3.5 求一个序列的最长子序列，使得最多修改一个数字使得这个子序列的为严格递增序列

题目描述

求一个序列的最长子序列，使得最多修改一个数字使得这个子序列的为严格递增序列。

状态转移方程

$dp[n][2]$ 定义为：

- $dp[i][0]$: 到目前为止最长严格递增子序列的长度；
- $dp[i][1]$: 到目前为止允许最多修改1位的严格递增子序列长度；

```

private static int getMaxLength(int[] arr){
    if (arr.length <= 2) return arr.length;
    int[][] dp = new int[arr.length][2];
    dp[0][0] = 1;
    dp[0][1] = 1;
    for (int i = 1; i < arr.length; i++) {
        dp[i][0] = dp[i-1][0]+1;
        if (arr[i] <= arr[i-1])
            dp[i][0]--;
        if (dp[i-1][0] == dp[i-1][1] && arr[i] <= arr[i-1]) // 说明前面还没有改
            dp[i][1] = dp[i][0] + 1;
            arr[i] = arr[i-1]+1;
        } else { // 说明前面已经改动或者 arr[i] <= arr[i-1]
            if (arr[i] > arr[i-1]) {
                // 判断前面是否已经改动
                dp[i][1] = dp[i-1][1]+1;
                if (dp[i-1][0] != dp[i-1][1])
                    dp[i][1]--;
            } else
                dp[i][1] = dp[i-1][1];
        }
    }
    return dp[arr.length-1][1];
}

```

2.3.6 115. Distinct Subsequences - Hard

Given two strings s and t, return the number of distinct subsequences of s which equals t.

A string's subsequence is a new string formed from the original string by deleting some (can be none) of the characters without disturbing the remaining characters' relative positions. (i.e., "ACE" is a subsequence of "ABCDE" while "AEC" is not).

It is guaranteed the answer fits on a 32-bit signed integer.

1. 解题思路与分析 这道题不是求两个字符串是匹配，而是判断 S 有多少种方式可以得到 T。但其实还是动态规划，我们一个定义二维数组 dp , $dp[i][j]$ 为字符串 $s(0,i)$ 变换到 $t(0,j)$ 的变换方法的个数。

如果 $S[i]==T[j]$, 那么 $dp[i][j] = dp[i-1][j-1] + dp[i-1][j]$

意思是：如果当前 $S[i]==T[j]$, 那么当前这个字符即可以保留也可以抛弃，所以变换方法等于保留这个字符的变换方法加上不用这个字符的变换方法，

$dp[i-1][j-1]$ 为保留这个字符时的变换方法个数, $dp[i-1][j]$ 表示抛弃这个字符时的变换方法个数。

如果 $S[i]!=T[i]$, 那么 $dp[i][j] = dp[i-1][j]$, 意思是如果当前字符不等, 那么就只能抛弃当前这个字符。

```

public int numDistinct(String ss, String tt) {
    int m = ss.length(), n = tt.length();
    char [] s = ("#" + ss).toCharArray();
    char [] t = ("#" + tt).toCharArray();
    int [][] dp = new int [m+1][n+1];
    dp[0][0] = 1;
    for (int j = 1; j <= n; j++) // 注意这两行初始状态的设置
        dp[0][j] = 0;
    for (int i = 1; i <= m; i++)
        dp[i][0] = 1;
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            if (s[i] == t[j])
                dp[i][j] = dp[i-1][j-1] + dp[i-1][j];
            else dp[i][j] = dp[i-1][j];
    return dp[m][n];
}

```

2.4 区间型 DP

2.4.1 2019. The Score of Students Solving Math Expression - Hard

有人说这是区间 dp，无感

You are given a string s that contains digits 0-9, addition symbols '+', and multiplication symbols '*' only, representing a valid math expression of single digit numbers (e.g., $3+5*2$). This expression was given to n elementary school students. The students were instructed to get the answer of the expression by following this order of operations:

Compute multiplication, reading from left to right; Then, Compute addition, reading from left to right. You are given an integer array answers of length n , which are the submitted answers of the students in no particular order. You are asked to grade the answers, by following these rules:

If an answer equals the correct answer of the expression, this student will be rewarded 5 points; Otherwise, if the answer could be interpreted as if the student applied the operators in the wrong order but had correct arithmetic, this student will be rewarded 2 points; Otherwise, this student will be rewarded 0 points. Return the sum of the points of the students.

1. 解题思路与分析

- 思路是记忆化搜索。先求一下正确答案，然后开始算所有可能得到的错误答案。枚举运算符，然后递归求解两边可能的答案，汇总成当前表达式可能得到的答案。用记忆化的方式避免重复计算。
- 时间复杂度 $O(l_s^3 + l_A)$ ，空间 $O(l_s^2)$ 。注意有 1000 这个限制，上面所说的复杂度的常数是 1000^2

$$1+2*3+4+5*6*7 = ?$$

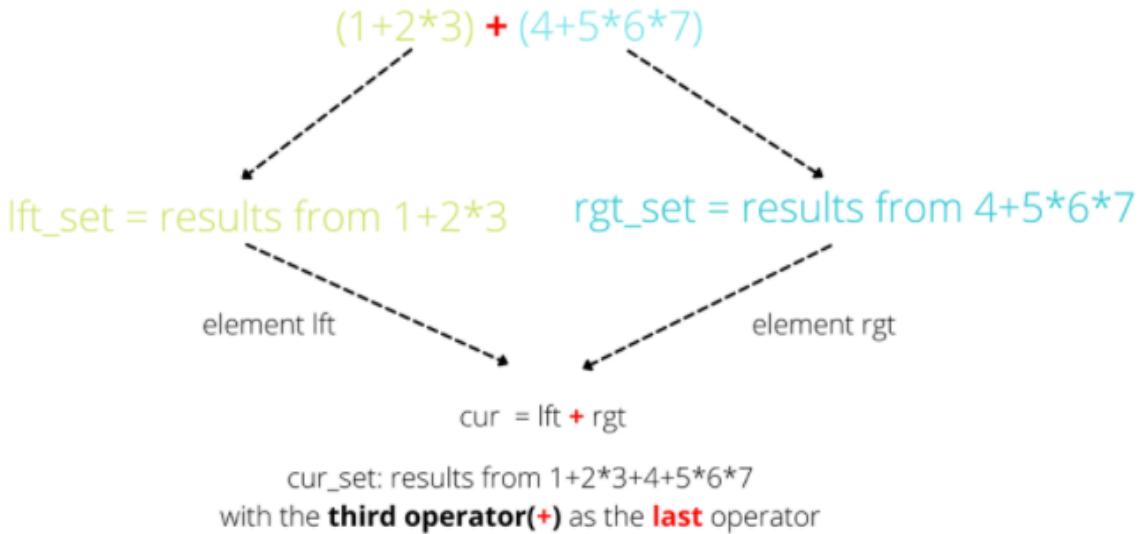
The possible **last** steps are:

- (1) **+** $(2*3+4+5*6*7)$
- $(1+2)$ ***** $(3+4+5*6*7)$
- $(1+2*3)$ **+** $(4+5*6*7)$
- $(1+2*3+4)$ **+** $(5*6*7)$
- $(1+2*3+4+5)$ ***** $(6*7)$
- $(1+2*3+4+5*6)$ ***** (7)

$$2*3+4+5*6*7 = ?$$

The possible **second last** steps are:

- (2) ***** $(3+4+5*6*7)$
- $(2*3)$ **+** $(4+5*6*7)$
- $(2*3+4)$ **+** $(5*6*7)$
- $(2*3+4+5)$ ***** $(6*7)$
- $(2*3+4+5*6)$ ***** (7)



```

private int compute(String t) {
    ArrayDeque<Integer> st = new ArrayDeque<>();
    char [] s = t.toCharArray();
    for (int i = 0; i < s.length; i++) {
        char c = s[i];
        if (Character.isDigit(c))
            if (i > 0 && s[i-1] == '*')
                st.push(st.pop() * (c - '0'));
            else st.push(c - '0');
    }
    int ans = 0;
    while (!st.isEmpty())
        ans += st.pop();
    return ans;
}
Set<Integer> dfs(String t, int l, int r, Set<Integer> [][] f) {
    if (f[l][r] != null) return f[l][r]; // 有记忆则调用记忆
    char [] s = t.toCharArray();
    int n = t.length(), v = 0;
    f[l][r] = new HashSet<>();
    if (l == r) {
        f[l][r].add(s[l] - '0');
        return f[l][r];
    }
    for (int i = l+1; i < r; i++)
        if (!Character.isDigit(s[i])) { // 递归求解左右两边可能算出的答案
            Set<Integer> left = dfs(t, l, i-1, f);
            Set<Integer> right = dfs(t, i+1, r, f);
            for (Integer va : left)
                for (Integer vb : right) {
                    if (s[i] == '*') v = va * vb;
                    else v = va + vb;
                    if (v >= 0 && v <= 1000) f[l][r].add(v);
                }
        }
    return f[l][r];
}
public int scoreOfStudents(String s, int [] num) {
    int m = num.length, res = compute(s), n = s.length(), ans = 0;
    Set<Integer> [][] f = new HashSet[n][n]; // 第一次见, 学习一下
    dfs(s, 0, n-1, f);
    Set<Integer> can = f[0][n-1]; // candidates: of wrong answers
    for (Integer v : num)
        if (v == res) ans += 5;
        else if (can.contains(v)) ans += 2;
    return ans;
}

```

2.4.2 312. Burst Balloons 区间型动态规划的典型代表仍是典型，最好不扔掉

You are given n balloons, indexed from 0 to $n - 1$. Each balloon is painted with a number on it represented by an array nums . You are asked to burst all the balloons. If you burst the i th balloon, you will get $\text{nums}[i - 1] * \text{nums}[i] * \text{nums}[i + 1]$ coins. If $i - 1$ or $i + 1$ goes out of bounds of the array, then treat it as if there is a balloon with a 1 painted on it. Return the maximum coins you can collect by bursting the balloons wisely.

```

public int maxCoins(int[] nums) {
    int n = nums.length;
    int [][] dp = new int [n+2][n+2];
    int [] arr = new int [n+2];
    System.arraycopy(nums, 0, arr, 1, n);
    arr[0] = arr[n+1] = 1; // [0, n+1] ==> [1, n]
    int j = 0;
    for (int len = 1; len <= n; len++) { // [1, n]
        for (int i = 1; i+len-1 <= n; i++) { // [1, n]
            j = i + len - 1;
            for (int k = i; k <= j; k++)
                dp[i][j] = Math.max(dp[i][j], dp[i][k-1] + dp[k+1][j] + arr[i-1]*arr[k]*arr[j+1]);
        }
    }
    return dp[1][n];
}

// 0   0   0   0   0   0
// 0   3   30  159  167  0
// 0   0   15  135  159  0
// 0   0   0   40   48   0
// 0   0   0   0   40   0
// 0   0   0   0   0   0
private int memorizedSearch(int [] arr, int x, int y) {
    if (dp[x][y] > 0) return dp[x][y];
    // if (x == y) return dp[x][y] = arr[x]; // 没有这些个边际条件
    // if (x == y-1)
    //     return dp[x][y] = arr[x] * arr[y] + Math.max(arr[x], arr[y]);
    int max = 0;
    for (int i = x; i <= y; i++) {
        max = Math.max(max, memorizedSearch(arr, x, i-1) + memorizedSearch(arr, i+1, y) + arr[x-1]*arr[i]*arr[y+1]);
    }
    return dp[x][y] = max;
}
int [][] dp;
int n;
public int maxCoins(int[] nums) {
    int n = nums.length + 2;
    dp = new int [n][n];
    int [] arr = new int [n];
    System.arraycopy(nums, 0, arr, 1, n-2);
    arr[0] = arr[n-1] = 1;
    return memorizedSearch(arr, 1, n-2);
}

```

2.5 多维数个数、数种类数的

2.5.1 1866. Number of Ways to Rearrange Sticks With K Sticks Visible - Hard

There are n uniquely-sized sticks whose lengths are integers from 1 to n . You want to arrange the sticks such that exactly k sticks are visible from the left. A stick is visible from the left if there are no longer sticks to the left of it.

For example, if the sticks are arranged [1,3,2,5,4], then the sticks with lengths 1, 3, and 5 are visible from the left. Given n and k , return the number of such arrangements. Since the answer may be large, return it modulo $10^9 + 7$.

```

// dp[i][j] 表示前面 i 根棍可以看到 j 根
// 设 dp[i][j] 表示从高度为 1, 2, ..., i 的木棍中, 高度逐渐递减地插入新的木棍, 从左侧看恰好看到 k 根木棍的方案数。
// 后面说看到 ith 根, 不是指从小到大的第 ith 根棍子, 而是指 ith 这个位置上的棍子
// 如果可以看到 ith 根的话, 那么数量为 dp[i-1][j-1]
// 如果看不到 ith 的话, 那么取前面 (i-1) 里面任意一个出来放在 ith 的最后, 接下来就是从前面 i-1 个棍子里面看到 j 根, 所以结果是 (i-1)* dp[i-1][j]
public int rearrangeSticks(int n, int k) {
    int mod = (int)1e9 + 7;
    long [][] dp = new long [n+1][k+1];
    dp[0][0] = 1;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= Math.min(n, k); j++)
            dp[i][j] = (dp[i-1][j-1] + (dp[i-1][j] * (i-1)) % mod) % mod;
    return (int)dp[n][k];
}

```

- dfs + memo

```

long mod = 1000_000_000 + 7;
long[][] dp;
public int rearrangeSticks(int n, int k) {
    dp = new long[n + 1][k + 1];
    long ans = dfs(n, k);
}

```

```

    return (int) (ans % mod);
}

long dfs(int n, int k) {
    if(n < k || k == 0) return 0;
    if(n == k) return 1;
    if(dp[n][k] != 0) return dp[n][k];
    long ans = 0;
    // instead of iterating for every stick
    // we are just multiplying number of ways with (n - 1)
    ans += ((n - 1) * dfs(n - 1, k)) % mod;
    ans %= mod;
    ans += dfs(n - 1, k - 1);
    ans %= mod;
    return dp[n][k] = ans;
}

```

2.5.2 1916. Count Ways to Build Rooms in an Ant Colony - Hard

You are an ant tasked with adding n new rooms numbered 0 to $n-1$ to your colony. You are given the expansion plan as a 0-indexed integer array of length n , prevRoom , where $\text{prevRoom}[i]$ indicates that you must build room $\text{prevRoom}[i]$ before building room i , and these two rooms must be connected directly. Room 0 is already built, so $\text{prevRoom}^1 = -1$. The expansion plan is given such that once all the rooms are built, every room will be reachable from room 0.

You can only build one room at a time, and you can travel freely between rooms you have already built only if they are connected. You can choose to build any room as long as its previous room is already built.

Return the number of different orders you can build all the rooms in. Since the answer may be large, return it modulo $109 + 7$.

对每个节点，可根据所有以其子节点为根的树的节点及排列数量，计算出以当前节点为根的树的节点及排列数量。

本题求解过程涉及较多前置知识点，包括排列组合、乘法逆元、快速乘方等

读者需要掌握如下进阶知识点才能解决本题：

- 排列数计算。假设有 a_0 个物品 0, a_1 个物品 1, ..., a_{n-1} 个物品 $n-1$, 我们需要将它们排成一排, 那么方案数为

$$\frac{(a_0 + a_1 + \dots + a_{n-1})!}{a_0! \cdot a_1! \cdot \dots \cdot a_{n-1}!}$$

- 乘法逆元。如果需要计算 $\frac{b}{a}$ 对 m 取模的值, b 和 a 均为表达式 (例如上面排列数的分子与分母) 并且它们实际的值非常大, 无法直接进行计算, 那么我们可以求出 a 在模 m 意义下的乘法逆元 a^{-1} , 此时有

$$\frac{b}{a} \equiv b \cdot a^{-1} \pmod{m}$$

这样将除法运算转化为乘法运算, 就可以在计算的过程中进行取模了。

乘法逆元的具体计算方法可以参考题解的最后一节「进阶知识点：乘法逆元」。

方法一：动态规划

提示 1

由于除了 0 号房间以外的每一个房间 i 都有唯一的 $prevRoom[i]$, 而 0 号房间没有 $prevRoom[0]$, 并且我们可以从 0 号房间到达每个房间, 即任意两个房间都是可以相互到达的, 因此:

如果我们把所有的 n 间房间看成 n 个节点, 任意的第 $i (i > 0)$ 号房间与 $prevRoom[i]$ 号房间之间连接一条有向边, 从 $prevRoom[i]$ 指向 i , 那么它们就组成了一棵以 0 号房间为根节点的树, 并且树上的每条有向边总是从父节点指向子节点。

提示 2

根据题目要求, 对于任意的 $i > 0$, 我们必须先构筑 $prevRoom[i]$, 再构筑 i , 而我们根据提示 1 构造的树中恰好有一条从 $prevRoom[i]$ 指向 i 的边, 因此:

题目中的要求与拓扑排序的要求是等价的。

也就是说, 构筑所有房间的不同顺序的数目, 等于我们构造的树的不同拓扑排序的方案数。

思路与算法

我们可以使用动态规划的方法求出方案数。

设 $f[i]$ 表示以节点 i 为根的子树的拓扑排序方案数，那么：

- 任意一种拓扑排序中的第一个节点一定是节点 i ；
- 假设节点 i 有子节点 $ch_{i,0}, ch_{i,1}, \dots, ch_{i,k}$ ，那么以 $ch_{i,0}, ch_{i,1}, \dots, ch_{i,k}$ 为根的这 $k+1$ 棵子树，它们均有若干种拓扑排序的方案。如果我们希望把这些子树的拓扑排序方案「简单地」合并到一起，可以使用乘法原理，即方案数为：

$$f[ch_{i,0}] \times f[ch_{i,1}] \times \dots \times f[ch_{i,k}]$$

乘法原理会忽略子树之间拓扑排序的顺序，所以我们还需要考虑这一层关系。记 $cnt[i]$ 表示以节点 i 为根的子树中节点的个数，那么除了拓扑排序中第一个节点为 i 以外，还剩余 $cnt[i] - 1$ 个位置。我们需要在 $cnt[i] - 1$ 个位置中，分配 $cnt[ch_{i,0}]$ 个给以 $ch_{i,0}$ 为根的子树，放置该子树的一种拓扑排序；分配 $cnt[ch_{i,1}]$ 个给以 $ch_{i,1}$ 为根的子树，放置该子树的一种拓扑排序。以此类推，分配的方案数乘以上述乘法原理得到的方案数，即为 $f[i]$ 。根据「前言」部分，我们知道分配的方案数为：

$$\frac{(cnt[i] - 1)!}{cnt[ch_{i,0}]! \times cnt[ch_{i,1}]! \times \dots \times cnt[ch_{i,k}]!}$$

因此，我们就可以得到状态转移方程：

$$f[i] = \left(\prod_{ch} f[ch] \right) \times \frac{(cnt[i] - 1)!}{\prod_{ch} cnt[ch]!}$$

细节

当节点 i 为叶节点时，它没有子节点，因此对应着 1 种拓扑排序的方案，即 $f[i] = 1$ 。

状态转移方程中存在除法，因此我们需要使用「前言」部分的乘法逆元将其转换为除法。

观察状态转移方程，它包含一些阶乘以及阶乘的乘法逆元，因此我们可以将它们全部预处理出来，这样就可以均摊 $O(1)$ 的时间在一对「父-子」节点之间进行状态转移。

```
// 快速计算 x^y 的乘方
public int quickMul(int x, int y) {
    long res = 1, cur = x;
    while (y > 0) {
        if ((y & 1) == 1)
            res = res * cur % mod;
        cur = cur * cur % mod;
        y >>= 1;
    }
    return (int)res;
}

// 深度优先搜索，返回以当前节点为根的子树节点个数 及 内部排列数
public int[] dfs(int idx) {
    if (!map.containsKey(idx)) return new int[] {1, 1}; // 子节点，节点个数及内部排列数均为 1
    int cnt = 1, res = 1; // 子树的节点个数、内部排列数
    for (Integer node : map.get(idx)) {
        int[] cur = dfs(node); // 递归得到子节点对应树的节点个数和排列数
        cnt += cur[0];
        res = (int)((long)res * cur[1] % mod * inv[cur[0]] % mod);
    }
    res = (int)((long)res * fac[cnt-1] % mod);
    return new int[] {cnt, res};
}

int mod = (int)1e9 + 7;
Map<Integer, List<Integer>> map = new HashMap<>();
int[] fac, inv;
public int waysToBuildRooms(int[] prevRoom) {
    int n = prevRoom.length;
    // 求阶乘数列及对应逆元
    this.fac = new int[n]; // fac[i]=i!
    this.inv = new int[n]; // inv[i]=i!^{(-1)}
    fac[0] = inv[0] = 1;
```

```

for (int i = 1; i < n; i++) {
    fac[i] = (int)((long)fac[i-1] * i % mod);
    inv[i] = quickMul(fac[i], mod - 2); // 费马小定理: (fac[i]^(-1))%mod = (fac[i]^(mod-2))%mod
}
// 记录各个节点与子节点之间的边
for (int i = 1; i < n; i++)
    map.computeIfAbsent(prevRoom[i], k->new ArrayList<>()).add(i);
// 动态规划得到总体顺序数量 x
return dfs(0)[1];
}

```

2.5.3 1987. Number of Unique Good Subsequences - Hard

You are given a binary string `binary`. A subsequence of `binary` is considered good if it is not empty and has no leading zeros (with the exception of "0").

Find the number of unique good subsequences of `binary`.

For example, if `binary` = "001", then all the good subsequences are ["0", "0", "1"], so the unique good subsequences are "0" and "1". Note that subsequences "00", "01", and "001" are not good because they have leading zeros. Return the number of unique good subsequences of `binary`. Since the answer may be very large, return it modulo $10^9 + 7$.

A subsequence is a sequence that can be derived from another sequence by deleting some or no elements without changing the order of the remaining elements.

```

public int numberOfUniqueGoodSubsequences(String binary) {
    int mod = (int)1e9 + 7;
    int n = binary.length(), preZoo = 0, preOne = 0, m = 1;
    long [] dp = new long [n+1];
    String s = "#" + binary;
    while (m <= n && s.charAt(m) == '0') m++;
    if (m == n+1) return 1;
    dp[m] = 1;
    preOne = m;
    preZoo = m-1;
    for (int i = m+1; i <= n; i++) {
        char c = s.charAt(i);
        int j = (c == '0' ? preZoo : preOne);
        dp[i] = (2 * dp[i-1] % mod - (j >= 1 ? dp[j-1] : 0) + mod) % mod;
        if (c == '0') preZoo = i;
        else preOne = i;
    }
    return (int)dp[n] + (s.indexOf("0") != -1 ? 1 : 0);
}

```

Intuition

An easier or harder version of 940. Distinct Subsequences II

Explanation

the subsequence 0 is tricky,

so firstly we only count the case without any leading 0

We count the number of subsequence that ends with 0 and ends with 1.

If we need 0,

we can append 0 to all existing `ends0 + ends1` subsequences,

and make up `ends0 + ends1` different subsequences that ends with 0.

ans these subsequences has length ≥ 2 ,

So `ends0 = ends0 + ends1`.

If we need 1, the same,

we can append 1 to all existing `ends0 + ends1` subsequence,

and make up `ends0 + ends1` different subsequence that ends with 1,

ans these subsequences has length ≥ 2 .

So `1` is not in them, `ends1 = ends0 + ends1 + 1`.

The result is `ends1 + ends2`, and don't forget about `0`.

The final result is `ends1 + ends2 + has0`.

Complexity

Time $O(n)$

Space $O(1)$

```
public int numberofUniqueGoodSubsequences(String binary) {
    int mod = (int)1e9 + 7;
    int endZoo = 0, endOne = 0, hasZoo = 0;
    for (int i = 0; i < binary.length(); i++) {
        if (binary.charAt(i) == '1')
            endOne = (endOne + endZoo + 1) % mod;
        else {
            endZoo = (endZoo + endOne) % mod;
            hasZoo = 1;
        }
    }
    return (endOne + endZoo + hasZoo) % mod;
}
```

- 还有一个没有看懂的

- <https://leetcode-cn.com/problems/number-of-unique-good-subsequences/solution/ju-yi-fan-san-by-average-1/>
- <https://leetcode-cn.com/problems/distinct-subsequences-ii/solution/dong-tai-gui-hua-cong-fen-xi-dai-mu-1/>

```
def numberofUniqueGoodSubsequences(self, binary: str) -> int:
    M = 10**9+7
    dp = [0]*10
    b = str(int(binary))
    l = len(binary) - len(b)
    if l > 0:
        dp[0] = 1
    for c in b:
        if dp[0] >= 1:
            dp[int(c)] = (sum(dp)) % M
        else:
            dp[int(c)] = (1 + sum(dp)) % M
    return sum(dp)%M
```

2.5.4 730. Count Different Palindromic Subsequences - Hard

Given a string s , return the number of different non-empty palindromic subsequences in s . Since the answer may be very large, return it modulo $10^9 + 7$.

A subsequence of a string is obtained by deleting zero or more characters from the string.

A sequence is palindromic if it is equal to the sequence reversed.

Two sequences a_1, a_2, \dots and b_1, b_2, \dots are different if there is some i for which $a_i \neq b_i$.

<code>count("bccb") = 2 * count("cc") + 2 = 6</code>	<code>"c", "cc"</code>
<code>count("cc") = 2 * count("") + 2 = 2</code>	<code>"c", "cc"</code>

<code>count("abccb") = count("abcc") + count("bccb") - count("bcc") = 4 + 6 - 3 = 7</code>	
<code>count("abcc") = count("abc") + count("bcc") - count("bc") = 3 + 3 - 2 = 4</code>	
<code>count("abc") = count("ab") + count("bc") - count("b") = 2 + 2 - 1 = 3</code>	
<code>count("ab") = count("a") + count("b") - count("") = 1 + 1 - 0 = 2</code>	
<code>count("bcc") = count("bc") + count("cc") - count("c") = 2 + 2 - 1 = 3</code>	
<code>count("bc") = count("b") + count("c") - count("") = 1 + 1 - 0 = 2</code>	

Base case:

```
count(x) = 1 if x in {"a", "b", "c", "d"}  
= 0 if x is empty
```

<code>count("bcbcb") = 2 * count("cbc") + 1 = 9</code>	<code>"b", "cbc", "c", "cc"</code>
<code>count("cbc") = 2 * count("b") + 2 = 4</code>	<code>"b", "cbc", "c", "cc"</code>

<code>count("bbcabbb") = 2 * count("bcab") - count("ca") = 10</code>	<code>"bcb", "bab", "b", "bb", "c", "a", "bbcb", "bbcb", "bbb", "bbb", "beb", "bab"</code>
<code>count("bcab") = 2 * count("ca") + 2 = 6</code>	<code>"c", "a", "bc", "bab", "b", "bb"</code>

`dp[i][j] = count of different palindromic subseq of $s[i] \sim s[j]$ (inclusive)`

```
dp[i][j] =  
    if  $s[i] \neq s[j]$ : # count("abcd")  
        dp[i][j - 1] + dp[i + 1][j] - dp[i + 1][j - 1], # count("abc") + count("bcd") - count("bc")  
  
    if  $s[i] == s[j]$ :  
        dp[i + 1][j - 1] * 2 + 2, s[i] not in  $s[i+1:j-1]$   
        dp[i + 1][j - 1] * 2 + 1, count( $s[i+1:j-1], s[i]$ ) == 1  
        dp[i + 1][j - 1] * 2 - dp[1 + 1][r - 1] l, r is the first/last pos of s[i] in  $s[i+1:j-1]$ 
```

```
private int dfs(char[] s, int i, int j) {  
    if (i > j) return 0;  
    if (i == j) return 1;  
    if (dp[i][j] > 0) return dp[i][j];  
    long ans = 0;  
    if (s[i] == s[j]) {  
        ans += dfs(s, i + 1, j - 1) * 2;  
        int l = i + 1;  
        int r = j - 1;  
        while (l <= r && s[l] == s[i]) ++l;  
        while (l <= r && s[r] == s[i]) --r;  
        if (l > r) ans += 2;  
        else if (l == r) ans += 1;  
        else ans -= dfs(s, l + 1, r - 1);  
    } else  
        ans = dfs(s, i, j - 1) + dfs(s, i + 1, j) - dfs(s, i + 1, j - 1);  
    return dp[i][j] = (int)((ans + mod) % mod);  
}  
private static final int mod = (int)1e9 + 7;
```

```

private int [][] dp;
public int countPalindromicSubsequences(String s) {
    int n = s.length();
    dp = new int[n][n];
    return dfs(s.toCharArray(), 0, n - 1);
}

```

- dp

```

public int countPalindromicSubsequences(String s) {
    int n = s.length();
    int mod = (int)1e9 + 7;
    char [] arr = s.toCharArray();
    long [][] dp = new long [n][n];
    for (int i = 0; i < n; i++)
        dp[i][i] = 1;
    for (int len = 1; len <= n; len++) {
        for (int i = 0; i+len < n; i++) {
            for (int j = i + len;
                 arr[i] == arr[j];
                 if (arr[i] == arr[j]) {
                    dp[i][j] = dp[i+1][j-1] * 2;
                    int l = i+1;
                    int r = j-1;
                    while (l <= r && arr[l] != arr[i]) ++l;
                    while (l <= r && arr[r] != arr[i]) --r;
                    if (l == r) dp[i][j] += 1;
                    else if (l > r) dp[i][j] += 2;
                    else dp[i][j] -= dp[l+1][r-1];
                } else dp[i][j] = dp[i][j-1] + dp[i+1][j] - dp[i+1][j-1];
            dp[i][j] = (dp[i][j] + mod) % mod;
        }
    }
    return (int)dp[0][n-1];
}

```

2.5.5 1125. Smallest Sufficient Team - Hard 这个题要多写几遍

In a project, you have a list of required skills $\text{req}_{\text{skills}}$, and a list of people. The i th person $\text{people}[i]$ contains a list of skills that the person has.

Consider a sufficient team: a set of people such that for every required skill in $\text{req}_{\text{skills}}$, there is at least one person in the team who has that skill. We can represent these teams by the index of each person.

For example, $\text{team} = [0, 1, 3]$ represents the people with skills people^1 , people^2 , and people^3 . Return any sufficient team of the smallest possible size, represented by the index of each person. You may return the answer in any order.

It is guaranteed an answer exists.

```

// 强行剪枝: 收集到的 size >= 目前的结果, 直接 return;
// 这题的思路就是先把 skill 和 set of people 建立好,
// 然后去用 skill set 做 backtracking 收集, 如果 temp team 的 size 大于结果, 直接 return, 否则 update 结果,
// 这里有个小 tricky 的地方, 就是如果 people 是新人, 加入之后 dfs, backtracking 的时候, 要判断如果是新人, 则 remove, 否则不 remove:
private void dfs(String[] req_skills, HashSet<Integer> team, int idx) {
    if (team.size() >= minTeamSize) return; // 强行剪枝: 收集到的 size >= 目前的结果, 直接 return;
    if (idx == req_skills.length) {
        minTeamSize = team.size();
        resTeam = new HashSet<Integer>(team);
        return;
    }
    boolean isNewPerson = false;
    for (int people : map.get(req_skills[idx])) {
        isNewPerson = team.add(people);
        dfs(req_skills, team, idx + 1);
        if (isNewPerson)
            team.remove(people);
    }
}
HashMap<String, Set<Integer>> map;
Set<Integer> resTeam;
int minTeamSize;
public int[] smallestSufficientTeam(String[] req_skills, List<List<String>> people) {
    minTeamSize = people.size();
    this.map = new HashMap<>();
    for (int i = 0; i < minTeamSize; i++)
        for (String skill: people.get(i))
            map.computeIfAbsent(skill, k -> new HashSet<Integer>()).add(i);
}

```

²DEFINITION NOT FOUND.

³DEFINITION NOT FOUND.

```

this.resTeam = new HashSet<Integer>();
dfs(req_skills, new HashSet<Integer>(), 0);
int[] res = new int[resTeam.size()];
int idx = 0;
for (int person : resTeam)
    res[idx++] = person;
return res;
}

```

- Java soution using Bit DP 10ms

```

public int[] smallestSufficientTeam(String[] req_skills, List<List<String>> people) {
    int n = req_skills.length, range = 1 << n, cur, idx;
    Map<String, Integer> idxMap = new HashMap<>();
    for (int i = 0; i < n; i++)
        idxMap.put(req_skills[i], i);
    long[] dp = new long[range]; // 每个 bit 位实际存了构成答案最小组的各成员的下标, 60 个人, long
    int[] cnt = new int[range];
    Arrays.fill(cnt, Integer.MAX_VALUE);
    cnt[0] = 0;
    for (int i = 0; i < people.size(); i++) {
        List<String> l = people.get(i);
        cur = 0;
        for (String skill : l)
            if (idxMap.containsKey(skill))
                cur |= 1 << idxMap.get(skill);
        for (int j = range-1; j > 0; j--) {
            int dx = (j & cur) ^ j; // 由其它人所构成的拥有 j 的这些种技能的子集/ j 的这些种技能可以由 j 一个人来替换 (其它可能需要很多人才能最终拥有这些技能)
            if (cnt[idx] != Integer.MAX_VALUE && cnt[j] > cnt[idx] + 1) {
                cnt[j] = cnt[idx] + 1;
                dp[j] = dp[idx] | (1L << i); // at most 60 people
            }
        }
    }
    int[] res = new int[cnt[range-1]];
    long preRes = dp[range-1]; // 5 people: 11111, 1111, 111, 11, 1
    int valIdx = 0;
    long val = 0;
    idx = 0;
    while (preRes != 0) {
        val = preRes & 1;
        if (val == 1) res[idx++] = valIdx;
        preRes >>= 1;
        valIdx++;
    }
    return res;
}

```

- DFS + Memorizaion (A real O($2^{skill} * people$) Solution) Java 8ms

- [https://leetcode.com/problems/smallest-sufficient-team/discuss/1011135/DFS-%2B-Memorizaion-\(A-real-O\(2skill-*-people\)-Solution\)-Java-8ms](https://leetcode.com/problems/smallest-sufficient-team/discuss/1011135/DFS-%2B-Memorizaion-(A-real-O(2skill-*-people)-Solution)-Java-8ms)

```

List<Integer> minComb;
int[] peopleSkillMasks;
Integer[] memo; // 这个方法确实快一点儿
int[] nextPerson;
int n;
public int[] smallestSufficientTeam(String[] req_skills, List<List<String>> people) {
    // 1. some preprocess to get bitmask for people skills
    this.n = req_skills.length;
    Map<String, Integer> skillToIdx = new HashMap<>();
    for (int i = 0; i < n; i++)
        skillToIdx.put(req_skills[i], i);
    this.peopleSkillMasks = new int[people.size()];
    for (int i = 0; i < peopleSkillMasks.length; i++) {
        int skillMask = 0;
        for (String skill : people.get(i))
            skillMask |= (1 << skillToIdx.get(skill));
        peopleSkillMasks[i] = skillMask;
    }
    // 2. dfs
    memo = new Integer[1 << n];
    nextPerson = new int[1 << n];
    dfs(0, 0);
    // 3. reconstruct the path
    int curSkillSet = 0;
    List<Integer> res = new ArrayList<>();
    while(curSkillSet != (1 << n) - 1) {
        res.add(nextPerson[curSkillSet]);
        curSkillSet |= (1 << curSkillSet);
    }
}

```

```

        curSkillSet |= peopleSkillMasks[nextPerson[curSkillSet]];
    }
    return res.stream().mapToInt(i->i).toArray();
}
// a very simple dfs with memo to compute all combinations of people.
// Use memorization to optimize the time complexity to O(2^skill * people) 2^skill for 2^skill node in the tree, people because each node has people comp
private int dfs(int curSkillSet, int startIdx) {
    if (curSkillSet == (1 << n) - 1) return 0;
    if (memo[curSkillSet] == null) {
        int res = Integer.MAX_VALUE / 2;
        int nextPersonIdx = -1;
        for (int i = startIdx; i < peopleSkillMasks.length; i++) {
            int withNewSkill = peopleSkillMasks[i] | curSkillSet;
            if (withNewSkill != curSkillSet) {
                int numPeople = dfs(withNewSkill, i+1) + 1;
                if (res > numPeople) {
                    res = numPeople;
                    nextPersonIdx = i;
                }
            }
        }
        memo[curSkillSet] = res;
        nextPerson[curSkillSet] = nextPersonIdx;
    }
    return memo[curSkillSet];
}

```

- Recursion + Memoization + bit mask , with Simple JAVA solution

- <https://leetcode.com/problems/smallest-sufficient-team/discuss/1487180/Recursion-%2B-Memoization-%2B-bit-mask-with-Simple-JAVA-solution>

上面的这些方法相对较偏，就暂时顾不上了

2.5.6 1575. Count All Possible Routes - Hard

You are given an array of distinct positive integers locations where locations[i] represents the position of city i. You are also given integers start, finish and fuel representing the starting city, ending city, and the initial amount of fuel you have, respectively.

At each step, if you are at city i, you can pick any city j such that $j \neq i$ and $0 \leq j < \text{locations.length}$ and move to city j. Moving from city i to city j reduces the amount of fuel you have by $|\text{locations}[i] - \text{locations}[j]|$. Please notice that $|x|$ denotes the absolute value of x.

Notice that fuel cannot become negative at any point in time, and that you are allowed to visit any city more than once (including start and finish).

Return the count of all possible routes from start to finish.

Since the answer may be too large, return it modulo $10^9 + 7$.

```

// 自顶向下（记忆化搜索）
// 每个 dfs 搜索当前状态为城市 i, 油量 f 到达终点的方案数。这样决策的时候就很直观：当前这个状态的方案数，由可去的城市的，且油量为剩余油量的到达终点方案数加起来。
// 初始化：每个状态都初始化为-1。
// 当走到终点时，这个状态的可走到终点的方案数 +1。
private int dfs(int[] arr, int end, int idx, int fu) {
    if (dp[idx][fu] != -1) return dp[idx][fu];
    dp[idx][fu] = 0;
    if (idx == end) {
        dp[idx][fu] += 1;
        dp[idx][fu] %= mod;
    }
    for (int i = 0; i < n; i++) {
        if (i == idx || Math.abs(arr[i] - arr[idx]) > fu) continue;
        dp[idx][fu] = (dp[idx][fu] + dfs(arr, end, i, fu - Math.abs(arr[i] - arr[idx]))) % mod;
    }
    return dp[idx][fu];
}
int mod = (int)1e9 + 7;
int[][] dp;
int n;
public int countRoutes(int[] locations, int start, int finish, int fuel) {
    n = locations.length;
    if (fuel < Math.abs(locations[start] - locations[finish])) return 0;
    dp = new int[n][fuel+1];
    for (int i = 0; i < n; i++)
        Arrays.fill(dp[i], -1);
    dfs(locations, finish, start, fuel);
    return dp[start][fuel];
}

```

```

}

// 自底向上
// 为什么想到动态规划：最优子结构：到达终点的方案数肯定由到达其他点的，不同油量的方案数求和。
// 如何定义状态：城市肯定在状态里，因为其他城市有不同的剩余油量的状态，且油量为 0 无法到达，也成为限制之一。所以油量也必须在状态里：
// d p ( i , f ) dp(i, f) 表示到达第 i 个城市，剩余油量为 f 的方案数。
// 状态转移：第 i 个城市，可以除本身外的城市转移过来，只要剩余的油量不小于所用的油量就够了，最后答案是求总共的个数，所以只要方案数相加就行：
// dp(i, fdist)=dp(i, fdist)+dp(k, f)(fdist>=0)
// 枚举顺序：每个城市肯定都要枚举一遍，因为还需要从另一个城市转移过来，所以除本身外的城市肯定还要再枚举一遍。
// 关键是油量的枚举，因为油量肯定是慢慢减少的，可以想到是逆序枚举，而且油量要放在最外层枚举。因为如果先枚举城市 i ii，再枚举城市 j jj，再枚举油量的话，只是不断更新
// dp：最优子结构：到达终点的方案数肯定由到达其他点的，不同油量的方案数求和
// 搜索：反过来 在第 i 个城市到达 fin 的方案数，也可以由其他的点到达 fin 的方案数转移过来，但是油量有限制，所以油量肯定在状态里
// 所以城市 和 剩余油量肯定在状态里
// dp(i, j) 表示到达第 i 个城市，剩余油量为 j 的方案数
// dp(i, j) = dp(i, j) + dp(k, j - dist)
public int countRoutes(int[] locations, int start, int finish, int fuel) {
    int n = locations.length;
    if (fuel < Math.abs(locations[start] - locations[finish])) return 0;
    int[][] dp = new int[n][fuel+1];
    dp[start][fuel] = 1; // 初始点且燃料满的点方案数为 1
    int leftFu = 0, mod = (int)1e9 + 7;
    for (int j = fuel; j >= 0; j--) { // fuel leftover
        for (int i = 0; i < n; i++) { // cur city
            for (int k = 0; k < n; k++) { // next city
                if (i == k) continue;
                leftFu = j - Math.abs(locations[i] - locations[k]);
                if (leftFu < 0) continue;
                dp[i][leftFu] = (dp[i][leftFu] + dp[k][j]) % mod; // 这里好别扭呀：想呀想呀
            }
        }
    }
    int ans = 0;
    for (int i = 0; i <= fuel; i++)
        ans = (ans + dp[finish][i]) % mod;
    return ans;
}

```

2.5.7 1012. Numbers With Repeated Digits - Hard

Given an integer n, return the number of positive integers in the range [1, n] that have at least one repeated digit.

题意：统计 1-N 中，满足每个位置都不同的数有几个。

思路：数位 DP。通过一个 1«10 的 mask 表示当前这个数，1-9 哪些数被用了。

比赛的时候，一直想通过一个 dfs 直接找到不重复的数，一直不对。

赛后发现，别人都是通过一个 dfs 找重复的数，然后总个数减去。

```

private int dfs(int len, int limit, int mask) { // 不重复数的个数
    if (len == 0) return 1;
    if (limit == 0 && dp[len][mask][limit] > 0) return dp[len][mask][limit]; // 记忆化部分
    int maxn = limit > 0 ? bit[len] : 9; // 求出最高可以枚举到哪个数字
    int ans = 0;
    for (int i = 0; i <= maxn; i++) // 当前位
        if ((mask & (1 << i)) == 0)
            if (mask == 0 && i == 0)
                ans += dfs(len - 1, (limit > 0 && i == maxn ? 1 : 0), mask); // 有前导 0，所以 0 不能统计，不更新 mask
            else ans += dfs(len - 1, (limit > 0 && i == maxn ? 1 : 0), mask | (1 << i)); // 更新 mask
    if (limit == 0) dp[len][mask][limit] = ans; // 如果没有限制，代表搜满了，可以记忆化，否则就不能
    return ans;
}
int[][][] dp;
int[] bit;
public int numDupDigitsAtMostN(int N) {
    int sum = N + 1;
    bit = new int[19];
    dp = new int[19][1 << 10][2];
    int idx = 0;
    while (N > 0) {
        bit[++idx] = N % 10;
        N /= 10;
    }
    return sum - dfs(idx, 1, 0);
}

```

- 这道题给了一个正整数 N，让返回所有不大于 N 且至少有一个重复数字的正整数的个数，题目中给的例子也可以很好的帮助我们理解。要求的是正整数的位数上至少要有一个重复数字，当然最简单暴力的方法就是从 1 遍历到 N，然后对于每个数字判断是否有重复数字，看了一眼题目难度 Hard，想都不用想，肯定是超时的。这道题需要更高效的解法，首先来想，若是直接求至少有一个重复数字的正整数，由于并不知道有多少个重复数字，可能 1 个，2 个，甚至全是重复数字，这样很难找到规律。有时候直接求一个问题不好求，可以考虑求其相反的情况，至少有一个重复数字反过来就是一

个重复数字都没有，所以这里可以求不大于 N 且一个重复数字都没有的正整数的个数，然后用 N 减去这个数字即为所求。好，接下来看怎么求，对于任意一个 N ，比如 7918，是个四位数，而所有的三位数，两位数，一位数，都一定比其小，所以可以直接求出没有重复数字的三位数，两位数，和一位数。比如三位数，由于百位上不能有 0，则只有 9 种情况，十位上可以有 0，则有 9 种情况，个位上则有 8 种情况，所以就是 $9 \times 9 \times 8$ 。可以归纳出没有重复数字的 n 位数的个数，最高位去除 0 还有 9 种，剩余的 $n-1$ 位则依次是 9, 8, 7... 则后面的 $n-1$ 位其实是个全排列，从 9 个数中取出 $n-1$ 个数字的全排列，初中就学过的。这里写一个全排列的子函数，求从 m 个数字中取 n 个数字的全排列，方便后面计算。算完这些后，还要来算符合题意的四位数，由于第一位是 7，若千位上是小于 7 的数字（共有 6 种，千位上不能是 0），则后面的百位，十位，个位又都可以全排列了，从 9 个数字中取 3 个数字的全排列，再乘以千位上小于 7 的 6 种情况。若当千位固定为 7，则百位上可以放小于 9 的数字（共有 8 种，百位不能放 7，但可以放 0），则后面的十位和个位都可以全排列了，从 8 个数字种取出 2 个数字的全排列，再乘以百位上小于 9 的 8 种情况。需要注意的是，遍历给定数字的各个位时，有可能出现重复数字，一旦出现了之后，则该 prefix 就不能再用了，因为已经不合题意了。所以要用一个 HashSet 来记录访问过的数字，一旦遇到重复数字后就直接 break 掉。最后还有一个小 trick 需要注意，由于 N 本身也需要计算进去，所以再计算的时候，使用 $N+1$ 进行计算的话，就可以把 N 这种情况算进去了

```

private int A(int m, int n) {
    return n == 0 ? 1 : A(m, n-1) * (m-n+1);
}
public int numDupDigitsAtMostN(int n) {
    List<Integer> digits = new ArrayList<>();
    Set<Integer> vis = new HashSet<>();
    for (int i = n+1; i > 0; i /= 10)
        digits.add(0, i % 10);
    int res = 0, m = digits.size();
    for (int i = 1; i < m; i++) res += 9 * A(9, i-1);
    for (int i = 0; i < m; i++) {
        for (int j = i > 0 ? 0 : 1; j < digits.get(i); ++j) {
            if (vis.contains(j)) continue;
            res += A(9-i, m-i-1);
        }
        if (vis.contains(digits.get(i))) break;
        vis.add(digits.get(i));
    }
    return n - res;
}

```

2.5.8 514. Freedom Trail - Hard

In the video game Fallout 4, the quest "Road to Freedom" requires players to reach a metal dial called the "Freedom Trail Ring" and use the dial to spell a specific keyword to open the door.

Given a string ring that represents the code engraved on the outer ring and another string key that represents the keyword that needs to be spelled, return the minimum number of steps to spell all the characters in the keyword.

Initially, the first character of the ring is aligned at the "12:00" direction. You should spell all the characters in key one by one by rotating ring clockwise or anticlockwise to make each character of the string key aligned at the "12:00" direction and then by pressing the center button.

At the stage of rotating the ring to spell the key character $key[i]$:

You can rotate the ring clockwise or anticlockwise by one place, which counts as one step. The final purpose of the rotation is to align one of ring's characters at the "12:00" direction, where this character must equal $key[i]$. If the character $key[i]$ has been aligned at the "12:00" direction, press the center button to spell, which also counts as one step. After the pressing, you could begin to spell the next character in the key (next stage). Otherwise, you have finished all the spelling.

- 解题思路分析: 这个图把钥匙中每个字母的出现位置记住了，以后拿去用不搜 dfs + 记忆数组

- 记录下所有字母对应的位置，这样在找字母相对位置的时候就不需要循环搜索了
- 采用递归的方法，找出当前字母对应的位置最小的步数：只需要把当前字母对应的所有位置找出来，然后计算最小值即可
- 下一个位置再次迭代计算即可

```

public int minLen(int len, int i, int j) {
    int min = Math.min(i, j);
    int max = Math.max(i, j);
    return Math.min(Math.abs(i - j), Math.abs(len + min - max));
}
public int helper(String ring, int i, String key, int j) {
    if (j >= n) return 0;

```

```

if (dp[i][j] > 0) return dp[i][j];
List<Integer> nextPos = map.get(key.charAt(j));
int min = Integer.MAX_VALUE;
for (int k = 0; k < nextPos.size(); k++)
    min = Math.min(min, helper(ring, nextPos.get(k), key, j+1) + minLen(m, nextPos.get(k), i) + 1);
dp[i][j] = min;
return dp[i][j];
}
Map<Character, List<Integer>> map = new HashMap<>(); // 这个图把钥匙中每个字母的出现位置记住了，以后拿去用不搜
int[][] dp;
int m, n;
public int findRotateSteps(String ring, String key) {
    m = ring.length();
    n = key.length();
    dp = new int[m][n];
    for (int i = 0; i < m; i++) {
        if (key.indexOf(ring.charAt(i)) == -1) continue;
        char c = ring.charAt(i);
        List<Integer> li = map.get(c);
        if (li == null) {
            li = new ArrayList<>();
            map.put(c, li);
        }
        li.add(i);
    }
    return helper(ring, 0, key, 0);
}

```

2. 解题思路分析动态规划

- 博主最先尝试的用贪婪算法来做，就是每一步都选最短的转法，但是 OJ 中总有些 test case 会引诱贪婪算法得出错误的结果，因为全局最优解不一定都是局部最优解，而贪婪算法一直都是在累加局部最优解，这也是为啥 DP 解法这么叼的原因。贪婪算法好想好实现，但是不一定能得到正确的结果。DP 解法难想不好写，但往往才是正确的解法，这也算一个 trade off 吧。
- 此题需要使用一个二维数组 dp，其中 $dp[i][j]$ 表示转动从 i 位置开始的 key 串所需要的最少步数（这里不包括 $spell$ 的步数，因为 $spell$ 可以在最后统一加上），此时表盘的 12 点位置是 $ring$ 中的第 j 个字符。不得不佩服这样的设计的确很巧妙，我们可以从 key 的末尾往前推，这样 $dp^{1,1}$ 就是我们所需要的结果，因为此时是从 key 的开头开始转动，而且表盘此时的 12 点位置也是 $ring$ 的第一个字符。现在我们来看如何找出递推公式，对于 $dp[i][j]$ ，我们知道此时要将 $key[i]$ 转动到 12 点的位置，而此时表盘的 12 点位置是 $ring[j]$ ，我们有两种旋转的方式，顺时针和逆时针，我们的目标肯定是要求最小的转动步数，而顺时针和逆时针的转动次数之和刚好为 $ring$ 的长度 n ，这样我们求出来一个方向的次数，就可以迅速得到反方向的转动次数。为了将此时表盘上 12 点位置上的 $ring[j]$ 转动到 $key[i]$ ，我们要将表盘转动一整圈，当转到 $key[i]$ 的位置时，我们计算出转动步数 $diff$ ，然后计算出反向转动步数，并取二者较小值为整个转动步数 $step$ ，此时我们更新 $dp[i][j]$ ，更新对比值为 $step + dp[i+1][k]$ ，这个也不难理解，因为 key 的前一个字符 $key[i+1]$ 的转动情况 suppose 已经计算好了，那么 $dp[i+1][k]$ 就是当时表盘 12 点位置上 $ring[k]$ 的情况的最短步数， $step$ 就是从 $ring[k]$ 转到 $ring[j]$ 的步数，也就是 $key[i]$ 转到 $ring[j]$ 的步数，用语言来描述就是，从 key 的 i 位置开始转动并且此时表盘 12 点位置为 $ring[j]$ 的最小步数 ($dp[i][j]$) 就等价于将 $ring[k]$ 转动到 12 点位置的步数 ($step$) 加上从 key 的 $i+1$ 位置开始转动并且 $ring[k]$ 已经在表盘 12 点位置上的最小步数 ($dp[i+1][k]$) 之和。
- 突然发现这不就是之前那道 Reverse Pairs 中解法一中归纳的顺序重现关系的思路吗，都做了总结，可换个马甲就又不认识了，泪目中。。。

```

public int findRotateSteps(String ring, String key) {
    int m = key.length();
    int n = ring.length();
    int [][] dp = new int[m+1][n];
    int diff = 0, step = 0;
    for (int i = m-1; i >= 0; i--) {
        for (int j = 0; j < n; j++) {
            dp[i][j] = Integer.MAX_VALUE;
            for (int k = 0; k < n; k++) {
                if (ring.charAt(k) == key.charAt(i)) {
                    diff = Math.abs(j - k);
                    step = Math.min(diff, n-diff);
                    dp[i][j] = Math.min(dp[i][j], step + dp[i+1][k]);
                }
            }
        }
    }
    return dp[0][0] + m;
}

```

3. 解题思路分析: dfs + 记忆数组

- 过程就是需要一步一步求 key 里面的每个字符。如果当前位置已经是对应到这个字符，那么直接按按钮就可以
- 如果当前位置不是，那么有两种旋转方式，顺时针或者逆时针，然后找到第一个字符就是在同一个方向上的最短距离，
- 因为在同一个方向上，即使后面有重复的字符，无论后面的字符在那里，遇到第一个符合条件的字符就按按钮一定是最优解。
- 但是在不同方向上就不一定了，有可能一个方向上当前字符距离更短，但是有可能后面的字符距离会更远，
 - 比如 ring=ABCDEFGBF, key=BG, 如果看第一个字符，那应该是顺时针，只需要转一格就到，逆时针需要转两格，
 - 但是顺时针第一步快了以后，后面到 G 会需要更长的步骤。而逆时针会比较快。
- 所以，基本的逻辑是每一步不能决定当前哪个方向是否是最优解，只有不断递归，把每步的两个方向全部尝试完到 key 结束才可以
- 当然，如果不做任何处理，这样做是要超时的（我开始就写了这样一个版本），一个直观的做法，就是在递归的基础上
 - 加一个记忆表，针对 ring 的位置 index 和 key 的 kindex 做记录，如果已经存在一个解了就可以直接返回结果
- 这个递归 +memorization 的解法，那一定存在一个 bottom up 的动态规划解法，这个后面再学习

```

private int helper(String s, String t, int i, int j) { // s: ring, t: key, i: idxRing, j: idxKey
    Map<Integer, Integer> locMap = mem.get(i);
    if (locMap != null)
        if (locMap.get(j) != null) return locMap.get(j);
    if (j == n) return 0;
    int step = 0, k = i;
    boolean foundK = false;
    for (; step <= m/2; ++step) {
        k = (i + step + m) % m;
        if (s.charAt(k) == t.charAt(j)) {
            foundK = true;
            break;
        }
    }
    int rstep = 0, x = i;
    boolean foundX = false;
    while (rstep <= m/2) {
        x = (i - rstep + m) % m;
        if (s.charAt(x) == t.charAt(j)) {
            foundX = true;
            break;
        }
        rstep++;
    }
    int min = Integer.MAX_VALUE;
    if (foundK) min = helper(s, t, k, j+1) + step + 1;
    if (foundX) min = Math.min(min, helper(s, t, x, j+1) + rstep + 1);
    if (locMap == null) {
        locMap = new HashMap<>();
        mem.put(i, locMap);
    }
    locMap.put(j, min);
    return min;
}
Map<Integer, Map<Integer, Integer>> mem = new HashMap<>();
int m, n;
public int findRotateSteps(String ring, String key) {
    m = ring.length();
    n = key.length();
    return helper(ring, key, 0, 0);
}

```

2.5.9 847. Shortest Path Visiting All Nodes

You have an undirected, connected graph of n nodes labeled from 0 to $n - 1$. You are given an array graph where $\text{graph}[i]$ is a list of all the nodes connected with node i by an edge. Return the length of the shortest path that visits every node. You may start and stop at any node, you may revisit nodes multiple times, and you may reuse edges.

```

public int shortestPathLength(int[][][] graph) {
    int n = graph.length;

```

```

int tar = 0, res = 0;
HashSet<String> s = new HashSet<>();
Queue<Pair<Integer, Integer>> q = new LinkedList<>();
for (int i = 0; i < n; i++) {
    int mask = (1 << i);
    tar |= mask;
    s.add(Integer.toString(mask) + "-" + Integer.toString(i));
    q.add(new Pair<>(mask, i));
}
while (!q.isEmpty()) {
    for (int i = q.size(); i > 0; i--) {
        Pair cur = q.remove();
        if ((int)cur.getKey() == tar) return res;
        for (int next : graph[(int)cur.getValue()]) {
            int path = (int)cur.getKey() | (1 << next);
            String str = Integer.toString(path) + "-" + Integer.toString(next);
            if (s.contains(str)) continue;
            s.add(str);
            q.add(new Pair<>(path, next));
        }
    }
    ++res;
}
return -1;
}

```

2.5.10 1931. Painting a Grid With Three Different Colors

You are given two integers m and n . Consider an $m \times n$ grid where each cell is initially white. You can paint each cell red, green, or blue. All cells must be painted. Return the number of ways to color the grid with no two adjacent cells having the same color. Since the answer can be very large, return it modulo $10^9 + 7$.

- lightweighted 轻巧点儿的解题方案: bitmask

```

// time O( (2^5) * 2 * N)
// SPACE O(N)
//     For m = 5, there are at most 48 valid states for a single column so we can handle it column by column.
//     We encode the color arrangement by bit mask (3 bit for a position) and use dfs to generate the all valid states.
//     Then for each column, we iterator all the states and check if its still valid with the previous column.
public void helper(int m, int pos, HashMap<Integer, Long> dic, int pre, int cur) {
    if (pos == m) {
        dic.put(cur, 1L);
        return;
    }
    //不需要 {1, 2, 4} {0, 1, 2} is ok 每个格 (实际占用 3 个 bit)
    for (int i = 0; i < 3; i++) {
        if (i == pre) continue;
        helper(m, pos + 1, dic, i, (cur << 3) | (1 << i)); // 每处理一格, 将当前状态左移 3 位? (实际每个格占用 3 个 bit 位) | 现在这个格的值? 这个, 我好昏呀
    }
}
static int mod = (int) 1e9 + 7;
public int colorTheGrid(int m, int n) {
    HashMap<Integer, Long> dic = new HashMap<>();
    helper(m, 0, dic, -1, 0); // 这应该就是我想找的精巧不占多少空间的 mask 了, 可是有点儿看不懂
    HashSet<Integer> set = new HashSet<>(dic.keySet());
    for (int i = 1; i < n; i++) { // 动态规划: 用两个图像滚动数组一样轮流记载得出答案
        HashMap<Integer, Long> tmp = new HashMap<>();
        for (int x: set) {
            for (int y: set) {
                if ((x & y) == 0) // 相邻涂色方案为有效方案
                    tmp.put(y, (tmp.getOrDefault(y, 0L) + dic.get(x)) % mod);
            }
        }
        dic = tmp;
    }
    long res = 0L;
    for (Long x: dic.values()) {
        res += x;
        res %= mod;
    }
    return (int) res;
}

```

- 比较传统一点儿的解法, 思路清晰

```

// 参考的答案里, 这个最逻辑简单、通俗大众易懂, 但稍显笨重, 两个图, 用一个链表来记忆一行的涂色方案, 如果有更精巧一点儿的 bitmask, 是我想找的答案
// https://leetcode.com/problems/painting-a-grid-with-three-different-colors/discuss/1334366/Easy-Java-comments-28ms-O(n*P*P)-complexity-memory-O(P)-where-P-is-the-number-of-colors
// 先预处理得到单行的所有有效涂色方案,
// 再进一步计算得到每种单行方案对应的有效邻行方案
// 在此基础上, 结合动态规划方法, 逐行求解各种涂色状态对应的方案总数, 最后统计得到总方案数。

```

```

public int colorTheGrid(int m, int n) {
// 获得单行所有涂色方案
    Map<Integer, List<Integer>> line = new HashMap<>(); // 3^m ways of paying one row
    int range = (int) Math.pow(3, m); // 用 0、1、2 表示各个网格的颜色, key 为方案对应的数值, value 为方案对应的数组
    for (int i = 0; i < range; i++) {
        List<Integer> list = new ArrayList<>(); // val val values (0, 1, 2) of every m cols into list
        int val = i;
        for (int j = 0; j < m; j++) {
            list.add(val % 3);
            val /= 3;
        }
        boolean valid = true; // 确认该数组中是否存在相邻位置颜色相同
        for (int j = 1; j < m; j++) {
            if (list.get(j-1) == list.get(j)) {
                valid = false;
                break;
            }
        }
        if (valid) line.put(i, list); // 相邻网格颜色均不同, 为有效方案, 加入哈希表
    }
// 预处理得到每种单行方案对应的有效邻行方案
    Map<Integer, List<Integer>> adj = new HashMap<>();
    Iterator it = line.entrySet().iterator();
    while (it.hasNext()) { // 3^m ways of paying one row
        Map.Entry entry = (Map.Entry) it.next();
        int va = (int) entry.getKey();
        List<Integer> lva = (List<Integer>) entry.getValue();
        adj.put(va, new ArrayList<Integer>());
        Iterator itb = line.entrySet().iterator();
        while (itb.hasNext()) { // 3^m ways of paying one row
            Map.Entry enb = (Map.Entry) itb.next();
            int vb = (int) enb.getKey();
            List<Integer> lvb = (List<Integer>) enb.getValue();
            boolean valid = true;
            for (int i = 0; i < m; i++)
                if (lva.get(i) == lvb.get(i)) {
                    valid = false;
                    break;
                }
            } // among 3^m ways of painting one row, how many is valid, and valid mask into adj.get(va);
            if (valid) adj.get(va).add(vb);
        }
    }
// 动态规划, 逐行求解方案数
    int mod = (int)(1e9+7);
    long[] dp = new long[range]; // 上一行各种涂色方案对应的总方法数
    for (int i = 0; i < range; i++) // 初始化
        dp[i] = line.containsKey(i) ? 1 : 0;
    for (int i = 1; i < n; i++) { // 从第二行开始动态规划
        long[] cur = new long[range]; // 新一行各种涂色方案对应的总方法数
        for (int j = 0; j < range; j++)
            if (adj.containsKey(j)) { // 该方案有效
                for (int v : adj.get(j)) // 遍历有效的相邻方案
                    cur[j] = (cur[j] + dp[v]) % mod; // 总方法数累加
            }
        System.arraycopy(cur, 0, dp, 0, range);
    }
    long ans = 0;
    for (int i = 0; i < range; i++)
        ans = (ans + dp[i]) % mod;
    return (int)ans;
}

```

2.5.11 313. Super Ugly Number

A super ugly number is a positive integer whose prime factors are in the array primes. Given an integer n and an array of integers primes, return the nth super ugly number. The nth super ugly number is guaranteed to fit in a 32-bit signed integer.

```

static class Node implements Comparable<Node> {
    private int index;
    private int val;
    private int prime;
    public Node(int index, int val, int prime) {
        this.index = index;
        this.val = val;
        this.prime = prime;
    }
    public int compareTo(Node other) {
        return this.val - other.val;
    }
    public int nthSuperUglyNumber(int n, int[] primes) {

```

```

final int [] arr = new int[n];
arr[0] = 1; // 1 is the first ugly number
final Queue<Node> q = new PriorityQueue<>();
for (int i = 0; i < primes.length; ++i)
    q.add(new Node(0, primes[i], primes[i]));
for (int i = 1; i < n; ++i) {
    Node node = q.peek(); // get the min element and add to arr
    arr[i] = node.val;
    do { // update top elements
        node = q.poll();
        node.val = arr[++node.index] * node.prime;
        q.add(node); // push it back
    } while (!q.isEmpty() && q.peek().val == arr[i]); // prevent duplicate
}
return arr[n - 1];
}

```

- 下面这种解法也很巧妙

```

public int nthSuperUglyNumber(int n, int[] primes) {
    int m = primes.length;
    int [] ans = new int[n]; // 存放 1-n 个 SuperUglyNumber
    ans[0] = 1; // 第一个 SuperUglyNumber 是 1
    int [] next = new int[m];
    for (int i=0; i < m; i++)
        next[i] = 0; // 初始化
    int cnt = 1, min = Integer.MAX_VALUE, tmp = 0;
    while (cnt < n) {
        min = Integer.MAX_VALUE;
        for (int i = 0; i < m; i++){
            tmp = ans[next[i]] * primes[i];
            min = Math.min(min, tmp);
        }
        for (int i = 0; i < m; i++)
            if (min == ans[next[i]] * primes[i])
                next[i]++;
        ans[cnt++] = min;
    }
    return ans[n-1];
}

```

2.5.12 1786. Number of Restricted Paths From First to Last Node - Dijkstra 算法

There is an undirected weighted connected graph. You are given a positive integer n which denotes that the graph has n nodes labeled from 1 to n , and an array edges where each $\text{edges}[i] = [u_i, v_i, \text{weight}_i]$ denotes that there is an edge between nodes u_i and v_i with weight equal to weight_i . A path from node start to node end is a sequence of nodes $[z_0, z_1, z_2, \dots, z_k]$ such that $z_0 = \text{start}$ and $z_k = \text{end}$ and there is an edge between z_i and z_{i+1} where $0 \leq i \leq k-1$. The distance of a path is the sum of the weights on the edges of the path. Let $\text{distanceToLastNode}(x)$ denote the shortest distance of a path between node n and node x . A restricted path is a path that also satisfies that $\text{distanceToLastNode}(z_i) > \text{distanceToLastNode}(z_{i+1})$ where $0 \leq i \leq k-1$. Return the number of restricted paths from node 1 to node n . Since that number may be too large, return it modulo $10^9 + 7$.

```

public void dijkstra(int n) {
    Queue<int []> q = new PriorityQueue<>((a, b) -> (a[1] - b[1]));
    q.add(new int []{n, 0});
    Arrays.fill(dist, Integer.MAX_VALUE);
    dist[n] = 0;
    int [] cur = null;
    int u = 0, d = 0;
    while (!q.isEmpty()) {
        cur = q.poll();
        u = cur[0];
        d = cur[1];
        if (dist[u] < d) continue;
        if (m.get(u) != null)
            for (int v : m.get(u).keySet())
                if (dist[v] > dist[u] + m.get(u).get(v)) {
                    dist[v] = dist[u] + m.get(u).get(v);
                    q.offer(new int []{v, dist[v]});
                }
    }
    private int dfs(int n, int i) {
        if (i == n) return 1;
        if (dp[i] != -1) return dp[i];
        long res = 0;

```

```

    if (m.get(i) != null) {
        for (int v : m.get(i).keySet()) {
            if (dist[i] > dist[v])
                res = (res + dfs(n, v)) % mod;
        }
    }
    return dp[i] = (int)res;
}
HashMap<Integer, Map<Integer, Integer>> m = new HashMap<>();
int mod = (int)(1e9+7);
int [] dist;
int [] dp;
public int countRestrictedPaths(int n, int[][][] edges) {
    for (int [] v : edges) {
        m.computeIfAbsent(v[0], k->new HashMap<>()).put(v[1], v[2]);
        m.computeIfAbsent(v[1], k->new HashMap<>()).put(v[0], v[2]);
    }
    dist = new int[n+1];
    dijkstra(n);
    dp = new int [n+1];
    Arrays.fill(dp, -1);
    return dfs(n, 1);
}

```

2.5.13 913. Cat and Mouse

A game on an undirected graph is played by two players, Mouse and Cat, who alternate turns. The graph is given as follows: $\text{graph}[a]$ is a list of all nodes b such that ab is an edge of the graph. The mouse starts at node 1 and goes first, the cat starts at node 2 and goes second, and there is a hole at node 0. During each player's turn, they must travel along one edge of the graph that meets where they are. For example, if the Mouse is at node 1, it must travel to any node in graph^2 . Additionally, it is not allowed for the Cat to travel to the Hole (node 0.) Then, the game can end in three ways: If ever the Cat occupies the same node as the Mouse, the Cat wins. If ever the Mouse reaches the Hole, the Mouse wins. If ever a position is repeated (i.e., the players are in the same position as a previous turn, and it is the same player's turn to move), the game is a draw. Given a graph, and assuming both players play optimally, return 1 if the mouse wins the game, 2 if the cat wins the game, or 0 if the game is a draw.

```

private int dfs(int [][] arr, int t, int i, int j) { // t: steps, i: mouse, j: cat, mouse goes first
    if (t == 2 * n) return 0;
    if (i == j) return dp[t][i][j] = 2;
    if (i == 0) return dp[t][i][j] = 1;
    if (dp[t][i][j] != -1) return dp[t][i][j];
    int tmp = 0;
    if (t % 2 == 0) { // mouse's turn
        boolean catWin = true;
        for (int k = 0; k < arr[i].length; k++) {
            tmp = dfs(arr, t+1, arr[i][k], j);
            if (tmp == 1) return dp[t][i][j] = 1;
            else if (tmp != 2) catWin = false;
        }
        if (catWin) return dp[t][i][j] = 2;
        else return dp[t][i][j] = 0;
    } else { // cat's turn, can NOT step on node # 0
        boolean mouseWin = true;
        for (int k = 0; k < arr[j].length; k++) {
            if (arr[j][k] == 0) continue;
            tmp = dfs(arr, t+1, i, arr[j][k]);
            if (tmp == 2) return dp[t][i][j] = 2;
            else if (tmp != 1) mouseWin = false;
        }
        if (mouseWin) return dp[t][i][j] = 1;
        else return dp[t][i][j] = 0;
    }
}
int [][] dp;
int n;
public int catMouseGame(int[][][] graph) {
    n = graph.length;
    dp = new int [2*n][n][n];
    for (int i = 0; i < 2*n; i++)
        for (int j = 0; j < n; j++)
            Arrays.fill(dp[i][j], -1);
    dfs(graph, 0, 1, 2);
    return dp[0][1][2];
}

```

2.5.14 1728. Cat and Mouse II

A game is played by a cat and a mouse named Cat and Mouse. The environment is represented by a grid of size $\text{rows} \times \text{cols}$, where each element is a wall, floor, player (Cat, Mouse), or food. Players are represented by the characters 'C'(Cat),'M'(Mouse). Floors are represented by the character '.' and can be walked on. Walls are represented by the character '#' and cannot be walked on. Food is represented by the character 'F' and can be walked on. There is only one of each character 'C', 'M', and 'F' in grid. Mouse and Cat play according to the following rules: Mouse moves first, then they take turns to move. During each turn, Cat and Mouse can jump in one of the four directions (left, right, up, down). They cannot jump over the wall nor outside of the grid. `catJump`, `mouseJump` are the maximum lengths Cat and Mouse can jump at a time, respectively. Cat and Mouse can jump less than the maximum length. Staying in the same position is allowed. Mouse can jump over Cat. The game can end in 4 ways: If Cat occupies the same position as Mouse, Cat wins. If Cat reaches the food first, Cat wins. If Mouse reaches the food first, Mouse wins. If Mouse cannot get to the food within 1000 turns, Cat wins. Given a $\text{rows} \times \text{cols}$ matrix grid and two integers `catJump` and `mouseJump`, return true if Mouse can win the game if both Cat and Mouse play optimally, otherwise return false.

```

private boolean dfs(String [] arr, int t, int i, int j) {
    if (dp[t][i][j] != null) return dp[t][i][j];
    if (t == m*n*2) return false;
    if (arr[i/n].charAt(i%n) == 'F') return true;
    if (arr[j/n].charAt(j%n) == 'F') return false;
    if (i == j) return false;
    int r = 0, c = 0;
    if (t % 2 == 0) { // mouse's turn 老鼠的: 只要它能赢一个状态就是赢了
        for (int [] d : dirs)
            for (int k = 0; k <= mj; k++) {
                r = i / n + d[0] * k;
                c = i % n + d[1] * k;
                if (r >= 0 && r < m && c >= 0 && c < n && arr[r].charAt(c) != '#') {
                    if (dfs(arr, t+1, r*n+c, j))
                        return dp[t][i][j] = true; // Mouse could win
                } else break;
            }
        return dp[t][i][j] = false;
    } else { // cat's turn: 但是当是猫的: 需要猫不能赢, 老鼠才能赢; 但是当猫哪怕是赢了只一局, 老鼠也就输了
        for (int [] d : dirs)
            for (int k = 0; k <= cj; k++) {
                r = j / n + d[0] * k;
                c = j % n + d[1] * k;
                if (r >= 0 && r < m && c >= 0 && c < n && arr[r].charAt(c) != '#') {
                    if (!dfs(arr, t+1, i, r*n+c)) // Can cat find a path that mouse loses in it?
                        return dp[t][i][j] = false; // Cat wins = mouse loose
                } else break; // 上面这一点儿狠重要
            }
        return dp[t][i][j] = true;
    }
}
int [][] dirs = {{1, 0}, {-1, 0}, {0, 1}, {0, -1}};
Boolean [][] dp;
int m, n, cj, mj;
public boolean canMouseWin(String[] grid, int catJump, int mouseJump) {
    m = grid.length;
    n = grid[0].length();
    cj = catJump;
    mj = mouseJump;
    dp = new Boolean [1001][m*n][m*n];
    int x = 0, y = 0;
    for (int i = 0; i < m; i++)
        for (int j = 0; j < n; j++)
            if (grid[i].charAt(j) == 'M')
                x = i * n + j;
            else if (grid[i].charAt(j) == 'C')
                y = i * n + j;
    return dfs(grid, 0, x, y);
}

```

2.5.15 810. Chalkboard XOR Game - Hard

You are given an array of integers `nums` represents the numbers written on a chalkboard.

Alice and Bob take turns erasing exactly one number from the chalkboard, with Alice starting first. If erasing a number causes the bitwise XOR of all the elements of the chalkboard to become 0, then that player loses. The bitwise XOR of one element is that element itself, and the bitwise XOR of no elements is 0.

Also, if any player starts their turn with the bitwise XOR of all the elements of the chalkboard equal to 0, then that player wins.

Return true if and only if Alice wins the game, assuming both players play optimally. There are three cases to consider:

Case 1- At the beginning of the game, XOR of all the elements are 0, then Alice wins before the game starts.

Case 2 - XOR!=0 and nums.length is even:
 Let's try to use proof by contradiction. $S=(x_1 \wedge x_2 \dots \wedge x_n)$
 Assume $s!=0$, let's try to find contradiction
 XOR s to both sides
 $s \wedge s = s \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 $s \wedge s = 0 \Rightarrow 0 = s \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 $0 = (s \wedge x_1) \wedge (s \wedge x_2) \dots \wedge (s \wedge x_n)$
 Now let's factor s from each bracket
 $0 = (s \wedge s \dots \wedge s) \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 Since the number of $x_1..x_n$ is even, the number of s in the left bracket is even, each number \wedge itself even times results to 0.
 $0 = 0 \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 $0 \wedge$ any number is itself so
 $0 = (x_1 \wedge x_2 \dots \wedge x_n) = s \Rightarrow 0 = s$
 You see that there is a contradiction (compare with initial assumption $s!=0$), at the beginning we assumed $s!=0$.
 Then our assumption is wrong. So, $s==0$ then Alice wins

Case 3- XOR!=0 and nums.length is odd:
 Let's try to use proof by contradiction here like the other case
 Assume $s!=0$, let's try to find contradiction
 XOR s to both sides
 $s \wedge s = s \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 $s \wedge s = 0 \Rightarrow 0 = s \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 $0 = (s \wedge x_1) \wedge (s \wedge x_2) \dots \wedge (s \wedge x_n)$
 Now let's factor s from each bracket
 $0 = (s \wedge s \dots \wedge s) \wedge (x_1 \wedge x_2 \dots \wedge x_n)$
 Since the number of $x_1..x_n$ is odd, the number of s in the left bracket is odd, each number \wedge itself odd times results to itself.
 $0 = s \wedge (x_1 \wedge x_2 \dots \wedge x_n) \Rightarrow 0 = s \wedge s$
 Any number XOR itself becomes zero
 $0 = s \wedge s = 0$
 You see here we couldn't find the contradiction

```
public boolean xorGame(int[] nums) {
    int xor = 0 ;
    for (int i : nums)
        xor = xor ^ i ;
    if (xor == 0 || (nums.length & 1) == 0)
        return true ;
    return false ;
}
```

- 硬瓣出来的：注意同猫老鼠游戏 2 一样，要回的是某一方赢与否，与 1 有点儿区别。

```
private boolean helper(int[] arr, int i, int xor) { // xor: the current leftover array xor result
    if (i == n) return (i % 2 == 0);
    if (dp[i] != null) return dp[i];
    if (xor == 0) return (i % 2 == 0); // to be noted
    int tmp = 0;
    if (i % 2 == 0) { // alice's turn
        for (int j = 0; j < n; j++) {
            if (arr[j] == -1) continue;
            if ((arr[j] ^ xor) == 0) continue;
            tmp = arr[j];
            arr[j] = -1;
            if (helper(arr, i+1, xor^tmp)) return dp[i] = true;
            arr[j] = tmp;
        }
        return dp[i] = false;
    } else { // bob's turn
        for (int j = 0; j < n; j++) {
            if (arr[j] == -1) continue;
            if ((arr[j] ^ xor) == 0) continue;
            tmp = arr[j];
            arr[j] = -1;
            if (!helper(arr, i+1, xor^tmp)) return dp[i] = false;
            arr[j] = tmp;
        }
        return dp[i] = true;
    }
}
Boolean[] dp; // alice win states
int n;
public boolean xorGame(int[] arr) {
    n = arr.length;
```

```

dp = new Boolean [n];
int [] xor = new int [n];
for (int i = 0; i < n; i++)
    xor[i] = (i == 0 ? 0 : xor[i-1]) ^ arr[i];
return helper(arr, 0, xor[n-1]); // i: turn
}

```

2.5.16 backpack III

```

public int backPackIII(int[] A, int[] V, int m) {
    int n = A.length;
    int [] dp = new int[m+1];
    for (int i = 1; i <= m; i++) {
        for (int j = 0; j < n; j++) {
            if (i - A[j] >= 0)
                dp[i] = Math.max(dp[i], dp[i-A[j]] + V[j]);
        }
    }
    return dp[m];
}

```

2.5.17 879. Profitable Schemes - Hard 0-1 背包问题

There is a group of n members, and a list of various crimes they could commit. The i th crime generates a profit $[i]$ and requires group $[i]$ members to participate in it. If a member participates in one crime, that member can't participate in another crime.

Let's call a profitable scheme any subset of these crimes that generates at least minProfit profit, and the total number of members participating in that subset of crimes is at most n .

Return the number of schemes that can be chosen. Since the answer may be very large, return it modulo $10^9 + 7$.

1. 解题思路与分析

Solution: DP

0-1 Knapsack problem, each task can be finished only once in each state.

state: (current_profit, people_used)

$dp[k][i][j] :=$ # of schemes to achieve i profit with j people by assigning first k tasks.

Init: $dp[0][0][0] = 1$

Transition:

$p, g = profit[k - 1], group[k - 1]$
 $dp[k][i][j] = dp[k-1][i][j]$
 $+ dp[k-1][i-p][j-g]$ if $j \geq g$ else 0
 $0 \leq i \leq p, 0 \leq j \leq G$

Ans: sums($dp[K][P]$)

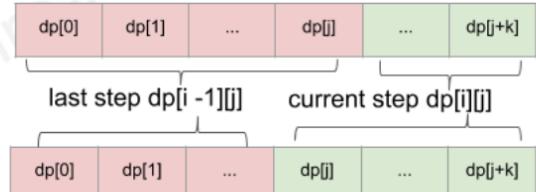
Time complexity: $O(KPG)$

Space complexity: $O(KPG) \rightarrow O(PG)$

Dimensionality reduction

If only $dp[i-1][*]$ is needed for $dp[i][*]$

1. Using a tmp array. [always applicable]
 - a. tmp = copy of dp / or create new
 - b. update tmp using dp
 - c. dp = tmp
2. Using a rolling array. [more efficient]
 - a. Iterate j in reverse order
 - b. Update $dp[j']$ using $dp[j]$ where $j' > j$



- 题目中说了结果可能非常大,要对一个超大数取余,看到这里,我们也就该明白为了不爆栈,只能用动态规划 Dynamic Programming 来做, LeetCode 里有好多题都是要对这个 $1e9+7$ 取余, 不知道为啥都是对这个数取余。Anyway, who cares, 还是来想想 dp 数组如何定义以及怎么推导状态转移方程吧。

首先来看分配黑帮资源时候都需要考虑哪些因素, 总共有三点, 要干几票买卖, 要用多少人, 能挣多少钱。所以我们需要一个三维的 dp 数组, 其中 $dp[k][i][j]$ 表示最多干 k 票买卖, 总共用了 i 个人, 获得利润为 j 的情况下分配方案的总数, 初始化 $dp^{1,1,1}$ 为 1。

现在来推导状态转移方程, 整个规划的核心是买卖, 总共买卖的个数是固定的, 每多干一票买卖, 可能的分配方法就可能增加, 但不可能减少的, 因为假如当前已经算出来做 $k-1$ 次买卖的分配方法总数, 再做一次买卖, 之前的分配方法不会减少, 顶多是人数不够, 做不成当前这票买卖而已, 所以我们的 $dp[k][i][j]$ 可以先更新为 $dp[k-1][i][j]$, 然后再来看这第 k 个买卖还能不能做, 我们知道假设这第 k 个买卖需要 g 个人, 能获得利润 p , 只有当我们现在的人数 i 大于等于 g

的时候，才有可能做这个任务，我们要用 g 个人来做任务 k 的话，那么其余的 $k-1$ 个任务只能由 $i-g$ 个人来做了，而且由于整个需要产生利润 j ，第 k 个任务能产生利润 p ，所以其余的 $k-1$ 个任务需要产生利润 $j-p$ ，由于利润不能是负值，所以我们还需要跟 0 比较，取二者最大值。

综上所述，若我们选择做任务 k ，则能新产生的分配方案的个数为 $dp[k-1][i-g][\max(0, j-p)]$ ，记得每次累加完要对超大数取余。最终我们需要将 $dp[n][i][P]$ ($0 \leq i \leq G$) 累加起来，因为我们不一定要全部使用 G 个人，只要能产生 P 的利润，用几个人都没关系，而 k 是表示最多干的买卖数，可能上并没有干到这么多，所以只需要累加人数这个维度即可。

```
public int profitableSchemes(int n, int minProfit, int[] group, int[] profit) { // 0-1 背包问题：每场罪恶在每个状态里最多只能存在一次
    int mod = (int)1e9 + 7, ans = 0;
    int m = group.length;
    int[][][] dp = new int [m+1][n+1][minProfit + 1];
    dp[0][0][0] = 1;
    for (int k = 1; k <= m; k++) {
        int p = profit[k-1], g = group[k-1];
        for (int i = 0; i <= n; i++) {
            for (int j = 0; j <= minProfit; j++) {
                dp[k][i][j] = dp[k-1][i][j];
                if (i >= g)
                    dp[k][i][j] = (dp[k][i][j] + dp[k-1][i-g][Math.max(0, j-p)]) % mod;
            }
        }
        for (int i = 0; i <= n; i++)
            ans = (ans + dp[m][i][minProfit]) % mod;
    }
    return ans;
}
```

2. 优化一下空间复杂度: 二维 dp Dimension reduction by using rolling array.

因为当前做的第 k 个任务，只跟前 $k-1$ 个任务的分配方案有关，所以并不需要保存所有的任务个数的分配方式。这样我们就节省了一个维度，但是需要注意的是，更新的时候 i 和 j 只能从大到小更新，这个其实也不难理解，因为此时 $dp[i][j]$ 存的是前 $k-1$ 个任务的分配方式，所以更新第 k 个任务的时候，一定要从后面开始覆盖，因为用到了前面的值，若从前的值开始更新的话，就不能保证用到的都是前 $k-1$ 个任务的分配方式，有可能用到的是已经更新过的值，就会出错。

```
public int profitableSchemes(int n, int minProfit, int[] group, int[] profit) { // 0-1 背包问题：每场罪恶在每个状态里最多只能存在一次
    int mod = (int)1e9 + 7, ans = 0;
    int m = group.length;
    int [][] dp = new int [n+1][minProfit + 1];
    dp[0][0] = 1;
    for (int k = 1; k <= m; k++) {
        int p = profit[k-1], g = group[k-1];
        for (int i = n; i >= g; i--) // i >=
            for (int j = minProfit; j >= 0; j--) // 保证了这一行覆盖原数组的正确性
                dp[i][j] = (dp[i][j] + dp[i-g][Math.max(0, j-p)]) % mod;
        for (int i = 0; i <= n; i++)
            ans = (ans + dp[i][minProfit]) % mod;
    }
    return ans;
}
```

3. 递归 + 记忆数组来做 todo: 改天补上

基本思想跟解法一没有太大的区别，递归的记忆数组其实跟迭代形式的 dp 数组没有太大的区别，作用都是保存中间状态从而减少大量的重复计算。这里稍稍需要注意下的就是递归函数中的 corner case，当 $k=0$ 时，则根据 j 的值来返回 0 或 1，当 j 小于等于 0，返回 1，否则返回 0，相当于修改了初始化值（之前都初始化为了整型最小值），然后当 j 小于 0 时，则 j 赋值为 0，因为利润不能为负值。然后就看若当前的 $memo[k][i][j]$ 已经计算过了，则直接返回即可，

- <https://www.cnblogs.com/grandyang/p/11108205.html>

2.5.18 377. Combination Sum IV 没能认出这个题目是考 DP

Given an array of distinct integers nums and a target integer target, return the number of possible combinations that add up to target. The answer is guaranteed to fit in a 32-bit integer.

```
public int combinationSum4(int[] nums, int target) {
    int n = nums.length;
    int [] dp = new int [target + 1];
    dp [0] = 1;
    for (int i = 1; i <= target; i++) {
        for (int j = 0; j < n; j++) {
            if (i - nums[j] >= 0)
                dp[i] += dp[i-nums[j]];
        }
    }
    return dp[target];
}
```

```

        }
    }
    return dp[target];
}

```

2.5.19 1049. Last Stone Weight II

You are given an array of integers stones where stones[i] is the weight of the ith stone. We are playing a game with the stones. On each turn, we choose any two stones and smash them together. Suppose the stones have weights x and y with $x \leq y$. The result of this smash is: If $x = y$, both stones are destroyed, and If $x \neq y$, the stone of weight x is destroyed, and the stone of weight y has new weight $y - x$. At the end of the game, there is at most one stone left. Return the smallest possible weight of the left stone. If there are no stones left, return 0.

```

public int lastStoneWeightII(int[] stones) {
    int n = stones.length;
    int sum = Arrays.stream(stones).sum();
    boolean[] dp = new boolean[sum+1];
    dp[0] = true;
    sum = 0;
    for (int v : stones) {
        sum += v;
        for (int i = sum; i >= v; i--)
            if (dp[i-v]) dp[i] = true;
    }
    for (int i = sum/2; i >= 0; i--)
        if (dp[i]) return sum - i * 2;
    return 0;
}

```

2.5.20 1449. Form Largest Integer With Digits That Add up to Target

Given an array of integers cost and an integer target. Return the maximum integer you can paint under the following rules: The cost of painting a digit (i+1) is given by cost[i] (0 indexed). The total cost used must be equal to target. Integer does not have digits 0. Since the answer may be too large, return it as string. If there is no way to paint any integer given the condition, return "0".

```

public String largestNumber(int[] cost, int target) {
    int n = cost.length;
    int [] dp = new int [target+1];
    Arrays.fill(dp, -1);
    dp[0] = 0;
    for (int i = 0; i < n; i++) {
        for (int j = cost[i]; j <= target; j++) {
            if (dp[j-cost[i]] >= 0)
                dp[j] = Math.max(dp[j], dp[j-cost[i]]+1);
        }
    }
    if (dp[target] < 0) return "0";
    char [] ans = new char[dp[target]]; // 采樱桃机器人数组路线那天可以想出来，今天这个路径居然没有想出来！
    int left = target;
    for (int i = 0; i < dp[target]; i++) {
        for (int j = n; j > 0; j--) {
            if (left >= cost[j-1] && dp[left] == dp[left-cost[j-1]] + 1) {
                ans[i] = (char)('0' + j);
                left -= cost[j-1];
                break;
            }
        }
    }
    return String.valueOf(ans);
}

```

2.5.21 516. Longest Palindromic Subsequence

Given a string s, find the longest palindromic subsequence's length in s. A subsequence is a sequence that can be derived from another sequence by deleting some or no elements without changing the order of the remaining elements.

```

public int longestPalindromeSubseq(String s) {
    int n = s.length();
    int [][] dp = new int [n][n];
    dp[n-1][n-1] = 1;

```

```

for (int i = n-2; i >= 0; i--) {
    dp[i][i] = 1;
    for (int j = i+1; j < n; j++) {
        if (s.charAt(i) == s.charAt(j))
            dp[i][j] = 2 + dp[i+1][j-1];
        else dp[i][j] = Math.max(dp[i+1][j], dp[i][j-1]);
    }
}
return dp[0][n-1];
}

```

2.5.22 1143. Longest Common Subsequence

Given two strings `text1` and `text2`, return the length of their longest common subsequence. If there is no common subsequence, return 0. A subsequence of a string is a new string generated from the original string with some characters (can be none) deleted without changing the relative order of the remaining characters. For example, "ace" is a subsequence of "abcde". A common subsequence of two strings is a subsequence that is common to both strings.

```

public int longestCommonSubsequence(String S, String T) {
    int m = S.length();
    int n = T.length();
    int [][] dp = new int [m+1][n+1];
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            if (S.charAt(i-1) == T.charAt(j-1)) dp[i][j] = dp[i-1][j-1] + 1;
            else dp[i][j] = Math.max(dp[i-1][j], dp[i][j-1]);
    return dp[m][n];
}

```

2.5.23 1092. Shortest Common Supersequence - Hard

Given two strings `str1` and `str2`, return the shortest string that has both `str1` and `str2` as subsequences. If there are multiple valid strings, return any of them.

A string `s` is a subsequence of string `t` if deleting some number of characters from `t` (possibly 0) results in the string `s`.

- 参考的标准答案:

```

public void longestCommonSubsequence(String S, String T) { // 标准模板, 记住
    int m = S.length();
    int n = T.length();
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            if (S.charAt(i-1) == T.charAt(j-1)) dp[i][j] = dp[i-1][j-1] + 1;
            else dp[i][j] = Math.max(dp[i-1][j], dp[i][j-1]);
}
int [][] dp;
public String shortestCommonSupersequence(String s, String t) {
    int m = s.length();
    int n = t.length();
    dp = new int [m+1][n+1];
    longestCommonSubsequence(s, t); // fill dp table
    int i = m, j = n;
    StringBuilder sb = new StringBuilder();
    while (i-1 >= 0 && j-1 >= 0) {
        if (s.charAt(i-1) == t.charAt(j-1)) {
            sb.append(s.charAt(i-1));
            --i;
            --j;
        } else {
            if (dp[i][j] == dp[i-1][j]) {
                sb.append(s.charAt(i-1));
                --i;
            } else {
                sb.append(t.charAt(j-1));
                --j;
            }
        }
    }
    if (i > 0) sb.append((new StringBuilder(s.substring(0, i))).reverse());
    if (j > 0) sb.append((new StringBuilder(t.substring(0, j))).reverse());
    return sb.reverse().toString();
}

```

- 自己写的

```

public String getLongestCommonSubsequence(String S, String T) { // 标准模板, 记住
    int m = S.length();
    int n = T.length();
    int [][] dp = new int [m+1][n+1];
    for (int i = 1; i <= m; i++)
        for (int j = 1; j <= n; j++)
            if (S.charAt(i-1) == T.charAt(j-1)) dp[i][j] = dp[i-1][j-1] + 1;
            else dp[i][j] = Math.max(dp[i-1][j], dp[i][j-1]);
    int i = m, j = n;
    StringBuilder sb = new StringBuilder();
    while (i-1 >= 0 && j-1 >= 0) {
        if (S.charAt(i-1) == T.charAt(j-1)) {
            sb.insert(0, S.charAt(i-1));
            --i;
            --j;
        } else {
            if (dp[i-1][j] >= dp[i][j-1]) --i;
            else --j;
        }
    }
    return sb.toString();
}
public String shortestCommonSupersequence(String s, String t) {
    int m = s.length();
    int n = t.length();
    int i = 0, j = 0;
    String sub = getLongestCommonSubsequence(s, t);
    String res = "";
    for (char c : sub.toCharArray()) {
        while (s.charAt(i) != c) {
            res += s.charAt(i);
            i++;
        }
        while (t.charAt(j) != c) {
            res += t.charAt(j);
            j++;
        }
        res += c;
        i++;
        j++;
    }
    return res + s.substring(i) + t.substring(j);
}

```

2.5.24 546. Remove Boxes - Hard

You are given several boxes with different colors represented by different positive numbers.

You may experience several rounds to remove boxes until there is no box left. Each time you can choose some continuous boxes with the same color (i.e., composed of k boxes, $k \geq 1$), remove them and get $k * k$ points.

Return the maximum points you can get.

```

// 定义 dp[l][r][k] 表示在 [l, r] 区间并且在后面前包含了 k 个与 boxes[r] 相同颜色的 boxes 的情况下, 可以获得的最大得分, 显然题目要求的就是 dp[0][boxes.size() - 1]
// 首先将 dp[l][r][k] 的值初始化为 dp[l][r - 1][0] + (k + 1)^2, 表示首先消除 l 到 r-1 之间的 boxes, 然后将 boxes[r] 连同后面的 k 个 boxes 一起消除。
// 然后就尝试对 dp[l][r][k] 进行更新了:
// 如果在 l 到 r-1 区间内有 boxes[i] 和 boxes[r] 相同的字符, 那么可以尝试首先将区间 [i + 1, r - 1] 消除, 这样 i 就和后面的 k + 1 个 boxes 连起来了,
// 其可以获得分数就是需要进一步计算的 dp[l][i][k + 1]。
private int dfs(int [] arr, int i, int j, int k) {
    if (i > j) return 0;
    if (dp[i][j][k] > 0) return dp[i][j][k];
    int res = dfs(arr, i, j-1, 0) + (k+1)*(k+1);
    for (int x = i; x < j; x++)
        if (arr[x] == arr[j]) {
            res = Math.max(res, dfs(arr, i, x, k+1) + dfs(arr, x+1, j-1, 0));
        }
    return dp[i][j][k] = res;
}
int [[[[]]] dp;
int n;
public int removeBoxes(int[] boxes) {
    n = boxes.length;
    dp = new int [n][n][n];
    return dfs(boxes, 0, n-1, 0);
}

```

2.5.25 1531. String Compression II - Hard

Run-length encoding is a string compression method that works by replacing consecutive identical characters (repeated 2 or more times) with the concatenation of the character and the number marking the count of the characters (length of the run). For example, to compress the string "aabccc" we replace "aa" by "a2" and replace "ccc" by "c3". Thus the compressed string becomes "a2bc3".

Notice that in this problem, we are not adding '1' after single characters.

Given a string s and an integer k. You need to delete at most k characters from s such that the run-length encoded version of s has minimum length.

Find the minimum length of the run-length encoded version of s after deleting at most k characters.

```

private int dfs(char[] s, int idx, int cnt) { // 求从下标 index 开始向后, 所有长度为 count 的子序列中, 编码后的最小长度
    if (cnt == 0) return 0;
    if (idx == n) return Integer.MAX_VALUE; // 当下标越界时还未找到长度为 count 的子序列
    if (dp[idx][cnt] > 0) return dp[idx][cnt];
    int min = Integer.MAX_VALUE, leftCnt = 0;
    boolean[] vis = new boolean[26];
    for (int i = idx; i < n; i++) {
        if (vis[s[i] - 'a']) continue; // 优化: 已处理过当前的字母, 跳过
        if (idx > 0 && s[i] == s[idx - 1]) continue;
        vis[s[i] - 'a'] = true;
        leftCnt = 0;
        for (int j = i; j < n; j++) {
            if (s[j] != s[i]) continue;
            leftCnt++;
            if (cnt - leftCnt < 0) break; // 如果左半部分长度大于子序列长度, 退出
            int right = dfs(s, j + 1, cnt - leftCnt);
            if (right == Integer.MAX_VALUE) continue;
            int left = String.valueOf(leftCnt).length();
            min = Math.min(min, left + right + (left == 1 && leftCnt == 1 ? 0 : 1));
        }
    }
    return dp[idx][cnt] = min;
}
int[][] dp;
int n;
public int getLengthOfOptimalCompression(String s, int k) {
    n = s.length();
    dp = new int[n][n - k + 1];
    return dfs(s.toCharArray(), 0, n - k);
}

```

1. 决策类 DP 总结

- <https://leetcode-cn.com/problems/string-compression-ii/solution/jie-ti-si-kao-guo-cheng-yu-jie-fa/>

2.5.26 1000. Minimum Cost to Merge Stones

There are n piles of stones arranged in a row. The ith pile has stones[i] stones. A move consists of merging exactly k consecutive piles into one pile, and the cost of this move is equal to the total number of stones in these k piles. Return the minimum cost to merge all piles of stones into one pile. If it is impossible, return -1.

```

public int mergeStones(int[] stones, int k) {
    int n = stones.length;
    if ((n - 1) % (k - 1) != 0) return -1;
    int[][] dp = new int[n][n];
    int[] pre = new int[n + 1];
    for (int i = 1; i <= n; i++)
        pre[i] = pre[i - 1] + stones[i - 1];
    int j = 0;
    for (int len = k; len <= n; len++) {
        for (int i = 0; i + len - 1 < n; i++) {
            j = i + len - 1;
            dp[i][j] = Integer.MAX_VALUE; // have to initialize it here !!!
            for (int x = i; x < j; x += k - 1)
                dp[i][j] = Math.min(dp[i][j], dp[i][x] + dp[x + 1][j]);
            if ((j - i) % (k - 1) == 0) // 如果总长度满足合并只剩一个数的条件, 则可以再合并一次
                dp[i][j] += pre[j + 1] - pre[i];
        }
    }
    return dp[0][n - 1];
}

```

2.5.27 1039. Minimum Score Triangulation of Polygon

You have a convex n-sided polygon where each vertex has an integer value. You are given an integer array values where values[i] is the value of the ith vertex (i.e., clockwise order). You will triangulate the polygon into n - 2 triangles. For each triangle, the value of that triangle is the product of the values of its vertices, and the total score of the triangulation is the sum of these values over all n - 2 triangles in the triangulation. Return the smallest possible total score that you can achieve with some triangulation of the polygon.

```
// 动态规划，递归可以使逻辑简单（本质还是动态规划）将多边形起
// 始位置设为 start, end, 用一个数组 dp 来记录任意起始位置的 score
// 为了计算 dp[start][end], 我们用一个 index k 在 start 到 end 之间遍
// 历 dp[start][end] = min(dp[start][k] + dp[k][end] + A[start]
// * A[k] * A[end]) 结果为 dp[0][n - 1] 注意：相邻的 dp[i][i + 1]
// = 0, 因为两条边无法组成三角形
private int dfs(int[] arr, int x, int y) {
    if (y - x < 2) return dp[x][y] = 0;
    if (dp[x][y] > 0) return dp[x][y];
    int min = Integer.MAX_VALUE;
    for (int i = x+1; i < y; i++)
        min = Math.min(min, dfs(arr, x, i) + dfs(arr, i, y) + arr[x]*arr[i]*arr[y]);
    return dp[x][y] = min;
}
int[][] dp;
int n;
public int minScoreTriangulation(int[] arr) {
    n = arr.length;
    dp = new int[n][n];
    return dfs(arr, 0, n-1);
}
```

2.5.28 375. Guess Number Higher or Lower II - Medium

We are playing the Guessing Game. The game will work as follows:

I pick a number between 1 and n.

You guess a number.

If you guess the right number, you win the game.

If you guess the wrong number, then I will tell you whether the number I picked is higher or lower, and you will continue guessing.

Every time you guess a wrong number x, you will pay x dollars. If you run out of money, you lose the game.

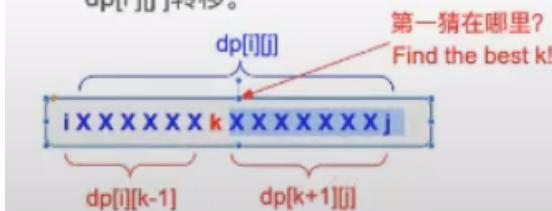
Given a particular n, return the minimum amount of money you need to guarantee a win regardless of what number I pick.

从1~N的数字里猜数。如果你猜一个数x，需要付钱x块，并且会得到反馈信息x是大是小。问最少付多少钱能保证猜中。

状态定义：照抄问题 $dp[i][j] \Rightarrow$ 最少付多少钱能保证猜中i~j里的数字。

状态的转移：

- 第一层循环是区间大小。
- 第二层循环是起始点。
- 千方百计将大区间的 $dp[i][j]$ 往小区间的 $dp[i][j]$ 转移。



```
for (int len=1; len<=N; len++)
    for (int i=1; i+len-1<=N; i++)
    {
        int j = i+len-1;
        for (int k=i; k<=j; k++)
            dp[i][j] = min(dp[i][j],
                           k + max(dp[i][k-1], dp[k+1][j]));
    }
}
Ans = dp[1][N]
注意边界条件：
dp[i][j] = 0 (i>=j)
```

```
private int dfs(int l, int r) {
    if (dp[l][r] > 0) return dp[l][r];
    if (l == r) return dp[l][r] = 0;
    if (l == r-1) return dp[l][r] = Math.min(l, r);
    int min = Integer.MAX_VALUE;
```

```

for (int i = l; i <= r; i++) {
    min = Math.min(min, i + Math.max((i == r ? i : dfs(i+1, r)), (i == l ? i : dfs(l, i-1))));
}
int [][] dp;
public int getMoneyAmount(int n) {
    dp = new int[n+1][n+1];
    return dfs(1, n);
}

```

2.5.29 1478. Allocate Mailboxes - Hard

Given the array houses and an integer k. where houses[i] is the location of the ith house along a street, your task is to allocate k mailboxes in the street.

Return the minimum total distance between each house and its nearest mailbox.

The answer is guaranteed to fit in a 32-bit signed integer.

解题思路分析：

对于如何安排邮箱位置，看到很多文章说应放在中位数的位置上，比如一共有 1, 2, 3, 4 这 4 间房屋，不论房屋间的距离是多少，如果只有一个邮箱的话，放在房间 2 处（3 也可以）最为合理。这个说法虽然正确，但实际上并不恰当。我们简单的讨论一下这个问题：

1. 当只有 1 栋房屋，1 个邮箱时，显然将邮箱放在房屋处最为合理，这时邮箱与房屋的距离为 0。
 2. 当有 2 栋房屋，1 个邮箱时，比如房屋 1 在坐标 0 处，房屋 2 在坐标 10 处，此时如果将邮箱放在坐标 0 的话，它与两栋房屋的距离和为 10。放在坐标 10 的情况下距离和也为 10。另外我们可以看出，不论邮箱放在两栋房屋之间的任意位置上，它与房屋的距离和都是 10。因此通过此例可得出，中位数的说法虽然正确，但并不全面，不过这不影响本题解题，对于本题，我们统一将邮箱安排在房屋位置上是为了方便计算，因此才得出中位数的说法（本例房屋 1 和 2 都可以看做是中位数）。
 3. 当有 3 栋房屋，1 个邮箱时，此时通过上面的例子可知，对于两侧的房子，将邮箱放在他们之间的任何位置对于结果没有任何影响，距离和都是两栋房子间的距离。但邮箱的位置会对中间的房子产生影响，因此将其放置在中间房子的坐标上最为合理，这样邮箱与中间房屋的距离为 0，可使得全局总距离最小。而中间的房子正是 3 个房屋的中位数。
 4. 当有 4 栋房屋，1 个邮箱时，与上例同理，对于两侧的房子，将邮箱放在他们之间的任何位置对于结果没有任何影响，因此邮箱可以考虑放在中间两个房屋的任何一个位置上。另外对于中间两个房屋，不论邮箱放置在其任何一个位置上，对于总距离都不会产生影响。
这相当于第 2 条。
-

因此我们可以得出结论，当有 N 栋房屋，1 个邮箱时，我们将邮箱放在房屋下标的中位数上最为合理。那么，如果有多个邮箱时该怎么办？其实也不难，本题最终可以理解为，我们将一个房屋数组分割为 K 个子数组（k 为邮筒个数），每一个子数组中放置一个邮筒，求最优分割方式。这就变为了经典的动态规划 DP 问题，对于 DP 问题我习惯采用递归加记忆数组的方式，本题我们也采用递归方式讲解。

首先建立一个递归函数，参数为当前子区间开始位置 index，以及剩余未分配邮筒个数 k。起始时，子区间开始位置为下标 0，邮筒个数为题目给定的整数 k。递归时，当前子区间的开始坐标是参数 index，结束坐标范围理论上可以是当前 index 到数组末尾为止，不过这里有一处可以优化，即要保证剩下的 k-1 个邮筒都能分配出去的话，还需要至少 k-1 个子区间，也就是说除了当前子区间外还至少需要 k-1 个房屋，因此当前子区间的结束坐标范围应该是当前 index 到 length-k 为止。我们从 index 循环至 length-k，分别作为当前子区间的结束位置 end。并通过中位数方式求出当前子区间 [index, end] 放置邮筒后的距离和（后文会给出方法）。然后将 end 加一作为下一个子区间的开始位置，同时 k 值减去一作为参数传入递归子问题中继续求解。递归函数的返回值加上当前子区间的距离和即是选择当前子区间范围后的一个结果 sum。循环完所有当前子区间的结束位置 end 之后，所有 sum 中的最小值即是最优方案，也是本层递归的返回值。

接下来再为递归加上一个记忆数组。记忆数组相当于动态规划中使用到的 DP 数组。由于递归函数中存在 2 个变量，因此我们需要使用一个 2 维数组来描述该递归函数，并记录它的返回值。

最后，上文中提到需要求解子区间内放置一个邮筒后所有房屋与邮筒的距离和。这个问题没有太好的方式，只能暴力累加每个房屋与中位数房屋所在位置的距离。为了提高效率，我们可以事先计算好所有区间（排列组合）内放置一个邮筒时的距离和，方便递归中使用，也避免重复运算。这里可能有人会提出质疑，既然递归方法中已经使用了记忆数组，目的就是防止重复计算，这里为什么还担心重复计算距离和呢？原因很简单，记忆数组是二维数组，即在两个条件都满足的情况下才会使用记忆数组中的数据，比如我们计算过以下标 5 作为子区间起点，并且当前还剩 2 个油桶的递归函数返回值为 x，即 memo^{4, 5}=x，再次遇到相同问题时我们可以直接返回 x。但是遇到 memo^{4, 2} 或者 memo^{4, 3} 时，我们尚未做出过计算，同样还会进入到递归函数内部，如果没有事前计算好下标 5 到 end (end 取值范围是 5 到 length-k) 的距离和的话，还要重复计算一遍。

对于上述问题，还有一个更好的优化方式即再建立一个保存距离和的记忆数组，计算一个距离和记录一个，方便下次使用。

```

private int getDist(int [] arr, int i, int j) { // 求区间 start 到 end 间放置邮筒后的距离和 i: left, j: right
    if (dist[i][j] > 0) return dist[i][j];
    int m = i + (j-i)/2, v = arr[m], sum = 0;

```

⁴DEFINITION NOT FOUND.

⁵DEFINITION NOT FOUND.

```

for (int k = i; k <= j; k++)
    sum += Math.abs(arr[k] - v);
return dist[i][j] = sum;
}

private int dfs(int[] arr, int idx, int k) { // idx: 待分割子区间的起始坐标; k: 待分割成的子区间的个数
    if (idx == n || idx == n-k) return 0;
    if (dp[idx][k] > 0) return dp[idx][k];
    if (k == 1) return dp[idx][k] = getDist(arr, idx, n-1);
    int res = Integer.MAX_VALUE;
    for (int i = idx; i < n-(k-1); i++)
        res = Math.min(res, getDist(arr, idx, i) + dfs(arr, i+1, k-1));
    return dp[idx][k] = res;
}

int[][] dp;
int[][] dist; // 这也是一种记忆数组优化
int n;
public int minDistance(int[] houses, int k) {
    n = houses.length;
    dist = new int[n][n];
    dp = new int[n][k+1];
    Arrays.sort(houses);
    return dfs(houses, 0, k);
}

```

2.5.30 486. Predict the Winner

You are given an integer array `nums`. Two players are playing a game with this array: player 1 and player 2. Player 1 and player 2 take turns, with player 1 starting first. Both players start the game with a score of 0. At each turn, the player takes one of the numbers from either end of the array (i.e., `nums[0]` or `nums[nums.length - 1]`) which reduces the size of the array by 1. The player adds the chosen number to their score. The game ends when there are no more elements in the array. Return true if Player 1 can win the game. If the scores of both players are equal, then player 1 is still the winner, and you should also return true. You may assume that both players are playing optimally.

博弈类题目，使用 `minMax` 思想，使自己分数最大化，对手分数尽量小，递归自顶向下求解。

该题不使用备忘机制同样能通过测试例，只不过耗时相对较长，单纯的比较取数后两 players 的分数差即可：`Math.max(nums[l] - getScore(nums, l + 1, r), nums[r] - getScore(nums, l, r - 1))`

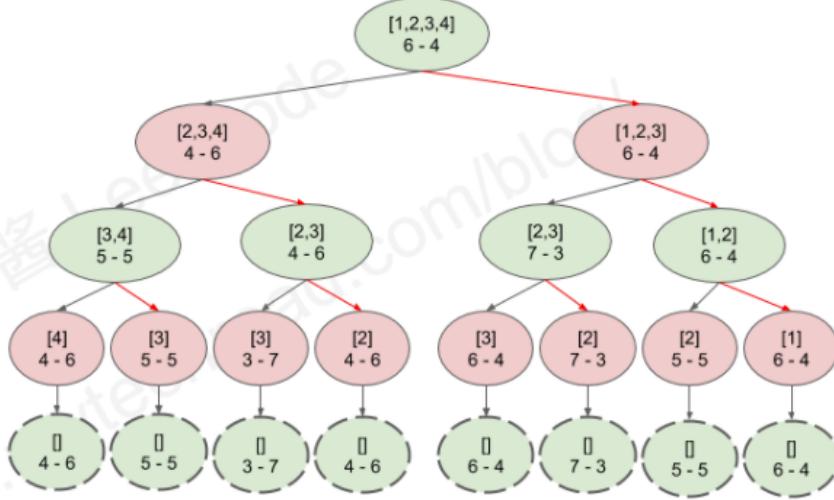
Solution: Recursion + Memorization

Min-Max Strategy:
Always pick the best, and your opponent will do the same thing!

No memory
TC: $T(n) = 2*T(n-1) + O(1) = O(2^n)$
SC: $O(n)$
Running time: 67 ms

With memory
Use `dp(i, j)` to store the best score that current player can achieve.

Time complexity: $O(n^2)$
Space complexity: $O(n^2+n)$
Running time: 5 ms



```

private int helper(int[] arr, int i, int j) {
    if (i == j) return arr[i];
    else return Math.max(arr[i] - helper(arr, i+1, j), arr[j] - helper(arr, i, j-1));
}

public boolean PredictTheWinner(int[] nums) {
    int n = nums.length;
    if (n == 1) return true;
    return helper(nums, 0, n-1) >= 0;
}

```

2.5.31 123. Best Time to Buy and Sell Stock III

You are given an array `prices` where `prices[i]` is the price of a given stock on the i th day. Find the maximum profit you can achieve. You may complete at most two transactions. Note: You may not engage in multiple transactions

simultaneously (i.e., you must sell the stock before you buy again).

```
// k 次交易 = k 个 non-overlapping subarray
// 以这个角度去想，无非就是从两个方向扫描。
// 利用 localMin / localMax 与当前元素的差值，去构造从左边/右边扫的 dp 数组。
// left[i] : 从最左面到 i 所能获得的最大利益（单次交易）
// right[i] : 从 i 到最右面所能获得的最大利益（单次交易）
public int maxProfit(int[] prices) {
    int n = prices.length;
    int [] left = new int [n];
    int [] right = new int[n];
    int locMin = prices[0];
    int globalMax = Integer.MIN_VALUE;
    for (int i = 1; i < n; i++) {
        globalMax = Math.max(globalMax, Math.max(0, prices[i] - locMin));
        locMin = Math.min(locMin, prices[i]);
        left[i] = globalMax;
    }
    int locMax = prices[n-1];
    globalMax = Integer.MIN_VALUE;
    for (int i = n-2; i >= 0; i--) {
        globalMax = Math.max(globalMax, Math.max(0, locMax - prices[i]));
        locMax = Math.max(locMax, prices[i]);
        right[i] = globalMax;
    }
    globalMax = 0;
    for (int i = 0; i < n-1; i++) {
        globalMax = Math.max(globalMax, left[i] + right[i+1]);
    }
    globalMax = Math.max(globalMax, left[n-1]);
    return globalMax;
}
```

2.5.32 188. Best Time to Buy and Sell Stock IV

You are given an integer array prices where prices[i] is the price of a given stock on the ith day, and an integer k. Find the maximum profit you can achieve. You may complete at most k transactions. Note: You may not engage in multiple transactions simultaneously (i.e., you must sell the stock before you buy again).

```
public int maxProfit(int k, int [] prices) {
    if (prices == null || prices.length == 0) return 0;
    int n = prices.length;
    int diff = 0;
    if (k >= n/2) {
        int res = 0;
        for (int i = 1; i < n; i++) {
            diff = prices[i] - prices[i-1];
            if (diff > 0) res += diff;
        }
        return res;
    }
    int [][] locMax = new int [n][k+1];
    int [][] gloMax = new int [n][k+1];
    for (int i = 1; i < n; i++) {
        diff = prices[i] - prices[i-1];
        for (int j = 1; j <= k && j * 2 <= i+1; j++) {
            locMax[i][j] = Math.max(locMax[i-1][j], gloMax[i-1][j-1]) + diff;
            gloMax[i][j] = Math.max(locMax[i][j], gloMax[i-1][j]);
        }
    }
    return gloMax[n-1][k];
}
```

2.5.33 714. Best Time to Buy and Sell Stock with Transaction Fee

You are given an array prices where prices[i] is the price of a given stock on the ith day, and an integer fee representing a transaction fee. Find the maximum profit you can achieve. You may complete as many transactions as you like, but you need to pay the transaction fee for each transaction. Note: You may not engage in multiple transactions simultaneously (i.e., you must sell the stock before you buy again).

```
public int maxProfit(int[] prices, int fee) {
    int n = prices.length;
    int [] sold = new int[n];
    int [] hold = new int[n];
    hold[0] = -prices[0];
    for (int i = 1; i < n; i++) {
```

```

        sold[i] = Math.max(sold[i-1], hold[i-1]+prices[i]-fee);
        hold[i] = Math.max(hold[i-1], sold[i-1]-prices[i]);
    }
    return sold[n-1];
}

```

2.5.34 309. Best Time to Buy and Sell Stock with Cooldown

You are given an array prices where prices[i] is the price of a given stock on the ith day. Find the maximum profit you can achieve. You may complete as many transactions as you like (i.e., buy one and sell one share of the stock multiple times) with the following restrictions: After you sell your stock, you cannot buy stock on the next day (i.e., cooldown one day). Note: You may not engage in multiple transactions simultaneously (i.e., you must sell the stock before you buy again).

- 感觉自己 DP 的能力还是太弱，越是这样越需要迎难而上。
- 这个题和 714. Best Time to Buy and Sell Stock with Transaction Fee 比较像。做题方法都是使用了两个数组：
- cash 该天结束手里没有股票的情况下，已经获得的最大收益
- hold 该天结束手里有股票的情况下，已经获得的最大收益
- 状态转移方程式这样的：
 - cash[i] 代表的是手里没有股票的收益，这种可能性是今天卖了或者啥也没干。max(昨天手里有股票的收益 + 今天卖股票的收益，昨天手里没有股票的收益)，即 $\max(\text{sell}[i - 1], \text{hold}[i - 1] + \text{prices}[i])$;
 - hold[i] 代表的是手里有股票的收益，这种可能性是今天买了股票或者啥也没干，今天买股票必须昨天休息。所以为 max(今天买股票是前天卖掉股票的收益-今天股票的价格，昨天手里有股票的收益)。即 $\max(\text{hold}[i - 1], \text{sell}[i - 2] - \text{prices}[i])$ 。
- 另外需要注意的是，题目说的是昨天卖了股票的话今天不能买，对于开始的第一天，不可能有卖股票的行为，所以需要做个判断。
- 该算法的时间复杂度是 $O(n)$ ，空间复杂度是 $O(n)$ 。

```

public int maxProfit(int[] prices) {
    int n = prices.length;
    int [] sold = new int [n];
    int [] hold = new int [n];
    hold[0] = -prices[0];
    for (int i = 1; i < n; i++) { // ith: do nothing, selling hold[i-1]
        sold[i] = Math.max((i >= 2 ? sold[i-1] : 0), hold[i-1] + prices[i]); // 今天卖了股票，或者今天什么也没有干
        hold[i] = Math.max(hold[i-1], (i >= 2 ? sold[i-2] : 0) - prices[i]); // 今天买了股票，或者今天什么也没有干
    }
    return Math.max(sold[n-1], hold[n-1]);
}

```

2.5.35 673. Number of Longest Increasing Subsequence

Given an integer array nums, return the number of longest increasing subsequences. Notice that the sequence has to be strictly increasing.

```

public int findNumberOfLIS(int[] nums) { // dynamic programming
    int n = nums.length;
    int [][] arr = new int[n][2];
    int maxLength = 1;
    for (int i = 0; i < n; i++)
        Arrays.fill(arr[i], 1);
    for (int i = 0; i < n; i++) {
        for (int j = i+1; j < n; j++) {
            if (nums[j] > nums[i]) {
                if (arr[i][0] + 1 > arr[j][0]) {
                    arr[j][0] = arr[i][0] + 1;
                    arr[j][1] = arr[i][1];
                    maxLength = Math.max(maxLength, arr[j][0]);
                } else if (arr[i][0] + 1 == arr[j][0])
                    arr[j][1] += arr[i][1];
            }
        }
    }
}

```

```

    }
    int cnt = 0;
    for (int i = 0; i < n; i++)
        if (arr[i][0] == maxLength) cnt += arr[i][1];
    return cnt;
}

```

2.5.36 1896. Minimum Cost to Change the Final Value of Expression - Hard

You are given a valid boolean expression as a string expression consisting of the characters '1','0','&' (bitwise AND operator),'|' (bitwise OR operator),'(',') and ','.

For example, "()1|1" and "(1)&()" are not valid while "1", "(((1))|(0))", and "1|(0&(1))" are valid expressions. Return the minimum cost to change the final value of the expression.

For example, if expression = "1|1|(0&0)&1", its value is $1|1|(0\&0)\&1 = 1|1|0\&1 = 1|0\&1 = 1\&1 = 1$. We want to apply operations so that the new expression evaluates to 0. The cost of changing the final value of an expression is the number of operations performed on the expression. The types of operations are described as follows:

Turn a '1' into a '0'. Turn a '0' into a '1'. Turn a '&' into a '|'. Turn a '|' into a '&'. Note: '&' does not take precedence over '|' in the order of calculation. Evaluate parentheses first, then in left-to-right order.

```

private int[] getMinOperations(int va, int vb, int ca, int cb, char sign) {
    if (sign == '&') {
        if (va == 1 && vb == 1)      // 1&1, 将其中一个 1 反转为 0
            return new int[] {1, Math.min(ca, cb)};
        else if (va == 0 && vb == 0) // 0&0, 将其中一个 0 反转为 1, 并将 & 反转为 |
            return new int[] {0, Math.min(ca, cb) + 1};
        else return new int[] {0, 1}; // 1&0, 将 & 反转为 |
    } else {
        if (va == 1 && vb == 1)      // 1|1, 将其中一个 1 反转为 0, 并将 | 反转为 &
            return new int[] {1, Math.min(ca, cb) + 1};
        else if (va == 0 && vb == 0) // 0|0, 将其中一个 0 反转为 1
            return new int[] {0, Math.min(ca, cb)};
        else return new int[] {1, 1}; // 1|0, 将 | 反转为 &
    }
}

public int minOperationsToFlip(String expression) {
    Stack<Integer> res = new Stack<>();
    Stack<Character> sgn = new Stack<>();
    Stack<Integer> cnt = new Stack<>();
    for (char c : expression.toCharArray()) {
        if (c == '(' || c == '&' || c == '|') {
            sgn.push(c);
            continue;
        } else if (c == ')') sgn.pop();
        else {
            res.push((int)(c - '0'));
            cnt.push(1);
        }
        if (res.size() > 1 && sgn.peek() != '(') {
            int[] loc = getMinOperations(res.pop(), res.pop(), cnt.pop(), cnt.pop(), sgn.pop());
            res.push(loc[0]); // expr results
            cnt.push(loc[1]); // min operations
        }
    }
    return cnt.peek();
}

```

2.5.37 823. Binary Trees With Factors

Given an array of unique integers, arr, where each integer arr[i] is strictly greater than 1. We make a binary tree using these integers, and each number may be used for any number of times. Each non-leaf node's value should be equal to the product of the values of its children. Return the number of binary trees we can make. The answer may be too large so return the answer modulo $10^9 + 7$.

```

public int numFactoredBinaryTrees(int[] arr) {
    int n = arr.length;
    Arrays.sort(arr);
    Map<Integer, Long> dp = new HashMap<>();
    int mod = 1_000_000_007;
    long res = 0;
    long max = 0;
    for (int i = 0; i < n; i++) {
        dp.put(arr[i], 1L);
        for (int j = 0; j < i; j++) {

```

```

        if (arr[i] % arr[j] == 0 && dp.containsKey(arr[i]/arr[j])) {
            max = dp.get(arr[i]) + dp.get(arr[j]) * dp.get(arr[i]/arr[j]);
            dp.put(arr[i], max % mod);
        }
    }
    res += dp.get(arr[i]);
    res %= mod;
}
return (int)(res % mod);
}

```

2.5.38 907. Sum of Subarray Minimums

Given an array of integers arr, find the sum of min(b), where b ranges over every (contiguous) subarray of arr. Since the answer may be large, return the answer modulo $10^9 + 7$.

```

public int sumSubarrayMins(int[] arr) {
    int n = arr.length;
    // for each A[i], find k <= i <= j, so that A[i] is the min from [k,j]
    // sum += A[i] * (i-k+1) * (j-i+1)
    // so we need to find the next min to the right and to the left
    // 这个过程可以简化为使用一个栈。对于被某个数从栈中弹出的数而言，它右侧第一个比它小的数就是这个数。所以我们可以对所有被弹出的数得到左侧的区间范围和右侧的区间范围。
    long [] right = new long [n]; // next smaller element index to the right
    long [] left = new long[n]; // next smaller element index to the left
    Stack<Integer> s = new Stack<>();
    for (int i = 0; i < n; i++) {
        while (!s.isEmpty() && arr[i] <= arr[s.peek()]) {
            right[s.pop()] = i-1;
        }
        s.push(i);
    }
    while (!s.isEmpty()) {
        right[s.pop()] = n-1;
    }
    s.clear();
    for (int i = n-1; i >= 0; i--) {
        while (!s.isEmpty() && arr[i] < arr[s.peek()])
            left[s.pop()] = i+1;
        s.push(i);
    }
    while (!s.isEmpty())
        left[s.pop()] = 0;
    long sum = 0;
    long leftsize = 0, rightsize = 0;
    for (int i = 0; i < n; i++) {
        leftsize = i - left[i] + 1;
        rightsize = right[i] - i + 1;
        sum += arr[i] * leftsize * rightsize;
        sum %= mod;
    }
    return (int)sum;
}

int mod = 1_000_000_007;
public int sumSubarrayMins(int[] arr) {
    int n = arr.length;
    long [] left = new long[n];
    long [] right = new long[n];
    long sum = 0;
    long cnt = 0;
    int j = 0;
    for (int i = 0; i < n; i++) { // 计算左边比自身大的数的个数
        cnt = 1;
        j = i-1;
        while (j >= 0 && arr[j] >= arr[i]) {
            cnt += left[j];
            j -= left[j];
        }
        left[i] = cnt;
    }
    // 就是因为计算了两个方向，所以对于数组里面有相同元素的情况下，需要特别考虑一下。
    // 不能重复计算，也不能漏掉，
    // 具体就是一个方向的时候用 <=，另外一个方向的时候用 <。这个在做的时候也 bug 了。
    for (int i = n-1; i >= 0; i--) { // 计算右边比自身大的数的个数
        cnt = 1;
        j = i+1;
        while (j < n && arr[j] > arr[i]) {
            cnt += right[j];
            j += right[j];
        }
        right[i] = cnt;
    }
}

```

```

    for (int i = 0; i < n; i++)
        sum += arr[i] * left[i] * right[i];
    return (int) (sum % mod);
}

```

2.5.39 1477. Find Two Non-overlapping Sub-arrays Each With Target Sum

Given an array of integers arr and an integer target. You have to find two non-overlapping sub-arrays of arr each with a sum equal target. There can be multiple answers so you have to find an answer where the sum of the lengths of the two sub-arrays is minimum. Return the minimum sum of the lengths of the two required sub-arrays, or return -1 if you cannot find such two sub-arrays.

```

// 找出数组中等于 target 的最小非重叠区间的长度, 用 dp[i] 表示当前 i 以及 i 之前的满足条件的最小区间长度, 状态更新规则为
//     dp[i]=min(dp[i-1],i-j+1) if sum[j,i]=target
// 答案更新规则
//     res=min(res,dp[j]+i-j+1)
public int minSumOfLengths(int[] arr, int target) {
    int n = arr.length;
    int [] dp = new int [n];
    Arrays.fill(dp, Integer.MAX_VALUE);
    int cur = 0, s = 0;
    int res = Integer.MAX_VALUE, minLen = Integer.MAX_VALUE;
    for (int i = 0; i < n; i++) {
        cur += arr[i];
        while (cur > target) {
            cur -= arr[s];
            s += 1;
        }
        if (cur == target) {
            int curLen = i - s + 1;
            if (s > 0 && dp[s-1] != Integer.MAX_VALUE)
                res = Math.min(res, curLen + dp[s-1]);
            minLen = Math.min(minLen, curLen);
        }
        dp[i] = minLen;
    }
    return res == Integer.MAX_VALUE ? -1 : res;
}

```

2.5.40 1771. Maximize Palindrome Length From Subsequences

You are given two strings, word1 and word2. You want to construct a string in the following manner: Choose some non-empty subsequence subsequence1 from word1. Choose some non-empty subsequence subsequence2 from word2. Concatenate the subsequences: subsequence1 + subsequence2, to make the string. Return the length of the longest palindrome that can be constructed in the described manner. If no palindromes can be constructed, return 0. A subsequence of a string s is a string that can be made by deleting some (possibly none) characters from s without changing the order of the remaining characters. A palindrome is a string that reads the same forward as well as backward.

```

public int longestPalindrome(String s, String t) { // 这个题目没有懂, 需要再好好看一下
    int m = s.length();
    int n = t.length();
    int mn = m + n;
    String st = s + t;
    int [][] dp = new int [mn][mn];
    for (int i = 0; i < mn; i++)
        dp[i][i] = 1;
    for (int l = 2; l <= mn; l++) {
        for (int i = 0, j = i+l-1; j < mn; i++, j++) { // 
            if (st.charAt(i) == st.charAt(j))
                dp[i][j] = dp[i+1][j-1] + 2;
            else dp[i][j] = Math.max(dp[i+1][j], dp[i][j-1]);
        }
    }
    int ans = 0;
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++) {
            if (s.charAt(i) == t.charAt(j))
                ans = Math.max(ans, dp[i][m+j]); //
        }
    }
    return ans;
}

```

2.5.41 907. Sum of Subarray Minimums

Given an array of integers arr, find the sum of min(b), where b ranges over every (contiguous) subarray of arr. Since the answer may be large, return the answer modulo 10⁹ + 7.

```

public int sumSubarrayMins(int[] arr) {
    int n = arr.length;
    // for each A[i], find k <= i <= j, so that A[i] is the min from [k,j]
    // sum += A[i] * (i-k+1) * (j-i+1)
    // so we need to find the next min to the right and to the left
    // 这个过程可以简化为使用一个栈。对于被某个数从栈中弹出的数而言，它右侧第一个比它小的数就是这个数。所以我们可以对所有被弹出的数得到左侧的区间范围和右侧的区间范围。
    long[] right = new long[n]; // next smaller element index to the right
    long[] left = new long[n]; // next smaller element index to the left
    Stack<Integer> s = new Stack<>();
    for (int i = 0; i < n; i++) {
        while (!s.isEmpty() && arr[i] <= arr[s.peek()]) {
            right[s.pop()] = i - 1;
        }
        s.push(i);
    }
    while (!s.isEmpty()) {
        right[s.pop()] = n - 1;
    }
    s.clear();
    for (int i = n - 1; i >= 0; i--) {
        while (!s.isEmpty() && arr[i] < arr[s.peek()])
            left[s.pop()] = i + 1;
        s.push(i);
    }
    while (!s.isEmpty())
        left[s.pop()] = 0;
    long sum = 0;
    long leftsize = 0, rightsize = 0;
    for (int i = 0; i < n; i++) {
        leftsize = i - left[i] + 1;
        rightsize = right[i] - i + 1;
        sum += arr[i] * leftsize * rightsize;
        sum %= mod;
    }
    return (int)sum;
}
int mod = 1_000_000_007;
public int sumSubarrayMins(int[] arr) {
    int n = arr.length;
    long[] left = new long[n];
    long[] right = new long[n];
    long sum = 0;
    long cnt = 0;
    int j = 0;
    for (int i = 0; i < n; i++) { // 计算左边比自身大的数的个数
        cnt = 1;
        j = i - 1;
        while (j >= 0 && arr[j] >= arr[i]) {
            cnt += left[j];
            j -= left[j];
        }
        left[i] = cnt;
    }
    // 就是因为计算了两个方向，所以对于数组里面有相同元素的情况下，需要特别考虑一下。
    // 不能重复计算，也不能漏掉，
    // 具体就是一个方向的时候用 <=，另外一个方向的时候用 <。这个在做的时候也 bug 了。
    for (int i = n - 1; i >= 0; i--) { // 计算右边比自身大的数的个数
        cnt = 1;
        j = i + 1;
        while (j < n && arr[j] > arr[i]) {
            cnt += right[j];
            j += right[j];
        }
        right[i] = cnt;
    }
    for (int i = 0; i < n; i++)
        sum += arr[i] * left[i] * right[i];
    return (int)(sum % mod);
}

```

2.5.42 689. Maximum Sum of 3 Non-Overlapping Subarrays

Given an integer array nums and an integer k, find three non-overlapping subarrays of length k with maximum sum and return them. Return the result as a list of indices representing the starting position of each interval (0-indexed). If there are multiple answers, return the lexicographically smallest one.

```

public int[] maxSumOfThreeSubarrays(int[] nums, int k) {
    int n = nums.length;
    int [] pre = new int [n+1];
    for (int i = 1; i <= n; i++) {
        pre[i] = pre[i-1] + nums[i-1];
    }
    // left[i] 表示在区间 [0, i] 范围内长度为 k 且和最大的子数组的起始位置
    // right[i] 表示在区间 [i, n - 1] 范围内长度为 k 且和最大的子数组的起始位置
    int [] left = new int [n];
    int [] right = new int [n];
    int [] res = new int [3];
    Arrays.fill(right, n-k);
    for (int i = k, total = pre[k]-pre[0]; i < n; i++) {
        if (pre[i+1] - pre[i+k] > total) {
            left[i] = i+k;
            total = pre[i+1] - pre[i+k];
        } else left[i] = left[i-1];
    }
    for (int i = n-1-k, total = pre[n]-pre[n-k]; i >= 0; i--) {
        if (pre[i+k] - pre[i] >= total) {
            right[i] = i;
            total = pre[i+k] - pre[i];
        } else right[i] = right[i+1];
    }
    int max = Integer.MIN_VALUE;
    for (int i = k; i <= n-2*k; i++) {
        int l = left[i-1];
        int r = right[i+k];
        int total = (pre[i+k]-pre[i]) + (pre[k+l]-pre[l]) + (pre[r+k] - pre[r]);
        if (max < total) {
            max = total;
            res = new int [] {l, i, r};
        }
    }
    return res;
}

```

2.5.43 363. Max Sum of Rectangle No Larger Than K

Given an $m \times n$ matrix matrix and an integer k , return the max sum of a rectangle in the matrix such that its sum is no larger than k . It is guaranteed that there will be a rectangle with a sum no larger than k .

```

public int maxSumSubmatrix(int[][] mat, int k) {
    int m = mat.length;
    int n = mat[0].length;
    if (m == 1 && n == 1) return mat[0][0];
    int [][] pre = new int [m][n];
    int res = Integer.MIN_VALUE;
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++) {
            int t = mat[i][j];
            if (i > 0) t += pre[i-1][j];
            if (j > 0) t += pre[i][j-1];
            if (i > 0 && j > 0) t -= pre[i-1][j-1];
            pre[i][j] = t;
            for (int r = 0; r <= i; r++) {
                for (int c = 0; c <= j; c++) {
                    int d = pre[i][j];
                    if (r > 0) d -= pre[r-1][j];
                    if (c > 0) d -= pre[i][c-1];
                    if (r > 0 && c > 0) d += pre[r-1][c-1];
                    if (d <= k) res = Math.max(res, d);
                }
            }
        }
    }
    return res;
}
// 把二维数组按行或列拆成多个一维数组，然后利用一维数组的累加和来找符合要求的数字,
// 这里用了 lower_bound 来加快的搜索速度，也可以使用二分搜索法来替代。
public int maxSumSubmatrix(int[][] mat, int target) {
    int row = mat.length;
    int col = mat[0].length;
    int res = Integer.MIN_VALUE;
    boolean key = col > row ? false : true;
    int m = Math.min(row, col);
    int n = Math.max(row, col);
    int [] pre = new int [n];
    TreeSet<Integer> ts = new TreeSet<>(); //用来保存当前高度下，长度为从 0 开始到 k 位置的矩形的结果。理解 set 的含义是解决此题的关键。
    Integer tmp = 0;
    for (int i = 0; i < m; i++) { // 找从第 i 行开始一直到第 0 行这 i+1 行的可能组成的矩形长度

```

```

Arrays.fill(pre, 0);
for (int j = i; j >= 0; j--) {
    ts.clear();
    ts.add(0);
    int curSum = 0;
    for (int k = 0; k < n; k++) {
        if (key)
            pre[k] += mat[k][j];
        else pre[k] += mat[j][k];
        curSum += pre[k];
        // * 因为要满足 (sum-set 中的元素) <=target,
        // * 而且 sum-set 中的元素的值要尽可能的大,
        // * 所以也就是再求小于等于 sum-target 中满足条件的元素的最小的一个
        // * 正好 TreeSet 中提供了这个方法 ceiling(), 可以很方便的找出这个元素
        tmp = ts.ceiling(curSum - target);
        if (tmp != null) res = Math.max(res, curSum - tmp);
        ts.add(curSum);
    }
}
return res;
}

```

2.5.44 805. Split Array With Same Average

You are given an integer array nums. You should move each element of nums into one of the two arrays A and B such that A and B are non-empty, and average(A) == average(B). Return true if it is possible to achieve that and false otherwise. Note that for an array arr, average(arr) is the sum of all the elements of arr over the length of arr.

```

public boolean splitArraySameAverage(int[] nums) {
    int n = nums.length;
    int m = n / 2;
    int sum = Arrays.stream(nums).sum();
    boolean poss = false;
    for (int i = 1; i <= m; i++)
        if (sum * i % n == 0) {
            poss = true;
            break;
        }
    if (!poss) return false;
    List<Set<Integer>> ls = new ArrayList<>();
    for (int i = 0; i <= m; i++)
        ls.add(new HashSet<Integer>());
    ls.get(0).add(0); // 这种构建子序列和的方法, 要学习一下
    for (int v : nums) // for each element in A, we try to add it to sums[i] by joining sums[i - 1]
        for (int i = m; i >= 1; i--)
            for (int t : ls.get(i-1))
                ls.get(i).add(t + v);
    for (int i = 1; i <= m; i++) {
        if (sum * i % n == 0 && ls.get(i).contains(sum * i / n))
            return true;
    }
    return false;
}

```

2.5.45 1981. Minimize the Difference Between Target and Chosen Elements

You are given an $m \times n$ integer matrix mat and an integer target. Choose one integer from each row in the matrix such that the absolute difference between target and the sum of the chosen elements is minimized. Return the minimum absolute difference. The absolute difference between two numbers a and b is the absolute value of $a - b$.

```

// DP + BitSet : 这里面有个小问题需要挑出来
// 使用一个 DP 数组存下当前行和之前行每行选一个数可能构成的和,
// 在本题中, 可以使用 BitSet (简介) 来存储之前行可以组成的和 (由于所有数的最大值为 70, 而行数最大也为 70, 故 BitSet 最大的位数即为 4900)。
// 对于当前行, 遍历 BitSet 已经 set 过的位 (即代表之前行可能组成的和), 然后加上当前数, set 新的和
// 最后遍历 BitSet, 求出当前位与 target 的最小值
public int minimizeTheDifference(int[][] mat, int target) {
    int m = mat.length;
    int n = mat[0].length;
    BitSet sum = new BitSet(); // 遍历每一行, 存下当前行和之前行可能组成的和
    for (int i = 0; i < n; i++) // 初始时存下第一行
        sum.set(mat[0][i]);
    for (int i = 1; i < m; i++) {
        BitSet newSum = new BitSet(); // 用来存新的和
        for (int j = 0; j < n; j++) {
            // 注意: 要遍历 BitSet 中的真实位, 请使用以下循环: previousSetBit() 方法 用于查找在指定的起始索引上或之前是否存在任何真位
        }
    }
}

```

```

        // for (int i = bs.length(); (i = bs.previousSetBit(i-1)) >= 0; ) {
        //     // operate on index i here
        // }
        for (int k = sum.length(); (k = sum.previousSetBit(k-1)) >= 0; ) {
            newSum.set(k+mat[i][j]);
        }
    }
    sum = newSum;
}
int ans = 4900;
for (int k = sum.length(); (k = sum.previousSetBit(k-1)) >= 0; ) {
    int diff = Math.abs(k - target);
    ans = Math.min(ans, diff);
}
return ans;
}

public int minimizeTheDifference(int[][] mat, int target) {
    int m = mat.length;
    int n = mat[0].length;
    int diff = Integer.MAX_VALUE, limit = 4900;
    int[] dp = new int[limit];
    for (int i = 0; i < n; i++) // 相当于手工实现 java BitSet
        dp[mat[0][i]] = 1;
    for (int i = 1; i < m; i++) {
        int[] tmp = new int[limit];
        for (int v = limit-1; v >= 0; v--) {
            if (dp[v] == 0) continue;
            for (int j = 0; j < n; j++) {
                if (v + mat[i][j] < limit)
                    tmp[v+mat[i][j]] = 1;
            }
        }
        System.arraycopy(tmp, 0, dp, 0, dp.length);
    }
    for (int i = 0; i < limit; i++)
        if (dp[i] > 0) diff = Math.min(diff, Math.abs(i-target));
    return diff; // min difference
}

```

2.5.46 805. Split Array With Same Average

You are given an integer array `nums`. You should move each element of `nums` into one of the two arrays `A` and `B` such that `A` and `B` are non-empty, and `average(A) == average(B)`. Return true if it is possible to achieve that and false otherwise. Note that for an array `arr`, `average(arr)` is the sum of all the elements of `arr` over the length of `arr`.

```

// 1) 如果一个长度为 n 的数组可以被划分为 A 和 B 两个数组，我们假设 A 的长度小于 B 并且 A 的大小是 k，那么: total_sum / n == A.sum / k == B.sum / (n - k)
// 2) 如果经过第一步的验证，发现确实有符合条件的 k，那么我们在第二步中，就试图产生 k 个子元素的所有组合，并且计算他们的和。这里的思路就有点类似于背包问题了，有了 sums[n - 1][k]，我们就检查 sums[n - 1][k] 中是否包含 (total_sum * k / n)。一旦发现符合条件的 k，就返回 true，否则就返回 false。
// 在递推公式中我们发现，sums[i][j] 仅仅和 sums[i - 1][j]，sums[i][j - 1] 有关，所以可以进一步将空间复杂度从 O(n^2*M)，降低到 O(n*M)，其中 M 是 n 中的所有元素之和
public boolean splitArraySameAverage(int[] nums) {
    int n = nums.length;
    int m = n / 2;
    int sum = Arrays.stream(nums).sum();
    boolean poss = false;
    for (int i = 1; i <= m; i++) {
        if (sum * i % n == 0) {
            poss = true;
            break;
        }
    }
    if (!poss) return false;
    List<Set<Integer>> ls = new ArrayList<>();
    for (int i = 0; i <= m; i++)
        ls.add(new HashSet<Integer>());
    ls.get(0).add(0); // 这种构建子序列和的方法，要学习一下
    for (int v : nums) { // for each element in A, we try to add it to sums[i] by joining sums[i - 1]
        for (int i = m; i >= 1; i--) {
            for (int t : ls.get(i-1)) {
                ls.get(i).add(t + v);
            }
        }
    }
    // System.out.println("ls.size(): " + ls.size());
    // for (int z = 0; z < ls.size(); ++z) {
    //     for (Integer x : ls.get(z))
    //         System.out.print(x + ", ");
    //     System.out.print("\n");
    //     System.out.print("\n ");
    // }
    for (int i = 1; i <= m; i++) {
        if (sum * i % n == 0 && ls.get(i).contains(sum * i / n))

```

```

        return true;
    }
    return false;
}
private boolean helper(int[] arr, int curSum, int cur, int start) {
    if (cur == 0) return curSum == 0;
    if (arr[start] > curSum / cur) return false;
    for (int i = start; i < arr.length - cur + 1; i++) {
        if (i > start && arr[i] == arr[i-1]) continue;
        if (helper(arr, curSum - arr[i], cur-1, i+1)) return true;
    }
    return false;
}
public boolean splitArraySameAverage(int[] nums) {
    int n = nums.length;
    int m = n / 2;
    int sum = Arrays.stream(nums).sum();
    boolean poss = false;
    for (int i = 1; i <= m; i++) {
        if (sum * i % n == 0) {
            poss = true;
            break;
        }
    }
    if (!poss) return false;
    Arrays.sort(nums);
    for (int i = 1; i <= m; i++) {
        if (sum * i % n == 0 && helper(nums, sum * i / n, i, 0)) return true;
    }
    return false;
}
bool splitArraySameAverage(vector<int>& A) { // https://www.cnblogs.com/grandyang/p/10285531.html
    int n = A.size(), m = n / 2, sum = accumulate(A.begin(), A.end(), 0);
    bool possible = false;
    for (int i = 1; i <= m && !possible; ++i) {
        if (sum * i % n == 0) possible = true;
    }
    if (!possible) return false;
    bitset<300001> bits[m + 1] = {1};
    for (int num : A) {
        for (int i = m; i >= 1; --i) {
            bits[i] |= bits[i - 1] << num;
        }
    }
    for (int i = 1; i <= m; ++i) {
        if (sum * i % n == 0 && bits[i][sum * i / n]) return true;
    }
    return false;
}
}

```

2.5.47 801. Minimum Swaps To Make Sequences Increasing - Hard

You are given two integer arrays of the same length `nums1` and `nums2`. In one operation, you are allowed to swap `nums1[i]` with `nums2[i]`.

For example, if `nums1 = [1,2,3,8]`, and `nums2 = [5,6,7,4]`, you can swap the element at $i = 3$ to obtain `nums1 = [1,2,3,4]` and `nums2 = [5,6,7,8]`. Return the minimum number of needed operations to make `nums1` and `nums2` strictly increasing. The test cases are generated so that the given input always makes it possible.

An array `arr` is strictly increasing if and only if $arr^1 < arr^2 < arr^5 < \dots < arr[arr.length - 1]$.

```

// 设 dp[0][i] 表示不交换 A[i] 和 B[i] 在下标 i 的交换次数
// 设 dp[1][i] 表示交换 A[i] 和 B[i] 在下标 i 的交换次数
// 可以看到交换与否只取决于前一个状态，可以将空间复杂度压缩到 O(1)
// 时间复杂度为 O(n)，空间复杂度为 O(1)
public int minSwap(int[] a, int[] b) {
    int n = a.length;
    int [][] dp = new int [2][n];
    for (int [] row : dp)
        Arrays.fill(row, Integer.MAX_VALUE);
    dp[0][0] = 0;
    dp[1][0] = 1;
    for (int i = 1; i < n; i++) {
        if (a[i] > a[i-1] && b[i] > b[i-1]) {
            dp[0][i] = Math.min(dp[0][i], dp[0][i-1]); // 不需要交换不用增加交换次数
            dp[1][i] = Math.min(dp[1][i], dp[1][i-1] + 1); // 如果要交换前一个也必须交换才能满足递增的条件
        }
        if (a[i] > b[i-1] && b[i] > a[i-1]) {
            dp[0][i] = Math.min(dp[0][i], dp[1][i-1]); // 表示 i - 1 位置发生交换
            dp[1][i] = Math.min(dp[1][i], dp[0][i-1] + 1); // 表示在 i - 1 不换的基础上，i 发生了交换
        }
    }
}

```

```

    return Math.min(dp[0][n-1], dp[1][n-1]);
}

```

2.5.48 837. New 21 Game - Medium

Alice plays the following game, loosely based on the card game "21".

Alice starts with 0 points and draws numbers while she has less than k points. During each draw, she gains an integer number of points randomly from the range [1, maxPts], where maxPts is an integer. Each draw is independent and the outcomes have equal probabilities.

Alice stops drawing numbers when she gets k or more points.

Return the probability that Alice has n or fewer points.

Answers within 10-5 of the actual answer are considered accepted.

```

// When the draws sum up to K, it stops, calculate the possibility K<=sum<=N.
// Think about one step earlier, sum = K-1, game is not ended and draw largest card W.
// K-1+W is the maximum sum could get when game is ended. If it is <= N, then for sure the possibility when games end ans sum <= N is 1.
// Because the maximum is still <= 1.
// Otherwise calculate the possibility sum between K and N.
// Let dp[i] denotes the possibility of that when game ends sum up to i.
// i is a number could be got equally from i - m and draws value m card.
// Then dp[i] should be sum of dp[i-W] + dp[i-W+1] + ... + dp[i-1], devided by W.
// We only need to care about previous W value sum, accumulate winSum, reduce the possibility out of range.
// Time Complexity: O(N).
// Space: O(N).
public double new21Game(int n, int k, int w) { // k : threshold
    if (k == 0 || n >= (k + w)) return 1.0;
    if (k > n) return 0;
    double[] dp = new double[n+1];
    dp[0] = 1.0;
    double winSum = 1;
    double res = 0;
    for (int i = 1; i <= n; i++) {
        dp[i] = winSum / w;
        if (i < k) winSum += dp[i];
        else res += dp[i];
        if (i >= w) winSum -= dp[i-w];
    }
    return res;
}

```

2.5.49 1105. Filling Bookcase Shelves - Medium

You are given an array books where books[i] = [thickness_i, height_i] indicates the thickness and height of the *i*th book. You are also given an integer shelfWidth.

We want to place these books in order onto bookcase shelves that have a total width shelfWidth.

We choose some of the books to place on this shelf such that the sum of their thickness is less than or equal to shelfWidth, then build another level of the shelf of the bookcase so that the total height of the bookcase has increased by the maximum height of the books we just put down. We repeat this process until there are no more books to place.

Note that at each step of the above process, the order of the books we place is the same order as the given sequence of books.

For example, if we have an ordered list of 5 books, we might place the first and second book onto the first shelf, the third book on the second shelf, and the fourth and fifth book on the last shelf. Return the minimum possible height that the total bookshelf can be after placing shelves in this manner.

```

// 求摆放前 i 本书需要的最小高度，首先需要求摆放前 i-1 本书需要的最小高度，以此类推，最初需要计算的是摆放第 0 本书需要的最小高度，也就是 0。
// 根据前 i-1 本书求前 i 本书需要的最小高度的思路是：
// 尝试将第 i 本书放在前 i-1 本书的下面
// 以及将前 i-1 本书的最后几本和第 i 本书放在同一层两种方案，看哪种方案高度更小就用哪种方案，依次求出摆放前 1, ..., n 本书需要的最小高度。
public int minHeightShelves(int[][] books, int shelfWidth) {
    int[] dp = new int[books.length + 1];
    for (int i = 1; i < dp.length; i++) { // 依次求摆放前 i 本书的最小高度
        int width = books[i-1][0];
        int height = books[i-1][1];
        dp[i] = dp[i-1] + height;
        // 将前 i - 1 本书从第 i - 1 本开始放在与 i 同一层，直到这一层摆满或者所有的书都摆好
        for (int j = i-1; j > 0 && width + books[j-1][0] <= shelfWidth; j--) {
            height = Math.max(height, books[j-1][1]); // 每层的高度由最高的那本书决定
            width += books[j-1][0];
            dp[i] = Math.min(dp[i], dp[j-1] + height); // 选择高度最小的方法
        }
    }
}

```

```

    }
    return dp[books.length];
}

```

2.5.50 1997. First Day Where You Have Been in All the Rooms - Medium

There are n rooms you need to visit, labeled from 0 to $n - 1$. Each day is labeled, starting from 0. You will go in and visit one room a day.

Initially on day 0, you visit room 0. The order you visit the rooms for the coming days is determined by the following rules and a given 0-indexed array nextVisit of length n :

Assuming that on a day, you visit room i , if you have been in room i an odd number of times (including the current visit), on the next day you will visit a room with a lower or equal room number specified by $\text{nextVisit}[i]$ where $0 \leq \text{nextVisit}[i] \leq i$; if you have been in room i an even number of times (including the current visit), on the next day you will visit room $(i + 1) \bmod n$. Return the label of the first day where you have been in all the rooms. It can be shown that such a day exists. Since the answer may be very large, return it modulo $10^9 + 7$.

```

public int firstDayBeenInAllRooms(int[] nextVisit) {
    int n = nextVisit.length, mod = (int)1e9 + 7;
    long[] dp = new long[n];
    dp[0] = 0;
    for (int i = 1; i < n; i++) {
        dp[i] = (2 * dp[i-1] % mod + mod - dp[nextVisit[i-1]] + 2) % mod;
    }
    return (int)dp[n-1];
}

```

2.5.51 943. Find the Shortest Superstring - Hard

Given an array of strings words , return the smallest string that contains each string in words as a substring. If there are multiple valid strings of the smallest length, return any of them.

You may assume that no string in words is a substring of another string in words .

- 深搜 + 记忆数组 + 裁枝

```

private void dfs(int[] tmp, int idx, int curCost, boolean[] vis, int path) {
    if (idx == n) {
        if (curCost > maxCost) {
            maxCost = curCost;
            best = Arrays.copyOf(tmp, n); // best = tmp.clone();
        }
        return;
    }
    for (int i = 0; i < n; i++) {
        if (!vis[i]) {
            int tmpCost = idx == 0 ? 0 : curCost + cost[tmp[idx-1]][i];
            int tmpPath = (path | (1 << i));
            int maxCost = dp[tmpPath][i];
            if (maxCost > 0 && tmpCost <= maxCost) continue;
            tmp[idx] = i;
            dp[tmpPath][i] = tmpCost; // need to remember the res
            vis[i] = true;
            dfs(tmp, idx+1, tmpCost, vis, tmpPath);
            vis[i] = false;
        }
    }
}
int n, maxCost = Integer.MIN_VALUE; // 最长公共子串的长度和，越大越好
int[][] dp;
int[][] cost;
int[] best;
public String shortestSuperstring(String[] words) {
    n = words.length;
    cost = new int[n][n];
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (i == j) continue;
            for (int k = 1; k <= words[i].length() && k <= words[j].length(); k++) {
                if (words[i].substring(words[i].length()-k).equals(words[j].substring(0, k)))
                    cost[i][j] = k;
            }
        }
    }
    dp = new int[1 << n][n];
    best = new int[n];
    dfs(new int[n], 0, 0, new boolean[n], 0);
    String res = words[best[0]];
}

```

```

    for (int i = 1; i < n; i++) {
        int costVal = cost[best[i-1]][best[i]];
        res += words[best[i]].substring(costVal);
    }
    return res;
}

```

- 动态规划

```

public String shortestSuperstring(String[] words) {
    int n = words.length;
    String [][] dp = new String [1 << n][n];
    int [][] overlap = new int [n][n];
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (i == j) continue;
            for (int k = Math.min(words[i].length(), words[j].length()); k > 0; k--) {
                if (words[i].substring(words[i].length()-k).equals(words[j].substring(0, k))) {
                    overlap[i][j] = k;
                    break;
                }
            }
        }
        for (int i = 0; i < n; i++)
            dp[1 << i][i] = words[i];
    }
    for (int mask = 1; mask < (1 << n); mask++) {
        for (int j = 0; j < n; j++) {
            if ((mask & (1 << j)) == 0) continue;
            for (int i = 0; i < n; i++) {
                if (i == j || (mask & (1 << i)) == 0) continue;
                String tmp = dp[mask ^ (1 << j)][i] + words[j].substring(overlap[i][j]);
                if (dp[mask][j] == null || tmp.length() < dp[mask][j].length() // str.isEmpty()
                    dp[mask][j] = tmp;
            }
        }
    }
    int last = (1 << n) - 1;
    String res = dp[last][0];
    for (int i = 1; i < n; i++)
        if (dp[last][i].length() < res.length())
            res = dp[last][i];
    return res;
}

```

2.5.52 964. Least Operators to Express Number - Hard

Given a single positive integer x , we will write an expression of the form $x \text{ (op1)} x \text{ (op2)} x \text{ (op3)} x \dots$ where each operator op1, op2, etc. is either addition, subtraction, multiplication, or division (+, -, *, or /). For example, with $x = 3$, we might write $3 * 3 / 3 + 3 - 3$ which is a value of 3.

When writing such an expression, we adhere to the following conventions:

The division operator (/) returns rational numbers. There are no parentheses placed anywhere. We use the usual order of operations: multiplication and division happen before addition and subtraction. It is not allowed to use the unary negation operator (-). For example, " $x - x$ " is a valid expression as it only uses subtraction, but " $-x + x$ " is not because it uses negation. We would like to write an expression with the least number of operators such that the expression equals the given target. Return the least number of operators used.

博主看了一会儿，发现没思路就直接放弃了，直奔论坛上找解法。这里直接参考 dongguafu 大神的解法吧，首先处理 edge cases，当 x 等于 target 的话，不用加任何运算符，返回 0 即可。若 x 大于 target，比如 $x=5$, $target=3$ ，我们其实可以迅速的求出运算符的个数，因为 5 比 3 大，要凑 3 就只能先变成 1，这里就有两种变法，一种是全部都变成 1，然后来凑 3，即 $5/5 + 5/5 + 5/5$ ，这时的运算符个数是 $target * 2 - 1$ ，因为加号的个数总是比除号少一个。另一种凑法就是 $5 - 5/5 - 5/5$ ，这时候的运算符个数是 $(x - target) * 2$ ，此时的加号和除号的个数相同，均为 x 和 target 的差值。

接下来就要处理 x 小于 target 的情况了，此时由于不知道 x 到底比 target 小多少，若差距太大的话，肯定不能用加号，所以应该先用乘号来让 x 变大，直到刚好大于等于 target 停止，并每次增加次数 cnt。若此时 sum 正好等于 target，太幸运了，直接返回 cnt。但通常情况下 sum 会大于 target，此时 $sum - target$ 的差值就需要另行计算了。这里差值跟 target 的大小关系又要分为两种情况来讨论，当 $sum - target < target$ 时，比如 $x=5$, $sum=25$, $target=15$ ，则 $sum - target = 10$ ，就是说现在已经乘到了 25，但需要再减去 10，这个差值 10 可以再次调用原函数来计算，此时新的 target 代入 10 即可，记得返回值要加上 cnt。当然之后还是要再计算一下另一种凑的方法，由于 sum 超过了 target，所以回退一个 x ，变成 sum / x ，此时小于 target，那么它们的差值 $target - (sum / x)$ 就可以通过再次调用函数来计算，注意这里加上 cnt 之后还要减去 1，因为回退了一个 x ，少了一个乘号。最终二者的较小值即为所求，记得要加上个 1，以为多加了个运算符，参见代码如下：

```

public int leastOpsExpressTarget(int x, int target) {
    if (x == target) return 0;

```

```
if (x > target) return Math.min(target*2-1, (x-target)*2);
int cnt = 0;
long sum = x;
while (sum < target) {
    sum *= x;
    ++cnt;
}
if (sum == target) return cnt;
int min = Integer.MAX_VALUE, max = Integer.MIN_VALUE;
// int tmp = sum - target; // -
if (sum - target < target)
    min = leastOpsExpressTarget(x, (int)(sum - target)) + cnt;
max = leastOpsExpressTarget(x, (int)(target - (sum / x))) + cnt - 1;
return Math.min(min, max) + 1; // -
}
```

- 和 race car 那道题类似。注意到，符号的添加就是对数字进行 $-x^i$ 的操作，最后要减到 0， $k = \log_2(t)$ ，有两种方式，可以先到 t 前面的数字， 2^k ，或者 t 后面的数字 $2^{(k+1)}$ 。

注意， $2^k - k$

看了花花酱的题解，感觉更像是 bfs。cost 小的点先扩展。这个再看一下

```

int leastOpsExpressTarget(int x, int target) {
    priority_queue<pair<int, int>, vector<pair<int, int>>, greater<pair<int, int>>> que;
    unordered_set<int> s;
    que.emplace(0, target);
    while(!que.empty()) {
        int cost = que.top().first;
        int t = que.top().second;
        que.pop();
        if (t == 0) return cost-1;
        if (s.count(t)) continue;
        s.insert(t);
        int k = log(t) / log(x);
        int l = t - pow(x, k);
        que.emplace(cost+(k == 0 ? 0 : k), l);
        int r = pow(x, k+1) - t;
        que.emplace(cost+k+l, r);
    }
    return -1;
}

```

2.5.53 1955. Count Number of Special Subsequences - Hard

A sequence is special if it consists of a positive number of 0s, followed by a positive number of 1s, then a positive number of 2s.

For example, [0,1,2] and [0,0,1,1,1,2] are special. In contrast, [2,1,0], 2 , and [0,1,2,0] are not special. Given an array `nums` (consisting of only integers 0, 1, and 2), return the number of different subsequences that are special. Since the answer may be very large, return it modulo $10^9 + 7$.

A subsequence of an array is a sequence that can be derived from the array by deleting some or no elements without changing the order of the remaining elements. Two subsequences are different if the set of indices chosen are different.

定义：

- $f[i][0]$ 表示前 i 项得到的全 0 子序列个数
- $f[i][1]$ 表示前 i 项得到的先 0 后 1 的子序列个数
- $f[i][2]$ 表示前 i 项得到的特殊子序列个数

遍历数组 $nums$, 对于 $f[i][j]$, 若 $j \neq nums[i]$, 则直接从前一项转移过来, 即 $f[i][j] = f[i - 1][j]$ 。

若 $j = nums[i]$ 则需要分类计算：

对于 $f[i][0]$, 当遇到 0 时, 有选或不选两种方案, 不选 0 时有 $f[i][0] = f[i - 1][0]$, 选 0 时, 可以单独组成一个子序列, 也可以与前面的 0 组合, 因此有 $f[i][0] = f[i - 1][0] + 1$, 两者相加得 $f[i][0] = 2 \cdot f[i - 1][0] + 1$ 。

对于 $f[i][1]$, 当遇到 1 时, 有选或不选两种方案, 不选 1 时有 $f[i][1] = f[i - 1][1]$, 选 1 时, 可以单独与前面的 0 组成一个子序列, 也可以与前面的 1 组合, 因此有 $f[i][1] = f[i - 1][1] + f[i - 1][0]$, 两者相加得 $f[i][1] = 2 \cdot f[i - 1][1] + f[i - 1][0]$ 。

$f[i][2]$ 和 $f[i][1]$ 类似, 有 $f[i][2] = 2 \cdot f[i - 1][2] + f[i - 1][1]$ 。

最后答案为 $f[n - 1][2]$ 。

代码实现时, 可以把第一维压缩掉。

```
public int countSpecialSubsequences(int[] arr) { // 去找个降维的参考一下
    int mod = (int)1e9 + 7;
    int n = arr.length;
    long [][] dp = new long [n][3];
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < 3; j++) {
            if (arr[i] != j) dp[i][j] = (i == 0 ? 0 : dp[i-1][j]);
            else {
                if (j == 0)
                    dp[i][j] = (i == 0 ? 0 : dp[i-1][j]) * 2 % mod + 1;
                else
                    dp[i][j] = ((i == 0 ? 0 : dp[i-1][j]) * 2 % mod + (i == 0 ? 0 : dp[i-1][j-1])) % mod;
            }
        }
    }
    return (int)dp[n-1][2];
}
```

2.5.54 446. Arithmetic Slices II - Subsequence - Hard

Given an integer array $nums$, return the number of all the arithmetic subsequences of $nums$.

A sequence of numbers is called arithmetic if it consists of at least three elements and if the difference between any two consecutive elements is the same.

For example, $[1, 3, 5, 7, 9]$, $[7, 7, 7, 7]$, and $[3, -1, -5, -9]$ are arithmetic sequences. For example, $[1, 1, 2, 5, 7]$ is not an arithmetic sequence. A subsequence of an array is a sequence that can be formed by removing some elements (possibly none) of the array.

For example, $[2,5,10]$ is a subsequence of $[1,2,1,2,4,1,5,10]$. The test cases are generated so that the answer fits in 32-bit integer.

1. 解题思路与分析 这道题是之前那道 Arithmetic Slices 的延伸, 那道题比较简单是因为要求等差数列是连续的, 而这道题让我们求的是等差数列的子序列, 可以跳过某些数字, 不一定非得连续, 那么难度就加大了, 但还是需要用动态规划 Dynamic Programming 来做。

好, 既然决定要用 DP 了, 那么首先就要确定 dp 数组的定义了, 刚开始我们可能会考虑使用一个一维的 dp 数组, 然后 $dp[i]$ 定义为范围为 $[0, i]$ 的子数组中等差数列的个数。定义的很简单, OK, 但是基于这种定义的状态转移方程却十分的难想。我们想对于 $(0, i)$ 之间的任意位置 j , 如何让 $dp[i]$ 和 $dp[j]$ 产生关联呢? 是不是只有 $A[i]$ 和 $A[j]$ 的差值 $diff$, 跟 $A[j]$ 之前等差数列的差值相同, 才会有关系, 所以差值 $diff$ 是一个很重要的隐藏信息 Hidden Information, 我们必须要在 dp 的定义中考虑进去。所以一维 dp 数组是罩不住的, 必须升维, 但是用二维 dp 数组的话, 差值 $diff$ 那一维的范围又是个问题, 数字的范围是整型数, 所以差值的范围也很大, 为了节省空间, 我们建立一个一维数组 dp , 数组里的元素

不是数字，而是放一个 `HashMap`，建立等差数列的差值和当前位置之前差值相同的数字个数之间的映射。我们遍历数组中的所有数字，对于当前遍历到的数字，又从开头遍历到当前数字，计算两个数字之差 `diff`，如果越界了不做任何处理，如果没越界，我们让 `dp[i]` 中 `diff` 的差值映射自增 1，因为此时 $A[i]$ 前面有相差为 `diff` 的 $A[j]$ ，所以映射值要加 1。然后我们看 `dp[j]` 中是否有 `diff` 的映射，如果有的话，说明此时相差为 `diff` 的数字至少有三个了，已经能构成题目要求的等差数列了，将 `dp[j][diff]` 加入结果 `res` 中，然后再更新 `dp[i][diff]`，这样等遍历完数组，`res` 即为所求。

我们用题目中给的例子数组 `[2, 4, 6, 8, 10]` 来看，因为 2 之前没有数字了，所以我们从 4 开始，遍历前面的数字，是 2，二者差值为 2，那么在 dp^2 的 `HashMap` 就可以建立 $2 \rightarrow 1$ 的映射，表示 4 之前有 1 个差值为 2 的数字，即数字 2。那么现在 $i=2$ 指向 6 了，遍历前面的数字，第一个数是 2，二者相差 4，那么在 dp^5 的 `HashMap` 就可以建立 $4 \rightarrow 1$ 的映射，第二个数是 4，二者相差 2，那么先在 dp^5 的 `HashMap` 建立 $2 \rightarrow 1$ 的映射，由于 dp^2 的 `HashMap` 中也有差值为 2 的映射， $2 \rightarrow 1$ ，那么说明此时至少有三个数字差值相同，即这里的 `[2, 4, 6]`，我们将 dp^2 中的映射值加入结果 `res` 中，然后当前 dp^5 中的映射值加上 dp^2 中的映射值。这应该不难理解，比如当 $i=3$ 指向数字 8 时， $j=2$ 指向数字 6，那么二者差值为 2，此时先在 dp^3 建立 $2 \rightarrow 1$ 的映射，由于 dp^5 中有 $2 \rightarrow 2$ 的映射，那么加上数字 8 其实新增了两个等差数列 `[2, 4, 6, 8]` 和 `[4, 6, 8]`，所以结果 `res` 加上的值就是 `dp[j][diff]`，即 2，并且 `dp[i][diff]` 也需要加上这个值，才能使得 dp^3 中的映射变为 $2 \rightarrow 3$ ，后面数字 10 的处理情况也相同，这里就不多赘述了，最终的各个位置的映射关系如下所示：

2	4	6	8	10
2->1	4->1	6->1	8->1	
2->2	4->1	6->1		
2->3	4->2			
2->4				

最终累计出来的结果是跟上面红色的数字相关，分别对应着如下的等差数列：

2->2:	[2, 4, 6]		
2->3:	[2, 4, 6, 8]	[4, 6, 8]	
4->2:	[2, 6, 10]		
2->4:	[2, 4, 6, 8, 10]	[4, 6, 8, 10]	[6, 8, 10]

- Both time and space complexities are $O(n^2)$

方法一：动态规划 + 哈希表

我们首先考虑至少有两个元素的等差子序列，下文将其称作弱等差子序列。

由于尾项和公差可以确定一个等差数列，因此我们定义状态 $f[i][d]$ 表示尾项为 $nums[i]$ ，公差为 d 的弱等差子序列的个数。

我们用一个二重循环去遍历 $nums$ 中的所有元素对 $(nums[i], nums[j])$ ，其中 $j < i$ 。将 $nums[i]$ 和 $nums[j]$ 分别当作等差数列的尾项和倒数第二项，则该等差数列的公差 $d = nums[i] - nums[j]$ 。由于公差相同，我们可以将 $nums[i]$ 加到以 $nums[j]$ 为尾项，公差为 d 的弱等差子序列的末尾，这对应着状态转移 $f[i][d] += f[j][d]$ 。同时， $(nums[i], nums[j])$ 这一对元素也可以当作一个弱等差子序列，故有状态转移

$$f[i][d] += f[j][d] + 1$$

由于题目要统计的等差子序列至少有三个元素，我们回顾上述二重循环，其中「将 $nums[i]$ 加到以 $nums[j]$ 为尾项，公差为 d 的弱等差子序列的末尾」这一操作，实际上就构成了一个至少有三个元素的等差子序列，因此我们将循环中的 $f[j][d]$ 累加，即为答案。

代码实现时，由于 $nums[i]$ 的范围很大，所以计算出的公差的范围也很大，我们可以将状态转移数组 f 的第二维用哈希表代替。

Define the type of the difference as Integer type instead of Long. This is because there is no valid arithmetic subsequence slice that can have difference out of the Integer value range. But we do need a long integer to filter out those invalid cases.

Preallocate the `HashMap` to avoid reallocation to deal with extreme cases.

Refrain from using lambda expressions inside loops.

```
public int numberOfArithmeticSlices(int[] arr) {
    int n = arr.length, ans = 0;
    Map<Integer, Integer>[] dp = new HashMap[n];
    dp[0] = new HashMap<>();
}
```

```

for (int i = 1; i < n; i++) {
    dp[i] = new HashMap<>();
    for (int j = 0; j < i; j++) {
        long diff = (long)arr[i] - arr[j];
        if (diff > Integer.MAX_VALUE || diff < Integer.MIN_VALUE) continue;
        int dif = (int)diff;
        dp[i].put(dif, dp[i].getOrDefault(dif, 0) + 1); // 这里先更新上
        if (dp[j].containsKey(dif)) {
            ans += dp[j].get(dif); // 更新结果
            dp[i].put(dif, dp[i].get(dif) + dp[j].get(dif)); // 再加上之前累积的
        }
    }
}
return ans;
}

```

2.5.55 639. Decode Ways II - Hard

A message containing letters from A-Z can be encoded into numbers using the following mapping:

'A' -> "1" 'B' -> "2" ... 'Z' -> "26" To decode an encoded message, all the digits must be grouped then mapped back into letters using the reverse of the mapping above (there may be multiple ways). For example, "11106" can be mapped into:

"AAJF" with the grouping (1 1 10 6) "KJF" with the grouping (11 10 6) Note that the grouping (1 11 06) is invalid because "06" cannot be mapped into 'F' since "6" is different from "06".

In addition to the mapping above, an encoded message may contain the '*' character, which can represent any digit from '1' to '9' ('0' is excluded). For example, the encoded message "1*" may represent any of the encoded messages "11", "12", "13", "14", "15", "16", "17", "18", or "19". Decoding "1*" is equivalent to decoding any of the encoded messages it can represent.

Given a string s consisting of digits and '*' characters, return the number of ways to decode it.

Since the answer may be very large, return it modulo $10^9 + 7$.

- 解题思路与分析 给定一个只含数字的长 n 的字符串 s，再给定一个对应规则，每个大写字母 ch 可以对应一个数字 ch - 'A' + 1。问该 s 有多少种不同的解码方式。s 中可能含有'*'，这个符号可以对应除了 0 以外的任意一位数。答案模 $10^9 + 7$ 后返回。

思路是动态规划。设 f[i] 是 s 的前缀的解码方式数，那么可以按照最后一位（或者两位）是解码成什么字母来分类进行累加。

```

public int numDecodings(String t) {
    int mod = (int)1e9 + 7;
    int n = t.length();
    char[] s = t.toCharArray();
    System.out.println(Arrays.toString(s));
    int[] dp = new int[Math.max(2, n+1)];
    dp[0] = 1;
    dp[1] = s[0] == '*' ? 9 : s[0] == '0' ? 0 : 1;
    for (int i = 2; i <= n; i++) {
        System.out.println("i: " + i);
        for (int j = 1; j <= 26; j++) { // 枚举 s 的长 i 前缀的末尾可以解码为哪个大写字母
            char c = s[i-1];
            if (j <= 9) { // 如果是要解码为 A 到 I，那么最后一个数字得单独解码
                if (c == '*' || c == '0') dp[i] += dp[i-1];
            } else { // 否则最后两个数字得一起解码
                char p = s[i-2];
                int x = j % 10, y = j / 10;
                if ((p == '*' || p == y + '0') && ((c == '*' && x != 0) || c == x + '0')) dp[i] += dp[i-2];
            }
            dp[i] %= mod;
        }
    }
    return dp[n];
}

```

- 解题思路与分析

定义 dp[i] 是 nums 前 i 个字符可以得到的解码种数，假设之前的字符串是 abcx，现在新加入了 y，则有以下 5 种情况：

如果 $x = '0'$, $y = '0'$, 无法解码, 返回 0;

如果只有 $x = '0'$, 则 y 只能单独放在最后, 不能与 x 合并(不能以 0 开头), 此时有: $dp[i] = dp[i-1]$

如果只有 $y = '0'$, 则 y 不能单独放置, 必须与 x 合并, 并且如果合并结果大于 26, 返回 0, 否则有: $dp[i] = dp[i-2]$

如果 $xy \leq 26$: 则 y 可以“单独”放在 $abcx$ 的每个解码结果之后后，并且如果 $abcx$ 以 x 单独结尾，此时可以合并 xy 作为结尾。而这种解码种数就是 abc 的解码结果。
 如果 $xy > 26$: 此时 x 又不能与 y 合并， y 只能单独放在 $dp[i]$ 的每一种情况的最后，此时有: $dp[i+1] = dp[i]$

```
public int numDecodings(String s) {
    char[] arr = s.toCharArray();
    int[] dp = new int[s.length() + 1];
    dp[0] = 1;
    dp[1] = arr[0] == '0' ? 0 : 1;
    if(s.length() <= 1) return dp[1];
    for(int i=2;i<=s.length();i++){
        int n = (arr[i-2]-'0')*10+(arr[i-1]-'0');
        if(arr[i-1]=='0' && arr[i-2]=='0'){
            return 0;
        }else if(arr[i-2]=='0'){
            dp[i] = dp[i-1];
        }else if(arr[i-1]=='0'){
            if(n>26) return 0;
            dp[i] = dp[i-2];
        }else if(n>26){
            dp[i] = dp[i-1];
        }else{
            dp[i] = dp[i-1]+dp[i-2];
        }
    }
    return dp[dp.length-1];
}
```

2.5.56 629. K Inverse Pairs Array - Hard

For an integer array $nums$, an inverse pair is a pair of integers $[i, j]$ where $0 \leq i < j < nums.length$ and $nums[i] > nums[j]$.

Given two integers n and k , return the number of different arrays consist of numbers from 1 to n such that there are exactly k inverse pairs. Since the answer can be huge, return it modulo $10^9 + 7$.

1. 解题思路与分析 比较容易辨别出来是一道 DP 的题目，但是确实算是比较难的了，下面是参考网上的代码之后我的理解。定义 $dp[n][k]$ 表示从 1 到 n 构成的数中含有 k 个逆序对的个数，则我们可以推导出 $dp[n][k]$ 和 $dp[n-1][i]$ 之间的递推关系：

如果我们把 n 放在最后一位，则所有的 k 个逆序对均来自于前 $n-1$ 个数所构成的逆序对，和 n 无关；

如果我们把 n 放在倒数第二位，则有 1 个逆序对和 n 有关，有 $k-1$ 个逆序对来自前 $n-1$ 个数所构成的逆序对；

.....

如果我们把 n 放在第一位，则有 $n-1$ 个逆序对和 n 有关， $k-(n-1)$ 个逆序对来自前 $n-1$ 个数所构成的逆序对。

所以: $dp[n][k] = dp[n-1][k] + dp[n-1][k-1] + dp[n-1][k-2] + \dots + dp[n-1][k+n-1] + dp[n-1][k-n+1]$ // A
 $dp[n][k+1] = dp[n-1][k+1] + dp[n-1][k] + dp[n-1][k-1] + dp[n-1][k-2] + \dots + dp[n-1][k+n+1]$ // B
 $dp[n][k+1] - dp[n][k] = dp[n-1][k+1] - dp[n-1][k-n+1]$; // B-A
 $dp[n][k+1] = dp[n][k] + dp[n-1][k+1] - dp[n-1][k-n+1]$; // 移项
// 将 k+1 替换成 k, 可以得到:
 $dp[n][k] = dp[n][k-1] + dp[n-1][k] - dp[n-1][k-n]$

把上面两个式子相减可以推导出: $dp[n][k+1] = dp[n][k] + dp[n-1][k+1] - dp[n-1][k+n-1]$ 。这样就可以写出代码了。

当然由于 $dp[n][k]$ 只和 $dp[n][x]$, $dp[n-1][x]$ 有关，所以该代码还可以进一步将空间复杂度从 $O(nk)$ 降低到 $O(k)$ 。时间复杂度是 $O(nk)$ 。

```
public int kInversePairs(int n, int k) {
    int mod = (int)1e9 + 7;
    long [][] dp = new long [n+1][k+1];
    dp[0][0] = 1;
    for (int i = 1; i <= n; i++) {
        dp[i][0] = 1;
        for (int j = 1; j <= k; j++) {
            dp[i][j] = dp[i][j-1] + dp[i-1][j];
            if (j >= i)
                dp[i][j] -= dp[i-1][j-i];
            dp[i][j] = (dp[i][j] + mod) % mod;
        }
    }
}
```

```

        }
    }
    return (int)dp[n][k];
}

```

2. 解题思路与分析

给定两个整数 n 和 k , $n \geq 1$ 并且 $k \geq 0$, 问恰好含 k 个逆序对的 n 全排列有多少个。

思路是动态规划。设 $f[n][k]$ 是恰有 k 个逆序对的全排列个数, 那么可以按照 n 被排在了哪儿来分类, 设全排列出的数组是 A , 如果 $A[n - 1] = n$, 那么 n 贡献的逆序对个数是 0, 所以此时个数是 $f[n - 1][k]$; 如果 $A[n - 2] = n$, 那么 n 贡献的逆序对个数是 1, 所以此时个数是 $f[n - 1][k - 1]$, 以此类推。所以有:

$$f[n][k] = \sum_{s=0}^{\min\{n-1,k\}} f[n-1][k-s]$$

初始条件 $f[0][0] = 1$, $f[0][s > 0] = 0$ 。而如果直接循环枚举 s 的话, 很多区间和都是重复计算的。考虑:

$$\begin{aligned} f[n][k+1] &= \sum_{s=0}^{\min\{n-1,k+1\}} f[n-1][k+1-s] \\ &= \sum_{s=-1}^{\min\{n-2,k\}} f[n-1][k-s] \\ &= f[n-1][k+1] + \sum_{s=0}^{\min\{n-2,k\}} f[n-1][k-s] \end{aligned}$$

也就是计算的时候, 我们维护 $f[n - 1]$ 这一行的长 $n - 1$ 的滑动窗口的和即可。代码如下:

```

public int kInversePairs(int n, int k) {
    int mod = (int)1e9 + 7;
    int [][] dp = new int [n+1][k+1];
    dp[0][0] = 1;
    for (int i = 1; i <= n; i++) {
        long sum = 0;
        for (int j = 0; j <= k; j++) {
            sum += dp[i-1][j];
            if (j >= i)
                sum -= dp[i-1][j-i];
            dp[i][j] = (int)(sum % mod);
        }
    }
    return (int)dp[n][k];
}

```

2.5.57 1787. Make the XOR of All Segments Equal to Zero - Hard

You are given an array nums and an integer k . The XOR of a segment $[\text{left}, \text{right}]$ where $\text{left} \leq \text{right}$ is the XOR of all the elements with indices between left and right , inclusive: $\text{nums}[\text{left}] \text{ XOR } \text{nums}[\text{left}+1] \text{ XOR } \dots \text{ XOR } \text{nums}[\text{right}]$.

Return the minimum number of elements to change in the array such that the XOR of all segments of size k is equal to zero.

1. 解题思路与分析

解题思路：

容易发现，最后得到的数组一定满足：

- $a_1 = a_{k+1} = a_{2k+1} = \dots$
- $a_2 = a_{k+2} = a_{2k+2} = \dots$
- \dots

因此可以考虑将同一组中的数放在一起进行考虑。

首先，我们对每一组分别进行计数。

接下来，我们用动态规划求解。 $dp[val]$ 表示处理到当前组时，异或值为 val 的最小修改次数。显然初值（考虑第一组之前）为：

- $dp[0] = 0$
- $dp[val] = \infty (val \neq 0)$

在转移时，我们有两种选择：

1. 将当前组的数全都改为同一个值，此时不管我们之前如何选择，都可以利用这一个值的选择得到任何一个异或值，那么我们自然应该选择之前的代价中最小的那个，此时的代价为 $size[i] + \min dp[val]$ ，新状态的异或值可以为 $[0, 2^D)$ 中的任意一个值。这一转移需要进行 K 次。
 2. 使用当前组中的某一个值 val' 。此时我们需要枚举之前的异或值 val （或枚举达到的目标值，二者是等价的），此时的代价为 $size[i] - cnt[i][val'] + dp[val]$ ，新状态的异或值为 $val \oplus val'$ 。这一转移最多需要进行 N 次。
 - 时间复杂度 $\mathcal{O}(2^D \cdot (K + N))$ 。其中 $D = 10$ 。
 - 空间复杂度 $\mathcal{O}(N + 2^D)$ 。
- 根据题目特点，最后所有长度为 k 的区间异或结果等于零，可推出得到的数组满足：

```
a1 = ak+1 = a2k+1 = ...
a2 = ak+2 = a2k+2 = ...
```

因此可将数组中的元素按上述规律，每间隔 k 个的数字为一组进行分组。

在此基础上，设计一个动态规划数组 $dp[j]$ ，表示到当前第 i 组为止，所有元素异或到对应数字 j 时的更改次数。则对第 i 组 $dp[j]$ 的状态转移方程可能为：

j 可由某一数值和当前组中的某个数 num 异或得到， $newDp[j] = dp[j \& num] + size[i]$ - 组中 num 的数量

j 可通过和任意数字异或得到， $newDp[j] = \text{前一 } dp \text{ 中最小的改变次数} + size[i]$

完成 k 个组的动态规划后， dp^1 就是所求的解。

```
public int minChanges(int[] arr, int k) {
    int n = arr.length;
    List<Map<Integer, Integer>> group = new ArrayList<>(); // 存储 k 个组、各组中各个数字数量
    int[] size = new int[k]; // 各组大小，它们会有可能不同吗？
    for (int i = 0; i < k; i++) {
        Map<Integer, Integer> map = new HashMap<>();
        for (int j = i; j < n; j += k) {
            map.put(arr[j], map.getOrDefault(arr[j], 0) + 1);
            size[i]++;
        }
        group.add(map);
    }
    int range = 1 << 10; // 题中 nums[i] < 2^10，为的是遍历所有可能更改值，以取最小
    int[] dp = new int[range]; // 当前组异或到对应数字时的更改次数
    Arrays.fill(dp, Integer.MAX_VALUE);
    dp[0] = 0;
    for (int i = 0; i < k; i++) {
        int minValue = Arrays.stream(dp).min().getAsInt();
        int[] newDp = new int[range]; // 遍历每一个子组，暴搜、取最小
        Arrays.fill(newDp, minValue + size[i]); // 变为当前组中不存在数字的改变次数：之前的最小改变次数 + 当前组元素个数
        for (int j = 0; j < range; j++) {
            if (map.getOrDefault(j, 0) > 0) {
                newDp[j] = minValue;
            } else {
                for (int num : map.keySet()) {
                    newDp[j ^ num] = Math.min(newDp[j ^ num], minValue + size[i]);
                }
            }
        }
        dp = newDp;
    }
    return dp[0];
}
```

```

    if (dp[j] == Integer.MAX_VALUE) continue; // 对第 i 组, 保证 dp[i] 是异或到 i 的最小改变数
    Iterator it = group.get(i).entrySet().iterator();
    while (it.hasNext()) {
        Map.Entry en = (Map.Entry)it.next();
        int num = (int)en.getKey(), v = (int)en.getValue();
        int xorNum = num ^ j;
        newDp[xorNum] = Math.min(newDp[xorNum], dp[j] + size[i] - v);
    }
}
dp = Arrays.copyOf(newDp, range); // 将遍历到当前组、累积较优解的 newDp 复制入全局最优解 dp 数组中
}
return dp[0];
}

```

2.5.58 1359. Count All Valid Pickup and Delivery Options - Hard

Given n orders, each order consist in pickup and delivery services.

Count all valid pickup/delivery possible sequences such that delivery(i) is always after of pickup(i).

Since the answer may be too large, return it modulo $10^9 + 7$.

1. 解题思路与分析

就是总共有 $2N$ 个位置，每次放一两个，还剩下多少个位置可以合理占用

```

public int countOrders(int n) {
    int mod = (int)1e9 + 7;
    int spots = n * 2;
    long ans = 1;
    for (int i = n; i >= 2; i--) {
        ans = (ans * spots * (spots - 1) / 2l) % mod;
        spots -= 2;
    }
    return (int)ans;
}

```

2.6 BitMask 掩码相关的

2.7 第二次刷 DP: 下面的题目只需要注意细节，思路没问题

2.7.1 198. House Robber - Medium

You are a professional robber planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security systems connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

Given an integer array nums representing the amount of money of each house, return the maximum amount of money you can rob tonight without alerting the police.

1. 解题思路与分析

- 对于每一个住户来说，只有两种选择：
 - 偷这一户人家的，下一家不要
 - 不偷这一家的，去偷下一家的

```

private int dfs(int[] arr, int idx) {
    if (idx >= n) return 0;
    if (dp[idx] > 0) return dp[idx];
    return dp[idx] = Math.max(dfs(arr, idx+2) + arr[idx], dfs(arr, idx+1)); // 不要忘记记忆——记住答案!!!
}
int[] dp;
int n;
public int rob(int[] a) {
    if (Arrays.stream(a).sum() == 0) return 0;
    n = a.length;
    dp = new int[n];
    return dfs(a, 0);
}

```

2.7.2 213. House Robber II

You are a professional robber planning to rob houses along a street. Each house has a certain amount of money stashed. All houses at this place are arranged in a circle. That means the first house is the neighbor of the last one. Meanwhile, adjacent houses have a security system connected, and it will automatically contact the police if two adjacent houses were broken into on the same night. Given an integer array nums representing the amount of money of each house, return the maximum amount of money you can rob tonight without alerting the police.

1. 解题思路与分析

- 与上一题不同的是：因为是环形数组，数组首尾相连，相邻的两户人家只能偷一家
- 问题就转化成了：我要么去偷 [0, n-2] 这些人家的，要么去偷 [1, n-1] 这些人家的，包括首尾
- 这成了上一题的稍微变形与套娃

```

private int dfs(int[] arr, int i, int j, int idx) {
    if (idx > j) return 0;
    if (dp[idx] != -1) return dp[idx];
    return dp[idx] = Math.max(dfs(arr, i, j, idx+2) + arr[idx], dfs(arr, i, j, idx+1));
}
int[] dp;
int n;
public int rob(int[] arr) {
    n = arr.length;
    if (n == 1) return arr[0];
    dp = new int[n];
    Arrays.fill(dp, -1);
    int max = dfs(arr, 1, n-1, 1);
    Arrays.fill(dp, -1); // 不要忘了 dp 数组的重置与重新计算
    // return Math.max(dfs(arr, 1, n-1, 1), dfs(arr, 0, n-2, 0)); // BUG: 因为 dp 数组不重置，会发生数据混淆，得到错误答案
    return Math.max(max, dfs(arr, 0, n-2, 0));
}

```

2.7.3 740. Delete and Earn: 一类题目，变形了你还认得出来吗？

You are given an integer array nums. You want to maximize the number of points you get by performing the following operation any number of times: Pick any $\text{nums}[i]$ and delete it to earn $\text{nums}[i]$ points. Afterwards, you must delete every element equal to $\text{nums}[i] - 1$ and every element equal to $\text{nums}[i] + 1$. Return the maximum number of points you can earn by applying the above operation some number of times.

1. 解题思路与分析

- 关键题意理解：对于 $\text{nums} = [2, 2, 3, 3, 3, 4]$ ，如果我们选了数字 3，则要删除每一个（所有）2 和 4，之后数组变为 [3, 3]，此时如果再选 3，并不会删除其他元素，剩余数组变为³，再选 3，数组变为空，返回结果是 $3+3+3=9$ 。
- 将上面的例子转化为 House Robber 的形式， $\text{nums}' = [0, 0, 22, 33, 4]$ ， $\text{nums}'[i]$ 代表值为 i 的数字的总数和（或者数字（下标） i 在原数组中出现的次数），每次若选择 nums' 数组的一个元素，则相邻元素不能在下一次被选中（包括其所有的重复计数），相当于被全部删去。转化后，用 House Robber 的思路来处理 nums' 数组即可。

```

private int dfs(int[] cnt, int i) {
    if (i >= N) return 0;
    if (dp[i] > 0) return dp[i];
    return dp[i] = Math.max(dfs(cnt, i+2) + cnt[i]*i, dfs(cnt, i+1));
}
int[] dp;
int N;
public int deleteAndEarn(int[] nums) {
    int n = nums.length, max = Arrays.stream(nums).max().getAsInt();
    N = max+1;
    int[] cnt = new int[N];
    for (Integer v : nums)
        cnt[v]++;
    dp = new int[N];
    int i = 0;
    while (i < N && cnt[i] == 0) i++;
    return dfs(cnt, i);
}

```

- dp 解

```

public int deleteAndEarn(int[] nums) {
    int n = nums.length, N = 10001;
    int [] cnt = new int [N];
    for (Integer v : nums)
        cnt[v] += v;
    int preOne = 0, preTwo = 0, ans = 0; // 相当于滚动数组，把一维 dp 压缩为两个位置
    for (Integer v : cnt) {
        ans = Math.max(preOne, v + preTwo);
        preTwo = preOne;
        preOne = ans;
    }
    return ans;
}

```

2.7.4 877. Stone Game

Alice and Bob play a game with piles of stones. There are an even number of piles arranged in a row, and each pile has a positive integer number of stones $piles[i]$. The objective of the game is to end with the most stones. The total number of stones across all the piles is odd, so there are no ties. Alice and Bob take turns, with Alice starting first. Each turn, a player takes the entire pile of stones either from the beginning or from the end of the row. This continues until there are no more piles left, at which point the person with the most stones wins. Assuming Alice and Bob play optimally, return true if Alice wins the game, or false if Bob wins.

```

// 使用 helper 函数表示 Alex 能比 Lee 多选的分数。可能比双函数更简洁易懂了。
// 记忆化递归的缺点：1. 有可能爆栈；2. 无法降维，而 DP 是可以降维的。
// 模板：
// dfs + memoization 模板
private int dfs(int [] arr, int l, int r) {
    if (l > r) return 0;
    if (dp[l][r] > 0) return dp[l][r]; // 走了来时的路，不需要重走，直接返回
    dp[l][r] = Math.max(arr[l] - dfs(arr, l+1, r), arr[r]-dfs(arr, l, r-1));
    return dp[l][r];
}
int [][] dp;
public boolean stoneGame(int[] piles) {
    int n = piles.length;
    dp = new int[n][n];
    return dfs(piles, 0, n-1) > 0;
}
// 动态规划解法比较难想，dp 数组的第 i 个位置表示的是从第 i 个石头到第 i+l-1 个石头之间最大的比对手得分。
// 使用的是一个长度变量和起始索引，计算每个位置开始的长度 1~N 长度的区间的 dp 状态。
public boolean stoneGame(int[] piles) {
    int n = piles.length;
    int [][] dp = new int[n][n];
    for (int i = n-1; i >= 0; i--) { // 最后一列
        dp[i][i] = piles[i]; // 填右上角
        for (int j = i+1; j < n; j++)
            dp[i][j] = Math.max(piles[i] - dp[i+1][j], piles[j]-dp[i][j-1]);
    }
    return dp[0][n-1] > 0;
}

```

2.7.5 1140. Stone Game II

Alice and Bob continue their games with piles of stones. There are a number of piles arranged in a row, and each pile has a positive integer number of stones $piles[i]$. The objective of the game is to end with the most stones. Alice and Bob take turns, with Alice starting first. Initially, $M = 1$. On each player's turn, that player can take all the stones in the first X remaining piles, where $1 \leq X \leq 2M$. Then, we set $M = \max(M, X)$. The game continues until all the stones have been taken. Assuming Alice and Bob play optimally, return the maximum number of stones Alice can get.

```

// dfs + memoization 模板
// 当次取的最优策略是限制下一次取的数量
private int dfs(int [] arr, int idx, int m) {
    if (dp[idx][m] > 0) return dp[idx][m];
    if (idx == n) return 0;
    if (idx >= n - 2 * m) {
        dp[idx][m] = suf[idx];
        return dp[idx][m];
    }
    int min = Integer.MAX_VALUE;
    for (int i = 1; i <= 2 * m; i++) // 选择限制对手得分最少的情况
        min = Math.min(min, dfs(arr, idx+i, Math.max(m, i)));
    dp[idx][m] = min;
    return dp[idx][m];
}

```

```

        dp[idx][m] = suf[idx] - min;
    return dp[idx][m];
}
int [][] dp;
int [] suf;
int n;
public int stoneGameII(int[] piles) {
    n = piles.length;
    dp = new int [n][2*n];
    suf = new int [n+1];
    for (int i = n-1; i >= 0; i--)
        suf[i] = suf[i+1] + piles[i];
    return dfs(piles, 0, 1);
}

```

2.7.6 2029. Stone Game IX - Medium

Alice and Bob continue their games with stones. There is a row of n stones, and each stone has an associated value. You are given an integer array stones , where $\text{stones}[i]$ is the value of the i th stone.

Alice and Bob take turns, with Alice starting first. On each turn, the player may remove any stone from stones. The player who removes a stone loses if the sum of the values of all removed stones is divisible by 3. Bob will win automatically if there are no remaining stones (even if it is Alice's turn).

Assuming both players play optimally, return true if Alice wins and false if Bob wins.

- 过程复制

```

private boolean dfs(int val, int one, int two, int zro) {
    if (val == 0) return true;
    String key = one + "-" + two + "-" + zro + "-" + val;
    if (dp.containsKey(key)) return dp.get(key);
    if (one == 0 && two == 0 && zro == 0) return n % 2 != 0; // 题目这个要求我可能没有理解清楚
    boolean res = false;
    if (!res && one > 0 && val != 2)
        if (!dfs((val+1) % 3, one-1, two, zro))
            res = true;
    if (!res && two > 0 && val != 1)
        if (!dfs((val + 2) % 3, one, two-1, zro))
            res = true;
    if (!res && zro > 0 && val != 0)
        if (!dfs((val+0) % 3, one, two, zro-1))
            res = true;
    dp.put(key, res);
    return res;
}
Map<String, Boolean> dp = new HashMap<>();
int n;
public boolean stoneGameIX(int[] stones) {
    n = stones.length;
    dp = new HashMap<String, Boolean>();
    int one = 0, two = 0, zro = 0;
    for (int v : stones)
        if (v % 3 == 0) zro++;
        else if (v % 3 == 1) one++;
        else two++; // zro % 2: each pair of numbers divisible by 3 cancel each other out, so only parity count of such numbers matters
    return dfs(3, one, two, zro % 2);
}

```

- 分析一下

Count the frequency of mod3 = 0,1,2.

```

Firstly, don't consider the multiples of 3.
Alice starts with mod3 = 1, Alice and Bob have to pick 1,1,2,1,2... in order.
Alice starts with mod3 = 2, Alice and Bob have to pick 2,2,1,2,1... in order.
If Alice starts with 1, then Alice needs 1 and Bob needs 2.
If 1 is much more than 2, then Bob is going to lose.
So if cnt[0] == 0, the result can be decided by Alice.
Then, consider the number of multiples of 3.
If cnt[0] is even,
Bob picks a 3, Alice can always picks one another.
the result won't be affected.
If cnt[0] is odd,
the final result will be reversed,
(unless the case Bob win for all numbers consumed)

```

Missing Case

[1,1,1,3] gave by @mittal582 and @qingqi_lei,
which can hack some solution.

Explanation

If $\text{cnt}[1] == 0$, Alice needs to start with $\text{mod}3 = 2$,
If $\text{cnt}[2] == 0$, Alice needs to start with $\text{mod}3 = 1$.
Alice can win if $\max(\text{cnt}[1], \text{cnt}[2]) > 2 \&& \text{cnt}[0] \% 2 > 0$,
for example [1,1,1,3].

If $\text{cnt}[0] \% 2 == 0$, easy case for Alice.

Alice can win in at least one of the two options, picking the less one.

Otherwise $\text{cnt}[0] \% 2 == 1$, this will reverse the result.

If $\text{abs}(\text{cnt}[1] - \text{cnt}[2]) > 2$,
Alice will pick $\text{mod}3=2$ if $\text{mod}3=2$ is more
Alice will pick $\text{mod}3=1$ if $\text{mod}3=1$ is more
If $\text{abs}(\text{cnt}[1] - \text{cnt}[2]) \leq 2$,
Alice will lose for no number remaining.

Complexity

Time $O(n)$

Space $O(1)$

```
public boolean stoneGameIX(int[] stones) {
    int[] cnt = new int[3];
    for (int a: stones)
        cnt[a % 3]++;
    if (Math.min(cnt[1], cnt[2]) == 0)
        return Math.max(cnt[1], cnt[2]) > 2 && cnt[0] % 2 > 0;
    return Math.abs(cnt[1] - cnt[2]) > 2 || cnt[0] % 2 == 0;
}
```

2.7.7 322. Coin Change

You are given an integer array coins representing coins of different denominations and an integer amount representing a total amount of money. Return the fewest number of coins that you need to make up that amount. If that amount of money cannot be made up by any combination of the coins, return -1. You may assume that you have an infinite number of each kind of coin.

```
public int coinChange(int[] coins, int amount) {
    if (amount == 0) return 0;
    int n = coins.length;
    int [] dp = new int [amount + 1];
    Arrays.fill(dp, amount + 1);
    dp[0] = 0;
    for (int i = 0; i <= amount; i++) {
        for (int v : coins) {
            if (i - v < 0) continue;
            dp[i] = Math.min(dp[i], dp[i-v] + 1);
        }
    }
    return dp[amount] == amount + 1 ? -1 : dp[amount];
}
```

Chapter 3

HashMap

3.1 数有多少个数组数不清楚

3.2 336. Palindrome Pairs - Hard

Given a list of unique words, return all the pairs of the distinct indices (i, j) in the given list, so that the concatenation of the two words $\text{words}[i] + \text{words}[j]$ is a palindrome.

3.2.1 解题思路与分析

- 利用字典 wmap 保存单词 -> 下标的键值对
- 遍历单词列表 words, 记当前单词为 word, 下标为 idx:

-
- 若当前单词 word 本身为回文, 则将空串下标 bidx 与 idx 加入答案
 - 若当前单词的逆序串在 words 中, 则将逆序串下标 ridx 与 idx 加入答案
 - 将当前单词 word 拆分为左右两半 left, right。
 - 若 left 为回文, 并且 right 的逆序串在 words 中, 则将 right 的逆序串下标 rridx 与 idx 加入答案
 - 若 right 为回文, 并且 left 的逆序串在 words 中, 则将 left 的逆序串下标 idxx 与 rridx 加入答案
-

情况分析清楚以后，针对每个情况逐个判断，然后提前建好 hashmap 以后判断就好。

细节部分是不要忘记空字符串的情况，然后针对整个字符串反过来的情况，不要有重复。

```
private boolean isPalindrome(String s, int i, int j) {
    while (i < j)
        if (s.charAt(i++) != s.charAt(j--)) return false;
    return true;
}
public List<List<Integer>> palindromePairs(String[] words) {
    Map<String, Integer> map = new HashMap<>();
    List<List<Integer>> ans = new ArrayList<>();
    for (int i = 0; i < words.length; i++)
        map.put(words[i], i);
    for (int i = 0; i < words.length; i++) {
        if (words[i].equals(""))
            for (int j = 0; j < words.length; j++) {
                String w = words[j];
                if (isPalindrome(w, 0, w.length()-1) && j != i) {
                    ans.add(List.of(i, j));
                    ans.add(List.of(j, i));
                }
            }
        continue;
    }
    StringBuilder sb = new StringBuilder(words[i]);
    sb.reverse();
    String bw = sb.toString();
    if (map.containsKey(bw)) {
        int res = map.get(bw);
        if (res != i) ans.add(List.of(i, res));
    }
    for (int j = 1; j < bw.length(); j++) {
        if (isPalindrome(bw, 0, j-1)) {
            String s = bw.substring(j);
```

```

        if (map.containsKey(s))
            ans.add(List.of(i, map.get(s)));
    }
    if (isPalindrome(bw, j, bw.length()-1)) {
        String s = bw.substring(0, j);
        if (map.containsKey(s))
            ans.add(List.of(map.get(s), i));
    }
}
return ans;
}

```

3.3 1392. Longest Happy Prefix - Hard

A string is called a happy prefix if it is a non-empty prefix which is also a suffix (excluding itself).

Given a string s, return the longest happy prefix of s. Return an empty string "" if no such prefix exists.

```

// 频繁的字符串操作 (substring 和 equals 操作) 会大幅消耗执行时间，也会导致 TLE 时间超时。
// 因此我们可以使用字符串的 hash 值方式来比较前后缀是否相同。这里我们需要普及一个知识点，任意一个字符串的 Hash 值的计算公式为：
// int hash=s[0]31^(n1)+s[1]31^(n2) +...+s[n2]31^1+s[n1]31^0
// 对于前缀 hash，每次长度加一后 hash 的变化应该是：
// hash = hash*31 + 新添头字符 ch
// 对于后缀 hash，每次长度加一后 hash 的变化应该是：
// hash = hash + 新添尾字符 *31^t (t 为后缀长度减 1)
// 这样我们每次只需要比较前缀 hash 与后缀 hash 是否相同即可。
public String longestPrefix(String s) {
    int n = s.length(), hashPre = 0, hashSuf = 0;
    int left = 0, right = n-1, pow = 1, maxLen = 0;
    String res = "";
    while (left < n-1) {
        hashPre = hashPre * 31 + s.charAt(left);
        hashSuf = hashSuf + s.charAt(right)*pow;
        if (hashPre == hashSuf) maxLen = left + 1;
        left++;
        right--;
        pow *= 31;
    }
    return maxLen == 0 ? "" : s.substring(0, maxLen);
}

```

3.3.1 KMP 算法

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
p ABCDABD\$ next=0	j 01234567

j	0	1	2	3	4	5	6	7
p	A	B	C	D	A	B	D	\$
nxt	0	0	0	0	0	1	2	0

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
p ABCDABD\$ next=0	j 01234567

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
p ABCDABD\$ next=2	j 01234567

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
p ABCDABD\$ next=0	j 01234567

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
w next=0 ABCDABD\$	j 01234567

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
w next=2 ABCDABD\$	j 01234567

1	2
i 01234567890123456789012	
s ABC ABCDAB ABCDABCDABDE	
w next=0 ABCDABD\$	j 01234567

j	0	1	2	3
p	A	A	A	\$
nxt	0	0	1	2

i 01234567
s AAAA~~AAAAA~~
p AAA\$
j 0123 next=2

i 01234567
s AAAA~~AAAAA~~
p AAA\$
j 0123 next=2

i 01234567
s AAAA~~AAAAA~~
p AA~~A~~\$
j 0123 next=2

i 01234567
s AAAA~~AA~~AA
p AA~~A~~\$
j 0123 next=2

i 01234567
s AAAA~~AA~~AA
p AA~~A~~\$
j 0123 next=2

i 01234567
s AAAA~~AA~~AA
p AA~~A~~\$
j 0123 next=2

KMP::Match

- Case 1: $s[i] == p[j]$, match
 - Action: $++i$, $++j$: try next pair
 - ABCABCD \Rightarrow ABCABCD \Rightarrow ABCABCD
 - ABC... ABC... ABC...
- Case 2: $s[i] \neq p[j]$, a mismatch happened
 - Key idea: use partial matching information
 - Let $q = p[0 \sim j-1]$ be the partial matched string
 - Action: i remains the same, if $j == 0$: $++i$ else: $j = \text{next}[j]$,
- Case 2.1: q is empty (e.g. $j = 0$)
 - $++i$, try next one
 - ABCABCD ABCABCD
 - CA CA

- Case 2.2: $\text{suffix}(q) \cap \text{prefix}(q) = \emptyset$
 - $\text{next}[j]$ must be 0
 - skip the partial match (no potential matches in between)

$$\begin{array}{c} \text{ABC}\text{A}\text{BCD} \\ \downarrow \quad \downarrow \\ \text{ABCD} \end{array} \Rightarrow \begin{array}{c} \text{ABC}\text{ABCD} \\ \text{ABCD} \end{array}$$
- Case 2.3: $\text{suffix}(q) \cap \text{prefix}(q) \neq \emptyset$
 - $\text{next}[j] = \text{len}(\text{LPS}(q))$
 - e.g. ABCDAB, AB is the longest suffix that is also a prefix
 - Skip the prefix AB and start matching from 'C'

$$\begin{array}{c} \text{ABCD}\text{ABC}\text{DABE} \\ \downarrow \quad \downarrow \\ \text{ABCDABD} \end{array} \Rightarrow \begin{array}{c} \text{ABCDAB} \\ \text{ABCDABD} \end{array}$$
- Case 2.4: $q = p$, we had a full match
 - Action: $++i$, $j = \text{next}[j]$, treat it as a mismatch

```
// straight forward KMP algorithm - 8ms - beats 94% time - O(N) time and O(N) space
public String longestPrefix(String s) {
    int i, x, N = s.length();
    int [] LPS = new int[N];
    LPS[0] = 0;
    for (i = 1; i < N; i++){
        x = LPS[i - 1];
        while (s.charAt(i) != s.charAt(x)){
            if (x == 0){
                x = -1;
                break;
            }
            x = LPS[x - 1];
        }
        LPS[i] = x + 1;
    }
    return s.substring(0, LPS[N - 1]);
}
```

```
string longestPrefix(string s) {
    vector<int> lps(s.size(), 0);
    size_t i = 0, j = 1;
    while (j < s.size()) {
        if (s[i] == s[j]) lps[j++] = (i++) + 1; // situ.1
        else if (i != 0) i = lps[i - 1]; // situ.2
        else lps[j++] = 0; // situ.3
    }
    return s.substr(0, lps.back());
}
```

3.4 1172. Dinner Plate Stacks - Hard

You have an infinite number of stacks arranged in a row and numbered (left to right) from 0, each of the stacks has the same maximum capacity.

Implement the DinnerPlates class:

DinnerPlates(int capacity) Initializes the object with the maximum capacity of the stacks capacity.
void push(int val) Pushes the given integer val into the leftmost stack with a size less than capacity.
int pop() Returns the value at the top of the rightmost non-empty stack and removes it from that stack, and returns -1 if all the stacks are empty.
int popAtStack(int index) Returns the value at the top of the stack with the given index index and removes it from that stack or returns -1 if the stack with that given index is empty.

```
Stack<Stack<Integer>> stacks = new Stack<>();
TreeSet<Integer> set = new TreeSet<>(); // set:
int capacity;
public DinnerPlates(int capacity) {
    this.capacity = capacity;
```

```

        stacks = new Stack<>();
    }
    public void push(int val) {
        if (set.size() != 0) {
            int idx = set.iterator().next();
            stacks.get(idx).push(val);
            if (stacks.get(idx).size() == capacity)
                set.remove(idx);
        } else {
            if (stacks.isEmpty() || stacks.peek().size() == capacity) {
                stacks.add(new Stack<>()); // 更高效一点儿?
                // stacks.push(new Stack<>());
                stacks.peek().add(val);
            } else stacks.peek().add(val);
        }
    }
    public int pop() {
        if (!stacks.isEmpty()) {
            int k = stacks.peek().pop();
            while (!stacks.isEmpty() && stacks.peek().isEmpty())
                set.remove(stacks.size()-1);
            stacks.pop();
        }
        return k;
    }
    return -1;
}
public int popAtStack(int index) {
    if (index >= stacks.size() || stacks.get(index).size() == 0)
        return -1;
    if (index == stacks.size()-1)
        return this.pop();
    set.add(index);
    return stacks.get(index).pop();
}
}

```

- 用了双端队列的一个方法

```

List<Deque<Integer>> stackList = new ArrayList<>();
TreeSet<Integer> pushIdxSet = new TreeSet<>();
TreeSet<Integer> popIdxSet = new TreeSet<>();
int capacity;
public DinnerPlates(int capacity) {
    stackList = new ArrayList<>();
    pushIdxSet = new TreeSet<>();
    popIdxSet = new TreeSet<>();
    this.capacity = capacity;
    stackList.add(new ArrayDeque<>());
    pushIdxSet.add(0);
}
public void push(int val) {
    int idx = pushIdxSet.first();
    if (stackList.get(idx).isEmpty())
        popIdxSet.add(idx);
    stackList.get(idx).offerLast(val);
    if (stackList.get(idx).size() == capacity) {
        if (idx == stackList.size() - 1) {
            stackList.add(new ArrayDeque<>());
            pushIdxSet.add(idx + 1);
        }
        pushIdxSet.remove(idx);
    }
}
public int pop() {
    if (popIdxSet.isEmpty()) return -1;
    int idx = popIdxSet.last();
    if (stackList.get(idx).size() == capacity)
        pushIdxSet.add(idx);
    int res = stackList.get(idx).pollLast();
    if (stackList.get(idx).isEmpty())
        popIdxSet.remove(idx);
    return res;
}
public int popAtStack(int index) {
    if (index >= stackList.size()) return -1;
    if (stackList.get(index).isEmpty()) return -1;
    if (stackList.get(index).size() == capacity)
        pushIdxSet.add(index);
    int res = stackList.get(index).pollLast();
    if (stackList.get(index).isEmpty())
        popIdxSet.remove(index);
    return res;
}

```

Chapter 4

Tree

4.1 979. Distribute Coins in Binary Tree

You are given the root of a binary tree with n nodes where each node in the tree has `node.val` coins. There are n coins in total throughout the whole tree.

In one move, we may choose two adjacent nodes and move one coin from one node to another. A move may be from parent to child, or from child to parent.

Return the minimum number of moves required to make every node have exactly one coin.

```
private int dfs(TreeNode r) { // 统计把自身, 左右子树都平衡, 需要移动的 coins 个数
    if (r == null) return 0;
    int left = dfs(r.left); // 左、右子树缺多少
    int right = dfs(r.right);
    res += Math.abs(left) + Math.abs(right); // 左, 右子树和自身都平衡需要的移动数
    return left + right + r.val - 1;
}
int res;
public int distributeCoins(TreeNode root) {
    res = 0;
    return res;
}
```

4.2 687. Longest Univalue Path

Given the root of a binary tree, return the length of the longest path, where each node in the path has the same value. This path may or may not pass through the root.

The length of the path between two nodes is represented by the number of edges between them.

- 此题与求二叉树的最长路径边长相似，只是此题要求是节点值相同的路径，也就是说在找最长路径的时候，还需要判断节点值，要是不相同，就重置为0，在此期间，我们使用一个全局变量来存储最长节点值相同路径的边长。

```
private int topDownTraverse(TreeNode r) {
    if (r == null) return 0;
    int left = topDownTraverse(r.left);
    int right = topDownTraverse(r.right);
    if (r.left == null || r.left.val != r.val) left = 0;
    if (r.right == null || r.right.val != r.val) right = 0;
    max = Math.max(max, left + right);
    return Math.max(left, right) + 1;
}
int max = 0;
public int longestUnivaluePath(TreeNode root) {
    if (root == null) return 0;
    topDownTraverse(root);
    return max;
}
```

4.3 652. Find Duplicate Subtrees

Given the root of a binary tree, return all duplicate subtrees.

For each kind of duplicate subtrees, you only need to return the root node of any one of them.
 Two trees are duplicate if they have the same structure with the same node values.

```

private String duplicate(TreeNode node) {
    if(node == null) return "X";
    String l = duplicate(node.left);
    String r = duplicate(node.right);
    String s = Integer.toString(node.val) + "-" + l + "-" + r;
    map.put(s, map.getOrDefault(s, 0)+1);
    if (map.get(s) == 2)
        list.add(node);
    return s;
}
HashMap<String, Integer> map = new HashMap<>();
ArrayList list = new ArrayList<>();
public List findDuplicateSubtrees(TreeNode root) {
    duplicate(root);
    return list;
}

```

- 看一下构造的图的效果图

```

1 -> root
2, 3,   ->
4, # | 2, 4,   ->
#.# | 4, # | #.# |  ->
#.## |  ->

map.size(): 4
3-2-4-X-X-4-X-X, 1
1-2-4-X-X-X-3-2-4-X-X-X-4-X-X, 1
2-4-X-X-X, 2
4-X-X, 3

res.size(): 2
TREE Level order traversal:
    4 -> root
    #.# |  ->

TREE Level order traversal:
    2 -> root
    4, # |  ->
#.## |  ->

```

- 一种 dfs 的写法

```

HashSet<String> set, added;
List<TreeNode> list;
public List<TreeNode> findDuplicateSubtrees(TreeNode root) {
    set = new HashSet();
    added = new HashSet();
    list = new ArrayList();
    StringBuilder ret = dfs(root);
    return list;
}
private StringBuilder dfs(TreeNode root){
    if (root == null) return null;
    StringBuilder sbL = dfs(root.left), sbR = dfs(root.right);
    if (sbL == null && sbR == null){
        sbL = new StringBuilder();
        sbL.append(root.val);
    } else if (sbL != null){
        sbL.append(" " + root.val);
        if (sbR != null){
            sbL.append(' ');
            sbL.append(sbR);
        } else sbL.append("\n");
    } else if (sbL == null){
        if (sbR != null){
            sbR.insert(0, " " + root.val);
            sbL = sbR;
        }
    }
    String temp = sbL.toString();
    if (set.contains(temp) && !added.contains(temp)){
        list.add(root);
        added.add(temp);
    }
}

```

```

    set.add(temp);
    return sbl;
}

```

- 这个跑起来很高效，可惜我看不懂。。。。以后再慢慢消化吧
- <https://leetcode.com/problems/find-duplicate-subtrees/discuss/1418487/Java-beats-99.5-in-time>

```

Map<Integer, Integer> count;           // frequency of each subtree represented in string
Map<List<Integer>, Integer> numberMap; // ** not hashset since it cannot reserve element order
List<TreeNode> ans;
int globalNumber = 1;
public List<TreeNode> findDuplicateSubtrees(TreeNode root) {
    count = new HashMap();
    numberMap = new HashMap();
    ans = new ArrayList();
    collect(root);
    return ans;
}
public int collect(TreeNode node) {
    if (node == null) return 0;
    int leftNumber = collect(node.left);
    int rightNumber = collect(node.right);
    List<Integer> numberExp = new ArrayList<>(); // construct expression
    numberExp.add(node.val);
    numberExp.add(leftNumber);
    numberExp.add(rightNumber);
    if (!numberMap.containsKey(numberExp)) { // update numberMap
        numberMap.put(numberExp, globalNumber);
        globalNumber++;
    }
    // check number frequency. if == 2, meaning duplication then add to result
    int rootNumber = numberMap.get(numberExp).intValue();
    count.put(rootNumber, count.getOrDefault(rootNumber, 0)+1);
    if (count.get(rootNumber) == 2) // not >=2, otherwise ans will have duplicated nodes
        ans.add(node);
    return rootNumber;
}

```

```

count.size(): 4
1, 3
2, 2
3, 1
4, 1
numberMap.size(): 4
2, 1, 0,
2
3, 2, 1,
3
1, 2, 3,
4
4, 0, 0,
1

```

Chapter 5

binary Search

5.1 LeetCode Binary Search Summary 二分搜索法小结

- <https://segmentfault.com/a/1190000016825704>
- <https://www.cnblogs.com/grandyang/p/6854825.html>

5.1.1 标准二分查找

```
public int search(int[] nums, int target) {  
    int left = 0;  
    int right = nums.length - 1;  
    while (left <= right) {  
        int mid = left + ((right - left) >> 1);  
        if (nums[mid] == target) return mid;  
        else if (nums[mid] > target)  
            right = mid - 1;  
        else  
            left = mid + 1;  
    }  
    return -1;  
}
```

循环终止的条件包括：

- 找到了目标值
- $left > right$ (这种情况发生于当 $left$, mid , $right$ 指向同一个数时, 这个数还不是目标值, 则整个查找结束。)

$left + ((right - left) \gg 1)$ 对于目标区域长度为奇数而言, 是处于正中间的, 对于长度为偶数而言, 是中间偏左的。因此左右边界相遇时, 只会是以下两种情况:

- $left/mid, right$ ($left$, mid 指向同一个数, $right$ 指向它的下一个数)
- $left/mid/right$ ($left$, mid , $right$ 指向同一个数)

即因为 mid 对于长度为偶数的区间总是偏左的, 所以当区间长度小于等于 2 时, mid 总是和 $left$ 在同一侧。

5.1.2 二分查找左边界

利用二分法寻找左边界是二分查找的一个变体, 应用它的题目常常有以下几种特性之一:

- 数组有序, 但包含重复元素
- 数组部分有序, 且不包含重复元素
- 数组部分有序, 且包含重复元素

1. 左边界查找类型 1

类型 1 包括了上面说的第一种，第二种情况。

既然要寻找左边界，搜索范围就需要从右边开始，不断往左边收缩，也就是说即使我们找到了 $\text{nums}[\text{mid}] == \text{target}$ ，这个 mid 的位置也不一定就是最左侧的那个边界，我们还是要向左侧查找，所以我们在 $\text{nums}[\text{mid}]$ 偏大或者 $\text{nums}[\text{mid}]$ 就等于目标值的时候，继续收缩右边界，算法模板如下：

```
public int search(int[] nums, int target) {
    int left = 0;
    int right = nums.length - 1;
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] < target)
            left = mid + 1;
        else
            right = mid;
    }
    return nums[left] == target ? left : -1;
}
```

返回值： $\text{nums}[\text{left}] == \text{target} ? \text{left} : -1$

与标准的二分查找不同：

首先，这里的右边界的更新是 $\text{right} = \text{mid}$ ，因为我们需要在找到目标值后，继续向左寻找左边界。

其次，这里的循环条件是 $\text{left} < \text{right}$ 。

因为在最后 left 与 right 相邻的时候， mid 和 left 处于相同的位置（前面说过， mid 偏左），则下一步，无论怎样， left , mid , right 都将指向同一个位置，如果此时循环的条件是 $\text{left} \leq \text{right}$ ，则我们需要再进入一遍循环，此时，如果 $\text{nums}[\text{mid}] < \text{target}$ 还好说，循环正常终止；否则，我们会令 $\text{right} = \text{mid}$ ，这样并没有改变 $\text{left}, \text{mid}, \text{right}$ 的位置，将进入死循环。

事实上，我们只需要遍历到 left 和 right 相邻的情况就行了，因为这一轮循环后，无论怎样， $\text{left}, \text{mid}, \text{right}$ 都会指向同一个位置，而如果这个位置的值等于目标值，则它就一定是最左侧的目标值；如果不等于目标值，则说明没有找到目标值，这也就是为什么返回值是 $\text{nums}[\text{left}] == \text{target} ? \text{left} : -1$ 。

```
public int searchInsert(int[] nums, int target) {
    int len = nums.length;
    if (nums[len - 1] < target) return len;
    int left = 0;
    int right = len - 1;
    while (left <= right) {
        int mid = (left + right) / 2;
        // 等于的情况最简单，我们应该放在第 1 个分支进行判断
        if (nums[mid] == target) {
            return mid;
        } else if (nums[mid] < target) {
            // 题目要我们返回大于或者等于目标值的第 1 个数的索引
            // 此时 mid 一定不是所求的左边界，
            // 此时左边界更新为 mid + 1
            left = mid + 1;
        } else {
            // 既然不会等于，此时 nums[mid] > target
            // mid 也一定不是所求的右边界
            // 此时右边界更新为 mid - 1
            right = mid - 1;
        }
    }
    // 注意：一定得返回左边界 left,
    // 理由是对于 [1,3,5,6], target = 2, 返回大于等于 target 的第 1 个数的索引，此时应该返回 1
    // 在上面的 while (left <= right) 退出循环以后，right < left, right = 0, left = 1
    // 根据题意应该返回 left,
    // 如果题目要求你返回小于等于 target 的所有数里最大的那个索引值，应该返回 right
    return left;
}
```

2. 左边界查找类型 2

左边界查找的第二种类型用于数组部分有序且包含重复元素的情况，这种条件下在我们向左收缩的时候，不能简单的令 $\text{right} = \text{mid}$ ，因为有重复元素的存在，这会导致我们有可能遗漏掉一部分区域，此时向左收缩只能采用比较保守的方式，代码模板如下：

```
public int search(int[] nums, int target) {
    int left = 0;
    int right = nums.length - 1;
```

```

while (left < right) {
    int mid = left + (right - left) / 2;
    if (nums[mid] < target)
        left = mid + 1;
    else if (nums[mid] > target)
        right = mid;
    else
        right--;
}
return nums[left] == target ? left : -1;
}

```

它与类型 1 的唯一区别就在于对右侧值的收缩更加保守。这种收缩方式可以有效地防止我们一下子跳过了目标边界从而导致了搜索区域的遗漏。

5.1.3 二分查找右边界

```

public int search(int[] nums, int target) {
    int left = 0;
    int right = nums.length - 1;
    while (left < right) {
        int mid = left + ((right - left) >> 1) + 1;
        if (nums[mid] > target)
            right = mid - 1;
        else
            left = mid;
    }
    return nums[right] == target ? right : -1;
}

```

- 循环条件: `left < right`
- 中间位置计算: `mid = left + ((right - left) >> 1) + 1`
- 左边界更新: `left = mid`
- 右边界更新: `right = mid - 1`
- 返回值: `nums[right] == target ? right : -1`

这里大部分和寻找左边界是对称着来写的，唯独有一点需要尤其注意——中间位置的计算变了，我们在末尾多加了 1。这样，无论对于奇数还是偶数，这个中间的位置都是偏右的。

对于这个操作的理解，从对称的角度看，寻找左边界的时候，中间位置是偏左的，那寻找右边界的时候，中间位置就应该偏右呗，但是这显然不是根本原因。根本原因是，在最后 `left` 和 `right` 相邻时，如果 `mid` 偏左，则 `left`, `mid` 指向同一个位置，`right` 指向它们的下一个位置，在 `nums[left]` 已经等于目标值的情况下，这三个位置的值都不会更新，从而进入了死循环。所以我们应该让 `mid` 偏右，这样 `left` 就能向右移动。这也就是为什么我们之前一直强调整查找条件，判断条件和左右边界的更新方式三者之间需要配合使用。

右边界的查找一般来说不会单独使用，如有需要，一般是需要同时查找左右边界。

5.1.4 二分查找左右边界

前面我们介绍了左边界和右边界，那么查找左右边界就容易很多了——只要分别查找左边界和右边界就行了。

5.1.5 二分查找极值

二分查找还有一种有趣的变体是二分查找极值点，之前我们使用 `nums[mid]` 去比较的时候，常常是和给定的目标值 `target` 比，或者和左右边界比较，在二分查找极值点的应用中，我们是和相邻元素去比，以完成某种单调性的检测。关于这一点，我们直接来看一个例子就明白了。

Find Peak Element

A peak element is an element that is greater than its neighbors.

Given an input array `nums`, where `nums[i] <= nums[i+1]`, find a peak element and return its index.

The array may contain multiple peaks, in that case return the index to any one of the peaks is fine.

You may imagine that `nums[-1] = nums[n] = -`.

这一题的有趣之处在于他要求求一个局部极大值点，并且整个数组不包含重复元素。所以整个数组甚至可以是无序的——你可能很难想象我们可以在一个无序的数组中直接使用二分查找，但是没错！我们确实可以这么干！谁要人家只要一个局部极大值即可呢。

```

public int findPeakElement(int[] nums) {
    int left = 0;
    int right = nums.length - 1;
    while (left < right) {
        int mid = left + ((right - left) >> 1);
        if (nums[mid] < nums[mid + 1])
            left = mid + 1;
        else
            right = mid;
    }
    return left;
}

```

```

public int binarySearch2(int[] nums, int target) {
    // left 和 right 都在数组下标范围内
    // [left, right]
    int left = 0;
    int right = nums.length - 1;
    // while 循环跳出的条件是 left > right
    // 所以如果没找到 target 的话，也不需要特判了
    while (left <= right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] == target) return mid;
        else if (nums[mid] < target)
            left = mid + 1;
        else
            right = mid - 1;
    }
    // 如果没找到就只能返回-1
    return -1;
}

// 模板二，适合判断当前 index 和 index + 1 之间的关系。
// right 指针一开始的定义是在数组下标范围外的, [left, right), 所以在需要移动 right 指针的时候不能写成 right = mid。这样会遗漏掉一些下标的判断。
public int binarySearch3(int[] nums, int target) {
    // right 不在下标范围内
    // [left, right)
    int left = 0;
    int right = nums.length;
    // while 循环跳出的条件是 left == right
    // 这个模板比较适合判断当前 index 和 index + 1 之间的关系
    // left < right, example, left = 0, right = 1
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] == target) return mid;
        else if (nums[mid] < target)
            left = mid + 1;
        else
            // 因为搜索范围是左闭右开所以这里不能-1
            right = mid;
    }
    // 最后的特判
    if (left != nums.length && nums[left] == target)
        return left;
    return -1;
}

// while 条件不满足的时候, left + 1 == right, 两下标应该指向某个下标 i 和 i + 1。这样如果有什么特殊的值需要判断, 应该不是 left 就是 right 了。
public int binarySearch1(int[] nums, int target) {
    // left 和 right 都在数组下标范围内
    // [left, right)
    int left = 0;
    int right = nums.length - 1;
    // 举例, start - 0, end = 3
    // 中间隔了起码有 start + 1 和 start + 2 两个下标
    // 这样跳出 while 循环的时候, start + 1 == end
    // 才有了最后的两个判断
    while (left + 1 < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] == target) return mid;
        else if (nums[mid] < target)
            left = mid;
        else
            right = mid;
    }
    // 特判
    if (nums[left] == target) return left;
    if (nums[right] == target) return right;
    // 如果没找到就只能返回-1
    return -1;
}

```

<u>Template #1:</u>	<u>Template #2:</u>	<u>Template #3:</u>
<pre>// Pre-processing ... left = 0; right = length - 1; while (left <= right) { mid = left + (right - left) / 2; if (nums[mid] == target) { return mid; } else if (nums[mid] < target) { left = mid + 1; } else right = mid - 1; } ... // right + 1 == left // No more candidate</pre>	<pre>// Pre-processing ... left = 0; right = length; while (left < right) { mid = left + (right - left) / 2; if (nums[mid] < target) { left = mid + 1; } else { right = mid; } } ... // left == right // 1 more candidate // Post-Processing</pre>	<pre>// Pre-processing ... left = 0; right = length - 1; while (left + 1 < right) { mid = left + (right - left) / 2; if (num[mid] < target) { left = mid; } else { right = mid; } } ... // left + 1 == right // 2 more candidates // Post-Processing</pre>

5.1.6 第一类：需查找和目标值完全相等的数

这是最简单的一类，也是我们最开始学二分查找法需要解决的问题，比如我们有数组 [2, 4, 5, 6, 9], target = 6，那么我们可以写出二分查找法的代码如下：

```
int find(vector<int>& nums, int target) {
    int left = 0, right = nums.size();
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] == target) return mid;
        else if (nums[mid] < target) left = mid + 1;
        else right = mid;
    }
    return -1;
}
```

会返回 3，也就是 target 的在数组中的位置。注意二分查找法的写法并不唯一，主要可以变动地方有四处：

第一处是 right 的初始化，可以写成 nums.size() 或者 nums.size() - 1。

第二处是 left 和 right 的关系，可以写成 left < right 或者 left <= right。

第三处是更新 right 的赋值，可以写成 right = mid 或者 right = mid - 1。

第四处是最后返回值，可以返回 left, right, 或 right - 1。

- 但是这些不同的写法并不能随机的组合，像博主的那种写法，

- 若 right 初始化为了 nums.size()，那么就必须用 left < right，而最后的 right 的赋值必须用 right = mid。
- 但是如果我们将 right 初始化为 nums.size() - 1，那么就必须用 left <= right，并且 right 的赋值要写成 right = mid - 1，不然就会出错。

所以博主的建议是选择一套自己喜欢的写法，并且记住，实在不行就带简单的例子来一步一步执行，确定正确的写法也行。

第一类应用实例：

Intersection of Two Arrays

5.1.7 第二类：查找第一个不小于目标值的数，可变形为查找最后一个小于目标值的数

这是比较常见的一类，因为我们要查找的目标值不一定会在数组中出现，也有可能是跟目标值相等的数在数组中并不唯一，而是有多个，那么这种情况下 nums[mid] == target 这条判断语句就没有必要存在。比如在数组 [2, 4, 5, 6, 9] 中查找数字 3，就会返回数字 4 的位置；在数组 [0, 1, 1, 1, 1] 中查找数字 1，就会返回第一个数字 1 的位置。我们可以使用如下代码：

```

int find(vector<int>& nums, int target) {
    int left = 0, right = nums.size();
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] < target) left = mid + 1;
        else right = mid;
    }
    return right;
}

```

最后我们需要返回的位置就是 `right` 指针指向的地方。在 C++ 的 STL 中有专门的查找第一个不小于目标值的数的函数 `lower_bound`

这一类可以轻松的变形为查找最后一个小于目标值的数，怎么变呢。我们已经找到了第一个不小于目标值的数，那么再往前退一位，返回 `right - 1`，就是最后一个小于目标值的数。

第二类应用实例：

[Heaters](#), [Arranging Coins](#), [Valid Perfect Square](#), [Max Sum of Rectangle No Larger Than K](#), [Russian Doll Envelopes](#)

第二类变形应用： [Valid Triangle Number](#)

5.1.8 第三类：查找第一个大于目标值的数，可变形为查找最后一个不大于目标值的数

这一类也比较常见，尤其是查找第一个大于目标值的数，在 C++ 的 STL 也有专门的函数 `upper_bound`。
`nums[mid] < target` 变成 `nums[mid] <= target`，就这一个小小的变化，其实直接就改变了搜索的方向，使得在数组中有很多跟目标值相同的数字存在的情况下，返回最后一个相同的数字的下一个位置。比如在数组 [2, 4, 5, 6, 9] 中查找数字 3，还是返回数字 4 的位置，这跟上面那查找方式返回的结果相同，因为数字 4 在此数组中既是第一个不小于目标值 3 的数，也是第一个大于目标值 3 的数，所以 make sense；在数组 [0, 1, 1, 1, 1] 中查找数字 1，就会返回坐标 5，通过对比返回的坐标和数组的长度，我们就知道是否存在这样一个大于目标值的数。参见下面的代码：

```

int find(vector<int>& nums, int target) {
    int left = 0, right = nums.size();
    while (left < right) {
        int mid = left + (right - left) / 2;
        if (nums[mid] <= target) left = mid + 1;
        else right = mid;
    }
    return right;
}

```

这一类可以轻松的变形为查找最后一个不大于目标值的数，怎么变呢。我们已经找到了第一个大于目标值的数，那么再往前退一位，返回 `right - 1`，就是最后一个不大于目标值的数。比如在数组 [0, 1, 1, 1, 1] 中查找数字 1，就会返回最后一个数字 1 的位置 4，这在有些情况下是需要这么做的。

第三类应用实例：

[Kth Smallest Element in a Sorted Matrix](#)

第三类变形应用示例：

[Sqrt\(x\)](#)

5.1.9 第四类：用子函数当作判断关系（通常由 `mid` 计算得出）

这是最令博主头疼的一类，而且通常情况下都很难。因为这里在二分查找法重要的比较大小的地方使用到了子函数，并不是之前三类中简单的数字大小的比较，比如 [Split Array Largest Sum](#) 那道题中的解法一，就是根据是否能分割数组来确定下一步搜索的范围。类似的还有 [Guess Number Higher or Lower](#) 这道题，是根据给定函数 `guess` 的返回值情况来确定搜索的范围。对于这类题目，博主也很无奈，遇到了只能自求多福了。

第四类应用实例：

[Split Array Largest Sum](#), [Guess Number Higher or Lower](#), [Find K Closest Elements](#), [Find K-th Smallest Pair Distance](#), [Kth Smallest Number in Multiplication Table](#), [Maximum Average Subarray II](#), [Minimize Max Distance to Gas Station](#), [Swim in Rising Water](#), [Koko Eating Bananas](#), [Nth Magical Number](#)

5.1.10 第五类：其他（通常 `target` 值不固定）

有些题目不属于上述的四类，但是还是需要用到二分搜索法，比如这道 [Find Peak Element](#)，求的是数组的局部峰值。由于是求的峰值，需要跟相邻的数字比较，那么 `target` 就不是一个固定的值，而且这道题的一定要注意的是 `right` 的初始化，一定要是 `nums.size() - 1`，这是由于算出了 `mid` 后，`nums[mid]` 要和 `nums[mid+1]` 比较，如果 `right` 初始化为 `nums.size()` 的话，`mid+1` 可能会越界，从而不能找到正确的值，同时 `while` 循环的终止条件必须是 `left < right`，不能有等号。

类似的还有一道 H-Index II, 这道题的 target 也不是一个固定值, 而是 len-mid, 这就很意思了, 跟上面的 $\text{nums}[\text{mid}+1]$ 有异曲同工之妙, target 值都随着 mid 值的变化而变化, 这里的 right 的初始化, 一定要是 $\text{nums.size()} - 1$, 而 while 循环的终止条件必须是 $\text{left} \leq \text{right}$, 这里又必须要有等号, 是不是很头大 -.-.!!!

其实仔细分析的话, 可以发现其实这跟第四类还是比较相似, 相似点是都很难 -.-.!!!, 第四类中虽然是用子函数来判断关系, 但大部分时候 mid 也会作为一个参数带入子函数进行计算, 这样实际上最终算出的值还是受 mid 的影响, 但是 right 却可以初始化为数组长度, 循环条件也可以不带等号, 大家可以对比区别一下~

第五类应用实例:

Find Peak Element

H-Index II

5.2 793. Preimage Size of Factorial Zeroes Function

Let $f(x)$ be the number of zeroes at the end of $x!$. Recall that $x! = 1 * 2 * 3 * \dots * x$ and by convention, $0! = 1$.

For example, $f(3) = 0$ because $3! = 6$ has no zeroes at the end, while $f(11) = 2$ because $11! = 39916800$ has two zeroes at the end. Given an integer k , return the number of non-negative integers x have the property that $f(x) = k$.

```
private long numberOfTrailingZeros(long v) {
    long cnt = 0;
    for (; v > 0; v /= 5)
        cnt += v / 5;
    return cnt;
}

public int preimageSizeFZF(int k) {
    long left = 0, right = 5l * (k + 1);
    while (left < right) {
        long mid = left + (right - left) / 2;
        long cnt = numberOfTrailingZeros(mid);
        if (cnt == k) return 5;
        if (cnt < k) left = mid + 1;
        else right = mid;
    }
    return 0;
}
```

- 下面这种解法是把子函数融到了 while 循环内, 使得看起来更加简洁一些, 解题思路跟上面的解法一模一样, 参见代码如下:

```
public int preimageSizeFZF(int k) {
    long left = 0, right = 5l * (k + 1);
    while (left < right) {
        long mid = left + (right - left) / 2, cnt = 0;
        for (long i = 5; mid / i > 0; i *= 5)
            cnt += mid / i;
        if (cnt == k) return 5;
        if (cnt < k) left = mid + 1;
        else right = mid;
    }
    return 0;
}
```

下面这种解法也挺巧妙的, 也是根据观察规律推出来的, 我们首先来看 x 为 1 到 25 的情况:

x:	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
f(x):	0 0 0 0 1 1 1 1 2 2 2 2 3 3 3 3 4 4 4 4 4 6
g(x):	0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 2

这里, $f(x)$ 是表示 $x!$ 末尾零的个数, 而 $g(x) = f(x) - f(x-1)$, 其实还可以通过观察发现, $f(x) = \sum(g(x))$.

再仔细观察上面的数字, 发现 $g(x)$ 有正值的时候都是当 x 是 5 的倍数的时候, 那么来专门看一下 x 是 5 的倍数时的情况吧:

x:	5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125
g(x):	1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 3

仔细观察上面的红色数字, $g(x)=1$ 时, 是 5 的倍数, $g(x)=2$ 时, 都是 25 的倍数, $g(x)=3$ 时, 是 125 的倍数, 那么就有:

```
g(x) = 0      if x % 5 != 0,
g(x) >= 1     if x % 5 == 0,
g(x) >= 2     if x % 25 == 0.
```

如果继续将上面的数字写下去，就可以发现规律， $g(x)$ 按照 1 1 1 1 x 的规律重复五次，第五次的时候 x 自增 1。再继续观察：

- 当 $x=25$ 时， $g(x)=2$ ，此时 $K=5$ 被跳过了。
- 当 $x=50$ 时， $g(x)=2$ ，此时 $K=11$ 被跳过了。
- 当 $x=75$ 时， $g(x)=2$ ，此时 $K=17$ 被跳过了。
- 当 $x=100$ 时， $g(x)=2$ ，此时 $K=23$ 被跳过了。
- 当 $x=125$ 时， $g(x)=3$ ，此时 $K=29, 30$ 被跳过了。

进一步，可以发现如下规律：

$$5(=1*5), 11(=6*1+5), 17(=6*2+5), 23(=6*3+5), 29(=6*4+5), 30(=6*5), 36(=31+5), 42(=31+6+5), 48(=31+6*2+5)$$

这些使得 x 不存在的 K ，出现都是有规律的，它们减去一个特定的基数 $base$ 后，都是余 5，而余 1, 2, 3, 4 的，都是返回 5。那么这个基数 $base$ ，实际是 1, 6, 31, 156, ...，是由 $base = base * 5 + 1$ ，不断构成的，通过这种不断对基数取余的操作，可以最终将 K 降为小于等于 5 的数，就可以直接返回结果了，参见代码如下：

```
public int preimageSizeFZF(int k) {
    if (k < 5) return 5;
    int base = 1;
    while (base * 5 + 1 <= k)
        base = base * 5 + 1;
    if (k / base == 5) return 0;
    return preimageSizeFZF(k % base);
}
```

5.3 2040. Kth Smallest Product of Two Sorted Arrays - Hard

Given two sorted 0-indexed integer arrays nums1 and nums2 as well as an integer k , return the k th (1-based) smallest product of $\text{nums1}[i] * \text{nums2}[j]$ where $0 \leq i < \text{nums1.length}$ and $0 \leq j < \text{nums2.length}$.

Thinking Process

- I can put the index pair for the two arrays in a priority queue and compute the answer gradually. However, the K can be up to 10^9 . This will lead to TLE.
- The element can be negative. Maybe I need to know the number of negative elements and handle 4 different combinations: (negative array 1, negative array 2), (negative array 1, positive array 2), (positive array 1, negative array 2), (positive array 1, positive array 2). At least, I can know the number of products of each combination and locate k -th product among them.
- Even though I know which combination the k -th product belongs to, it doesn't guarantee I can use the priority queue approach. I need another hint.
- Continue with above, I think I need some way to eliminate some number of products step by step to reach the goal.
- Since the array is sorted, if I turn my attention on $\text{nums1}[i] \times \text{nums2}[j]$, I can know there are $j + 1$ products which are less than or equal to $\text{nums1}[i] \times \text{nums2}[j]$ that are generated by $\text{nums1}[i]$. Then I realize that I should try the binary search.

Algorithm

- Binary search the answer
- For each $\text{nums1}[i]$, count the number of products that are less than or equal to the current guess

```
private static long INF = (long)1e10;
public long kthSmallestProduct(int[] a, int[] b, long k) {
    int m = a.length, n = b.length;
    long lo = -INF-1, hi = INF + 1;
    while (lo < hi) {
        long mid = lo + (hi - lo) / 2, cnt = 0;
        for (int i : a) { // 对于数组 a 中的每一个数与 b 中元素的乘积，数 <=mid 的个数，二分搜索
            if (i >= 0) {
                int l = 0, r = n-1, p = 0;
                while (l <= r) {
                    int c = l + (r - l) / 2;
                    long mul = i * (long)b[c];
                    if (mul <= mid) {
                        p = c + 1;
                        l = c + 1;
                    } else r = c - 1;
                }
                cnt += p;
            } else { // i < 0
                if (i * b[0] <= mid) cnt++;
            }
        }
        if (cnt < k) lo = mid + 1;
        else hi = mid;
    }
    return lo;
}
```

```

int l = 0, r = n-1, p = 0;
while (l <= r) {
    int c = l + (r - l) / 2;
    long mul = i * (long)b[c];
    if (mul <= mid) {
        p = n - c; // i < 0, 数右边 <= mid 的个数
        r = c - 1;
    } else l = c + 1;
}
cnt += p;
}
if (cnt >= k) hi = mid;
else lo = mid + 1;
}
return lo;
}

```

- 另一种相当于换汤不换药的写法

```

private static long INF = (long)1e10;
public long kthSmallestProduct(int[] a, int[] b, long k) {
    long lo = -INF, hi = INF;
    while (lo < hi) {
        long mid = lo + hi + 1 >> 1;
        if (f(a, b, mid) < k) lo = mid;
        else hi = mid - 1;
    }
    return lo;
}
private long f(int[] a, int[] b, long mid) {
    long cnt = 0;
    for (int v : a) {
        int l = 0, r = b.length;
        if (v < 0) {
            while (l < r) {
                int m = l + r >> 1;
                if ((long)v * b[m] >= mid) l = m + 1;
                else r = m;
            }
            cnt += b.length - l;
        } else { // v >= 0
            while (l < r) {
                int m = l + r >> 1;
                if ((long)v * b[m] < mid) l = m + 1;
                else r = m;
            }
            cnt += l;
        }
    }
    return cnt;
}

```

Chapter 6

字符串

6.1 总结

- Rabin-Karp Rolling Hash 算法

6.2 KMP 算法

6.3 1392. Longest Happy Prefix - Hard : Use Longest Prefix Suffix (KMP-table) or String Hashing.

A string is called a happy prefix if it is a non-empty prefix which is also a suffix (excluding itself).

Given a string s, return the longest happy prefix of s. Return an empty string "" if no such prefix exists.

6.3.1 解题思路与分析: KMP 算法

给定一个字符串，其长度为 k 的前缀和长度为 k 后缀有可能相等，问最大的 k 对应的那个前缀是什么。

题目本质上是求 KMP 算法中的 next 数组 (KMP 有个原始版本和优化版本，这里指的是原始版本。对于原始版本， $n[i]$ 表示 $s[0:i-1]$ 中最长相等前后缀的长度，规定 $n[0] = -1$)。

我们考虑 $n[i]$ 和 $n[i-1]$ 的递推关系。

首先 $n[1] = 0$ ，我们是可以确定的。对于 $n[i-1]$ ，假设 $n[i-1] = t$ ，意味着 $s[0:t-1] = s[i-t-1:i-2]$ ：

1、若 $s[i-1] = s[t]$ ，那么就有 $s[0:t] = s[i-t-1:i-1]$ ，所以 $n[i] \geq t+1$ 。但如果 $n[i] > t+1$ 的话，就会导致 $s[0:t+1] = s[i-t-2:i-1]$ ，从而有 $s[0:t] = s[i-t-2:i-2]$ ，这与 $n[i-1] = t$ 矛盾了，所以有 $n[i] = t+1$ 。

2、若 $s[i-1] \neq s[t]$ ，根据 next 数组的定义， $s[i-1]$ 要继续和 $s[n[t]]$ 进行比较，如果不等，则继续和 $s[n[n[t]]]$ 进行比较，如此这样下去，直到与 $s[i-1] = s[n^k[t]]$ 相等为止，相等后， $n[i] = n^k[t] + 1$ ；若一直都不等，则 $n[i] = n[0] + 1 = -1 + 1 = 0$ ，也是对的。证明与 1 类似。代码如下：

```
public String longestPrefix(String ss) {
    int n = ss.length();
    char [] s = ss.toCharArray();
    int [] lps = new int [n];
    for (int i = 1, j = 0; i < n; i++) {
        while (j > 0 && s[i] != s[j])
            j = lps[j-1];
        if (s[i] == s[j])
            lps[i] = ++j;
    }
    return ss.substring(0, lps[n-1]);
}
```

6.3.2 rolling hash

Time complexity: O(n) / worst case: O(n²)

Space complexity: O(1)

```
public String longestPrefix(String s) { // 容易出错, KMP 写起来比较简单
    int n = s.length(), hashPre = 0, hashSuf = 0;
    int left = 0, right = n-1, pow = 1, maxLen = 0;
    String res = "";
    while (left < n-1) {
        hashPre = hashPre * 31 + s.charAt(left);
        hashSuf = hashSuf + s.charAt(right)*pow;
        if (hashPre == hashSuf) maxLen = left + 1;
        left++;
        right--;
        pow *= 31;
    }
    return maxLen == 0 ? "" : s.substring(0, maxLen);
}
```

6.4 1910. Remove All Occurrences of a Substring - Medium 可用 KMP 算法

Given two strings s and $part$, perform the following operation on s until all occurrences of the substring $part$ are removed:

Find the leftmost occurrence of the substring $part$ and remove it from s . Return s after removing all occurrences of $part$.

A substring is a contiguous sequence of characters in a string.

```
public String removeOccurrences(String s, String part) {
    if (!s.contains(part)) return s;
    int n = s.length();
    int m = part.length();
    while (s.contains(part)) {
        int idx = s.indexOf(part);
        s = s.substring(0, idx) + (idx+m-1 == n-1 ? "" : s.substring(idx+m));
    }
    return s;
}
```

- 有人用了 KMP: 字符串匹配可以用 KMP 算法, 由于 p 始终不变, 可以先算一下 p 的 next 数组, 然后每次从 s 中找 p 的第一次出现, 删去之, 再重复进行这个过程

```
private int[] buildNext(String s) { // 找与每个位置字符不同的下一个字母的 idx
    int[] next = new int[s.length()];
    for (int i = 0, j = next[0] = -1; i < s.length()-1; ) {
        if (j == -1 || s.charAt(i) == s.charAt(j)) {
            i++;
            j++;
            next[i] = s.charAt(i) != s.charAt(j) ? j : next[j];
        } else j = next[j];
    }
    return next;
}
private int kmp(String s, String p, int[] next) { // 像是夹生饭, 半生不熟的
    for (int i = 0, j = 0; i < s.length(); ) {
        if (j == -1 || s.charAt(i) == p.charAt(j)) {
            i++;
            j++;
        } else j = next[j];
        if (j == p.length()) return i-j;
    }
    return -1;
}
public String removeOccurrences(String s, String part) {
    int[] next = buildNext(part);
    int idx = -1;
    while ((idx = kmp(s, part, next)) != -1)
        s = s.substring(0, idx) + s.substring(idx + part.length());
    return s;
}
```

6.5 467. Unique Substrings in Wraparound String - Medium Rabin-Karp Rolling Hash 算法

We define the string s to be the infinite wraparound string of "abcdefghijklmnopqrstuvwxyz", so s will look like this:

"...zabcdefghijklmnopqrstuvwxyzabcdefghijklmnopqrstuvwxyzabcdefghijklmnopqrstuvwxyzabcd....". Given a string p , return the number of unique non-empty substrings of p are present in s .

这道题说有一个无限长的封装字符串，然后又给了我们另一个字符串 p ，问我们 p 有多少非空子字符串在封装字符串中。我们通过观察题目中的例子可以发现，由于封装字符串是 26 个字符按顺序无限循环组成的，那么满足题意的 p 的子字符串要么是单一的字符，要么是按字母顺序的子字符串。这道题遍历 p 的所有子字符串会 TLE，因为如果 p 很大的话，子字符串很多，会有大量的满足题意的重复子字符串，必须要用到 trick，而所谓技巧就是一般来说你想不到的方法。我们看 $abcd$ 这个字符串，以 d 结尾的子字符串有 $abcd$, bcd , cd , d , 那么我们可以发现 bcd 或者 cd 这些以 d 结尾的字符串的子字符串都包含在 $abcd$ 中，那么我们知道以某个字符结束的最大字符串包含其他以该字符结束的字符串的所有子字符串，说起来很拗口，但是理解了我上面举的例子就行。那么题目就可以转换为分别求出以每个字符 (a-z) 为结束字符的最长连续字符串就行了，我们用一个数组 cnt 记录下来，最后在求出数组 cnt 的所有数字之和就是我们要的结果啦，

```
public int findSubstringInWraproundString(String p) {
    int n = p.length();
    int [] arr = new int [n];
    int [] cnt = new int [26];
    for (int i = 0; i < n; i++)
        arr[i] = p.charAt(i) - 'a';
    int res = 0, maxLen = 0;
    for (int i = 0; i < n; i++) {
        if (i > 0 && (arr[i-1]+1) % 26 == arr[i]) // 判断前一个位置上的字符比现位字符小 1
            ++maxLen;
        else maxLen = 1;
        cnt[arr[i]] = Math.max(cnt[arr[i]], maxLen);
    }
    for (int i = 0; i < 26; i++)
        res += cnt[i];
    return res;
}
```

6.6 1044. Longest Duplicate Substring - Hard

Given a string s , consider all duplicated substrings: (contiguous) substrings of s that occur 2 or more times. The occurrences may overlap.

Return any duplicated substring that has the longest possible length. If s does not have a duplicated substring, the answer is "".

```
public static int base = 26; // 256
public static int mod = (1 << 31) - 1;
public static boolean match(String str1, String str2) {
    assert str1.length() == str2.length();
    for (int i = 0; i < str1.length(); i++)
        if (str1.charAt(i) != str2.charAt(i))
            return false;
    return true;
}
private String search(String s, int v) { // v: substring length
    int n = s.length();
    long hash = 0L, mp = 1L;           // to avoid overflow, long long long
    Map<Long, List<Integer>> map = new HashMap<>();
    for (int j = 0; j < v; j++) {
        hash = ((hash * base) + s.charAt(j)) % mod;
        if (j >= 1)
            mp = mp * base % mod;      // 先乘好准备好了，准备着备用
    }
    map.computeIfAbsent(hash, k->new ArrayList<>()).add(0);
    for (int i = 1; i+v <= n; i++) {
        hash = ((hash - s.charAt(i-1) * mp % mod + mod) % mod * base % mod + s.charAt(i+v-1)) % mod; // mod
        if (map.containsKey(hash))
            for (int idx : map.get(hash))
                if (match(s.substring(i, i+v), s.substring(idx, idx+v)))
                    return s.substring(i, i+v);
        map.computeIfAbsent(hash, k->new ArrayList<>()).add(i);
    }
    return null;
}
public String longestDupSubstring(String s) {
    int n = s.length();
    int l = 0, r = n;
    String res = "";
    while (l <= r) {
        int m = l + (r-l) / 2;
        String tmp = search(s, m);
        if (tmp == null) r = m-1;
        else {
```

```

        if (tmp.length() > res.length())
            res = tmp;
        l = m+1;
    }
}
return res;
}

```

6.7 1156. Swap For Longest Repeated Character Substring - Medium

You are given a string text. You can swap two of the characters in the text.

Return the length of the longest substring with repeated characters.

给你一个字符串，如何找最长的重复子串，博主会数连续相同的字符，若此时有一个不同字符出现了，只要后面还有相同的字符，就会继续数下去，因为有一次交换的机会，什么时候停止呢，当再次出现不同字符的时候就停止，或者是当前统计个数等于该字符出现的总个数时也停止，因为得到的结果不可能超过某个字符出现的总个数。所以可以先统计每个字符的出现次数，然后开始遍历字符，对于每个遍历到的字符，都开始数之后跟其相等的字符，新建变量 j, cnt, 和 diff，当 j 小于 n，且当前字符和比较字符相同，或者 diff 等于 0，且 cnt 小于比较字符出现的总个数时进行遍历，若当前遍历到的字符和要比较的字符不相等，说明该使用交换操作了，diff 自增 1，此时将 i 更新为 j-1，这是一个优化操作，可以避免一些不必要的计算，下次从这个位置往后统计，也相当于重置了 diff。还有就是这个 cnt 小于字符出现总个数这个条件卡的非常好，即便下一个还是相同字符，也不能再统计了，因为最后的这个相同字符可能是用来交换前面的断点位置的。每次用统计出来的 cnt 更新结果 res，但是一个方向的遍历可能无法应对所有情况，比如“acbaaa”，若只是从前往后遍历，那么最终只能得到 3，而正确的答案是 4，因为可以将 b 和第一个 a 交换，所以还需要从后往前进行一次相同的操作，这样才能得到正确的答案，参见代码如下：

```

public int maxRepOpt1(String s) { // O(n^2)
    int n = s.length(), ans = 0;
    Map<Character, Integer> charCnt = new HashMap<>();
    for (char c : s.toCharArray())
        charCnt.put(c, charCnt.getOrDefault(c, 0) + 1);
    for (int i = 0; i < n; i++) {
        char cur = s.charAt(i);
        int j = i, cnt = 0, dif = 0;
        while (j < n && (cur == s.charAt(j) || dif == 0) && cnt < charCnt.get(cur)) {
            if (cur != s.charAt(j)) {
                ++dif;
                i = j-1; // exchanged once, i moves to be the repeated sequence tail
            }
            ++cnt;
            ++j;
        }
        ans = Math.max(ans, cnt);
    }
    for (int i = n-1; i >= 0; i--) {
        char cur = s.charAt(i);
        int j = i, cnt = 0, dif = 0;
        while (j >= 0 && (cur == s.charAt(j) || dif == 0) && cnt < charCnt.get(cur)) {
            if (cur != s.charAt(j)) {
                ++dif;
                i = j+1;
            }
            ++cnt;
            --j;
        }
        ans = Math.max(ans, cnt);
    }
    return res;
}

```

- O(N) 解法

上面的解法严格来说还是平方级的，再来看一种线性时间的解法，可能比较难想，由于这里需要关注的是相同字符的出现位置，所以可以将所有相同的字符的位置都放到一个数组中，那么这里就建立一个字符与其出现位置数组之间的映射。由于题目中限制了只有英文字母，所以可以按照每个字母进行遍历，直接遍历每个字符的位置数组，这里新建变量 cnt, cnt2, 和 mx，其中 cnt 统计的是连续字母的个数，cnt2 相当于一个临时变量，当使用交换操作时，保存之前的 cnt 值，mx 为二者之和。在遍历完某个字母位置数组之后，最后看一下若该字母出现总个数大于 mx，则说明交换后的字母还没有统计进去，不管之前有没有使用交换操作，都需要加上这个额外的一个，参见代码如下：

```

public int maxRepOpt1(String s) { // O(n^2)
    int n = s.length(), ans = 0;
    Map<Character, List<Integer>> idxMap = new HashMap<>();
    for (int i = 0; i < n; i++)

```

```

idxMap.computeIfAbsent(s.charAt(i), k -> new ArrayList<>().add(i));
for (char c = 'a'; c <= 'z'; c++) {
    if (!idxMap.containsKey(c)) continue;
    int cnt = 1, cntb = 0, max = 0;
    List<Integer> idxs = idxMap.get(c);
    for (int i = 1; i < idxs.size(); i++) {
        if (idxs.get(i) == idxs.get(i-1) + 1) // aa
            ++cnt;
        else {
            cntb = (idxs.get(i) == idxs.get(i-1) + 2) ? cnt : 0; // aba ?
            cnt = 1;
        }
        max = Math.max(max, cnt + cntb);
    }
    ans = Math.max(ans, max + (idxs.size() > max ? 1 : 0)); // aaaaabaaaaaca 多于两个重复子段，中间替换字符可是是相同的
}
return ans;
}

```

算法

(动态规划) $O(n)$

1. 我们分别考察每种字符所能得到的最大答案。
2. 假设我们当前考察的是字符 c 。我们通过遍历一次数组的方式，找到单字符就是 c 的情况下的最长单重复子串。
3. 设 cnt 为字符串中 c 出现的次数； $f(i)$ 表示以 i 结尾且没有替换过不是 c 的字符时的最长但重复子串； $g(i)$ 表示以 i 结尾且替换过不是 c 的字符时的最长但重复子串。
4. 当 $c == \text{text}[i]$ 时，累加 cnt ， $f(i) = f(i-1) + 1$, $g(i) = g(i-1) + 1$ ，也就是都可以沿着上一次的结果往后累加 1 的长度。
5. 如果 $c != \text{text}[i]$ ，则 $g(i) = f(i-1) + 1$ ，表示我们强制替换 $\text{text}[i]$ 的字符为 c （注意不是交换），所以要从没有替换过的 f 数组转移； $f(i) = 0$ ，表示如果不替换，那么长度只能归 0。交换只是替换的一种特殊形式，我们都先假设总能交换。如果不能交换，如 aabaaa 我们无法把中间的 b 用一个新的 a 交换时，可以通过答案不能超过 cnt 来特判。
6. 答案为 $\min(cnt, \max(f(i), g(i)))$ ，也就是所有的 f 和 g 的最大值，但结果不能超过 cnt 。
7. 这里的 f 和 g 数组因为之和上一位有关，所以可以化简为变量。

时间复杂度

- 枚举每种字符时，只需要遍历数组一次，故时间复杂度为 $O(n)$ 。

空间复杂度

- 动态规划 • 优化后，只需要常数个变量，故空间复杂度为 $O(1)$ 。

```

private int solve(char c, String s) {
    int n = s.length(), max = 0;
    int f = 0, g = 0, cnt = 0;
    for (int i = 0; i < n; i++) {
        if (c == s.charAt(i)) {
            f++;
            g++;
            cnt++;
        } else {
            g = f + 1;
            f = 0;
        }
        max = Math.max(max, Math.max(f, g));
    }
    return Math.min(max, cnt);
}
public int maxRepOpt1(String s) {
    int n = s.length(), ans = 0;
    for (char i = 'a'; i <= 'z'; i++)
        ans = Math.max(ans, solve(i, s));
    return ans;
}

```

6.8 395. Longest Substring with At Least K Repeating Characters - Medium

Given a string s and an integer k, return the length of the longest substring of s such that the frequency of each character in this substring is greater than or equal to k.

```
// 由于字母只有 26 个，而整型 mask 有 32 位，足够用了，  
// 每一位代表一个字母，如果为 1，表示该字母不够 k 次，如果为 0 就表示已经出现了 k 次，这种思路真是太聪明了，  
// 隐约记得这种用法在之前的题目中也用过，但是博主并不能举一反三（沮丧脸：( )，还得继续努力啊。  
// 遍历字符串，对于每一个字符，都将其视为起点，然后遍历到末尾，增加 HashMap 中字母的出现次数，如果其小于 k，将 mask 的对应位改为 1，如果大于等于 k，将 mask 对应位改  
// 然后看 mask 是否为 0，是的话就更新 res 结果，然后把当前满足要求的子字符串的起始位置 j 保存到 max_idx 中，等内层循环结束后，将外层循环变量 i 赋值为 max_idx+1，继  
public int longestSubstring(String s, int k) { // O(N^2)  
    int n = s.length(), res = 0, i = 0;  
    while (i + k <= n) {  
        int[] m = new int[26];  
        int mask = 0, maxIdx = i;  
        for (int j = i; j < n; j++) {  
            int t = s.charAt(j) - 'a';  
            m[t]++;  
            if (m[t] < k) mask |= (1 << t);  
            else mask &= (~(1 << t));  
            if (mask == 0) {  
                res = Math.max(res, j - i + 1);  
                maxIdx = j;  
            }  
        }  
        i = maxIdx + 1;  
    }  
    return res;  
}
```

- 双指针 sliding window O(N)

```
public int longestSubstring(String s, int k) {  
    int n = s.length(), res = 0;  
    for (int cnt = 1; cnt <= 26; cnt++) {  
        int start = 0, i = 0, uniqueCnt = 0;  
        int[] charCntr = new int[26];  
        while (i < n) {  
            boolean valid = true;  
            if (charCntr[s.charAt(i++) - 'a']++ == 0) ++ uniqueCnt;  
            while (uniqueCnt > cnt)  
                if (--charCntr[s.charAt(start++) - 'a'] == 0) --uniqueCnt;  
            for (int j = 0; j < 26; j++)  
                if (charCntr[j] > 0 && charCntr[j] < k) valid = false;  
            if (valid) res = Math.max(res, i - start);  
        }  
    }  
    return res;  
}
```

- 分治：分而治之

```
public int longestSubstring(String s, int k) { // str.split("[dkfldjf]")  
    int n = s.length();  
    if (n < k) return 0;  
    if (n == k && s.chars().distinct().count() == 1) return k;  
    int[] cnt = new int[26];  
    for (int i = 0; i < n; i++)  
        cnt[s.charAt(i) - 'a']++;  
    if (Arrays.stream(cnt).max().getAsInt() < k) return 0;  
    StringBuilder sb = new StringBuilder("[");  
    for (int i = 0; i < 26; i++)  
        if (cnt[i] < k && cnt[i] != 0)  
            sb.append((char)(i + 'a'));  
    sb.append(']');  
    if (sb.length() == 2) return n;  
    String[] sa = s.split(sb.toString()); // str.split("[-*=/]") pay attention to the format  
System.out.println(Arrays.toString(sa));  
    int max = 0;  
    for (int i = 0; i < sa.length; i++)  
        max = Math.max(max, longestSubstring(sa[i], k));  
    return max;  
}  
public int longestSubstring(String s, int k) { // 人工手动拆分  
    int n = s.length(), maxIdx = 0, res = 0;  
    int[] cnt = new int[128];  
    boolean valid = true;  
    for (char c : s.toCharArray())  
        cnt[c]++;
}

```

```

for (int i = 0; i < n; i++) {
    if (cnt[s.charAt(i)] < k) {
        res = Math.max(res, longestSubstring(s.substring(maxIdx, i), k));
        valid = false;
        maxIdx = i+1;
    }
}
return valid ? n : Math.max(res, longestSubstring(s.substring(maxIdx, n), k));
}

```

6.9 1830. Minimum Number of Operations to Make String Sorted - Hard

排列组合费小马快速幂

You are given a string s (0-indexed). You are asked to perform the following operation on s until you get a sorted string:

Find the largest index i such that $1 \leq i < s.length$ and $s[i] < s[i - 1]$. Find the largest index j such that $i \leq j < s.length$ and $s[k] < s[i - 1]$ for all the possible values of k in the range $[i, j]$ inclusive. Swap the two characters at indices $i - 1$ and j . Reverse the suffix starting at index i . Return the number of operations needed to make the string sorted. Since the answer can be too large, return it modulo $10^9 + 7$.

6.9.1 解题思路与分析

题中每次对字符串 s 执行的操作，是将其变为由当前字母组成的前一字典序的字符串。因此求最少操作次数，等价于求解该字符串在由当前字母组成的所有排列中的字典序；

求比当前字符串 s 小的排列个数，可通过排列组合公式计算得到；

排列组合公式中的阶乘逆元取模，可通过费马小定理，转化为对模数的乘方进行计算；

可通过快速乘方算法，进一步提高对乘方的计算效率。

```

private int quickmul(int base, int exp) { // 快速乘方算法
    long ans = 1L;
    while (exp > 0) {
        if ((exp & 1) == 1) // 指数是奇数，就先乘一次 base
            ans = ans * base % mod;
        base = (int)((long)base * base % mod); // 底平方 (指数变偶数之后)
        exp >>= 1; // 指数除 2，快速计算
    }
    return (int)ans;
}
int mod = (int)1e9 + 7;
public int makeStringSorted(String t) {
    int n = t.length();
    char [] s = t.toCharArray();
    int [] cnt = new int [26]; // 记录剩余字符串中各字母个数
    Arrays.fill(cnt, 0);
    for (int i = 0; i < n; i++)
        cnt[s[i]-'a']++;
    int [] fact = new int [n+1];
    int [] finv = new int [n+1];
    fact[0] = 1;
    finv[0] = 1;
    for (int i = 1; i <= n; i++) {
        fact[i] = (int)((long)fact[i-1] * i % mod); // fact[i] = i! % mod
        finv[i] = quickmul(fact[i], mod - 2); // 费马小定理计算乘法逆元，facinv[i] = (i!) ^ -1 % mod
    }
    long ans = 0L;
    for (int i = 0; i < n-1; i++) {
        int lessCnt = 0; // 比当前位置小的字母总数
        for (int j = 0; j < s[i]-'a'; j++)
            lessCnt += cnt[j];
        long upper = (long)lessCnt * fact[n-1-i] % mod; // 排列公式分子
        for (int j = 0; j < 26; j++)
            upper = upper * finv[cnt[j]] % mod;
        ans = (ans + upper) % mod;
        cnt[s[i]-'a']--; // 指针右移
    }
    return (int)ans;
}

```

6.10 1960. Maximum Product of the Length of Two Palindromic Substrings

- Hard 马拉车算法 todo

You are given a 0-indexed string s and are tasked with finding two non-intersecting palindromic substrings of odd length such that the product of their lengths is maximized.

More formally, you want to choose four integers i, j, k, l such that $0 \leq i \leq j < k \leq l < s.length$ and both the substrings $s[i\dots j]$ and $s[k\dots l]$ are palindromes and have odd lengths. $s[i\dots j]$ denotes a substring from index i to index j inclusive.

Return the maximum possible product of the lengths of the two non-intersecting palindromic substrings.

A palindrome is a string that is the same forward and backward. A substring is a contiguous sequence of characters in a string.

6.10.1 解题思路与分析

这个马拉车，好像还是很半生不熟，要好好理解消化一下

[https://leetcode.com/problems/maximum-product-of-the-length-of-two-palindromic-substrings/discuss/1393288/Java-100-O\(n\)-time-O\(n\)-space-using-Manacher's-algorithm](https://leetcode.com/problems/maximum-product-of-the-length-of-two-palindromic-substrings/discuss/1393288/Java-100-O(n)-time-O(n)-space-using-Manacher's-algorithm)

```

private static int[] manacherOdd(String str) {
    int n = str.length();
    char[] s = str.toCharArray();
    int [] ans = new int [n];
    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int len = i > r ? 1 : Math.min(ans[l+r-i], r-i+1);
        int maxLen = Math.min(i, n-1-i);
        int x = i - len, y = i + len;
        while (len <= maxLen && s[x--] == s[y++]) len++;
        ans[i] = len--;
        if (i + len > r) {
            l = i - len;
            r = i + len;
        }
    }
    return ans;
}
public long maxProduct(String s) { // 这个马拉车，得多写几遍
    int n = s.length();
    int [] d = manacherOdd(s);
    int [] l = new int [n], r = new int [n];
    for (int i = 0; i < n; i++) {
        l[i+d[i]-1] = Math.max(l[i+d[i]-1], 2 * d[i]-1);
        r[i-d[i]+1] = 2 * d[i] - 1;
    }
    for (int i = n-2, j = n-1; i >= 0; i--, j--)
        l[i] = Math.max(l[i], l[j]-2);
    for (int i = 1, j = 0; i < n; i++, j++)
        r[i] = Math.max(r[i], r[j]-2);
    for (int i = 1, j = 0; i < n; i++, j++)
        l[i] = Math.max(l[i], l[j]);
    for (int i = n-2, j = n-1; i >= 0; i--, j--)
        r[i] = Math.max(r[i], r[j]);
    long ans = 1;
    for (int i = 1; i < n; i++)
        ans = Math.max(ans, (long)l[i-1] * r[i]);
    return ans;
}

```

6.10.2 DP 解：要再好好理解消化一下

Chapter 7

Segment Tree 与 Binary Index Tree 线段树与 树状数组

线段树 (segment tree)，顾名思义，是用来存放给定区间 (segment, or interval) 内对应信息的一种数据结构。与树状数组 (binary indexed tree) 相似，线段树也用来处理数组相应的区间查询 (range query) 和元素更新 (update) 操作。与树状数组不同的是，线段树不止可以适用于区间求和的查询，也可以进行区间最大值，区间最小值 (Range Minimum/Maximum Query problem) 或者区间异或值的查询。

对应于树状数组，线段树进行更新 (update) 的操作为 $O(\log n)$ ，进行区间查询 (range query) 的操作也为 $O(\log n)$ 。

7.1 1157. Online Majority Element In Subarray - Hard

Design a data structure that efficiently finds the majority element of a given subarray.

The majority element of a subarray is an element that occurs threshold times or more in the subarray.

Implementing the MajorityChecker class:

MajorityChecker(int[] arr) Initializes the instance of the class with the given array arr. int query(int left, int right, int threshold) returns the element in the subarray arr[left...right] that occurs at least threshold times, or -1 if no such element exists.

```
private class Node {
    private int bgn;
    private int end;
    private int val;
    private int cnt; // sum
    private Node left;
    private Node right;
    public Node(int bgn, int end, int val, int cnt) {
        this.bgn = bgn;
        this.end = end;
        this.val = val;
        this.cnt = cnt;
        this.left = null;
        this.right = null;
    }
    public Node(int bgn, int end, int val, int cnt, Node left, Node right) {
        this.bgn = bgn;
        this.end = end;
        this.val = val;
        this.cnt = cnt;
        this.left = left;
        this.right = right;
    }
}
public void update(int index, int val) {
    updateTree(root, index, val);
}
public int cntRange(int left, int right) {
    return cntRangeFromTree(root, left, right);
}
private int cntRangeFromTree(Node r, int i, int j) {
    if (r == null || i < r.bgn || i > r.end) return 0;
    else if (i <= r.bgn && j >= r.end) return r.cnt;
    else return cntRangeFromTree(r.left, i, j) + cntRangeFromTree(r.right, i, j);
}
private void updateTree(Node r, int i, int va) {
    if (r == null || i < r.bgn || i > r.end) return;
```

```

else if (r.bgn == r.end && r.bgn == i) r.cnt = va;
else {
    updateTree(r.left, i, va);
    updateTree(r.right, i, va);
    int cnt = 0;
    if (r.left != null) cnt += r.left.cnt;
    if (r.right != null) cnt += r.right.cnt;
    r.cnt = cnt;
}
}

private Node buildTree(int[] arr, int i, int j) {
    if (i > j) return null;
    else if (i == j)
        return new Node(i, i, arr[i], 1);
    else {
        int mid = i + (j-i)/2;
        Node left = buildTree(arr, i, mid);
        Node right = buildTree(arr, mid+1, j);
        if (left.val == right.val)
            return new Node(i, j, left.val, left.cnt + right.cnt, left, right);
        else {
            if (left.cnt > right.cnt)
                return new Node(i, j, left.val, left.cnt-right.cnt, left, right);
            else return new Node(i, j, right.val, right.cnt-left.cnt, left, right);
        }
    }
}

// 排序数组中 第一个大于 tar 的下标
int upper_bound(List<Integer> list, int tar) {
    int l = 0, r = list.size();
    while (l < r) {
        int mid = l + (r-l)/2;
        if (list.get(mid) <= tar) l = mid+1;
        else r = mid;
    }
    return l;
}

// 排序数组中 第一个大于等于 tar 的下标
int lower_bound(List<Integer> list, int tar) {
    int l = 0, r = list.size()-1;
    while (l < r) {
        int mid = l + (r-l)/2;
        if (list.get(mid) < tar) l = mid+1;
        else r = mid;
    }
    return l;
}

/***
 * 构建线段树
 * @param arr 被构建数组
 * @param l 构建节点的左值 表示查询区域左边界
 * @param r 构建节点的右值 表示查询区域右边界
 * @return 以构建完成的线段树节点
 */
private SegTreeNode buildTree(int[] arr, int l, int r) {
    if (l > r) return null;
    // 初始化一个线段树节点
    SegTreeNode root = new SegTreeNode(l, r);
    // 叶子节点
    if (l == r) {
        // 众数就是当前值 计数为 1
        root.val = arr[l]; root.count = 1;
        return root;
    }

    int mid = (l+r)/2;
    // 构建左子节点
    root.left = buildTree(arr, l, mid);
    // 构建右子节点
    root.right = buildTree(arr, mid+1, r);
    // 整合父节点
    makeRoot(root);
    return root;
}

/***
 * 整合一个父节点
 * @param root 被整合节点
 */
private void makeRoot(SegTreeNode root) {
    if (null == root) return;
    // 如果该节点有左子节点 该节点的值"先"等于左子节点
    if (root.left != null) {
        root.val = root.left.val;
        root.count = root.left.count;
    }
}

```

```

// 如果该节点还有右子节点 融合父节点和子节点
if (root.right != null) {
    if (root.val == root.right.val)
        root.count = root.count + root.right.count;
    else {
        if (root.count >= root.right.count)
            root.count = root.count - root.right.count;
        else {
            root.val = root.right.val;
            root.count = root.right.count - root.count;
        }
    }
}
/***
 * 查询线段树
 * @param root 被查询节点
 * @param l 需要查询的范围左边界
 * @param r 需要查询的范围右边界
 */
private void searchSegTree(Node root, int l, int r) {
    if (root == null || l > r) return;
    if (root.bgn > r || root.end < l) return;

    // 当查询边界 覆盖 节点边界 该节点就是查询区域
    if (root.bgn >= l && root.end <= r) {
        if (key == root.val) cnt += root.cnt;
        else if (cnt <= root.cnt) {
            key = root.val;
            cnt = root.cnt - cnt;
        } else cnt = cnt - root.cnt;
        return;
    }

    int mid = (root.end + root.bgn)/2;
    // root.bgn <= l <= mid 左节点也可以是查询区域
    if (l <= mid) // 这两个查询条件再好好想想 !!!!!!!!
        searchSegTree(root.left, l, r);
    // mid+1 <= r <= root.end 右节点也可以是查询区域
    if (r >= mid+1)
        searchSegTree(root.right, l, r);
}

// https://books.halfrost.com/leetcode/ChapterFour/1100~1199/1157.Online-Majority-Element-In-Subarray/ 也有一个直观图
// https://www.cnblogs.com/slowbirdoflsh/p/11381565.html 思路比较清晰
HashMap<Integer, List<Integer>> idx = new HashMap<>();
private Node root;
int key = 0, cnt = 0;
public MajorityChecker(int[] arr) {
    root = buildTree(arr, 0, arr.length-1);
    levelPrintTree(root);
    idx = new HashMap<>();
    for (int i = 0; i < arr.length; i++) {
        if (!idx.containsKey(arr[i]))
            idx.put(arr[i], new ArrayList<>());
        idx.get(arr[i]).add(i);
    }
}
public int countRangeSum(int[] nums, int lower, int upper) {
    MajorityChecker mc = new MajorityChecker(nums);
}
public int query(int left, int right, int threshold) {
    // 初始化 所查询众数 key 及辅助判断的计数 cnt
    key = 0; cnt = 0;
    // 查询线段树
    searchSegTree(root, left, right);
    // 如果查询区域没有众数 即 key 没被更改
    // 或者
    // 所查询出来的众数 在原数组中根本没有超出阈值的能力
    System.out.println("key: " + key);
    System.out.println("(idx.get(key) == null): " + (idx.get(key) == null));

    if (key == 0 || idx.get(key).size() < threshold) return -1;

    // 上确界 排序数组中 第一个大于 right 的下标
    int r = upper_bound(idx.get(key), right);
    // 下确界 排序数组中 第一个大于等于 left 的下标
    int l = lower_bound(idx.get(key), left);
    cnt = r - l;
    return cnt >= threshold ? key : -1;
}

```

7.2 1825. Finding MK Average - Hard

You are given two integers, m and k , and a stream of integers. You are tasked to implement a data structure that calculates the MKAverage for the stream.

The MKAverage can be calculated using these steps:

If the number of the elements in the stream is less than m you should consider the MKAverage to be -1 . Otherwise, copy the last m elements of the stream to a separate container. Remove the smallest k elements and the largest k elements from the container. Calculate the average value for the rest of the elements rounded down to the nearest integer. Implement the MKAverage class:

MKAverage(int m, int k) Initializes the MKAverage object with an empty stream and the two integers m and k . void addElement(int num) Inserts a new element num into the stream. int calculateMKAverage() Calculates and returns the MKAverage for the current stream rounded down to the nearest integer.

```
// 根据题意需要找到前 k 大的数，又需要求区间和，就自然想到线段树。写起来较不容易出错。
// 维护 2 个线段树数组，一个记录数的个数，一个记录区间值，
// 注意一般线段树中 [s, e] 指固定的区间，这里类似线段树求第 k 小的数，所以 [s, e] 指第 s 小的值到第 e 小的值的区间。
LinkedList<Integer> q;
int[] cnt;
long[] sum;
int m, k;
public MKAverage(int m, int k) {
    q = new LinkedList<>();
    cnt = new int[400001]; // space: 4N
    sum = new long[400001];
    this.m = m;
    this.k = k;
}
public void addElement(int num) {
    if (q.size() == m) {
        int v = q.pollFirst();
        insert(1, 0, 100000, v, -1);
    }
    insert(1, 0, 100000, num, 1);
    q.addLast(num);
}
public int calculateMKAverage() {
    if (q.size() < m) return -1;
    int s = k+1, e = m-k;
    return (int)(query(1, 0, 100000, s, e)/(m-2*k));
}
void insert(int idx, int l, int r, int v, long d) {
    cnt[idx] += d;
    sum[idx] += d*v;
    if (l == r) return;
    int m = l + (r-l)/2;
    if (v <= m)
        insert(idx<<1, l, m, v, d); // 向左子树查询
    else
        insert(idx<<1|1, m+1, r, v, d); // 向右子树查询
}
long query(int idx, int l, int r, int s, int e){//线段中第 s 个到第 e 个
    if (l == r) //起始和结束最多出现 2 次此情况
        int c = e-s+1;
        return (long)c*l;
    } else if (cnt[idx] == e-s+1) {
        return sum[idx];
    } else {
        int m = (l+r)/2;
        int c1 = cnt[idx<<1];
        int c2 = cnt[idx<<1|1];
        if (c1 >= e){
            return query(idx<<1, l, m, s, e);
        } else if (c1 >= s){
            return query(idx<<1, l, m, s, c1)+query(idx<<1|1, m+1, r, 1, e-c1);
        } else { //c1<
            return query(idx<<1|1, m+1, r, s-c1, e-c1);
        }
    }
}
```

- 数状数组的解法：另外第一次看到别人二分 + 树状数组也能求前 k 大的值。

```
// We can have a queue to maintain m elements
// Use two Fenwick tree, 1 for count and 1 for prefix sum
// Do 2 times binary search for the first k elements and the last k elements by using the count from our first fenwick tree
// We can get the sum by subtracting the sum of first k elements and sum of last k element by using our second fenwick tree
Queue<Integer> q = new LinkedList<>();
```

```

FenWick fone, ftwo;
int [] cnt = new int [100010];
long sum = 0;
int m,k;
public MKAverage(int m, int k) {
    this.m = m;
    this.k = k;
    long A [] = new long [100010];
    long B [] = new long [100010];
    fone = new FenWick(A);
    ftwo = new FenWick(B);
}
public void addElement(int num) {
    q.add(num);
    sum += num;
    fone.update(num, 1);
    ftwo.update(num, num);
    cnt[num]++;
}
public int calculateMKAverage() {
    if (q.size() < m) return -1;
    while (q.size() > m) {
        int cur = q.poll();
        cnt[cur]--;
        sum -= cur;
        fone.update(cur, -1);
        ftwo.update(cur, -cur);
    }
    // binary search for the first k (there may be duplicated)
    int l = 0, r = cnt.length-1;
    int i = -1, j = -1; // pos1, pos2
    while (l <= r) { // 二分查找总计数
        int m = (r + l) / 2;
        long count = fone.sumRange(0, m);
        if (count >= k) {
            i = m;
            r = m - 1;
        } else l = m+1;
    }
    // binary search for the last k (there may be duplicated)
    l = 0;
    r = cnt.length-1;
    while (l <= r) {
        int m = l + (r-l)/2;
        long count = ftwo.sumRange(m, cnt.length-1);
        if (count >= k) {
            j = m;
            l = m + 1;
        } else r = m-1;
    }
    long sum1 = ftwo.sumRange(0, i);
    long sum2 = ftwo.sumRange(j, cnt.length-1);
    long cnt1 = fone.sumRange(0, i);
    long cnt2 = fone.sumRange(j, cnt.length-1);
    if (cnt1 > k)
        sum1 -= i*(cnt1-k);
    if (cnt2 > k)
        sum2 -= j*(cnt2-k);
    long remain = sum - sum1 - sum2; // 总和，减去两边最小最大各 K 个数的和
    return (int)(remain / (m-2*k));
}
class FenWick {
    long tree []; //1-index based
    long A [];
    long arr[];
    public FenWick(long [] A) {
        this.A = A;
        arr = new long [A.length];
        tree = new long [A.length + 1];
    }
    public void update(int i, int v) {
        arr[i] += v;
        i++;
        while (i < tree.length) {
            tree[i] += v;
            i += (i & -i); // 这是的原理细节再回去复习一下
        }
    }
    public long sumRange(int i, int j) {
        return pre(j+1)-pre(i);
    }
    public long pre(int i) {
        long sum = 0;
        while (i > 0) {
            sum += tree[i];
            i -= i & -i;
        }
    }
}

```

```
i -= (i & -i);  
}  
return sum;  
}  
}
```

Chapter 8

树结构：各种新型数据结构

8.1 968. Binary Tree Cameras

You are given the root of a binary tree. We install cameras on the tree nodes where each camera at a node can monitor its parent, itself, and its immediate children. Return the minimum number of cameras needed to monitor all nodes of the tree.

```
// 对于每个节点，有一下三种 case:  
// case (1): 如果它有一个孩子，且这个孩子是叶子（状态 0），则它需要摄像头，res ++，然后返回 1，表示已经给它装上了摄像头。  
// case (2): 如果它有一个孩子，且这个孩子是叶子的父节点（状态 1），那么它已经被覆盖，返回 2。  
// case (0): 否则，这个节点无孩子，或者说，孩子都是状态 2，那么我们将这个节点视为叶子来处理。  
// 由于 dfs 最终返回后，整棵树的根节点的状态还未处理，因此需要判断，若根节点被视为叶子，需要在其上加一个摄像头。  
private int dfs(TreeNode r) {  
    // 空节点不需要被覆盖，归入情况 2  
    if (r == null) return 2; // do not need cover  
    int left = dfs(r.left); // 递归求左右孩子的状态  
    int right = dfs(r.right);  
    // 获取左右孩子状态之后的处理  
    // 有叶子孩子，加摄像头，归入情况 1  
    if (left == 0 || right == 0) {  
        res ++;  
        return 1;  
    }  
    // 孩子上有摄像头，说明此节点已被覆盖，情况 2;  
    if (left == 1 || right == 1) return 2;  
    return 0;  
}  
int res = 0;  
public int minCameraCover(TreeNode root) {  
    // 若根节点被视为叶子，需要在其上加一个摄像头  
    return (dfs(root) == 0 ? 1 : 0) + res;  
}
```

8.2 1696. Jump Game VI

You are given a 0-indexed integer array `nums` and an integer `k`. You are initially standing at index 0. In one move, you can jump at most `k` steps forward without going outside the boundaries of the array. That is, you can jump from index `i` to any index in the range $[i + 1, \min(n - 1, i + k)]$ inclusive. You want to reach the last index of the array (index `n - 1`). Your score is the sum of all `nums[j]` for each index `j` you visited in the array. Return the maximum score you can get.

```
public int maxResult(int[] nums, int k) { // O(N) DP with double ended queue  
    int n = nums.length;  
    int [] dp = new int[n];  
    ArrayDeque<Integer> q = new ArrayDeque<>();  
    for (int i = 0; i < n; i++) {  
        while (!q.isEmpty() && q.peekFirst() < i - k) // 头大尾小  
            q.removeFirst();  
        dp[i] = nums[i] + (q.isEmpty() ? 0 : dp[q.peekFirst()]);  
        while (q.size() > 0 && dp[q.peekLast()] <= dp[i])  
            q.removeLast();  
        q.addLast(i);  
    }  
    return dp[n-1];  
}
```

```

public int maxResult(int[] nums, int k) { // BigO: O (NlogN)
    int n = nums.length;
    int [] dp = new int[n];
    Queue<int []> q = new PriorityQueue<>(Comparator.comparingInt(e -> -e[0]));
    for (int i = 0; i < n; i++) {
        while (!q.isEmpty() && q.peek()[1] + k < i)
            q.poll();
        dp[i] = nums[i] + (q.isEmpty() ? 0 : q.peek()[0]);
        q.add(new int[] {dp[i], i});
    }
    return dp[n-1];
}

```

8.3 Create Sorted Array through Instructions

Given an integer array instructions, you are asked to create a sorted array from the elements in instructions. You start with an empty container nums. For each element from left to right in instructions, insert it into nums. The cost of each insertion is the minimum of the following: The number of elements currently in nums that are strictly less than instructions[i]. The number of elements currently in nums that are strictly greater than instructions[i]. For example, if inserting element 3 into nums = [1,2,3,5], the cost of insertion is min(2, 1) (elements 1 and 2 are less than 3, element 5 is greater than 3) and nums will become [1,2,3,3,5]. Return the total cost to insert all elements from instructions into nums. Since the answer may be large, return it modulo 109 + 7

```

// https://blog.csdn.net/qq_28033719/article/details/112506925
private static int N = 100001;
private static int [] tree = new int [N]; // 拿元素值作为 key 对应 tree 的下标值
public int lowbit(int i) {
    return i & -i;
}
public void update(int i, int v) { // 更新父节点
    while (i <= N) {
        tree[i] += v;
        i += lowbit(i);
    }
}
public int getSum(int i) { // 得到以 i 为下标 1-based 的所有子、叶子节点的和，也就是 [1, i] 的和，1-based
    int ans = 0;
    while (i > 0) {
        ans += tree[i];
        i -= lowbit(i);
    }
    return ans;
}
public int createSortedArray(int[] instructions) {
    int n = instructions.length;
    long res = 0;
    Arrays.fill(tree, 0);
    for (int i = 0; i < n; i++) {
        //      严格小于此数的个数 严格大于此数的个数: 为总个数 (不含自己) - 小于自己的个数
        res += Math.min(getSum(instructions[i]-1), i-getSum(instructions[i]));
        update(instructions[i], 1);
    }
    return (int)(res % ((int)Math.pow(10, 9) + 7));
}

```

8.4 1932. Merge BSTs to Create Single BST: 这棵树我曾投入巨大热情，可是总不过，好缺德

You are given n BST (binary search tree) root nodes for n separate BSTs stored in an array trees (0-indexed). Each BST in trees has at most 3 nodes, and no two roots have the same value. In one operation, you can:

Select two distinct indices i and j such that the value stored at one of the leaves of trees[i] is equal to the root value of trees[j]. Replace the leaf node in trees[i] with trees[j]. Remove trees[j] from trees. Return the root of the resulting BST if it is possible to form a valid BST after performing n - 1 operations, or null if it is impossible to create a valid BST.

A BST (binary search tree) is a binary tree where each node satisfies the following property:

Every node in the node's left subtree has a value strictly less than the node's value. Every node in the node's right subtree has a value strictly greater than the node's value. A leaf is a node that has no children.

```

public TreeNode canMerge(List<TreeNode> trees) {
    final int size = trees.size();

```

```

final Map<Integer, TreeNode> roots = new HashMap<>(size);
for (final TreeNode node : trees)
    roots.put(node.val, node);
for (final TreeNode node : trees) {
    if (roots.containsKey(node.val)) { // 这里判断：是因为接下来 buildTree 会将可以合并的子树键值对删除并回收利用建大树了
        final TreeNode root = buildTree(roots, node);
        roots.put(root.val, root); // update root node
    }
}
if (roots.size() != 1) return null; // 无法合并所有的子树
final TreeNode root = roots.values().iterator().next(); // 只有这一颗树根
return isValid(root, Integer.MIN_VALUE, Integer.MAX_VALUE) ? root : null;
}

private TreeNode buildTree(Map<Integer, TreeNode> roots, TreeNode node) { // 用 recursion 把所有需要/可以合并的子树建成一棵完整大树，方法很传神
    final TreeNode next = roots.remove(node.val); // map.remove() 返回值：如果存在 key，则删除并返回 value；如果不存在则返回 null
    if (next != null) {
        if (next.left != null) node.left = buildTree(roots, next.left);
        if (next.right != null) node.right = buildTree(roots, next.right);
    }
    return node;
}

private boolean isValid(TreeNode node, int min, int max) { // 这些个递归写得很传功力，要活学活用到出神入化。。。。。。
    if (node == null) return true;
    final int value = node.val;
    if (value <= min || value >= max) return false;
    return isValid(node.left, min, value) && isValid(node.right, value, max);
}

```

Chapter 9

Trie

应用 Trie 树最直观的定义就是 LinkedList of HashMap。所以 Trie 和 HashMap 都可以用来查询某个单词是否在字典当中。我们需要知道他们的优缺点。优点：支持字符级别的查询，比如说我们需要在 matrix 当中通过 traverse 构造单词，那么这个单词是一个一个字符形成的，我们可以在 traverse 的每一步去检验当前路径是否可以形成 valid word。另外，对于含有 regex 符号的字符串，我们需要一个字符一个字符的考虑，这种情况下我们也需要通过 trie 去查找。节省空间，相同的 prefix 只存一遍，而 HashMap 需要存很多遍。缺点：实现起来较麻烦，大部分题目使用 Trie 都是 overkill，所以除非需要支持字符级别的查询，否则 HashMap 更好。操作：三个操作：insert search startWith 其中 insert 记得把最后一个 node 标记为 isEnd = true。其中 search 和 startWith 都可以通过同一个 searchHelper helper method 来实现，我们只需要 return 最后一个 node 就可以，如果 isEnd == true，那么说明找到一个完整的单词，否则至少找到了 prefix。别忘了使用 trie 的第一步是 preprocess，把字典里的所有 word 加入到 trie 树当中。题目

9.1 440. K-th Smallest in Lexicographical Order - Hard

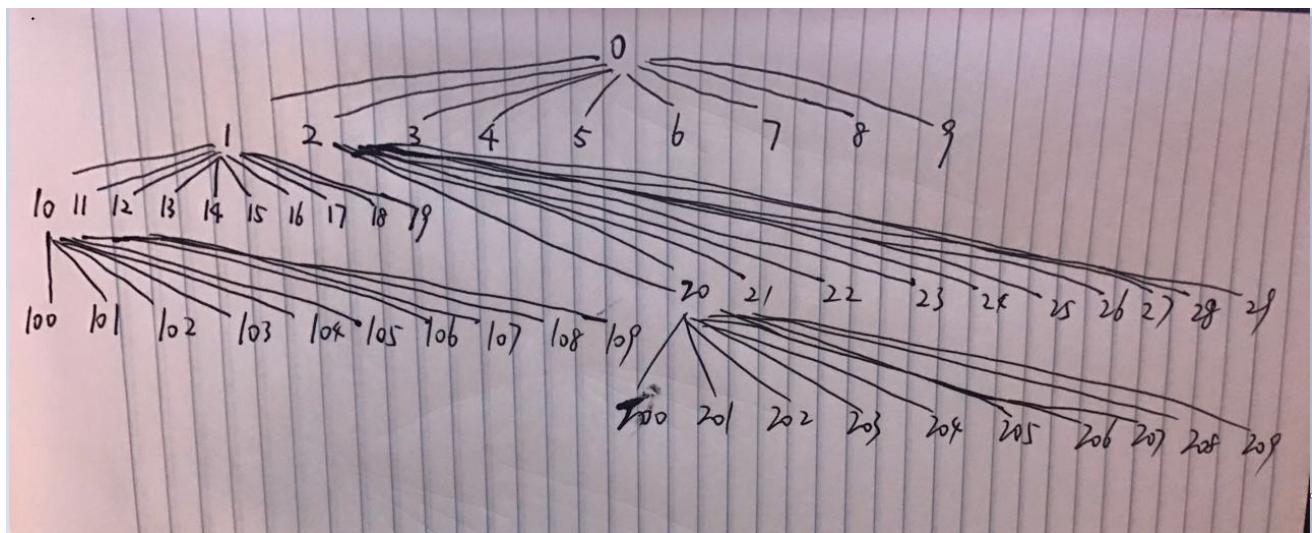
Given two integers n and k, return the kth lexicographically smallest integer in the range [1, n].

9.1.1 解题思路与分析

就像 dfs 时当我们需要两个字符串，遍历字符串，我们并不需要看的去遍历字符串，我们只要移动下标就可以了

这里我们并不需要真的去建和遍历这样一个字典，我们只要理清数字个数之间的关系就可以了

还要一个典型案例，把它找出来。。。 todo



Initially, image you are at node 1 (variable: curr),
the goal is move (k - 1) steps to the target node x. (subtract steps from k after moving)
when k is down to 0, curr will be finally at node x, there you get the result.

we don't really need to do a exact k steps preorder traverse of the denary tree, **the idea is to calculate the steps between curr and curr + 1 (neighbor nodes in same level), in order to skip some unnecessary moves.**

Main function

Firstly, calculate how many steps curr need to move to curr + 1.

1. if the steps $\leq k$, we know we can move to curr + 1, and narrow down k to $k - \text{steps}$.
2. else if the steps $> k$, that means the curr + 1 is actually behind the target node x in the preorder path, we can't jump to curr + 1. What we have to do is to move forward only 1 step (curr * 10 is always next preorder node) and repeat the iteration.

calSteps function

1. how to calculate the steps between curr and curr + 1?

Here we come up a idea to calculate by level.

Let n1 = curr, n2 = curr + 1.

n2 is always the next right node beside n1's right most node (who shares the same ancestor "curr")
(refer to the pic, 2 is right next to 1, 20 is right next to 19, 200 is right next to 199).

2. so, if $n2 \leq n$, what means n1's right most node exists, we can simply add the number of nodes from n1 to n2 to steps.

3. else if $n2 > n$, what means n (the biggest node) is on the path between n1 to n2, add $(n + 1 - n1)$ to steps.

4. organize this flow to "steps += Math.min(n + 1, n2) - n1; n1 *= 10; n2 *= 10;"

```
private int calSteps(int n, long n1, long n2) { // n1 和 n2 得是 long 类型的, int 会产生溢出, 不能通过这个案例: 输入 n=681692778, k=351251360, 预期结果 =4161
    int steps = 0;
    while (n1 <= n) {
        steps += Math.min(n2, n+1) - n1;
        n1 *= 10;
        n2 *= 10;
    }
    return steps;
}

public int findKthNumber(int n, int k) {
    int cur = 1; //根据题意, 第一个数是 1
    --k;          //第一个是 1, 所以再找出 k-1 个数后就知道第 k 个数是多少了
    while (k > 0) {
        int steps = calSteps(n, cur, cur+1);
        if (steps <= k) { //横向扩展, 相当于 +steps,
            cur += 1;
            k -= steps;
        } else {           //steps > k; 纵向扩展, 相当于 +1
            cur *= 10;
            k -= 1;
        }
    }
    return cur;
}
```

9.2 208. Implement Trie (Prefix Tree)

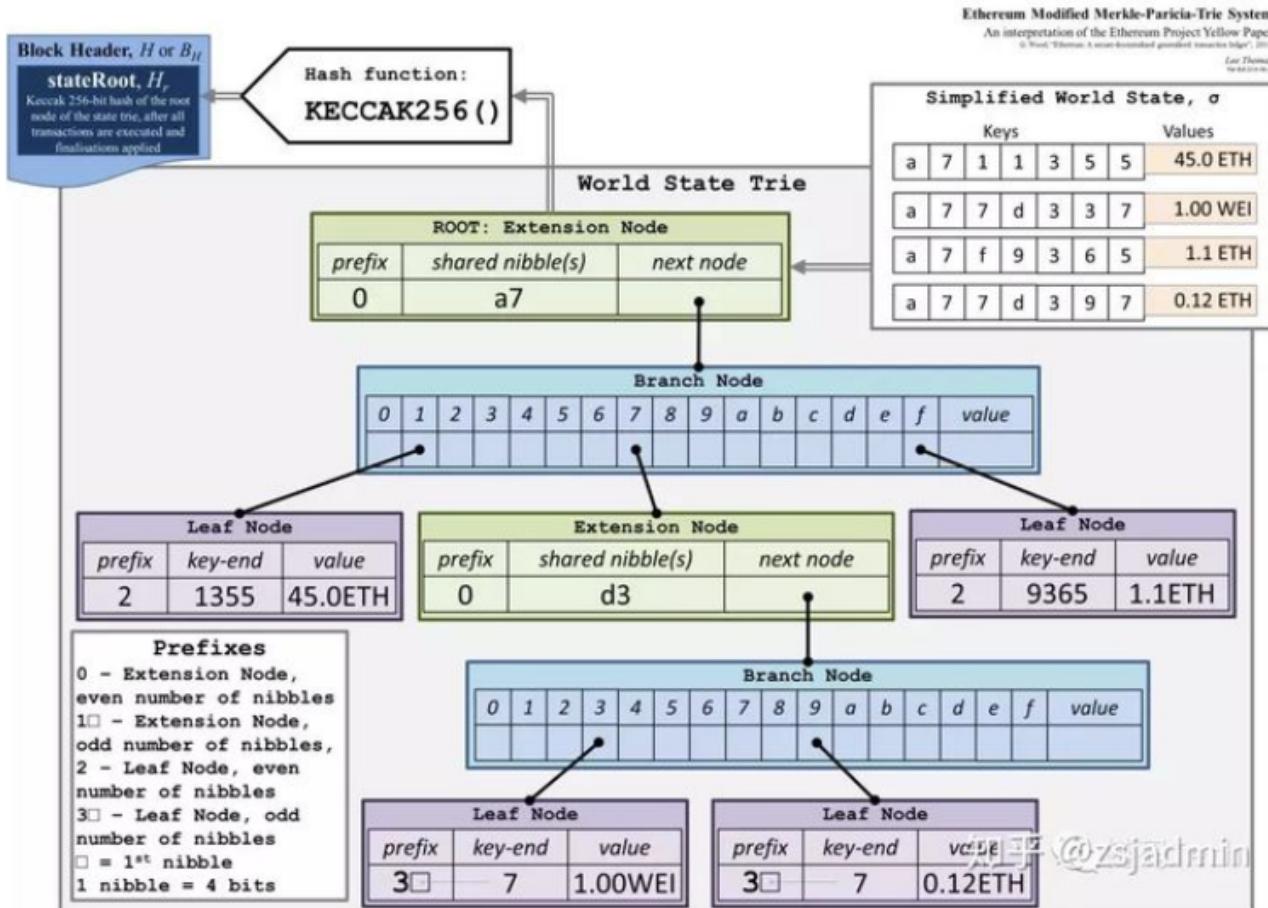
9.3 212. Word Search II

9.4 211. Add and Search Word - Data structure design (Facebook 店面)

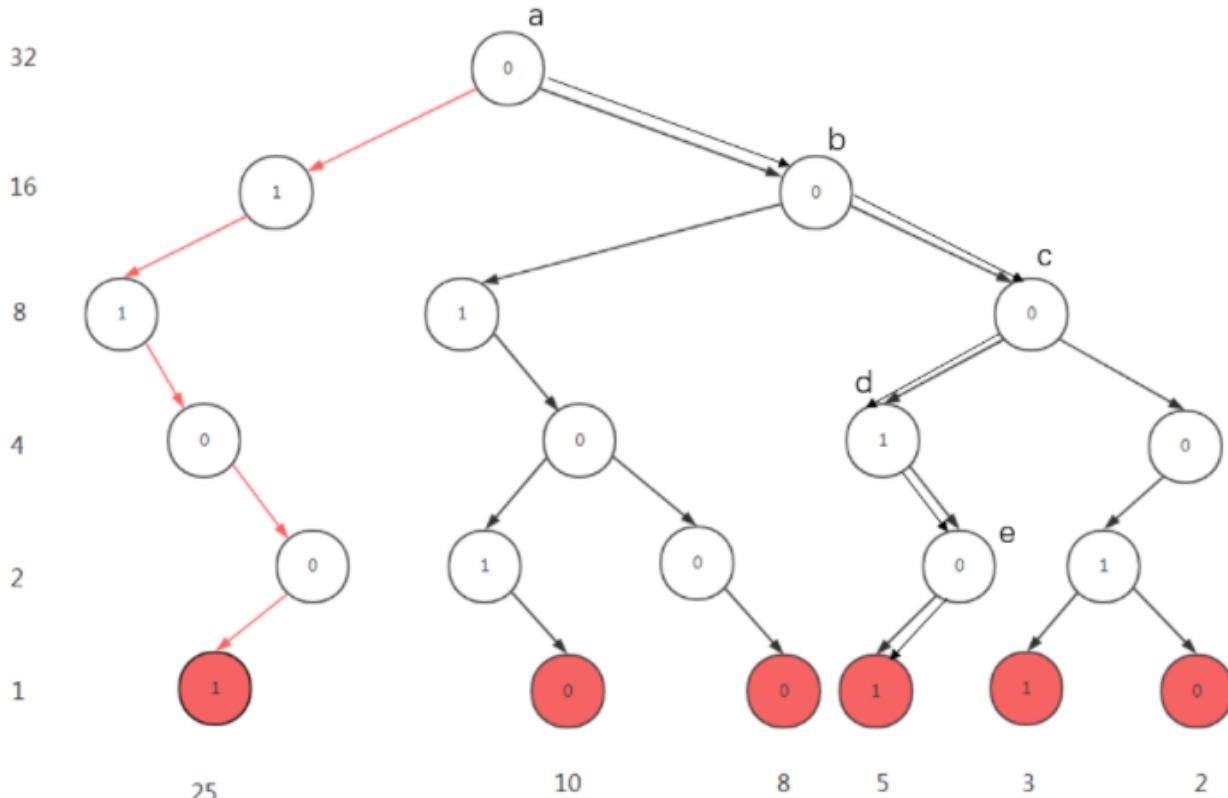
9.5 14. Longest Common Prefix (这道题可以稍作改编, 比如说 string list 会经常 update, 会经常 query, 那这时很明显用 trie 更好)

9.6 421. Maximum XOR of Two Numbers in an Array

Given an integer array nums, return the maximum result of $\text{nums}[i] \text{ XOR } \text{nums}[j]$, where $0 \leq i \leq j < n$.



以样例 [3, 10, 5, 25, 2, 8] 为例，建完树之后可以得到下图：



左儿子为 1 的分支，右儿子为 0 的分支。

然后依次枚举每个数，在 Trie 树中找到与它异或结果最大的数。

这一步可以贪心来做：

从高位到低位，依次在 Trie 树中遍历，每次尽量走到与当前位不同的分支，这样可以使得找到的数与当前数在当前二进制位的异或结果是 1，从而可以得到尽量大的结果。

如上图所示，我们用 25 来举例说明，它的二进制表示是 (11001)：

最初指针在根节点(编号是 a 的点)，我们从 25 的二进制表示的最高位开始枚举；
 由于最高位是 1，我们走到 0 分支，走到 b 点；
 次高位是 1，我们继续往右儿子走，走到 c 点；
 下一位是 0，我们往左走，走到 d 点；
 下一位是 0，我们希望往左走，但发现左儿子不存在，所以只能往右走，走到 e 点；
 最后一位是 1，我们希望往右走，但发现右儿子不存在，所以只能往左走，最终走到 5；
 所以和 25 异或值最大的数是 5， $25 \wedge 5 = 28$ 。

```
public class Trie {
    private class Node { // 这我自己写的乱代码，贴在这里很不相关，也需要先测试一下
        public int val;
        public boolean isExist;
        public Node[] next;
        public Node(boolean isExist) {
            this.isExist = isExist;
            next = new Node[2];
            val = 0;
        }
        public Node() { this(false); }
        public Node(int va) {
            this(true);
            this.val = va;
        }
    }
    private Node root;
    public Trie() { root = new Node(); }
    public void insert(int va) {
        Node cur = root;
        for (int i = 31; i >= 0; i--) {
            int tmp = (va >> i) & 1;
            if (cur.next[tmp] == null)
                cur.next[tmp] = new Node();
            cur = cur.next[tmp];
        }
        cur.isExist = true;
    }
    public int search(int va) {
        int max = 0;
        Node cur = root;
        for (int i = 31; i >= 0; i--) {
            int t = (va >> i) & 1;
            if (cur.next[t^1] != null) {
                max += (1 << i);
                cur = cur.next[t^1];
            } else cur = cur.next[t&1];
        }
        return max;
    }
}
```

9.6.1 另一种位操作法

- 学到了异或操作的一个重要性质： $a^b = c$ ，则有 $a^c = b$ ，且 $b^c = a$ ；

我们还需要用上一个异或的特性，假设 a 和 b 产生了最终的答案 max，即 $a \wedge b = x$ ，那么根据异或的特性， $a \wedge x = b$ 。同理，a 和 b 的最高位（前 n 位）也有相同的性质。

先以最高位为例子，我们可以把所有的数字的最高位放到一个 HashSet 里面，然后使用 1 与 set 里面的所有数字进行异或，如果得出的结果仍然在 set 里面，那么最终结果的最高位必然为 1，否则为 0。也即，先假定结果为 1，然后与 set 中所有数字异或，假定 a 与 1 异或得到结果 b ($a \wedge 1 = b$)，而 b 仍然在 set 里面，那么说明 set 中有两个数字异或能得到 1 ($a \wedge b = 1$)。否则，set 中没有两个数字能够异或得到 1，那么最终结果的最高位为 1 的假设失败，说明最终结果的最高位为 0。以此类推可以得到第二位、第三位。。。的数字。

再做一下推广，我们将所有数字的前 N 位放到一个 HashSet 里面，然后使用之前 N-1 位得到的最大值前缀 prefix 与 set 里面的所有数字进行异或，如果得出的结果仍然在 set 中，那么第 N 位必然为 1，否则为 0。

举个例子，给定数组 [14, 11, 7, 2]，二进制表示分别为 [1110, 1011, 0111, 0010]。题目说了，数字最长不会超过 32 位，所以应从 i = 31 开始，但是所有数字中最位 4 位数，简单起见，我直接从最高位 i=3 开始

[14, 11, 7, 2]
 [1110, 1011, 0111, 0010]

```

1. i = 3, set = {1000, 0000} => max = 1000
2. i = 2, set = {1100, 1000, 0100, 0000} => max = 1100
3. i = 1, set = {1110, 1010, 0110, 0010} => max = 1100
4. i = 0, set = {1110, 1011, 0111, 0010} => max = 1100

```

```

public int findMaximumXOR(int[] nums) { // 这种解法没有用到上面的这个 trie 呀
    int n = nums.length;
    int mask = 0, max = 0;
    HashSet<Integer> s = new HashSet<>();
    for (int i = 31; i >= 0; --i) { // i == 31 时
        mask = mask | 1 << i;      // 为获取前 n 位的临时变量
        for (int va : nums)
            s.add(va & mask);     // 将所有数字的前 n 位放入 set 中
        int tmp = max | (1 << i); // 假定第 n 位为 1, 前 n-1 位 max 为之前迭代求得
        for (Integer va : s)
            if (s.contains(va ^ tmp)) { // 查看 `b` 是否在 // i == 31, (va^tmp): -2147483648
                max = tmp;           // b 存在, 第 n 位为 1
                break;
            }
        s.clear();
    }
    return max;
}
// 此解法时间复杂度为 O(32n)=O(n), 空间复杂度上, 我们使用了一个 HashSet 用于存储所有数字, 因此空间复杂度是 O(n)

```

9.7 1617. Count Subtrees With Max Distance Between Cities - Hard

There are n cities numbered from 1 to n . You are given an array edges of size $n-1$, where $\text{edges}[i] = [\text{ui}, \text{vi}]$ represents a bidirectional edge between cities ui and vi . There exists a unique path between each pair of cities. In other words, the cities form a tree.

A subtree is a subset of cities where every city is reachable from every other city in the subset, where the path between each pair passes through only the cities from the subset. Two subtrees are different if there is a city in one subtree that is not present in the other.

For each d from 1 to $n-1$, find the number of subtrees in which the maximum distance between any two cities in the subtree is equal to d .

Return an array of size $n-1$ where the d th element (1-indexed) is the number of subtrees in which the maximum distance between any two cities is equal to d .

Notice that the distance between the two cities is the number of edges in the path between them.

- So apparently the brute-force approach passed this question. I guess for future contests, I should really pay attention to the input size...

```

public int[] countSubgraphsForEachDiameter(int n, int[][][] edges) {
    int[] res = new int[n-1];
    List<List<int>> subsets = new ArrayList<>();
    generateSubsets(edges, new ArrayList<int>(), subsets, 0);
    for (List<int> subset : subsets)
        solve(subset, res);
    return res;
}
private void solve(List<int> subset, int[] res) {
    if (!isValidGraph(subset)) return;
    Map<Integer, List<Integer>> graph = new HashMap<>();
    for (int[] eg : subset) {
        graph.computeIfAbsent(eg[0], k -> new ArrayList<>().add(eg[1]));
        graph.computeIfAbsent(eg[1], k -> new ArrayList<>().add(eg[0]));
    }
    int max = 1;
    for (Integer key : graph.keySet()) {
        if (graph.get(key).size() == 1) {
            int[] longest = new int[] {1}; // 减少 global 变量的数量
            Set<Integer> vis = new HashSet<>();
            vis.add(key);
            dfs(graph, vis, key, longest, 0);
            max = Math.max(max, longest[0]);
        }
    }
    res[max - 1]++;
}
private void dfs(Map<Integer, List<Integer>> graph, Set<Integer> vis, int idx, int[] longest, int level) {
    longest[0] = Math.max(longest[0], level);
    for (Integer node : graph.get(idx))
        if (vis.add(node)) // Set.add(element) return false if it contains element already

```

```

        dfs(graph, vis, node, longest, level + 1);
    }

    private boolean isValidGraph(List<int []> subset) {
        Set<Integer> nodes = new HashSet<>();
        for (int [] cur : subset) {
            nodes.add(cur[0]);
            nodes.add(cur[1]);
        }
        return nodes.size() - 1 <= subset.size();
    }

    private void generateSubsets(int [][] arr, List<int []> cur, List<List<int []>> res, int idx) {
        if (idx == arr.length) return; // arr.length <= 15, 用回溯法直接生成 subsets, 但是这是相对耗时的操作
        for (int i = idx; i < arr.length; i++) {
            cur.add(arr[i]);
            res.add(new ArrayList<>(cur));
            generateSubsets(arr, cur, res, i+1);
            cur.remove(cur.size()-1);
        }
    }
}

```

- 另一种位操作法

```

// One way in which we can find the diameter of a tree is using DFS, just like if our tree is represented using tree nodes instead of as graph
// 1. Make a call to DFS from any node as root, lets say 1 as root
// 2. Maintain a global max parameter
// 3. For each call to dfs, of all current node's children (excluding parent)
//     find top two distances from current node to any leaf reachable from current node
// 4. Sum of these top two distances froms the longes path passing through current node to all its children. Update if this path is maximum
// 5. return 1 + top distance for this dfs call. Need to add 1 since,
//     max length of path that can be reached from current ndoe is current ndoe + max distance reachable from current ndoes's children
int ans = 0, vis = 0;
int [] res;
public int [] countSubgraphsForEachDiameter(int n, int[][][] edges) {
    res = new int [n-1];
    ans = 0; vis = 0;
    Map<Integer, List<Integer>> graph = new HashMap<>();
    for (int [] i : edges) { // if our node is 5, we store it as 1 << 4 which is 2^4
        graph.computeIfAbsent(1 << (i[0]-1), ArrayList::new).add(1 << (i[1]-1));
        graph.computeIfAbsent(1 << (i[1]-1), ArrayList::new).add(1 << (i[0]-1));
    }
    int range = (1 << n) - 1; // (int)Math.pow(2, n) - 1;
    for (int subset = 3; subset <= range; subset++) {
        boolean isPowerOf2 = subset != 0 && (subset & (subset - 1)) == 0; // is power of 2
        if (isPowerOf2) continue; // Single node subtrees can be excluded.
        ans = 0; vis = 0;
        dfs(graph, subset, Integer.highestOneBit(subset), -1); // Integer.highestOneBit(subset): subset: 0b1100, highest: 0b1000
        if (vis == subset) // If visited is not equal to our current subset, all nodes are not reachable.
            res[ans - 1]++; // In otherwords is not a proper subtree, hence dont include in the answer
    }
    return res;
}

// we can pass any node in this subset as root for dfs, we are passing node represented with highest set bit as root
// pass -1 as parent of root, since it has no root
private int dfs(Map<Integer, List<Integer>> graph, int subset, int cur, int pre) {
    if ((subset & cur) == 0) return 0;
    vis = vis | cur;
    int fstMax = 0, sndMax = 0;
    for (Integer next : graph.get(cur)) {
        if (next == pre) continue;
        int dist = dfs(graph, subset, next, cur);
        if (dist > fstMax) {
            sndMax = fstMax;
            fstMax = dist;
        } else sndMax = Math.max(sndMax, dist);
    }
    ans = Math.max(ans, fstMax + sndMax); // top two distances from this node c
    // top distance this cur node to any leaf is topdistance from c's children + 1. Adding 1 since we need to include cur node
    return 1 + fstMax; // 这里要再想一下 ???
}

```

9.8 1938. Maximum Genetic Difference Query - Hard 离线算法、离线思维、批量处理、顺序无关

There is a rooted tree consisting of n nodes numbered 0 to $n - 1$. Each node's number denotes its unique genetic value (i.e. the genetic value of node x is x). The genetic difference between two genetic values is defined as the bitwise-XOR of their values. You are given the integer array parents , where $\text{parents}[i]$ is the parent for node i . If node x is the root of the tree, then $\text{parents}[x] == -1$.

You are also given the array queries where queries[i] = [nodei, vali]. For each query i, find the maximum genetic difference between vali and pi, where pi is the genetic value of any node that is on the path between nodei and the root (including nodei and the root). More formally, you want to maximize vali XOR pi.

Return an array ans where ans[i] is the answer to the ith query.

```

// 可以从根节点开始，对整棵树进行一次深度优先遍历，即：
// 当我们第一次遍历到某一节点 ii 时，我们将 ii 放入「数据结构」中；
// 当我们遍历完所有节点 ii 的子节点，即将回溯到 ii 的父节点前，我们将 ii 从「数据结构」中移除。
// 这样一来，我们就可以通过「离线」的思想将每一个询问在遍历到节点 \textit{val\_ival} 时进行求解。这是因为，如果当前正在遍历节点 \textit{val\_ival}
// 那么数据结构中就存放着所有从根节点到节点 \textit{val\_ival} 的路径上的所有节点。
// 此时，我们只需要找出数据结构中使得 p_i \oplus \textit{val\_ip} 达到最大值的节点 p_ip 即可。
// 而深度优先搜索过程中，当前入队的部分正是该节点及其所有层级的父节点，因此可结合 DFS 方法进行离线搜索。
// 对最大异或值的计算，可结合字典树方法进行。
// 本题需涉及对字典树中数值的删除操作，为简化代码，可在字典树的节点中设计一个计数器，记录当前该节点对应的数字个数，从而避免删除实际节点。
public class Trie {
    static final int H = 18; // 树高度，本题 val<=2*10^5<2^18
    Trie[] next;
    int cnt; // 当前节点对应的数值个数，简化删除操作
    public Trie() {
        this.next = new Trie[2];
        this.cnt = 0;
    }
    public void insert(int va) { // 插入数值
        Trie r = this;
        for (int i = H-1; i >= 0; i--) {
            int bit = (va >> i) & 1;
            if (r.next[bit] == null)
                r.next[bit] = new Trie();
            r = r.next[bit];
            r.cnt++;
        }
    }
    private void removeVal(int v) { // 删除数值
        Trie r = this;
        for (int i = H-1; i >= 0; i--) {
            int bit = (v >> i) & 1;
            r = r.next[bit];
            r.cnt--;
        }
    }
    public int search(int va) { // 针对数值查询当前字典树对应的最大异或值
        Trie r = this;
        int max = 0;
        for (int i = H-1; i >= 0; i--) {
            int bit = (va >> i) & 1 ^ 1;
            if (r == null) return -1;
            if (r.next[bit] != null && r.next[bit].cnt > 0) {
                max += (1 << i);
                r = r.next[bit];
            } else
                r = r.next[bit] ^ 1;
        }
        return max;
    }
    private void dfs(int idx) { // 深度优先搜索
        trie.insert(idx); // 当前节点加入字典树
        if (queVal.containsKey(idx)) // 处理针对当前节点的查询
            for (int i = 0; i < queVal.get(idx).size(); i++)
                ans[queId.get(idx).get(i)] = trie.search(queVal.get(idx).get(i));
        if (tree.containsKey(idx)) // 当前节点存在子节点
            for (int n : tree.get(idx))
                dfs(n);
        trie.removeVal(idx); // 从字典树中删除当前节点
    }
}

Map<Integer, List<Integer>> tree; // 树中各个节点对应的子节点
Map<Integer, List<Integer>> queVal; // 树中各个节点对应的查询值
Map<Integer, List<Integer>> queId; // 树中各个节点对应的 queries 下标
Trie trie; // 字典树根节点
int[] ans;
public int[] maxGeneticDifference(int[] parents, int[][] queries) {
    int n = parents.length, m = queries.length, root = -1;
    this.tree = new HashMap<>();
    for (int i = 0; i < n; i++) { // 记录树中各个节点对应的子节点
        if (parents[i] != -1) { // Note: 当作有向树图来处理 !!!
            tree.computeIfAbsent(parents[i], k -> new ArrayList<>());
            tree.get(parents[i]).add(i);
        } else root = i;
    }
    this.queVal = new HashMap<>();
    this.queId = new HashMap<>();
    for (int i = 0; i < m; i++) {
        int nid = queries[i][0], val = queries[i][1];
        queId.put(nid, i);
        queVal.put(nid, new ArrayList<>());
    }
}

```

```

        queVal.computeIfAbsent(nid, k -> new ArrayList<>()).add(val);
        queId.computeIfAbsent(nid, k -> new ArrayList<>()).add(i);
    }
    this.ans = new int [m];
    this.trie = new Trie();
    dfs(root);
    return ans;
}

```

复杂度分析

时间复杂度: $O((n+q) \log C)$ $O((n+q)\log C)$, 其中 q 是数组 queries 的长度, $\log C = 18$ $\log C=18$ 是本题中最大的数的二进制表示的位数。在深度优先遍历的过程中, 访问的节点个数为 n , 每个节点需要 $O(\log C)$ $O(\log C)$ 的时间在一开将其加入字典树以及回溯前将其从字典树中移除。对于数组 queries 中的每一个询问, 我们需要 $O(\log C)$ $O(\log C)$ 的时间得到答案。因此总时间复杂度为 $O((n+q) \log C)$ $O((n+q)\log C)$ 。

空间复杂度: $O(n \log C + q)$ $O(n\log C+q)$ 。我们需要 $O(n)$ $O(n)$ 的空间存储树本身, $O(n \log C)$ $O(n\log C)$ 的空间存储字典树, $O(q)$ $O(q)$ 的空间存储将询问进行离线, 分配到每个节点上。

9.9 472. Concatenated Words - Hard

Given an array of strings words (without duplicates), return all the concatenated words in the given list of words.

A concatenated word is defined as a string that is comprised entirely of at least two shorter words in the given array.

Example 1:

```

Input: words = ["cat", "cats", "catsdogcats", "dog", "dogcatsdog", "hippopotamuses", "rat", "ratcatdogcat"]
Output: ["catsdogcats", "dogcatsdog", "ratcatdogcat"]
Explanation: "catsdogcats" can be concatenated by "cats", "dog" and "cats";
"dogcatsdog" can be concatenated by "dog", "cats" and "dog";
"ratcatdogcat" can be concatenated by "rat", "cat", "dog" and "cat".

```

- 切记: dfs 深搜 + 记忆

```

// 切记: dfs 深搜 + 记忆 // Trie with memo, Time: o(m*2^n)
public class Trie {
    boolean isWord;
    Trie [] children;
    public Trie() {
        isWord = false;
        children = new Trie[26];
    }
    public void insert(String word) {
        Trie cur = root;
        for (int i = 0; i < word.length(); i++) {
            char c = word.charAt(i);
            if (cur.children[c-'a'] == null)
                cur.children[c-'a'] = new Trie();
            cur = cur.children[c-'a'];
        }
        cur.isWord = true;
    }
    public boolean isConcatenated(String word, int idx, int cnt, HashMap<Integer, Boolean> memo) {
        if (memo.containsKey(idx)) return memo.get(idx);
        if (idx == word.length()) {
            memo.put(idx, cnt > 1);
            return cnt > 1;
        }
        Trie cur = root;
        for (int i = idx; i < word.length(); i++) {
            char c = word.charAt(i);
            if (cur.children[c-'a'] == null) {
                memo.put(idx, false);
                return false;
            } else {
                cur = cur.children[c-'a'];
                if (cur.isWord && isConcatenated(word, i+1, cnt+1, memo)) {
                    memo.put(idx, true);
                    return true;
                }
            }
        }
        memo.put(idx, false);
        return false;
    }
}

```

```

}
Trie root = new Trie();
public List<String> findAllConcatenatedWordsInADict(String[] words) {
    for (String word : words)
        insert(word);
    List<String> res = new ArrayList<>();
    for (String word : words)
        if (isConcatenated(word, 0, 0, new HashMap<Integer, Boolean>()))
            res.add(word);
    return res;
}

```

- 一种稍微优化了一下方法，逻辑就相对复杂一点儿，参考一下

```

public class Trie { // Trie with memo, Time: o(m*2^n)
    boolean isKey;
    Trie [] child;
    public Trie() {
        this.isKey = false;
        child = new Trie[26];
    }
    public void insert(String s) {
        int [] memo = new int [s.length()];
        Trie p = this;
        char [] sArr = s.toCharArray();
        boolean added = false;
        for (int i = 0; i < sArr.length; i++) {
            char c = sArr[i];
            if (p.child[c-'a'] == null)
                p.child[c-'a'] = new Trie();
            p = p.child[c-'a'];
            if (p.isKey && isConcatenated(s, i+1, 0, memo) && !added) {
                res.add(s);
                added = true;
            }
        }
        p.isKey = true;
    } // 这么看来，我还没能透彻理解 dfs 深搜中的重复，什么时候应该拥有记忆!!!!
    public boolean isConcatenated(String s, int start, int cnt, int [] memo) {
        if (start == s.length() && cnt > 0) return true;
        if (memo[start] != 0) return memo[start] == 1;
        Trie p = this;
        char [] sArr = s.toCharArray();
        for (int i = start; i < sArr.length; i++) {
            char c = sArr[i];
            Trie cur = p.child[c-'a'];
            if (cur == null) {
                memo[start] = -1;
                return false;
            } else {
                if (cur.isKey && isConcatenated(s, i+1, cnt+1, memo)) {
                    memo[start] = 1;
                    return true;
                }
                p = cur;
            }
        }
        memo[start] = -1;
        return false;
    }
} // Sort the words based on length
// Use trie to store words: while adding, checking if it is concatenated
// While checking, use dfs + memo
List<String> res = new ArrayList<>();
public List<String> findAllConcatenatedWordsInADict(String[] words) {
    Arrays.sort(words, (x, y) -> Integer.compare(x.length(), y.length()));
    Trie tree = new Trie();
    for (String word : words)
        tree.insert(word);
    return res;
}

```

9.10 1948. Delete Duplicate Folders in System - Hard

Due to a bug, there are many duplicate folders in a file system. You are given a 2D array paths, where paths[i] is an array representing an absolute path to the ith folder in the file system.

For example, ["one", "two", "three"] represents the path "/one/two/three". Two folders (not necessarily on the same level) are identical if they contain the same non-empty set of identical subfolders and underlying subfolder structure.

The folders do not need to be at the root level to be identical. If two or more folders are identical, then mark the folders as well as all their subfolders.

For example, folders "/a" and "/b" in the file structure below are identical. They (as well as their subfolders) should all be marked:

```
/a
/a/x
/a/x/y
/a/z
/b
/b/x
/b/x/y
/b/z
```

However, if the file structure also included the path "/b/w", then the folders "/a" and "/b" would not be identical. Note that "/a/x" and "/b/x" would still be considered identical even with the added folder.

Once all the identical folders and their subfolders have been marked, the file system will delete all of them. The file system only runs the deletion once, so any folders that become identical after the initial deletion are not deleted.

Return the 2D array ans containing the paths of the remaining folders after deleting all the marked folders. The paths may be returned in any order.

```
public class Node {
    String name;
    Map<String, Node> children = new HashMap<>();
    private String hashCode = null;
    public Node (String name) {
        this.name = name;
    }
    public void add(List<String> path) {
        Node cur = this;
        for (String file : path) {
            if (!cur.children.containsKey(file))
                cur.children.put(file, new Node(file));
            cur = cur.children.get(file);
        }
    }
    public String getHashCode() {
        if (hashCode == null)
            hashCode = computeHash();
        return hashCode;
    }
    private String computeHash() {
        StringBuilder sb = new StringBuilder();
        List<Node> nodes = new ArrayList<>();
        for (Node n : children.values())
            nodes.add(n);
        if (nodes.size() == 0) return null;
        nodes.sort((a, b) -> a.name.compareTo(b.name));
        for (Node n : nodes) {
            sb.append('(');
            sb.append(n.name + n.getHashCode());
            sb.append(')');
        }
        return sb.toString();
    }
}
private void getGoodFiles(Node node, Map<String, Integer> occurs, List<String> cur, List<List<String>> ans) {
    if (occurs.containsKey(node.getHashCode()) && occurs.get(node.getHashCode()) > 1) return;
    cur.add(node.name);
    ans.add(new ArrayList<>(cur));
    for (Node n : node.children.values())
        getGoodFiles(n, occurs, cur, ans);
    cur.remove(cur.size()-1);
}
private void findOccurs(Node node, Map<String, Integer> occurs) {
    String key = node.getHashCode();
    if (key != null)
        occurs.put(key, occurs.getOrDefault(node.getHashCode(), 0) + 1);
    for (Node n : node.children.values())
        findOccurs(n, occurs);
}
Node root;
public List<List<String>> deleteDuplicateFolder(List<List<String>> paths) {
    root = new Node("");
    for (List<String> path : paths)
        root.add(path);
    Map<String, Integer> occurs = new HashMap<>();
    findOccurs(root, occurs);
    List<List<String>> ans = new ArrayList<>();
```

```

    for (Node n : root.children.values())
        getGoodFiles(n, occurs, new ArrayList<>(), ans);
    return ans;
}

```

9.11 792. Number of Matching Subsequences - Medium

Given a string s and an array of strings words , return the number of $\text{words}[i]$ that is a subsequence of s .

A subsequence of a string is a new string generated from the original string with some characters (can be none) deleted without changing the relative order of the remaining characters.

For example, "ace" is a subsequence of "abcde".

```

// 我们需要使用每个字典中的单词去和 S 比较，看它是否是 S 的子序列。不过这种比较非常耗时，因此我们需要对 S 进行一下预处理。
// 首先定义一个二维数组 arr[1][1], 其中 arr[i][j] 代表距离 S 中第 i 位字符最近的 j 字符的位置。
// 换句话说，我们需要遍历一边字符串，记录下字符串 S 每一位上的字符，在它右侧距离它最近的 a-z 分别在哪。
public int numMatchingSubseq(String s, String[] words) {
    int n = s.length();
    int [][] arr = new int [n][26]; // 预处理用的数组
    for (int i = n-2; i >= 0; i--) { // 预处理
        arr[i] = Arrays.copyOf(arr[i+1], 26);
        arr[i][s.charAt(i+1)-'a'] = i+1;
    }
    int res = 0, idxAtS = 0, idx = 0, cur = 0;
    for (String v : words) { // 比较每一个单词
        idxAtS = 0; // 对应 S 的下标
        idx = 0; // 当前单词下标
        if (v.charAt(0) == s.charAt(0)) { // 如果当前单词首字符等于 S 首字符
            idx++; // 当前单词下标加一
            if (v.length() == 1) res++; // 如果当前单词长度只有 1，说明当前单词已经遍历结束，结果加一
        }
        while (idx < v.length()) { // 继续比较单词接下来的字符，在 S 中是否存在
            cur = v.charAt(idx) - 'a';
            if (arr[idxAtS][cur] == 0) break; // 如果 indexAtS 之后不存在 c, 当前单词不合法
            idxAtS = arr[idxAtS][cur]; // 将 indexAtS 更新为 c 在 S 中的位置
            if (++idx == v.length()) res++; // index 加一，如果 index 为单词最后一位，代表单词中所有字符均在 S 中找到
        }
    }
    return res;
}

```

Chapter 10

Bit Manipulations

10.1 基本概念：原码、反码、与补码（对负数尤其重要）

10.1.1 1、原码：

一个正数，按照绝对值大小转换成的二进制数；一个负数按照绝对值大小转换成的二进制数，然后最高位补 1，称为原码。比如 00000000 00000000 00000000 00000101 是 5 的原码。

10000000 00000000 00000000 00000101 是 -5 的原码。

备注：

比如 byte 类型，用 2^8 是 0 - 255 了；如果有符号，最高位表示符号，0 为正，1 为负，那么，正常的理解就是 -127 至 +127 了。这就是原码了，值得一提的是，原码的弱点，有 2 个 0，即 +0 和 -0（10000000 和 00000000）；还有就是，进行异号相加或同号相减时，比较笨蛋，先要判断 2 个数的绝对值大小，然后进行加减操作，最后运算结果的符号还要与大的符号相同；于是，反码产生了。

10.1.2 2、反码

正数的反码与原码相同，负数的反码为对该数的原码除符号位外各位取反 [每一位取反 (除符号位)]。

取反操作指：原为 1，得 0；原为 0，得 1。（1 变 0；0 变 1）

比如：正数 00000000 00000000 00000000 00000101 的反码还是 00000000 00000000 00000000 00000101

负数 10000000 00000000 00000000 00000101 的反码则是 11111111 11111111 11111111 11111010。

反码是相互的，所以也可称：10000000 00000000 00000000 00000101 和 11111111 11111111 11111111 11111010 互为反码。

备注：还是有 +0 和 -0，没过多久，反码就成为了过滤产物，也就是，后来补码出现了。

10.1.3 3、补码

正数的补码与原码相同，负数的补码为对该数的原码除符号位外各位取反，然后在最后一位加 1。

比如：10000000 00000000 00000000 00000101 的补码是：11111111 11111111 11111111 11111010。

那么，补码为：

11111111 11111111 11111111 11111010 + 1 = 11111111 11111111 11111111 11111011

备注：1、从补码求原码的方法跟原码求补码是一样的，也可以通过完全逆运算来做，先减一，再取反。

2、补码却规定 0 没有正负之分

所以，-5 在计算机中表达为：11111111 11111111 11111111 11111011。转换为十六进制：0xFFFFFFFF。

10.2 数组中不重复的两个元素

给定一个整数数组 `nums`，其中恰好有两个元素只出现一次，其余所有元素均出现两次。找出只出现一次的那两个元素。

输入：[1,2,1,3,2,5]

输出：[3,5]

复制代码将所有元素进行异或运算，得到两个不重复元素的异或值，也就是这两个元素中不相同的部分为 1 的数， $n \& (-n)$ 得到 n 的位级表示中最低的那一位 1，这个 1 只可能来自两个不重复元素中的一个（就算重复数的二进制数中也有可能包含这个 1，但通过 $x \hat{=} num$ 的异或操作便消除）。

isolate rightmost 1-bit
which is different for
x and y

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

 $x = 1$

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

 $y = 2$

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

 $a = 3$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

 $a = 3$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

bitmask =
bitmask ^ x ^ y ^ a ^ a

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

diff =
bitmask & (-bitmask)

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

10.3 371. Sum of Two Integers

Given two integers a and b, return the sum of the two integers without using the operators + and

一开始自己想的如果两个数都是正数，那么很简单，运用 XOR \wedge 去找出所有的单一的 1。接着运用 AND & 去找出所有重复的 1；重复的 1 就相当于 carryover，需要进位。然后运动 $<<$ 把重复的 1 给进位就可以了，最后直接 OR 一下就等于答案（这是错的，需要每次循环来判断新的进位）。但是发现这个是能运用于两个正数，研究来研究去，不会算带负数的，所以放弃网上找答案。

发现答案不和我的两个正数之和算法一样嘛！唯一不同的就是答案是把算出的答案继续带回 function 直到 carry 等于 0；

通过例子来看一下：

a = 5, b = 1:

a: 101

b: 001

根据我最初的算法：（错误的）

sum = a \wedge b = 100

carry = a & b = 001 这里这个 1 就是需要进位的

carry = 001 \ll 1 = 010

最后把 sum 100 和 carry 010 OR 一下就等于 110 = 6。

但是答案的做法却是把 sum 和 carry 在带回 function 继续算直至 carry = 0，我们来看一下例子：

a = 5, b = 1:

a = 101

b = 001

sum = 100

carry = 010

带回

a = 100

b = 010

sum = 110

carry = 000 这里等于 0 了，所以结束，我的理解是，答案的做法是把 carryover 带回去，和 sum 比较，如果这一次没有继续需要进位的数字了，就可以结束，否则继续下一轮；换一句话就是，答案是把每一轮的 sum 和 carryover 拿出来，下一轮继续加一起看一看有没有新的需要进位的地方，所以明显我之前的做法是错的，我只考虑了一轮而已，实际上是每一轮都有可能有新的需要进位的地方。

那新的问题又来了，为啥负数也可以，这里的负数是 2's complement:

比如说 $-5 = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1011$

为何-5 是这样：首先把上面的 bits -1

11111 11111 11111 11111 11111 11111 11111 11111 1010

然后再 flip 一下

带负数的两数之和，有点麻烦就是有那么多 1，所以利用了自己的想象力来帮助自己理解：（不知道对不对）

举个例子：

$$a = -5, b = 15$$

把每一个 sum (a) 和 carry (b) 打出来是这样的：

我们可以看到最后是 10，在我理解，有负数的情况下，我们需要把负数的那些 1 都过滤一下，所以循环的次数会多很多，相对于正数来说。

通过上面规律，每次 a 都是减少它的 1 的数量，每次 b 都是增多它的 0 的数量，直到 a 的 1 过滤完， b 的 0 达到极限，便结束了，得到正确答案。

利用想象力的时候到了，这就相当于 $a = -5$ $b = 15$ 在一个横坐标上，每一次 a 向右走一格， b 像左走一格，或者说是负数向右走，正数向左走，直到一个最小的负数走到 0，那么另外一个数就是答案。

```

public int getSum(int a, int b) {
    if (b == 0) return a;
    int sum = a ^ b;
    int carry = (a & b) << 1;
    return getSum(sum, carry);
}
public int getSum(int a, int b) { // (-1, 1) 过不了
    if (b == 0) return a;
    int carryOver = 0;
    while (b != 0) { // 这里是 b != 0, b > 0 对负数不成立
        carryOver = (a & b);
        a = a ^ b;
        b = (carryOver << 1);
    }
    return a;
}

```

10.4 201. Bitwise AND of Numbers Range

Given two integers left and right that represent the range [left, right], return the bitwise AND of all numbers in this range, inclusive.

```

public int rangeBitwiseAnd(int left, int right) {
    if (left == 0) return 0;
    if (left == 1 && right == Integer.MAX_VALUE) return 0;
    if (Integer.toBinaryString(left).length() != Integer.toBinaryString(right).length()) return 0;
    if (left == right) return left;
    int ans = left;
    for (int i = left+1; i <= right; i++) {
        ans &= i;
        if (ans == 0 || i == Integer.MAX_VALUE) return ans;
    }
    return ans;
}

```

10.5 1835. Find XOR Sum of All Pairs Bitwise AND - Hard

The XOR sum of a list is the bitwise XOR of all its elements. If the list only contains one element, then its XOR sum will be equal to this element.

For example, the XOR sum of [1,2,3,4] is equal to 1 XOR 2 XOR 3 XOR 4 = 4, and the XOR sum of 3 is equal to 3. You are given two 0-indexed arrays arr1 and arr2 that consist only of non-negative integers.

Consider the list containing the result of arr1[i] AND arr2[j] (bitwise AND) for every (i, j) pair where $0 \leq i < \text{arr1.length}$ and $0 \leq j < \text{arr2.length}$.

Return the XOR sum of the aforementioned list.

```

// Think about (a&b) ^ (a&c). Can you simplify this expression?
// It is equal to a&(b^c).
// Then, (A[i]&B[0])^(A[i]&B[1])... = A[i]&(B[0]^B[1]^...arr[2]...).
// Let bXorSum = (B[0]^B[1]^B[2]...),
// aXorSum = (A[0]^A[1]^A[2]...) so the final answer is
// (bXorSum&A[0]) ^ (bXorSum&A[1]) ^ (bXorSum&A[2]) ^ ... = bXorSum & aXorSum.
public int getXORSum(int[] a, int[] b) {
    int m = a.length;
    int n = b.length;
    int aXorSum = a[0], bXorSum = b[0];
    for (int i = 1; i < m; i++)
        aXorSum ^= a[i];
    for (int i = 1; i < n; i++)
        bXorSum ^= b[i];
    return aXorSum & bXorSum;
}

```

10.6 982. Triples with Bitwise AND Equal To Zero 平生不识 **TwoSum**, 刷尽 **LeetCode** 也枉然

Given an integer array nums, return the number of AND triples.

An AND triple is a triple of indices (i, j, k) such that:

$0 \leq i < \text{nums.length}$ $0 \leq j < \text{nums.length}$ $0 \leq k < \text{nums.length}$ $\text{nums}[i] \& \text{nums}[j] \& \text{nums}[k] == 0$, where $\&$ represents the bitwise-AND operator.

```
// ‘平生不识 TwoSum, 刷尽 LeetCode 也枉然’ 还好不至于哭死呀。。。。。。
public int countTriplets(int[] arr) {
    Map<Integer, Integer> m = new HashMap<>();
    int v = 0, res = 0;
    for (int i = 0; i < arr.length; i++) {
        for (int j = 0; j < arr.length; j++) {
            v = arr[i] & arr[j];
            m.put(v, m.getOrDefault(v, 0) + 1);
        }
    }
    for (int i = 0; i < arr.length; i++)
        for (int k : m.keySet())
            if ((arr[i] & k) == 0) res += m.get(k);
    return res;
}
public int countTriplets(int[] arr) { // 这种方法执行起来效率更高一点儿
    int res = 0, v = 0;
    int[] cnt = new int[1 << 16];
    Arrays.fill(cnt, -1);
    for (int a : arr) {
        for (int b : arr) {
            v = a & b;
            if (cnt[v] == -1) {
                cnt[v] = 0;
                for (int c : arr)
                    if ((v & c) == 0) ++cnt[v];
            }
            res += cnt[v];
        }
    }
    return res;
}
```

10.7 187. Repeated DNA Sequences - Medium

The DNA sequence is composed of a series of nucleotides abbreviated as 'A', 'C', 'G', and 'T'.

For example, "ACGAATTCCG" is a DNA sequence. When studying DNA, it is useful to identify repeated sequences within the DNA.

Given a string s that represents a DNA sequence, return all the 10-letter-long sequences (substrings) that occur more than once in a DNA molecule. You may return the answer in any order.

有人说上面 native 方法超时是因为字符串存储浪费了太多的空间和时间，因此可以考虑用整数存储，即二进制方法。这个思路非常简单，这里一共有四个字母：A, C, G, T。我们转换整数的思路如下：

```
A = 00□, C = 01□, G = 10□, T = 11□
int key = 0, key = key << 2 | code(A|C|G|T)□
```

这样我们就很容易把一个字符串转换为整数了，上面公式不清楚的话，可以直接看转换代码：

```
private static int hashCode(String s) {
    int hash = 0;
    for (int i = 0; i < s.length(); i++)
        hash = hash << 2 | mapInteger(s.charAt(i));
    return hash;
}
private static int mapInteger(char c) {
    switch (c) {
        case 'A': return 0;
        case 'C': return 1;
        case 'G': return 2;
        case 'T': return 3;
        default: return 0;
    }
}
public List<String> findRepeatedDnaSequences(String s) {
    List<String> res = new ArrayList<>();
    if (s == null || s.length() == 0) return res;
    Set<Integer> si = new HashSet<>();
    for (int i = 0; i <= s.length() - 10; i++) {
        String substr = s.substring(i, i+10);
        Integer key = hashCode(substr);
        if (si.contains(key) && !res.contains(substr))
            res.add(substr);
        else si.add(key);
    }
}
```

```
    }  
    return res;  
}
```

10.8 1915. Number of Wonderful Substrings - Medium

A wonderful string is a string where at most one letter appears an odd number of times.

For example, "ccjjc" and "abab" are wonderful, but "ab" is not. Given a string `word` that consists of the first ten lowercase English letters ('a' through 'j'), return the number of wonderful non-empty substrings in `word`. If the same substring appears multiple times in `word`, then count each occurrence separately.

A substring is a contiguous sequence of characters in a string.

```
public long wonderfulSubstrings(String word) {
    int n = word.length(), mask = 0, cur = 0;
    long res = 0, cnt = 0;
    Map<Integer, Integer> m = new HashMap<>();
    m.put(0, 1);
    for (int i = 0; i < n; i++) {
        mask ^= (1 << (word.charAt(i) - 'a'));
        res += m.getOrDefault(mask, 0);
        m.put(mask, m.getOrDefault(mask, 0) + 1);
        for (int j = 0; j < 10; j++) {
            cur = mask ^ (1 << j);
            res += m.getOrDefault(cur, 0);
        }
    }
    return res;
}
```

10.9 782. Transform to Chessboard- Hard

You are given an $n \times n$ binary grid board. In each move, you can swap any two rows with each other, or any two columns with each other.

Return the minimum number of moves to transform the board into a chessboard board. If the task is impossible, return -1.

A chessboard board is a board where no 0's and no 1's are 4-directionally adjacent.

我们发现对于长度为奇数的棋盘，各行的 0 和 1 个数不同，但是还是有规律的，每行的 1 的个数要么为 $n/2$ ，要么为 $(n+1)/2$ ，这个规律一定要保证，不然无法形成棋盘。

还有一个很重要的规律，我们观察题目给的第一个例子，如果我们只看行，我们发现只有两种情况 0110 和 1001，如果只看列，只有 0011 和 1100，我们发现不管棋盘有多长，都只有两种情况，而这两种情况下各位上是相反的，只有这样的矩阵才有可能转换为棋盘。那么这个规律可以衍生出一个规律，就是任意一个矩形的四个顶点只有三种情况，要么四个 0，要么四个 1，要么两个 0 两个 1，不会有其他的情况。那么四个顶点亦或在一起一定是 0，所以我们判断只要亦或出了 1，一定是对的，直接返回-1。之后我们来统计首行和首列中的 1 个数，因为我们要让其满足之前提到的规律。统计完了首行首列 1 的个数，我们判断如果其小于 $n/2$ 或者大于 $(n+1)/2$ ，那么一定无法转为棋盘。我们还需要算下首行和首列跟棋盘位置的错位的个数，虽然 01010 和 10101 都可以是正确的棋盘，我们先默认跟 10101 比较好，之后再做优化处理。

最后的难点就是计算最小的交换步数了，这里要分 n 的奇偶来讨论。如果 n 是奇数，我们必须得到偶数个，为啥呢，因为我们之前统计的是跟棋盘位置的错位的个数，而每次交换行或者列，会修改两个错位，所以如果是奇数就无法还原为棋盘。举个例子，比如首行是 10001，如果我们跟棋盘 10101 比较，只有一个错位，但是我们是无法通过交换得到 10101 的，所以我们必须要交换得到 01010，此时的错位是 4 个，而我们通过 $n - \text{rowDiff}$ 正好也能得到 4，这就是为啥我们需要偶数个错位。如果 n 是偶数，那么就不会出现这种问题，但是会出现另一个问题，比如我们是 0101，这本身就是正确的棋盘排列了，但是由于我们默认是跟 1010 比较，那么我们会得到 4 个错位，所以我们应该跟 $n - \text{rowDiff}$ 比较取较小值。列的处理跟行的处理完全一样。最终我们把行错位个数跟列错位个数相加，再除以 2，就可以得到最小的交换次数了，之前说过了每交换一次，可以修复两个错位，参见代码如下：

```
public int movesToChessboard(int[][] bd) { // bd: board
    int n = bd.length, rowSum = 0, colSum = 0, rowDiff = 0, colDiff = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if ((bd[0][0] ^ bd[i][0] ^ bd[0][j] ^ bd[i][j]) > 0) return -1;
        }
    }
    for (int i = 0; i < n; i++) {
        rowSum += bd[0][i];
        colSum += bd[i][0];
        rowDiff += bd[i][0] == i % 2 ? 1 : 0; //
        colDiff += bd[0][i] == i % 2 ? 1 : 0; //
    }
}
```

```

}
if (rowSum < n/2 || rowSum > (n+1)/2) return -1;
if (colSum < n/2 || colSum > (n+1)/2) return -1;
if (n % 2 == 1) {
    if (rowDif % 2 == 1) rowDif = n - rowDif;
    if (colDif % 2 == 1) colDif = n - colDif;
} else {
    rowDif = Math.min(rowDif, n - rowDif);
    colDif = Math.min(colDif, n - colDif);
}
return (rowDif + colDif) / 2;
}

```

- 方法一：分维度计算【通过】

思路

首先需要思考的是一次交换之后，棋盘会发生什么变化。为了简单起见，这里用交换列来做例子。在对任意两列进行交换之后，可以看到列交换是不会改变任意两行之间的状态的，简单的来说如果这两行原本就相同，列交换之后这两行依旧相同，如果这两行本来就不同，列交换之后也还是不同。由于最终的棋盘只有两种不同的行，最初的棋盘也一定只有两种不同的行，否则不管怎么做列交换都不会得到最终的棋盘。

之后再来看棋盘行的规律，棋盘有两种行，这两种行每一位都互相不同。同时对于每一行来说，一定有一半为 1，一半为 0（如果长度为奇数，会多一个 1 或多一个 0）。对于棋盘的列也是同样的规律。

可以观察到，先换行再换列跟先换列再换行结果是一样的。在这里先将所有的行调到正确的位置，再将所有的列调到正确的位置。

考虑到只有两种不同的行，可以分别用 0, 1 对其表示。要达成最终的棋盘实际上等价于将棋盘的行表示成 0, 1 相隔的状态。假设在将棋盘的行用 0, 1 表示之后得到数组为 [0, 1, 1, 1, 0, 0]，那么只需求这个数组变成 [0, 1, 0, 1, 0, 1] 和 [1, 0, 1, 0, 1, 0] 的代价，之后取其中最小的代价就好了。同理，对列也是如此，这就将二维问题变成了两个一维问题。

算法

首先需要确认是否有且只有两种行（列）存在，且这两种行（列）的 0, 1 排布合法，如果不符合条件直接返回 -1。之后需要生成理想的行（列）的状态（即 0, 1 相隔的数组排列），对于每种理想状态，计算其与初始状态之间变换的代价。举个例子，对于 [0, 1, 1, 1, 0, 0] 初始状态来说，有两种理想状态，分别是 [0, 1, 0, 1, 0, 1] 和 [1, 0, 1, 0, 1, 0]，对于 [0, 1, 1, 1, 0] 初始状态只有一种理想状态 [1, 0, 1, 0, 1]。

在 Java 实现中，用整型来表示每行。之后将其与 0b010101010101...01 进行异或来计算初始状态转换到理想状态的代价。为了代码简洁，这里统一使用 0xAAAAAAA 和 0x55555555，为了不引入额外的转换代价，还需要根据行的长度 N 生成 0b00...0011...11 掩码与结果做与运算。

```

public int movesToChessboard(int[][] board) {
    int N = board.length;
    // count[code] = v, where code is an integer
    // that represents the row in binary, and v
    // is the number of occurrences of that row
    Map<Integer, Integer> count = new HashMap();
    for (int[] row: board) {
        int code = 0;
        for (int x: row)
            code = 2 * code + x;
        count.put(code, count.getOrDefault(code, 0) + 1);
    }
    int k1 = analyzeCount(count, N);
    if (k1 == -1) return -1;
    // count[code], as before except with columns
    count = new HashMap();
    for (int c = 0; c < N; ++c) {
        int code = 0;
        for (int r = 0; r < N; ++r)
            code = 2 * code + board[r][c];
        count.put(code, count.getOrDefault(code, 0) + 1);
    }
    int k2 = analyzeCount(count, N);
    return k2 >= 0 ? k1 + k2 : -1;
}
public int analyzeCount(Map<Integer, Integer> count, int N) {
    // Return -1 if count is invalid
    // Otherwise, return number of swaps required
    if (count.size() != 2) return -1;
    List<Integer> keys = new ArrayList(count.keySet());
    int k1 = keys.get(0), k2 = keys.get(1);
    // If lines aren't in the right quantity
    if (!(count.get(k1) == N/2 && count.get(k2) == (N+1)/2) ||
        !(count.get(k2) == N/2 && count.get(k1) == (N+1)/2))
        return -1;
    // If lines aren't opposite
    if ((k1 ^ k2) != (1<<N) - 1)

```

```

    return -1;
int Nones = (1 << N) - 1;
int ones = Integer.bitCount(k1 & Nones); // bitCount 统计二进制中 1 的个数
int cand = Integer.MAX_VALUE;
if (N%2 == 0 || ones * 2 < N) // zero start
    cand = Math.min(cand, Integer.bitCount(k1 ^ 0xAAAAAAAA & Nones) / 2);
if (N%2 == 0 || ones * 2 > N) // ones start
    cand = Math.min(cand, Integer.bitCount(k1 ^ 0x55555555 & Nones) / 2);
return cand;
}

```

10.10 1803. Count Pairs With XOR in a Range - Hard

Given a (0-indexed) integer array nums and two integers low and high, return the number of nice pairs.

A nice pair is a pair (i, j) where $0 \leq i < j < \text{nums.length}$ and $\text{low} \leq (\text{nums}[i] \text{ XOR } \text{nums}[j]) \leq \text{high}$.

- 1. 暴力算法

直接暴力计算，利用 $\text{num}^{\text{num}2=i}$ 等效于 $\text{num}^i=\text{num}2$ 的特点，先统计当前各个数字出现的次数，再将当前数字和 $[\text{low}, \text{high}]$ 范围内的数字进行异或运算，将其结果对应的出现次数相加。

```

public int countPairs(int[] arr, int low, int high) {
    int[] freq = new int[20002]; // nums[i]<=20000
    for (int v : arr)
        freq[v]++;
    int val = 0, res = 0;
    for (int v : arr) {
        for (int i = low; i <= high; i++) {
            val = v ^ i; // num ^ i = num2 <=> num ^ num2 = i
            if (val <= 20000)
                res += freq[val]; // v^val=i 中当前 v 对应 val 出现的次数
        }
        freq[v]--;
    }
    return res;
}

```

- 2. 字典树 (Trie 树)

(1) 思路在上述算法的基础上，结合字典树方法快速统计。在依次将 nums 中数字加入字典树的同时，搜索和该数字异或值在 $[0, \text{high}]$ 和 $[0, \text{low} - 1]$ 范围内数字 $\text{num}2$ 的个数并相减，就是符合异或值为 $[\text{low}, \text{high}]$ 区间内的数字个数。

```

public class Trie {
    static final int H = 14; // 2^15=32768, 15 位二进制足够计算
    int cnt;
    Trie[] next;
    public Trie() {
        this.cnt = 0;
        this.next = new Trie[2];
    }
    public void insert(int va) { // 插入数值
        Trie r = this;
        for (int i = H; i >= 0; i--) {
            int bit = (va >> i) & 1;
            if (r.next[bit] == null)
                r.next[bit] = new Trie();
            r = r.next[bit];
            r.cnt++;
        }
    }
    public int search(Trie r, int digit, int v, int range) { // 搜索和 v 异或值在 [0, range] 范围内的数字 num2 的个数
        if (r == null) return 0;
        if (digit < 0) return r.cnt;
        int vb = (v >> digit) & 1; // v 和 range 在该位的值
        int vr = (range >> digit) & 1;
        if (vr == 1) { // range 在该位为 1
            if (vb == 0) // num 在该位为 0, num2 该位为 0 的部分全部满足, 为 1 的部分继续判断
                return (r.next[0] == null ? search(r.next[1], digit-1, v, range) : r.next[0].cnt + search(r.next[1], digit-1, v, range));
            else // v 在该位为 1, num2 该位为 1 的部分全部满足, 为 0 的部分继续判断
                return (r.next[1] == null ? search(r.next[0], digit-1, v, range) : r.next[1].cnt + search(r.next[0], digit-1, v, range));
        }
        return search(r.next[vb], digit-1, v, range); // range 在该位 vr 为 0, num2 该位必须和 num 一致
    }
    private Trie root;
}

```

```

public int countPairs(int[] arr, int low, int high) {
    int n = arr.length, maxHeight = 14; // 2^15=32768, 15 位二进制足够计算
    int res = 0;
    root = new Trie();
    for (int v : arr) {
        res += root.search(root, maxHeight, v, high) - root.search(root, maxHeight, v, low-1); // 这里的脑袋好难转呀...
        root.insert(v);
    }
    return res;
    // for (int v : arr)
    //     root.insert(v);
    // for (int v : arr)
    //     res += root.search(root, maxHeight, v, high) - root.search(root, maxHeight, v, low-1);
    // return res / 2; // 如果按这种写法, 就得 / 2, 智商呢?!!!
}

```

10.11 1734. Decode XORed Permutation - Medium

There is an integer array perm that is a permutation of the first n positive integers, where n is always odd.

It was encoded into another integer array encoded of length $n - 1$, such that $\text{encoded}[i] = \text{perm}[i] \text{ XOR } \text{perm}[i + 1]$.
For example, if $\text{perm} = [1, 3, 2]$, then $\text{encoded} = [2, 1]$.

Given the encoded array, return the original array perm. It is guaranteed that the answer exists and is unique.

结合 n 为奇数的特点, 先对 encoded 数组中下标为奇数的元素进行异或, 得到第 2 到 n 个数的异或值;

因为整数数组是前 n 个正整数的排列, 再对 1 到 n 进行异或, 得到全部数的异或值;

上述二者进行异或即可得到第 1 个数, 然后依次求解获得其他数字, 得到原始数组。

```

public int[] decode(int[] encoded) {
    int n = encoded.length + 1;
    int xor = 0, vFrom2 = 0;
    for (int i = 1; i < n-1; i += 2) // 记录第 2 到 n 个数的异或值
        vFrom2 = vFrom2 ^ encoded[i]; // (a[1]^a[2])^(a[3]^a[4])^...^(a[n-2]^a[n-1])
    for (int i = 1; i <= n; i++) // a[0]^a[1]^a[2]^...^a[n-1]
        xor ^= i;
    int [] arr = new int [n];
    arr[0] = xor ^ vFrom2;
    for (int i = 1; i < n; i++)
        arr[i] = arr[i-1] ^ encoded[i-1];
    return ans;
}

```

10.12 957. Prison Cells After N Days - Medium

There are 8 prison cells in a row and each cell is either occupied or vacant.

Each day, whether the cell is occupied or vacant changes according to the following rules:

If a cell has two adjacent neighbors that are both occupied or both vacant, then the cell becomes occupied. Otherwise, it becomes vacant. Note that because the prison is a row, the first and the last cells in the row can't have two adjacent neighbors.

You are given an integer array cells where $\text{cells}[i] = 1$ if the i th cell is occupied and $\text{cells}[i] = 0$ if the i th cell is vacant, and you are given an integer n .

Return the state of the prison after n days (i.e., n such changes described above).

```

Input: cells = [0,1,0,1,1,0,0,1], N = 7
Output: [0,0,1,1,0,0,0,0]
Explanation: The following table summarizes the state of the prison on each day:
Day 0: [0, 1, 0, 1, 1, 0, 0, 1]
Day 1: [0, 1, 1, 0, 0, 0, 0, 0]
Day 2: [0, 0, 0, 1, 1, 1, 0, 0]
Day 3: [0, 1, 1, 0, 0, 1, 0, 0]
Day 4: [0, 0, 0, 0, 1, 0, 0, 0]
Day 5: [0, 1, 1, 0, 1, 0, 0, 0]
Day 6: [0, 0, 1, 0, 1, 1, 0, 0]
Day 7: [0, 0, 1, 1, 0, 0, 0, 0]

```

博主最开始做的时候, 看题目标记的是 Medium, 心想应该不需要啥特别的技巧, 于是就写了一个暴力破解的, 但是超时了 Time Limit Exceeded。给了一个超级大的 N , 不得不让博主怀疑是否能够直接遍历 N , 又看到了本题的标签是 Hash Table, 说明了数组的状态可能是会有重复的, 就是说可能是有一个周期循环的, 这样就完全没有必要每次都算一遍。正确的做法的应该是建立状态和当前 N 值的映射, 一旦当前计算出的状态在 HashMap 中出现了, 说明周期找到了, 这样就可以通过取余来

快速的缩小 N 值。为了使用 `HashMap` 而不是 `TreeMap`, 这里首先将数组变为字符串, 然后开始循环 N, 将当前状态映射为 N-1, 然后新建了一个长度为 8, 且都是 0 的字符串。更新的时候不用考虑首尾两个位置, 因为前面说了, 首尾两个位置一定会变为 0。更新完成了后, 便在 `HashMap` 查找这个状态是否出现过, 是的话算出周期, 然后 N 对周期取余。最后再把状态字符串转为数组即可, 参见代码如下:

```
vector<int> prisonAfterNDays(vector<int>& cells, int N) {
    vector<int> res;
    string str;
    for (int num : cells) str += to_string(num);
    unordered_map<string, int> m;
    while (N > 0) {
        m[str] = N--;
        string cur(8, '0');
        for (int i = 1; i < 7; ++i) {
            cur[i] = (str[i - 1] == str[i + 1]) ? '1' : '0';
        }
        str = cur;
        if (m.count(str)) {
            N %= m[str] - N;
        }
    }
    for (char c : str) res.push_back(c - '0');
    return res;
}
```

下面的解法使用了 `TreeMap` 来建立状态数组和当前 N 值的映射, 这样就不用转为字符串了, 写法是简单了一点, 但是运行速度下降了许多, 不过还是在 OJ 许可的范围之内, 参见代码如下:

```
vector<int> prisonAfterNDays(vector<int>& cells, int N) {
    map<vector<int>, int> m;
    while (N > 0) {
        m[cells] = N--;
        vector<int> cur(8);
        for (int i = 1; i < 7; ++i) {
            cur[i] = (cells[i - 1] == cells[i + 1]) ? 1 : 0;
        }
        cells = cur;
        if (m.count(cells)) {
            N %= m[cells] - N;
        }
    }
    return cells;
}
```

- 下面这种解法是看 lee215 大神的帖子中说的这个循环周期是 1, 7, 或者 14, 知道了这个规律后, 直接可以在开头就对 N 进行缩小处理, 取最大的周期 14, 使用 $(N-1) \% 14 + 1$ 的方法进行缩小, 至于为啥不能直接对 14 取余, 是因为首尾可能会初始化为 1, 而一旦 N 大于 0 的时候, 返回的状态首尾一定是 0。为了不使得正好是 14 的倍数的 N 直接缩小为 0, 所以使用了这么个小技巧, 参见代码如下:

```
vector<int> prisonAfterNDays(vector<int>& cells, int N) {
    for (N = (N - 1) % 14 + 1; N > 0; --N) {
        vector<int> cur(8);
        for (int i = 1; i < 7; ++i) {
            cur[i] = (cells[i - 1] == cells[i + 1]) ? 1 : 0;
        }
        cells = cur;
    }
    return cells;
}
```

```
public int[] prisonAfterNDays(int[] arr, int n) {
    int m = 8, cnt = 0;
    int[] tmp = arr.clone();
    while (cnt < (n % 14 == 0 ? 14 : n % 14)) {
        Arrays.fill(tmp, 0);
        for (int i = 1; i < m-1; i++)
            tmp[i] = 1 - (arr[i-1] ^ arr[i+1]);
        arr = tmp.clone();
        ++cnt;
    }
    return arr;
}
```

- 还有一个大神级的思路

since N might be pretty large, so we can't starting from times 1 to times N , No matter what the rules are, the states might be reappear after a certain times of proceeding(because we have fixed number of different states.)

but for different initial state, it might take different steps to reach back to this same state.

so we need to calculate the length of that. and based on N , we can get what we want after N steps.

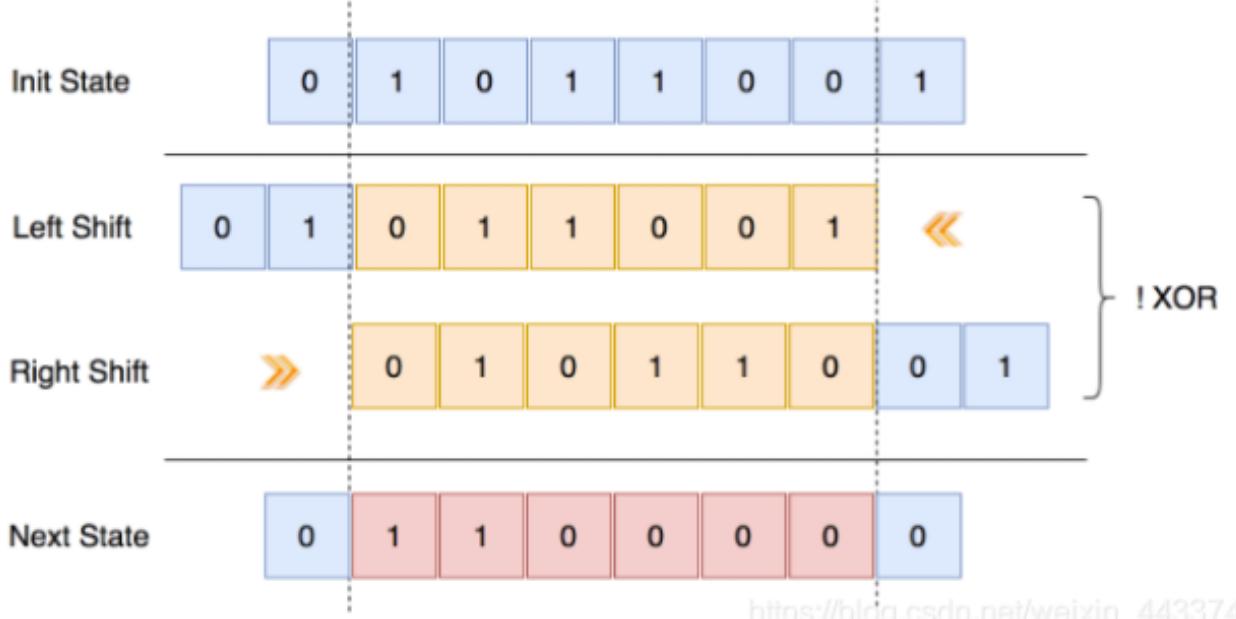
This is the method called fast-forward.

and if the number of possible states is very large, say 10^{10} , and it's even larger than N , then calculate the length of repetitive pattern is not acceptable. but in this problem, there will be 2^8 number of possible states. so we can calculate the length of cycle.

however, think twice about it. each time we need to check if this is a repetitive pattern of initial state. this is time consuming.

Solution2:

we have a better solution, in stead of change each digit at a time for each transaction, we use bit map, based on the follow rule:



https://hlcdr.csrdn.net/weizin_4423744

```

public int[] prisonAfterNDays(int[] cells, int N) {
    HashMap<Integer, Integer> seen = new HashMap<>();
    boolean isFastForwarded = false;
    // step 1). convert the cells to bitmap
    int stateBitmap = 0x0;
    for (int cell : cells) {
        stateBitmap <= 1;
        stateBitmap = (stateBitmap | cell);
    }
    // step 2). run the simulation with hashmap
    while (N > 0) {
        if (!isFastForwarded) {
            if (seen.containsKey(stateBitmap)) {
                // the length of the cycle is seen[state_key] - N
                N %= seen.get(stateBitmap) - N;
                isFastForwarded = true;
            } else
                seen.put(stateBitmap, N);
        }
        // check if there is still some steps remained,
        // with or without the fast forwarding.
        if (N > 0) {
            N -= 1;
            stateBitmap = this.nextDay(stateBitmap);
        }
    }
    // step 3). convert the bitmap back to the state cells
    int ret[] = new int[cells.length];
    for (int i = cells.length - 1; i >= 0; i--) {
        ret[i] = (stateBitmap & 0x1);
        stateBitmap = stateBitmap >> 1;
    }
    return ret;
}
protected int nextDay(int stateBitmap) {
    stateBitmap = ~(stateBitmap << 1) ^ (stateBitmap >> 1);
}

```

```
// set the head and tail to zero
stateBitmap = stateBitmap & 0x7e;
return stateBitmap;
}
```

Chapter 11

Bit Masks

11.1 总结一下

对于一个含有 N 个元素的集合，其总共包含个子集，因此有个掩码的可能，每一个掩码表示一个子集。事实上，每一个掩码就是一个用二进制表示的整数，比如 1001 就是 9。

Bitmasking 是为每个掩码分配一个值（即为每个子集分配一个值），然后使用已经计算出的掩码值来计算新掩码的值。通常，我们的主要目标是为整个集合（即掩码 11111111）计算值。

要计算子集 X 的值，我们要么以各种可能的方式删除元素，并将获得的子集的值，来计算 X 的值或解。这意味着的值必须已经计算过，因此我们需要考虑掩码计算的先后顺序。

最容易想到就是自然序：按相应数字的递增顺序遍历并计算掩码所对应的解。同样，我们一般从空的子集 X 开始，然后以各种可能的方式添加元素，并使用解已知的子集的值来计算 X 的值/解。

掩码常见的操作和表示：`bit(i, mask)` 表示取掩码的第 i 位 `count(mask)` 表示掩码中非零位的个数 `first(mask)` 表示掩码中最低非零位的数目 `set(i, mask)` 表示设置掩码中的第 i 位 `check(i, mask)` 表示检查掩码中的第 i 位

而在基于状态压缩的动态规划中，我们常用到以下四种计算操作：

- 若当前状态为 S ，对 S 有下列操作。
 - 判断第 i 位是否为 0: $(S \& (1 \ll i)) == 0$, 意思是将 1 左移 i 位与 S 进行与运算后，看结果是否为零。
 - 将第 i 位设置为 1: $S | (1 \ll i)$, 意思是将 1 左移 i 位与 S 进行或运算。
 - 将第 i 位设置为 0: $S \& \sim(1 \ll i)$, 意思是将 S 与第 i 位为 0, 其余位为 1 的数进行与运算;
 - 取第 i 位的值: $S \& (1 \ll i)$

11.2 1659. Maximize Grid Happiness - Hard

You are given four integers, m , n , `introvertsCount`, and `extrovertsCount`. You have an $m \times n$ grid, and there are two types of people: introverts and extroverts. There are `introvertsCount` introverts and `extrovertsCount` extroverts.

You should decide how many people you want to live in the grid and assign each of them one grid cell. Note that you do not have to have all the people living in the grid.

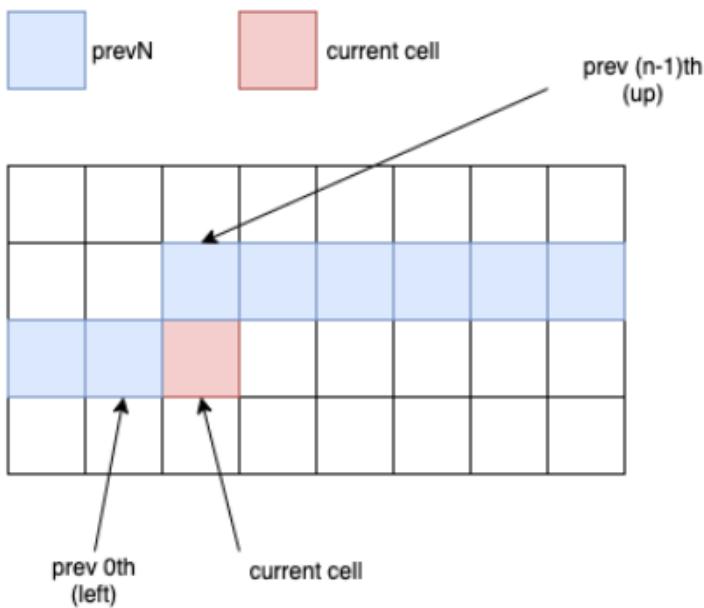
The happiness of each person is calculated as follows:

Introverts start with 120 happiness and lose 30 happiness for each neighbor (introvert or extrovert). Extroverts start with 40 happiness and gain 20 happiness for each neighbor (introvert or extrovert). Neighbors live in the directly adjacent cells north, east, south, and west of a person's cell.

The grid happiness is the sum of each person's happiness. Return the maximum possible grid happiness.

Key Notes:

- Just DFS+Memo
- For each cell we have 3 states -> **0: empty, 1: intro, 2: extro**
 - Only previous N bits matter (previous 1 is **left**, previous N is **up**)
 - Naturally, we use **Ternary** to construct our prevN bit mask
 - There are totally $3^5 = 243$ combinations, meaning the prevN value is bounded by 243.
- For each cell: try all possibilities: 0, 1 or 2
 - 0: do nothing, move our prevN bit mask with new 0 bit.
 - 1: happiness 120, move our prevN bit mask with new 1 bit.
 - 2: happiness 40, move our prevN bit mask with new 2 bit.
 - Check **up** and **left** cells to determine how many extra happiness need to be **added/subtracted**.



```

public int getMaxGridHappiness(int m, int n, int introvertsCount, int extrovertsCount) {
    return helper(m, n, 0, 0, introvertsCount, extrovertsCount, 0, new Integer[m][n][introvertsCount+1][extrovertsCount+1][243]);
}
private int get(int preN, int i) { // Ternary get ith bit value (0, 1 or 2)
    preN /= ((int) Math.pow(3, i));
    return preN % 3;
}
private int set(int curRow, int value) { // Ternary set new-coming bit to value
    return (curRow * 3 + value) % 243;
}
private int helper(int m, int n, int x, int y, int inCnt, int exCnt, int preN, Integer[][][][][] dp) {
    if (y == n) { // advance pointer
        y = 0;
        x++;
    }
    if (inCnt == 0 && exCnt == 0) return 0;
    if (x == m) return 0;
    if (dp[x][y][inCnt][exCnt][preN] != null) return dp[x][y][inCnt][exCnt][preN];
    int res = helper(m, n, x, y+1, inCnt, exCnt, set(preN, 0), dp); // leave the cell empty
    int up = get(preN, n-1); // get up bit -> which is at (n - 1)th
    int left = get(preN, 0); // get left bit -> which is at (0)th
    if (inCnt > 0) {
        int cur = preN;
        cur = set(cur, 1); // set new-coming bit to 1
        int addOn = 120;
        if (x-1 >= 0 && up != 0) { // up: 1 or 2
            addOn -= 30;
            if (up == 1) addOn -= 30;
            else addOn += 20;
        }
        if (y-1 >= 0 && left != 0) {
            addOn -= 30;
            if (left == 1) addOn -= 30;
            else addOn += 20;
        }
    }
}

```

```

        else addOn += 20;
    }
    res = Math.max(res, helper(m, n, x, y+1, inCnt-1, exCnt, cur, dp) + addOn);
}
if (exCnt > 0) {
    int cur = preN;
    cur = set(cur, 2); // set new-coming bit to 1
    int addOn = 40 ;
    if (x-1 >= 0 && up != 0) { // up: 1 or 2
        addOn += 20;
        if (up == 1) addOn -= 30;
        else addOn += 20;
    }
    if (y-1 >= 0 && left != 0) {
        addOn += 20;
        if (left == 1) addOn -= 30;
        else addOn += 20;
    }
    res = Math.max(res, helper(m, n, x, y+1, inCnt , exCnt-1, cur, dp) + addOn);
}
return dp[x][y][inCnt][exCnt][preN] = res;
}

```

The maximum score of any sub grid is determined by following variables:

1. Introverts count.
2. Extroverts count.
3. The starting cell of the sub grid.
4. The introverts/extroverts placement status of previous N cells where n is the total column of the grid.

For 4, we only need to store last N cells because the above cell is N away and the left cell is one away from current cell if we flatten the grid to a 1d array.

```

xxxx
xxxC

```

e.g. We only need four previous cells to calculate the score for Cell C.

So we can use a rolling string to store the state. E.g. if current state is "1111" and we pick extrovert for current cell then the state becomes "1112".

Put all pieces together, we have following as the overall state and use a map to store the maximum score for any particular state:
idx + PrevNCellstate + Introverts Count + Extroverts Count

Time complexity: $m * n * 3^n * \text{IntroCount} * \text{ExtroCount}$

Space complexity: $m * n * 3^n * \text{IntroCount} * \text{ExtroCount}$

```

public int getMaxGridHappiness(int m, int n, int introvertsCount, int extrovertsCount) {
    Map<String, Integer> memo = new HashMap<>();
    return helper("0".repeat(n), 0, m, n, introvertsCount, extrovertsCount, memo);
}
private int helper(String state, int idx, int m, int n, int inCnt, int exCnt, Map<String, Integer> memo) {
    if (inCnt == 0 && exCnt == 0 || idx == m*n) return 0;
    String key = idx + state + inCnt + exCnt;
    if (memo.containsKey(key)) return memo.get(key);
    int i = idx / n, j = idx % n;
    int max = 0;
    if (inCnt > 0) { // case 1: place an introvert in this cell if possible.
        int curVal = 120;
        if (i > 0) curVal = calScore(state.charAt(0)-'0', 1, curVal);
        if (j > 0) curVal = calScore(state.charAt(state.length()-1)-'0', 1, curVal);
        max = Math.max(max, helper(state.substring(1)+"1", idx+1, m, n, inCnt-1, exCnt, memo) + curVal);
    }
    if (exCnt > 0) { // case 2: place an extrovert in this cell if possible.
        int curVal = 40;
        if (i > 0) curVal = calScore(state.charAt(0)-'0', 2, curVal);
        if (j > 0) curVal = calScore(state.charAt(state.length()-1)-'0', 2, curVal);
        max = Math.max(max, helper(state.substring(1)+"2", idx+1, m, n, inCnt, exCnt-1, memo) + curVal);
    }
    // case 3: Do not place any person.
    max = Math.max(max, helper(state.substring(1)+"0", idx+1, m, n, inCnt, exCnt, memo)); // 不要忘记这种选择
    memo.put(key, max);
    return max;
}
private int calScore(int i, int j, int v) {
    if (i == 1 && j == 1) return v - 60;
    if (i == 2 && j == 2) return v + 40;
    if (i == 1 && j == 2 || i == 2 && j == 1) return v - 10;
}

```

```

    return v;
}

```

- 还有一种其它语言写的，行与行之间以行为单位进行动态规划状态转移的，可以搜出来参考一下

11.3 1494. Parallel Courses II - Hard

You are given an integer n , which indicates that there are n courses labeled from 1 to n . You are also given an array relations where $\text{relations}[i] = [\text{prevCourse}_i, \text{nextCourse}_i]$, representing a prerequisite relationship between course prevCourse_i and course nextCourse_i : course prevCourse_i has to be taken before course nextCourse_i . Also, you are given the integer k .

In one semester, you can take at most k courses as long as you have taken all the prerequisites in the previous semester for the courses you are taking.

Return the minimum number of semesters needed to take all courses. The testcases will be generated such that it is possible to take every course.

```

public int minNumberOfSemesters(int n, int [][] relations, int k) {
    int [] pre = new int [n]; // bitmask representing prerequirements
    for (int i = 0; i < relations.length; i++) {
        int u = relations[i][0] - 1;
        int v = relations[i][1] - 1;
        pre[v] |= (1 << u);
    }
    int range = (1 << n);
    int [] cnt = new int [range]; // 所有的状态只数一遍位数
    for (int i = 0; i < range; i++)
        cnt[i] = Integer.bitCount(i);
    int [] dp = new int [range]; // dp[state] = minimum semesters to complete all the courses of 'state'.
    Arrays.fill(dp, n);
    dp[0] = 0;
    for (int i = 0; i < range; i++) {
        int available = 0;
        for (int j = 0; j < n; j++)
            if ((i & pre[j]) == pre[j]) // 可是包含了已经选过的课程
                available |= (1 << j); // Can study course j next, since all required courses have been studied.
        available &= ~i; // Don't want to study those already studied courses.
        int nextCourses = available;
        while (nextCourses > 0) {
            if (cnt[nextCourses] <= k)
                dp[i | nextCourses] = Math.min(dp[i | nextCourses], dp[i] + 1);
            nextCourses = (nextCourses - 1) & available;
        }
    }
    return dp[range-1];
}

```

- 另一种用以 $++$ 写的递归的

算法

(状态压缩动态规划) $O(n \cdot 2^n + 3^n)$

1. 统计出每个课程的直接依赖课程的二进制掩码，记为 `pre`。
2. 设状态 $f(S)$ 表示完成掩码 S 的课程所需要的最少学期数。
3. 初始时， $f(0) = 0$ ，其余为正无穷。
4. 转移时，对于某个已经修完的课程掩码 S_0 ，求出 S_1 ，表示当前可以选择的新课程（新课程需要满足依赖条件），从 S_1 中通过递归回溯选出不多于 k 门课程，其掩码记为 S ，转移

$$f(S_0 \text{ bit or } S) = \min(f(S_0 \text{ bit or } S), f(S_0) + 1).$$
5. 最终答案为 $f((1 \ll n) - 1)$ 。

时间复杂度

- 状态数有 $O(2^n)$ 个，采用向后转移的方式，每个状态需要 $O(n)$ 的时间计算 S_1 ，同时枚举 S_1 的合法子集。
- 容易证明子集共有 $O(3^n)$ 个。
- 故总时间复杂度为 $O(n \cdot 2^n + 3^n)$ ，由于合法子集数目非常少，采用递归回溯枚举子集不会出现不合法的子集，则复杂度的常数很小。

空间复杂度

- 需要 $O(2^n)$ 的额外空间存储 `pre` 数组，系统栈和动态规划的状态。

```

void solve(int i, int s, int k, int n, int s0, int s1, vector<int> &f) {
    if (k == 0 || i == n) {
        f[s0 | s] = min(f[s0 | s], f[s0] + 1);
        return;
    }
    solve(i + 1, s, k, n, s0, s1, f);
    if ((s1 >> i) & 1)
        solve(i + 1, s | 1 << i, k - 1, n, s0, s1, f);
}
int minNumberOfSemesters(int n, vector<vector<int>> &dependencies, int k) {
    vector<int> pre(n, 0);
    for (const auto &v : dependencies)
        pre[v[1] - 1] |= 1 << (v[0] - 1);
    vector<int> f(1 << n, INT_MAX);
    f[0] = 0;
    for (int s0 = 0; s0 < (1 << n); s0++) {
        if (f[s0] == INT_MAX)
            continue;
        int s1 = 0;
        for (int i = 0; i < n; i++)
            if (!((s0 >> i) & 1) && ((pre[i] & s0) == pre[i]))
                s1 |= 1 << i;
        solve(0, 0, k, n, s0, s1, f);
    }
    return f[(1 << n) - 1];
}

```

11.4 1815. Maximum Number of Groups Getting Fresh Donuts - Hard

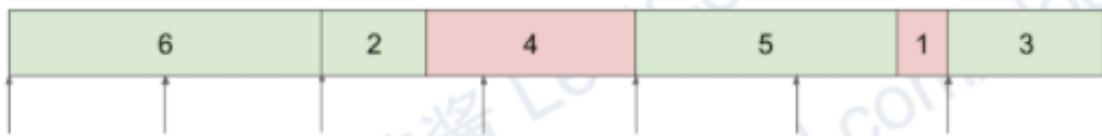
There is a donuts shop that bakes donuts in batches of batchSize. They have a rule where they must serve all of the donuts of a batch before serving any donuts of the next batch. You are given an integer batchSize and an integer array groups, where $\text{groups}[i]$ denotes that there is a group of $\text{groups}[i]$ customers that will visit the shop. Each customer will get exactly one donut.

When a group visits the shop, all customers of the group must be served before serving any of the following groups. A group will be happy if they all get fresh donuts. That is, the first customer of the group does not receive a donut that was left over from the previous group.

You can freely rearrange the ordering of the groups. Return the maximum possible number of happy groups after rearranging the groups.

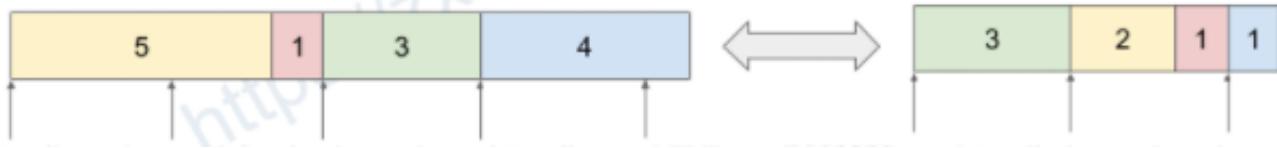
[6,2,4,5,1,3]

4 groups start with a new batch => ans = 4



Key observation:

1. If groups have sizes that are factors of batch size, we can put it in the front and all of them will be happy. And it is optimal.
2. For groups of sizes that are larger than batch size, we can use $n \% k$ as the new group size which will not affect the final results.



```

private int dfsBackTracking(int[] cnt, int batchSize, int lastGroup, int leftOverGroups) { // DFS+ 记忆化搜索求最多开心组数
    if (leftOverGroups == 0) return 0;
    String key = Arrays.toString(cnt); // 剩余组情况生成 String 作为哈希表的键, this is so called HASHING
    if (dp.containsKey(key)) return dp.get(key);
    int res = 0;
    for (int i = 1; i < batchSize; i++) {
        if (cnt[i] == 0) continue;
        --cnt[i];
        res = Math.max(res, dfsBackTracking(cnt, batchSize, lastGroup+i, leftOverGroups-1) + (lastGroup % batchSize == 0 ? 1 : 0));
        ++cnt[i];
    }
    dp.put(key, res);
    return res;
}
Map<String, Integer> dp;
int n;
public int maxHappyGroups(int batchSize, int[] groups) { // group size 30 is too large for backtracking WITHOUT modifications
    n = groups.length;
    int[] cnt = new int[batchSize];
    for (int v : groups)
        cnt[v % batchSize]++;
    dp = new HashMap<>();
    return dfsBackTracking(cnt, batchSize, 0, n-cnt[0]) + cnt[0];
}
    
```

- 一点儿稍微的优化，以减少回溯 backtracking 的耗时

```

private int dfsBackTracking(int[] cnt, int batchSize, int lastGroup, int leftOverGroups) { // DFS+ 记忆化搜索求最多开心组数
    if (leftOverGroups == 0) return 0;
    String key = Arrays.toString(cnt); // 剩余组情况生成 String 作为哈希表的键, this is so called HASHING
    if (dp.containsKey(key)) return dp.get(key);
    int res = 0;
    for (int i = 1; i < batchSize; i++) {
        if (cnt[i] == 0) continue;
        --cnt[i];
        res = Math.max(res, dfsBackTracking(cnt, batchSize, lastGroup+i, leftOverGroups-1) + (lastGroup % batchSize == 0 ? 1 : 0));
        ++cnt[i];
    }
    dp.put(key, res);
    return res;
}
Map<String, Integer> dp;
int n;
public int maxHappyGroups(int batchSize, int[] groups) { // group size 30 is too large for backtracking WITHOUT modifications
    n = groups.length;
    int[] cnt = new int[batchSize];
    for (int v : groups)
        cnt[v % batchSize]++;
    // take out the 2 remainder's min groups if their sum is batchSize.
    // it still works but slow without this step
    // Note: < batchSize / 2 to avoid when i is batchSize / 2 it will subtract itself
    
```

```

int res = cnt[0], min = 0;
for (int i = 1; i < batchSize/2; i++) {
    min = Math.min(cnt[i], cnt[batchSize-i]);
    cnt[i] -= min;
    cnt[batchSize-i] -= min;
    res += min;
}
dp = new HashMap<>();
return res + dfsBackTracking(cnt, batchSize, 0, n-cnt[0]);
}

```

- 压缩方法里的参数

```

private int dfsBackTracking(int[] cnt, int batchSize, int s) { // DFS+ 记忆化搜索
    String key = Arrays.toString(cnt);
    if (dp.containsKey(key)) return dp.get(key);
    int res = 0;
    for (int i = 1; i < batchSize; i++) {
        if (cnt[i] == 0) continue;
        --cnt[i];
        res = Math.max(res, dfsBackTracking(cnt, batchSize, (s+i) % batchSize) + (s == 0 ? 1 : 0));
        ++cnt[i];
    }
    dp.put(key, res);
    return res;
}
Map<String, Integer> dp;
int n;
public int maxHappyGroups(int batchSize, int[] groups) {
    n = groups.length;
    int[] cnt = new int[batchSize];
    for (int v : groups)
        cnt[v % batchSize]++;
    int res = cnt[0], min = 0;
    for (int i = 1; i < batchSize/2; i++) {
        min = Math.min(cnt[i], cnt[batchSize-i]);
        cnt[i] -= min;
        cnt[batchSize-i] -= min;
        res += min;
    }
    dp = new HashMap<>();
    return res + dfsBackTracking(cnt, batchSize, 0);
}

```

- 另一种 DP 超时的做法，掌握这个方法

```

// Time complexity: O(n*2n) TLE TLE TLE
// Space complexity: O(2n) TLE TLE TLE
public int maxHappyGroups(int batchSize, int[] groups) {
    int n = groups.length;
    int[] dp = new int[1 << n];
    int s = 0;
    for (int mask = 0; mask < 1 << n; mask++) {
        s = 0;
        for (int i = 0; i < n; i++)
            if ((mask & (1 << i)) >= 1)
                s = (s + groups[i]) % batchSize;
        for (int i = 0; i < n; i++)
            if ((mask & (1 << i)) == 0)
                dp[mask | (1 << i)] = Math.max(dp[mask | (1 << i)], dp[mask] + (s == 0 ? 1 : 0));
    }
    return dp[(1 << n) - 1];
}

```

11.5 691. Stickers to Spell Word - Hard

We are given n different types of stickers. Each sticker has a lowercase English word on it.

You would like to spell out the given string target by cutting individual letters from your collection of stickers and rearranging them. You can use each sticker more than once if you want, and you have infinite quantities of each sticker.

Return the minimum number of stickers that you need to spell out target. If the task is impossible, return -1.

Note: In all test cases, all words were chosen randomly from the 1000 most common US English words, and target was chosen as a concatenation of two random words.

- 【位图法】因为待匹配串 target 的数量最多是 15 个，因此其子集的数量最多有 2^{15}

- 一个，而 int 类型占用四个字节，能够容纳标识所有 target 的子集。所以我们可以将 target 的子集映射到 int 的整型数中。
- 【int 与 target 子集之间的映射关系】将 int 类型分解为二进制的形式后，有些位置为 0，有些位置为 1. 表明在 target 中哪些位置的字符是否保留（1 表示保留）。
- 【动态规划】dp 中存储的是得到子集 i, 需要的最少的单词的数量。

```

public int minStickers(String[] stickers, String target) {
    int m = target.length(), n = 1 << m;
    int [] dp = new int [1 << m];
    Arrays.fill(dp, Integer.MAX_VALUE);
    dp[0] = 0;
    int cur = 0;
    for (int i = 0; i < n; i++) {
        if (dp[i] == Integer.MAX_VALUE) continue;
        for (String s : stickers) {
            cur = i; // 关键代码 (下面: 在 i 上面加入一个单词后的效果)
            for (char c : s.toCharArray()) // for each char in the sticker, try apply it on a missing char in the subset of target
                for (int j = 0; j < m; j++) {
                    if (target.charAt(j) == c && ((cur >> j) & 1) == 0) {
                        cur |= 1 << j; // 在 cur 中相应位置, 加入 c, 形成新的集合。
                        break;
                    }
                }
            dp[cur] = Math.min(dp[cur], dp[i]+1); // 判断是否需要替换原来 cur 中的值。
        }
    }
    return dp[n-1] == Integer.MAX_VALUE ? -1 : dp[n-1];
}

```

- 另一种解法

```

private int helper(String s) {
    if (dp.containsKey(s)) return dp.get(s);
    int ans = Integer.MAX_VALUE;
    int [] tar = new int [26];
    for (char c : s.toCharArray())
        tar[c-'a']++;
    for (int i = 0; i < m; i++) {
        if (map[i][s.charAt(0)-'a'] == 0) continue;
        StringBuilder sb = new StringBuilder();
        for (int j = 0; j < 26; j++) {
            if (tar[j] > 0)
                for (int k = 0; k < Math.max(0, tar[j]-map[i][j]); k++)
                    sb.append((char)('a'+j));
        }
        int tmp = helper(sb.toString());
        if (tmp != -1) ans = Math.min(ans, 1+tmp);
    }
    dp.put(s, ans == Integer.MAX_VALUE ? -1 : ans);
    return dp.get(s);
}
Map<String, Integer> dp;
int [][] map;
int m;
public int minStickers(String[] stickers, String target) {
    m = stickers.length;
    map = new int [m][26];
    dp = new HashMap<>();
    for (int i = 0; i < m; i++)
        for (char c : stickers[i].toCharArray())
            map[i][c-'a']++;
    dp.put("", 0);
    return helper(target);
}

```

- 上面的这个，因为使用了图，以及必要的优化，性能还比较好

什么叫状态压缩？其实就是要用二进制数来表示动态规划状态转移过程中的状态。

什么时候应该状态压缩？状态压缩的题目，一般都会有非常明显的标志：如果你看到有一个参数的数值小于 20，同时这道题目中有涉及到是否选取、是否使用这样的二元状态，那么这道题目很可能就是一道状态压缩的题目。

本题中的标志就是 target 的长度不超过 15。于是，我们可以用一个二进制数表示 target 的每一位是否已经获取到。

后得到的状态对应的二进制数一定大于它的父状态。所以我们可以很自然地从 000...000 这一状态开始，一直遍历到 111...111（目标状态）。对于每一个状态，我们遍历所有的 stickers，看它能够更新出怎样的状态。

为了减少计算量，预处理得到了每一个 sticker 包含的每一种小写字母的个数。

这里讲的 `++` 的状态优化，可以参考一下

<https://leetcode-cn.com/problems/stickers-to-spell-word/solution/zhuang-tai-ya-su-dpji-you-hua-by-luc>

```

int INF = std::numeric_limits<int>::max();
int minStickers(vector<string>& stickers, string target) {
    vector<int> dp(1 << 15, INF);
    int n = stickers.size(), m = target.size();
    vector<vector<int>> cnt(n, vector<int>(26));
    for (int i = 0; i < n; ++i)
        for (char c : stickers[i])
            cnt[i][c - 'a']++;

    dp[0] = 0;
    for (int i = 0; i < (1 << m); ++i) {
        if (dp[i] == INF)
            continue;
        for (int k = 0; k < n; ++k) {
            int nxt = i;
            vector<int> left(cnt[k]);
            for (int j = 0; j < m; ++j) {
                if (nxt & (1 << j))
                    continue;
                if (left[target[j] - 'a'] > 0) {
                    nxt += (1 << j);
                    left[target[j] - 'a']--;
                }
            }
            dp[nxt] = min(dp[nxt], dp[i] + 1);
        }
    }
    return dp[(1 << m) - 1] == INF ? -1 : dp[(1 << m) - 1];
}

```

如何优化？

上面的代码通过了测试，但时间和空间消耗均无法让人满意。让我们思考一下问题出在哪里。

考虑有 `hello` 和 `world`，目标状态是 `helloworld`。我们从 `0000000000` 开始时，既考虑了使用 `hello`，也考虑了使用 `world`。这样就更新出了 `1111100000` 和 `0000011111` 两个状态。我们会发现，它们其实是殊途同归的。第一次选 `hello`，第二次就要选 `world`；第一次选 `world`，第二次就要选 `hello`。由于我们只需要计算使用贴纸的数量，先后顺序其实并不重要，这两个状态其实是重复的。

如何消除这一重复？我们可以增加一重限制。每次从当前状态开始更新时，我们只选择包含了当前状态从左边开始第一个没有包含的字母的那些贴纸。比如说在上面的例子中，在 `0000000000` 状态下，我们将只会选择 `hello`，不会选择 `world`（没有包含 `h`）。这样就去除了顺序导致的重复状态。

为了实现这一优化，我们预处理得到了 `can` 数组，记录包含每一个字母的贴纸序号。

```

int INF = std::numeric_limits<int>::max();
int minStickers(vector<string>& stickers, string target) {
    vector<int> dp(1 << 15, INF);
    int n = stickers.size(), m = target.size();
    vector<vector<int>> cnt(n, vector<int>(26));
    vector<vector<int>> can(26);
    for (int i = 0; i < n; ++i)
        for (char c : stickers[i]) {
            int d = c - 'a';
            cnt[i][d]++;
            if (can[d].empty() || can[d].back() != i)
                can[d].emplace_back(i);
        }

    dp[0] = 0;
    for (int i = 0; i < (1 << m) - 1; ++i) {
        if (dp[i] == INF)
            continue;
        int d;
        for (int j = 0; j < m; ++j) {
            if (!(i & (1 << j))) {
                d = j;
                break;
            }
        }
        d = target[d] - 'a';
        for (int k : can[d]) {
            int nxt = i;
            vector<int> left(cnt[k]);
            for (int j = 0; j < m; ++j) {
                if (nxt & (1 << j))
                    continue;
                if (left[target[j] - 'a'] > 0) {

```

```

        nxt += (1 << j);
        left[target[j] - 'a']--;
    }
}
dp[nxt] = min(dp[nxt], dp[i] + 1);
}
return dp[(1 << m) - 1] == INF ? -1 : dp[(1 << m) - 1];
}

```

11.6 1986. Minimum Number of Work Sessions to Finish the Tasks

There are n tasks assigned to you. The task times are represented as an integer array tasks of length n , where the i th task takes $\text{tasks}[i]$ hours to finish. A work session is when you work for at most sessionTime consecutive hours and then take a break. You should finish the given tasks in a way that satisfies the following conditions: If you start a task in a work session, you must complete it in the same work session. You can start a new task immediately after finishing the previous one. You may complete the tasks in any order. Given tasks and sessionTime , return the minimum number of work sessions needed to finish all the tasks following the conditions above.

The tests are generated such that sessionTime is greater than or equal to the maximum element in $\text{tasks}[i]$.

```

private void dfs(int[] arr, int t, int i, int cnt) { // cnt: sessionCnt
    if (cnt > res) return;
    if (i < 0) {
        res = Math.min(res, cnt);
        return;
    }
    for (int j = 0; j < cnt; j++) {
        if (sessions[j] + arr[i] <= t) { // 把当前 task 放入旧的 sessions 里
            sessions[j] += arr[i];
            dfs(arr, t, i-1, cnt);
            sessions[j] -= arr[i];
        }
    }
    sessions[cnt] += arr[i]; // 把当前 task 放入新的 sessions 里
    dfs(arr, t, i-1, cnt + 1);
    sessions[cnt] -= arr[i];
}
int[] sessions;
int n, res;
public int minSessions(int[] tasks, int sessionTime) {
    n = tasks.length;
    res = n;
    sessions = new int[n];
    Arrays.sort(tasks);
    dfs(tasks, sessionTime, n-1, 0);
    return res;
}

```

- 另一种写法

```

private int[] getMin(int[] a, int[] b) { // 这个题最近需要再写一遍
    if (a[0] > b[0]) return b;
    if (a[0] < b[0]) return a;
    if (a[1] > b[1]) return b;
    return a;
}
// dp[mask] = {a, b} where
// a - minimum number of session
// b - minimum time of last session
// The idea is to go through all tasks who belong to mask and optimally choose the last task 't' that was added to last session.
public int minSessions(int[] tasks, int sessionTime) {
    int n = tasks.length;
    int[][] dp = new int[1 << n][2]; // 在 [1, 1 << n) 范围内枚举每一个 mask 计算其包含的时间的总和
    dp[0][0] = 1;
    dp[0][1] = 0;
    for (int i = 1; i < 1 << n; i++) {
        dp[i][0] = Integer.MAX_VALUE;
        dp[i][1] = 0;
        int sum = 0;
        for (int t = 0; t < n; t++) {
            if ((i & (1 << t)) == 0) continue;
            int[] pre = dp[(1 << t) ^ i];
            if (pre[1] + tasks[t] <= sessionTime)
                dp[i] = getMin(dp[i], new int[] {pre[0], pre[1] + tasks[t]});
            else dp[i] = getMin(dp[i], new int[] {pre[0]+1, tasks[t]});
        }
    }
}

```

```

    }
    return dp[(1 << n) -1][0];
}

```

11.7 1655. Distribute Repeating Integers - Hard

You are given an array of n integers, nums , where there are at most 50 unique values in the array. You are also given an array of m customer order quantities, quantity , where $\text{quantity}[i]$ is the amount of integers the i th customer ordered. Determine if it is possible to distribute nums such that:

The i th customer gets exactly $\text{quantity}[i]$ integers, The integers the i th customer gets are all equal, and Every customer is satisfied. Return true if it is possible to distribute nums according to the above conditions.

```

private boolean backTracking(int[] arr, int[] quantity, int idx) {
    if (idx < 0) return true;
    Set<Integer> vis = new HashSet<>();
    for (int i = 0; i < arr.length; i++) {
        if (arr[i] < quantity[idx] || vis.contains(arr[i])) continue; // 去重
        vis.add(arr[i]);
        arr[i] -= quantity[idx];
        if (backTracking(arr, quantity, idx-1)) return true;
        arr[i] += quantity[idx];
    }
    return false;
}
public boolean canDistribute(int[] nums, int[] quantity) {
    Map<Integer, Integer> map = new HashMap<>();
    for (Integer v : nums)
        map.put(v, map.getOrDefault(v, 0) + 1);
    int[] arr = new int[map.size()];
    int i = 0;
    for (Integer val : map.values()) arr[i++] = val;
    Arrays.sort(quantity); // decreasing frequency: 是一种剪枝优化
    return backTracking(arr, quantity, quantity.length-1);
}

```

- 不用 map 的操作相对快一点儿

```

public boolean canDistribute(int[] nums, int[] quantity) {
    int[] counts = new int[1001]; // compress the states first
    int n = 0;
    for(int i: nums) {
        counts[i]++;
        if (counts[i] == 1) n++;
    }
    nums = new int[n];
    int j = 0;
    for (int i: counts)
        if (i > 0) nums[j++] = i;
    return distribute(nums, quantity, 0);
}
private boolean distribute(int[] nums, int[] quantity, int idx) {
    if (idx == quantity.length) return true;
    int q = quantity[idx];
    Set<Integer> used = new HashSet<>();
    for(int j = 0; j < nums.length; j++) {
        int k = nums[j];
        if (k < q || used.contains(k)) continue;
        nums[j] -= q;
        used.add(k); // Avoid duplicates. TLE without it.
        if (distribute(nums, quantity, idx+1)) return true;
        nums[j] += q;
    }
    return false;
}

```

11.8 956. Tallest Billboard: 这个题是最高挑战

You are installing a billboard and want it to have the largest height. The billboard will have two steel supports, one on each side. Each steel support must be an equal height. You are given a collection of rods that can be welded together. For example, if you have rods of lengths 1, 2, and 3, you can weld them together to make a support of length 6. Return the largest possible height of your billboard installation. If you cannot support the billboard, return 0.

```
// // https://blog.csdn.net/luke2834/article/details/89457888 // 这个题目要多写几遍
public int tallestBillboard(int[] rods) { // 写得好神奇呀
    int n = rods.length;
    int sum = Arrays.stream(rods).sum();
    System.out.println("sum: " + sum);
    int [][] dp = new int [2][(sum + 1) << 1]; // (sum + 1) * 2
    for (int i = 0; i < 2; i++)
        Arrays.fill(dp[i], -1);
    dp[0][sum] = 0;
    for (int i = 0; i < n; i++) {
        int cur = i & 1, next = (i & 1) ^ 1; // 相当于滚动数组: [0, 1]
        for (int j = 0; j < dp[cur].length; j++) {
            if (dp[cur][j] == -1) continue;
            dp[next][j] = Math.max(dp[cur][j], dp[next][j]); // update to max
            dp[next][j+rods[i]] = Math.max(dp[next][j+rods[i]], dp[cur][j] + rods[i]);
            dp[next][j-rods[i]] = Math.max(dp[next][j-rods[i]], dp[cur][j] + rods[i]);
        }
    }
    return dp[rods.length & 1][sum] >> 1; // dp[n&1][sum] / 2
}
```

- 这里详细纪录一下生成过程，记住这个方法

```
int [] a = new int [] {1, 2, 3};
sum: 6
i: 0
-1, -1, -1, -1, -1, 0, -1, -1, -1, -1, -1, -1,
-1, -1, -1, -1, -1, 1, 0, 1, -1, -1, -1, -1, -1,
i: 1
-1, -1, -1, 3, 2, 3, 0, 3, 2, 3, -1, -1, -1, -1,
-1, -1, -1, -1, -1, 1, 0, 1, -1, -1, -1, -1, -1,
i: 2
-1, -1, -1, 3, 2, 3, 0, 3, 2, 3, -1, -1, -1, -1,
6, 5, 6, 3, 6, 5, 6, 5, 6, 3, 6, 5, 6, -1,
r: 3
```

- 另一种写法

```
// 1. 所有的状态全集: dp[len][sum+1], len = length of array, sum = sum of the array, 代表两边共有的高度
// 2. state transfer:
//     a. 忽略当前 dp[i][j] = max(dp[i-1][j], dp[i][j])
//     b. 加入到 higher 一侧 dp[i][j+h] = max(dp[i][j+h], dp[i-1][j])
//     c. 加入到 lower 一侧 lower = abs(j-h); dp[i][lower] = max(dp[i][lower], dp[i][j] + min(j, h)); 其中 min(j, h) 为新增高度
private void dfs(int [] arr, int idx) {
    int cur = arr[idx];
    if (dp[idx][cur] != -1) return;
    if (idx == 0) {
        dp[idx][cur] = 0; // add
        dp[idx][0] = 0; // ignore
        return;
    }
    dfs(arr, idx-1);
    int lower = 0;
    for (int i = 0; i < dp[idx].length-cur; i++) {
        if (dp[idx-1][i] < 0) continue;
        dp[idx][i] = Math.max(dp[idx][i], dp[idx-1][i]); // 1: ignore
        dp[idx][i+cur] = Math.max(dp[idx][i+cur], dp[idx-1][i]); // 2: add to higher
        lower = Math.abs(i - cur); // 3. add to lower
        dp[idx][lower] = Math.max(dp[idx][lower], dp[idx-1][i] + Math.min(i, cur));
    }
}
int [][] dp;
int n;
public int tallestBillboard(int[] rods) {
    int n = rods.length;
    int sum = Arrays.stream(rods).sum();
    dp = new int [n][sum+1];
    for (int i = 0; i < n; i++)
        Arrays.fill(dp[i], -1);
    dfs(rods, n-1);
    return dp[n-1][0];
}
```

- 这里详细纪录一下生成过程，记住这个方法

```
int [] a = new int [] {1, 2, 3};
i: 0
0, -1, -1, -1, -1, -1, -1,
```

```

0, -1, 0, -1, -1, -1, -1, -1,
-1, -1, -1, -1, -1, -1, -1, -1,
i: 1
0, 0, -1, -1, -1, -1, -1, -1,
0, 1, 0, 0, -1, -1, -1, -1,
-1, -1, -1, -1, -1, -1, -1, -1,
i: 0
0, 0, -1, -1, -1, -1, -1, -1,
0, 1, 0, 0, -1, -1, -1, -1,
0, -1, -1, 0, -1, -1, -1, -1,
i: 1
0, 0, -1, -1, -1, -1, -1, -1,
0, 1, 0, 0, -1, -1, -1, -1,
0, 1, 2, 0, 1, -1, -1, -1,
i: 2
0, 0, -1, -1, -1, -1, -1, -1,
0, 1, 0, 0, -1, -1, -1, -1,
0, 2, 2, 0, 1, 0, -1, -1,
i: 3
0, 0, -1, -1, -1, -1, -1, -1,
0, 1, 0, 0, -1, -1, -1, -1,
3, 2, 2, 0, 1, 0, 0, -1,
r: 3

```

```

// 定义一个数对键值: (i, j): i 表示两个子序列的累加和差值的绝对值, j 表示这个差值下, 子序列中累加和的最大值, 定义一个 dp 的 map 存放前 m 个数的所有子序列的累加和之差
// 新建一个 HashMap temp 用于存放第 m 个数对之前子序列累加和只差的状态转移结果
// 对于新到来的 rod, 只能够有 3 中情况:
// 1. rod 不加入任何列表:
//    从 dp 中拿出每个子序列的差值 k 的累加和最大值 v1, 每个结果与 temp 中相应 k 的累加和最大值 v2 比较, 如果 v1>v2, 那么更新 temp 中 k 对应的最大累加和
// 2. rod 加入累加和较大的序列
//    从 dp 中拿出每个子序列的差值 k 的累加和最大值 v1, 并加上 rod, 这时差值变成 k + rod, 累加和最大值变成 v1+ rod, 每个结果与 temp 中相应 k + rod 的累加和最大值 v2 比较, 如果 v1+v2>v2, 那么更新 temp 中 k + rod 对应的最大累加和
// 3. rod 加入累加和较小的序列
//    从 dp 中拿出每个子序列的差值 k 的累加和最大值 v1, 累加和较小的子序列加上了 rod, 那么和累加和较大的子序列之差为 k-rod, k-rod 为负数时, 说明累加和较小
public int tallestBillboard(int[] rods) {
    int n = rods.length;
    Map<Integer, Integer> dp = new HashMap<>();
    dp.put(0, 0);
    for (int rod : rods) {
        System.out.println("\nrod: " + rod);
        Map<Integer, Integer> tmp = new HashMap<>();
        dp.forEach((k, v) -> {
            if (tmp.getOrDefault(k, -1) < v) tmp.put(k, v);
            if (tmp.getOrDefault(k+rod, -1) < v+rod) tmp.put(k + rod, v+rod);
            int dis = k - rod;
            int larger = Math.max(v, v-dis);
            dis = Math.abs(dis);
            if (tmp.getOrDefault(dis, -1) < larger) tmp.put(dis, larger);
        });
        dp = tmp;
    }
    return dp.get(0);
}

```

11.9 1681. Minimum Incompatibility

You are given an integer array `nums` and an integer `k`. You are asked to distribute this array into `k` subsets of equal size such that there are no two equal elements in the same subset.

A subset's incompatibility is the difference between the maximum and minimum elements in that array.

Return the minimum possible sum of incompatibilities of the k subsets after distributing the array optimally, or return -1 if it is not possible.

A subset is a group integers that appear in the array with no particular order.

- Java O(k^n) solution with early termination (9ms 98%)

This problem is asking us to do the reversal of "merging k sorted lists into one sorted list".

In other words, considering we are "distributing a sorted list to k sorted lists".

The time complexity is $O(k^n)$ since each number can have k choices.

```
private void backTracking(int [] arr, int k, int idx, int total) {  
    if (total >= min) return; // early termination  
    if (idx == n) {  
        min = total; // With early termination, Math.min() is no longer needed.  
        return;  
    }  
    for (int i = 0; i < dp.size(); i++) {
```

```

    LinkedList<Integer> bucket = dp.get(i);
    int dist = 0;
    if (bucket.size() < n/k && bucket.peekLast() < arr[idx]) {
        dist = arr[idx] - bucket.peekLast(); // .....
        bucket.addLast(arr[idx]);
        backTracking(arr, k, idx+1, total + dist);
        bucket.removeLast();
    }
}
if (dp.size() < k) { // 记住这个分组，总是有可以多分出一个组的情况需要考虑到
    LinkedList<Integer> bucket = new LinkedList<>();
    bucket.add(arr[idx]);
    dp.addLast(bucket);
    backTracking(arr, k, idx+1, total);
    dp.removeLast();
}
int min = Integer.MAX_VALUE;
LinkedList<LinkedList<Integer>> dp;
int n;
public int minimumIncompatibility(int[] arr, int k) {
    n = arr.length;
    dp = new LinkedList<>();
    Arrays.sort(arr);
    backTracking(arr, k, 0, 0);
    return min == Integer.MAX_VALUE ? -1 : min;
}

```

- Optimized version (9ms): Replacing `LinkedList` with `int[]`, where `int[]{length, tail element}` represents a sorted list/bucket since we only need to remember the length and the tail element of each sorted list.

```

int ans = Integer.MAX_VALUE;
void helper(int[] nums, int s, int[][] buckets, int idx, int size, int total) {
    if(total >= ans) return; //early termination
    if (s == nums.length) {
        ans = total; // With early termination, Math.min() is no longer needed.
    } else {
        // distribute current number to an existing bucket
        for (int i=0; i<idx; i++) {
            if (buckets[i][0] < size && buckets[i][1] < nums[s]) {
                int distance = nums[s] - buckets[i][1];
                int last = buckets[i][1];
                buckets[i][0]++;
                buckets[i][1] = nums[s];
                helper(nums, s+1, buckets, idx, size, total+distance);
                buckets[i][0]--;
                buckets[i][1] = last;
            }
        }
        // distribute current number to an empty bucket
        if (buckets.length > idx) {
            buckets[idx][0] = 1;
            buckets[idx][1] = nums[s];
            helper(nums, s+1, buckets, idx+1, size, total);
            buckets[idx][0] = 0;
        }
    }
}
public int minimumIncompatibility(int[] nums, int k) {
    Arrays.sort(nums);
    helper(nums, 0, new int[k][2], 0, nums.length/k, 0);
    return ans == Integer.MAX_VALUE?-1: ans;
}

```

11.10 1994. The Number of Good Subsets

You are given an integer array `nums`. We call a subset of `nums` good if its product can be represented as a product of one or more distinct prime numbers.

For example, if `nums` = [1, 2, 3, 4]: [2, 3], [1, 2, 3], and [1, 3] are good subsets with products $6 = 2 \cdot 3$, $6 = 2 \cdot 3$, and $3 = 3$ respectively. [1, 4] and ¹ are not good subsets with products $4 = 2 \cdot 2$ and $4 = 2 \cdot 2$ respectively. Return the number of different good subsets in `nums` modulo 109 + 7.

A subset of `nums` is any array that can be obtained by deleting some (possibly none or all) elements from `nums`. Two subsets are different if and only if the chosen indices to delete are different.

¹DEFINITION NOT FOUND.

11.10.1 分析一下

- The value range from 1 to 30. If the number can be reduced to 20, an algorithm runs $O(2^{20})$ should be sufficient. So I should factorialize each number to figure out how many valid number within the range first.
- There are only 18 valid numbers (can be represented by unique prime numbers)
- Represent each number by a bit mask - each bit represent the prime number
- The next step should be that categorize the input - remove all invalid numbers and count the number of 1 as we need to handle 1 separately.
- The problem is reduced to a math problem and I simply test all the combinations - $O(18*2^{18})$
- If 1 exists in the input, the final answer will be result * $(1 \ll \text{number}_{of\ one}) \% \text{mod}$.
- <https://leetcode.com/problems/the-number-of-good-subsets/discuss/1444183/Java-Bit-Mask-%2B-DP-Solution>

```

static int mod = (int) 1e9 + 7;
static int[] map = new int[31];
static {
    int[] prime = new int[] {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}; // length: 10
    for (int i = 2; i <= 30; ++i) {
        if (i % 4 == 0 || i % 9 == 0 || i == 25) continue;
        int mask = 0;
        for (int j = 0; j < 10; ++j)
            if (i % prime[j] == 0)
                mask |= 1 << j;
        map[i] = mask;
    }
}
public int numberOfGoodSubsets(int[] nums) {
    int n = nums.length, one = 0;
    int[] dp = new int[1024], cnt = new int[31]; // 1024 ?
    dp[0] = 1;
    for (int i : nums) {
        if (i == 1) one++;
        else if (map[i] != 0) cnt[i]++;
    }
    for (int i = 0; i < 31; ++i) {
        if (cnt[i] == 0) continue;
        for (int j = 0; j < 1024; ++j) {
            if ((j & map[i]) != 0) continue; // 含有某个公共质因子 val 子集的统计数 * 当前 val 的重复次数
            dp[j | map[i]] = (int) ((dp[j | map[i]] + dp[j] * (long) cnt[i]) % mod);
        }
    }
    long res = 0;
    for (int i : dp) res = (res + i) % mod;
    res--; // 应该是减去一个 1 吧
    if (one != 0) res = res * pow(one) % mod;
    return (int) res;
}
private long pow(int n) { // 快速幂
    long res = 1, m = 2;
    while (n != 0) {
        if ((n & 1) == 1) res = (res * m) % mod;
        m = m * m % mod;
        n >>= 1;
    }
    return res;
}

```

- 另一种方法参考一下，没有使用到快速幂，稍慢一点儿
- For each number n from 1 to 30, you can decide select it or not.
 - 1 - select any times, full permutation $\text{pow}(2, \text{cnt})$
 - 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 - select 0 or one time
 - 6, 10, 14, 15, 21, 22, 26, 30 - select if the prime factors of n not yet selected
 - others - can not select

```

long f(int n, long mask) {
    if (n > 30) return mask == 0 ? 0L : 1L;
    long rst = f(n + 1, mask) % MOD;

```

```

if (n == 2 || n == 3 || n == 5 || n == 7 || n == 11 || n == 13 || n == 17 || n == 19 || n == 23 || n == 29)
    rst = (rst + cnts[n] * f(n + 1, mask | (1 << n))) % MOD;
else if (n == 6)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 3)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 3))) % MOD;
else if (n == 10)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 5)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 5))) % MOD;
else if (n == 14)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 7)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 7))) % MOD;
else if (n == 22)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 11)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 11))) % MOD;
else if (n == 26)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 13)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 13))) % MOD;
else if (n == 15)
    if ((mask & (1 << 3)) == 0 && (mask & (1 << 5)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 3) | (1 << 5))) % MOD;
else if (n == 21)
    if ((mask & (1 << 3)) == 0 && (mask & (1 << 7)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 3) | (1 << 7))) % MOD;
else if (n == 30)
    if ((mask & (1 << 2)) == 0 && (mask & (1 << 3)) == 0 && (mask & (1 << 5)) == 0)
        rst = (rst + cnts[n] * f(n + 1, mask | (1 << 2) | (1 << 3) | (1 << 5))) % MOD;
return rst;
}
int MOD = 1_000_000_007;
long[] cnts = new long[31];
public int numberofGoodSubsets(int[] nums) {
    for (int n : nums) cnts[n]++;
    long rst = f(1, 0L);
    for (int i = 0; i < cnts[1]; i++) // 没有快速幂, 稍慢
        rst = rst * 2 % MOD;
    return (int) rst;
}

```

- // Speed up using frequency array. $O(30*1024 + N)$ = linear time

- 看不懂:<https://leetcode.com/problems/the-number-of-good-subsets/discuss/1444661/Java-DP-%2B-Bitmask-or->

```

private static final long MOD=(long) (1e9+7);
private static long add(long a, long b){ a+=b; return a<MOD?a:a-MOD;}
private static long mul(long a, long b){ a*=b; return a<MOD?a:a%MOD;}
private static long pow(long a, long b) {
    //a %= MOD;
    //b%=(MOD-1); //if MOD is prime
    long res = 1;
    while (b > 0) {
        if ((b & 1) == 1)
            res = mul(res, a);
        a = mul(a, a);
        b >>= 1;
    }
    return add(res, 0);
}
public int numberofGoodSubsets(int[] nums) {
    int N = nums.length, i;
    int[] primes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
    int[] mask = new int[31];
    int[] freq = new int[31];
    for (int x : nums) freq[x]++;
    for (i = 1; i <= 30; i++)
        for (int j = 0; j < primes.length; j++)
            if (i % primes[j] == 0) {
                if ((i / primes[j]) % primes[j] == 0) {
                    mask[i] = 0;
                    break;
                }
                mask[i] |= (int) pow(2, j);
            }
    long[] dp = new long[1024];
    dp[0] = 1;
    for (i = 1; i <= 30; i++) {
        if (mask[i] == 0) continue;
        for (int j = 0; j < 1024; j++)
            if ((mask[i] & j) == 0 && dp[j] > 0)
                dp[(mask[i] | j)] = add(dp[(mask[i] | j)], mul(dp[j], freq[i]));
    }
    long ans = 0;
    for (i = 1; i < 1024; i++) ans = add(ans, dp[i]);
    ans = mul(ans, pow(2, freq[1]));
}

```

```

ans = add(ans, 0);
return (int) ans;
}

```

- Java 预处理 + 暴搜
- <https://leetcode-cn.com/problems/the-number-of-good-subsets/solution/java-yu-chu-li-bao-sou-by-liin-n/>
- 暂略

11.11 1349. Maximum Students Taking Exam - Hard

Given a $m * n$ matrix seats that represent seats distributions in a classroom. If a seat is broken, it is denoted by '#' character otherwise it is denoted by '.' character.

Students can see the answers of those sitting next to the left, right, upper left and upper right, but he cannot see the answers of the student sitting directly in front or behind him. Return the maximum number of students that can take the exam together without any cheating being possible..

Students must be placed in seats in good condition.

```

public int maxStudents(char[][] seats) {
    int m = seats.length;
    int n = seats[0].length;
    int range = 1 << n, mask = 0;
    int [] rowMax = new int [m+1]; // 相比于我的第一行，他是先生成了每一行的 mask base
    for (int i = 0; i < m; i++) {
        mask = 0;
        for (int j = 0; j < n; j++)
            mask = mask * 2 + (seats[i][j] == '.' ? 1 : 0);
        rowMax[i+1] = mask;
    }
    int [][] dp = new int [m+1][range];
    for (int i = 0; i <= m; i++)
        Arrays.fill(dp[i], -1);
    dp[0][0] = 0;
    // 如果想要满足限制条件 2，则需要第 i 排可能的 bitmask 与第 i - 1 排可能的 bitmask 进行检测
    // // upper left and upper right are valid or not
    // (mask >> 1) & prev_mask
    // mask & (prev_mask >> 1)
    // dp[r - 1][prev_mask] is valid
    // 基于以上的分析，动规方程可以归纳为以下
    // dp[r][mask] = max(dp[r][mask], dp[r - 1][prev_mask] + bit_count(mask))
    for (int i = 1; i <= m; i++)
        for (int curMask = 0; curMask <= rowMax[i]; curMask++)
            if ((curMask & rowMax[i]) == curMask && (curMask & (curMask >> 1)) == 0) // 现行所有的有效掩码：既不会坐墙上，也左右无人
                for (int preMask = 0; preMask < range; preMask++)
                    if (dp[i-1][preMask] != -1 && (curMask & (preMask >> 1)) == 0 && ((curMask >> 1) & preMask) == 0)
                        dp[i][curMask] = Math.max(dp[i][curMask], dp[i-1][preMask] + Integer.bitCount(curMask));
    int max = 0;
    for (int i = 0; i < range; i++)
        max = Math.max(max, dp[m][i]);
    return max;
}

```

- 自己写的，bug 还有找出来。。。

```

private boolean isValid(char [][] arr, int v, int idx) { // 自己写的，不知道自己写的错哪里了，改天把它找出来
    for (int i = 0; i < n; i++)
        if (((v >> i) & 1) == 1 && arr[idx][i] != '.') return false;
    return true;
}
int m, n;
public int maxStudents(char[][] seats) {
    m = seats.length;
    n = seats[0].length;
    int range = 1 << n;
    int [][] dp = new int [m+1][range];
    for (int i = 0; i <= m; i++)
        Arrays.fill(dp[i], -1);
    dp[0][0] = 0;
    for (int i = 1; i < m; i++) {
        for (int k = 0; k < range; k++) { // cur mask == k
            if (!isValid(seats, k, i-1)) continue;
            for (int j = 0; j < range; j++) { // pre mask == j

```

```

        if (dp[i-1][j] == -1) continue;
        if (((k >> 1) & j) == 0 && (k & (j >> 1)) == 0)
            dp[i][k] = Math.max(dp[i][k], dp[i-1][j] + Integer.bitCount(k));
    }
}
int max = 0;
for (int i = 1; i < range; i++)
    max = Math.max(max, dp[m][i]);
return max;
}

```

11.12 1434. Number of Ways to Wear Different Hats to Each Other - Hard

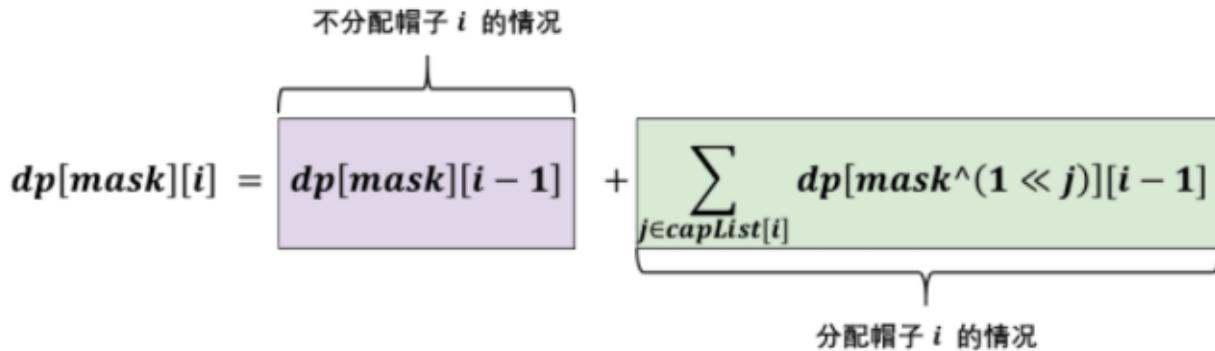
There are n people and 40 types of hats labeled from 1 to 40.

Given a list of list of integers hats , where $\text{hats}[i]$ is a list of all hats preferred by the i -th person.

Return the number of ways that the n people wear different hats to each other.

Since the answer may be too large, return it modulo $10^9 + 7$.

当前 $i = 2$, $mask = 011$, 要将帽子 2 分配给人 $j = 1$, 我们就应该保证人 $j = 1$ 的前一个状态的第 j 位为 0, 也就是说, 其当前尚未分配帽子, 只有在没有分配到帽子且可以戴帽子的时候才能将帽子 i 分配给 j , 所有我们要考虑问题的子状态 ($i = i-1 = 1$ $mask = 011 \wedge 010 = 001$), 也就是说 $dp[3][2] = dp[3][2] + dp[1][1] = 0 + 1 = 1$, 注意 $dp[][]$ 二维数组初始化均为 0, 大胆推广一下, $dp[mask][i] += dp[mask \wedge (1 \ll j)][i-1]$, 其中 $j \in capList[i]$ 。再结合不分配帽子 i 的情况, 则状态转移方程为:



这里需要对 $mask \wedge (1 \ll j)$ ** 进行说明, $1 \ll j$ ** 就表示左移 j 位, 含义是第 j 个人戴帽子的二进制表示, 比如 $dp[3][2]$ 中, $capList[2]=\{1\}$, $j = 1$, $1 \ll j = 010$; 而 $mask \wedge (1 \ll j)$ 则表示 $mask$ 的第 j 位为 0 的情况, 即第 j 个人当前未戴帽子 i 的状态, 比如 $010 \wedge 011 = 001$, 我们发现异或的作用就是将第 j 位为 1 的情况变成 0, 而其他位置保持不变。

则 $i = 2$ 时的 DP Table 如下所示:

		mask							
		000	001	010	011	100	101	110	111
i		0	1	0	0	0	0	0	0
		1	1	1	0	0	0	0	0
2	1	1	1	1	0	0	0	0	0
3	1	1	1	1	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0
5									
...
100									

```
public int numberWays(List<List<Integer>> hats) {
    int n = hats.size(), range = 1 << n, mod = (int)1e9 + 7;
    List<Integer> ids = new ArrayList<>();
    Map<Integer, List<Integer>> hm = new HashMap<>(); // hats map
```

```

for (int i = 0; i < n; i++) {
    for (int j = 0; j < hats.get(i).size(); j++) {
        hm.computeIfAbsent(hats.get(i).get(j), k -> new ArrayList<>());
        hm.get(hats.get(i).get(j)).add(i);
        if (!ids.contains(hats.get(i).get(j))) ids.add(hats.get(i).get(j));
    }
}
int [][] dp = new int [range][ids.size()+1];
dp[0][0] = 1; // 回到 dp[0][0] 状态时为 1 个方案数!
for (int i = 1; i <= ids.size(); i++) {
    for (int mask = 0; mask < range; mask++) {
        dp[mask][i] = dp[mask][i-1]; // 1. 第 i 个帽子不分配的情况
        int size = hm.get(ids.get(i-1)).size();
        for (int j = 0; j < size; j++) // 2. 第 i 个帽子分给第 j 个人的情况
            if ((mask & (1 << hm.get(ids.get(i-1)).get(j))) != 0) {
                dp[mask][i] += dp[mask ^ (1 << hm.get(ids.get(i-1)).get(j))][i-1]; // 第 i 个帽子可以是由右边异或的状态转移过来的
                dp[mask][i] %= mod;
            }
    }
}
return dp[range-1][ids.size()];
}

```

11.13 1595. Minimum Cost to Connect Two Groups of Points - Hard 自顶向下 (dfs + 记忆数组); 自底向上: DP table

You are given two groups of points where the first group has size1 points, the second group has size2 points, and $\text{size1} \geq \text{size2}$.

The cost of the connection between any two points are given in an $\text{size1} \times \text{size2}$ matrix where $\text{cost}[i][j]$ is the cost of connecting point i of the first group and point j of the second group. The groups are connected if each point in both groups is connected to one or more points in the opposite group. In other words, each point in the first group must be connected to at least one point in the second group, and each point in the second group must be connected to at least one point in the first group.

Return the minimum cost it takes to connect the two groups.

```

// Straightforward top-down DP for the first group. At the same time, we track which elements from the second group were connected in mask.
// After finishing with the first group, we detect elements in group 2 that are still disconnected,
// and connect them with the "cheapest" node in the first group.
private int dfs(List<List<Integer>> arr, int i, int mask, int [] minCost) { // 自顶向下, 需要记忆
    if (dp[i][mask] != null) return dp[i][mask];
    // if (i == m && Integer.bitCount(mask) == n) return 0; // 这行可要可不要
    if (i == m) {
        int res = 0;
        for (int j = 0; j < n; j++)
            if ((mask & (1 << j)) == 0) res += minCost[j];
        return dp[i][mask] = res;
    }
    int res = Integer.MAX_VALUE;
    for (int j = 0; j < n; j++) // 只有暴力查找尝试了所有可能性, 才是全局最优解
        res = Math.min(res, dfs(arr, i+1, mask | (1 << j), minCost) + arr.get(i).get(j));
    return dp[i][mask] = res;
}
Integer [][] dp; // (number of points assigned in first group, bitmask of points assigned in second group).
int m, n;
public int connectTwoGroups(List<List<Integer>> cost) {
    m = cost.size();
    n = cost.get(0).size();
    dp = new Integer [m+1][1 << n]; // 右边点组过程中共有  $1 << n$  种状态, 但是如何知道记住右边的点分别是与左边哪个点连接起来的呢?
    int [] minCost = new int [n]; // 对右边的每个点, 它们分别与左边点连通, 各点所需的小花费
    Arrays.fill(minCost, Integer.MAX_VALUE);
    for (int j = 0; j < n; j++)
        for (int i = 0; i < m; i++)
            minCost[j] = Math.min(minCost[j], cost.get(i).get(j));
    return dfs(cost, 0, 0, minCost);
}

```

- 动态规划, 用二进制压缩状态, 注意分析几种情况, 就能推出来正确的状态转移方程。

```

public int connectTwoGroups(List<List<Integer>> cost) {
    int m = cost.size();
    int n = cost.get(0).size();
    int [][] dp = new int [m][1 << n]; // 右边点组过程中共有  $1 << n$  种状态, 但是如何知道记住右边的点分别是与左边哪个点连接起来的呢?
    for (int i = 0; i < m; i++)
        Arrays.fill(dp[i], Integer.MAX_VALUE/2);
    for (int i = 0; i < m; i++) { // 暴力求解所有值取最小
        for (int j = 0; j < 1 << n; j++) {

```

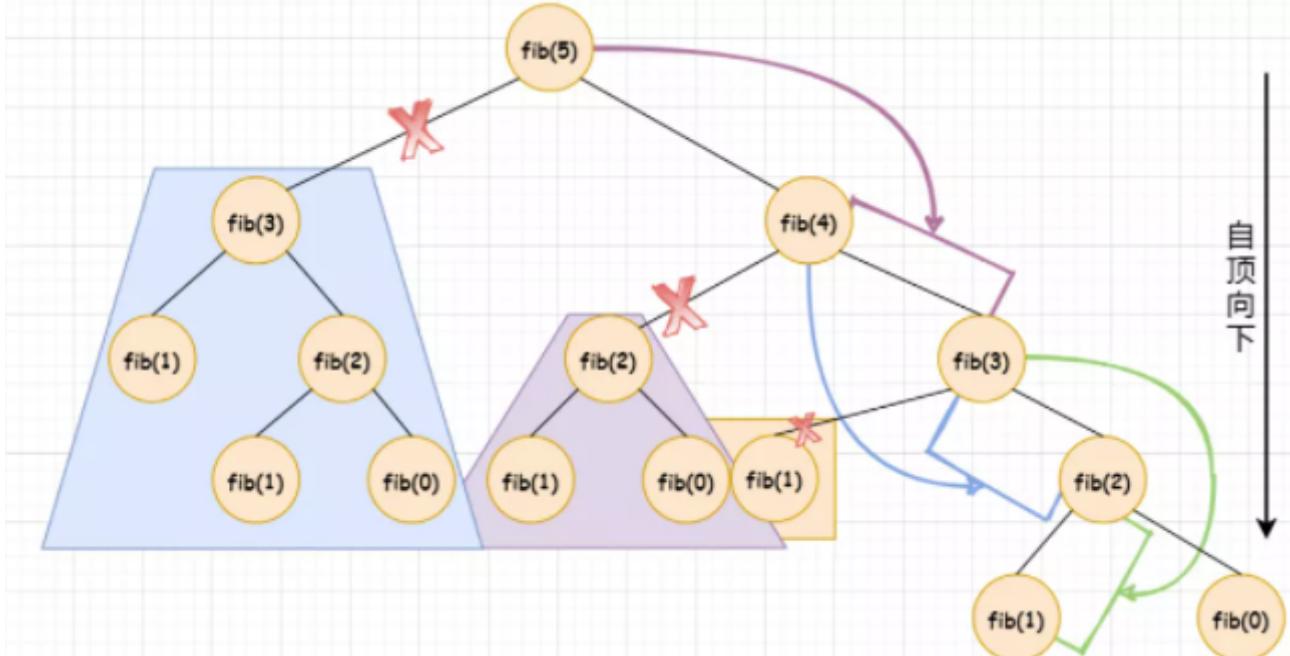
```

        for (int k = 0; k < n; k++) {
            if (i > 0 && dp[i-1][j^(1 << k)] != Integer.MAX_VALUE/2)
                dp[i][j] = Math.min(dp[i][j], cost.get(i).get(k) + dp[i-1][j ^ (1 << k)]);
            if (i > 0 && dp[i-1][j] != Integer.MAX_VALUE/2)
                dp[i][j] = Math.min(dp[i][j], cost.get(i).get(k) + dp[i-1][j]);
            if (i == 0 && (j ^ (1 << k)) == 0) dp[i][j] = cost.get(i).get(k);
            else if (dp[i][j^(1 << k)] != Integer.MAX_VALUE/2)
                dp[i][j] = Math.min(dp[i][j], cost.get(i).get(k) + dp[i][j ^ (1 << k)]);
        }
    }
    return dp[m-1][(1 << n)-1];
}

```

11.13.1 自顶向下与自底向上

以前写 dfs 的时候总是会忘记记忆数组，现在明白为什么需要记忆以避免重复操作



在两种方法的比较里，自顶向下最下层的操作就转化为自底向上最底层的相关处理，注意两种方法里的不同与相互转换

11.14 1755. Closest Subsequence Sum - Hard 分成两半: 由 $O(2^N)$ 降为 $O(N \log N)$

You are given an integer array `nums` and an integer `goal`.

You want to choose a subsequence of `nums` such that the sum of its elements is the closest possible to `goal`. That is, if the sum of the subsequence's elements is `sum`, then you want to minimize the absolute difference $\text{abs}(\text{sum} - \text{goal})$.

Return the minimum possible value of $\text{abs}(\text{sum} - \text{goal})$.

Note that a subsequence of an array is an array formed by removing some elements (possibly all or none) of the original array.

```

public int minAbsDifference(int[] arr, int goal) {
    int n = arr.length;
    List<Integer> lsum = new ArrayList<>();
    List<Integer> rsum = new ArrayList<>();
    lsum.add(0);
    rsum.add(0);
    for (int i = 0; i <= n/2; i++) { // 这种生成和的方式, i < n/2
        int size = lsum.size();
        for (int j = 0; j < size; j++)
            lsum.add(lsum.get(j) + arr[i]);
    }
    for (int i = n/2+1; i < n; i++) { // int i = n/2 同样可以, 只是左右大小的细微差别
        int size = rsum.size();
        for (int j = 0; j < size; j++)
            rsum.add(rsum.get(j) + arr[i]);
    }
}

```

```

    }
    TreeSet<Integer> rightSumSet = new TreeSet<>(rsum);
    Set<Integer> leftSumSet = new HashSet<>(lsum);
    int ans = Math.abs(goal);
    for (int v : leftSumSet) {
        int b = goal - v;
        Integer lower = rightSumSet.floor(b); // 对 treeset 的这几个函数总是记不住
        Integer higher = rightSumSet.ceiling(b);
        if (lower != null)
            ans = Math.min(ans, Math.abs(goal-v-lower));
        if (higher != null)
            ans = Math.min(ans, Math.abs(goal-v-higher));
    }
    return ans;
}

```

- 另一种我都怀疑是不是自己写出来的，居然会忘了。。。。。

```

// 要把这种工具方法像写 binarySearch 一样随手拈来随手就敲才行：仍然超时，最后两个不过
public void getSum(List<Integer> li, int [] arr, int sum, int l, int r) {
    if (l >= r) { // /
        li.add(sum);
        return;
    }
    getSum(li, arr, sum + arr[l], l+1, r); // choose and add idx l to sum
    getSum(li, arr, sum, l+1, r); // skip idx l, move directly to next element
}
public int minAbsDifference(int[] arr, int goal) {
    int n = arr.length, m = arr.length / 2;
    List<Integer> l = new ArrayList<>();
    List<Integer> r = new ArrayList<>();
    getSum(l, arr, 0, 0, m); // m
    getSum(r, arr, 0, m, n); // m ? 这里反而想不明白了？
    Collections.sort(l);
    Collections.sort(r);
    int i = 0, j = r.size()-1, cur = 0;
    int minDiff = Integer.MAX_VALUE;
    while (i < l.size() && j >= 0) {
        cur = l.get(i) + r.get(j) - goal;
        if (cur > 0) {
            minDiff = Math.min(minDiff, cur);
            j--;
        } else if (cur < 0) {
            minDiff = Math.min(minDiff, -cur);
            i++;
        }
    }
    else return 0;
}
return minDiff;
}

```

- Horowitz and Sahni's Subset Sum | comments | links
 - <https://leetcode.com/problems/closest-subsequence-sum/discuss/1055432/Java-252ms-or-47.4-MB-or-Horowitz-and-Sahni's-Subset-Sum-or-comments-or-links>
- 好像还有一个类似提交 python 关于子集的位操作的 java 方法，回头再找来参考一下

11.15 2035. Partition Array Into Two Arrays to Minimize Sum Difference - Hard 上一题：分成两半的套娃题

You are given an integer array nums of $2 * n$ integers. You need to partition nums into two arrays of length n to minimize the absolute difference of the sums of the arrays. To partition nums, put each element of nums into one of the two arrays.

Return the minimum possible absolute difference.

```

public int minimumDifference(int[] nums) {
    int n = nums.Length;
    int sum = Arrays.stream(nums).sum();
    TreeSet<Integer>[] sets = new TreeSet[n/2+1]; // 数组，而不是 hashMap，这个应该关系不是很大
    for (int i = 0; i < (1 << (n / 2)); ++i) { // 一次遍历，而不是 n 次遍历
        int curSum = 0;
        int m = 0; // element Cnts
        for (int j = 0; j < n / 2; ++j)

```

```

    if ((i & (1<<j)) != 0) {
        curSum += nums[j]; // 左半部分
        m++;
    }
    if (sets[m] == null) sets[m] = new TreeSet<Integer>();
    sets[m].add(curSum);
}

int res = Integer.MAX_VALUE;
for (int i = 0; i < (1 << (n / 2)); ++i) {
    int curSum = 0;
    int m = 0;
    for (int j = 0; j < n / 2; ++j)
        if ((i & (1<<j)) != 0) {
            curSum += nums[n/2 + j]; // 遍历计算右半部分的和：边遍历，边解决问题
            m++;
        }
    int target = (sum - 2 * curSum) / 2;
    Integer left = sets[n/2-m].floor(target), right = sets[n/2-m].ceiling(target);
    if (left != null)
        res = Math.min(res, Math.abs(sum - 2 * (curSum + left.intValue())));
    if (right != null)
        res = Math.min(res, Math.abs(sum - 2 * (curSum + right.intValue())));
    if (res == 0) return 0;
}
return res;
}

```

- 做题的时候有想起 tallest billboard，但因为有负数，就去想别的了，却忘记了上面那个一分为二的经典题型

Chapter 12

sliding window

12.1 数 subarray 个数 (满足某些特定要求的子数组个数) 问题：感觉傻傻永远数不清楚！列几个题，牢记一下

12.2 930. Binary Subarrays With Sum

Given a binary array nums and an integer goal, return the number of non-empty subarrays with a sum goal. A subarray is a contiguous part of the array.

```
public int numSubarraysWithSum(int[] arr, int goal) {
    int n = arr.length, res = 0, leftCnt = 0, j = 0, sum = 0;
    for (int i = 0; i < n; i++) {
        sum += arr[i];
        while (j < i && sum > goal) sum -= arr[j++];
        if (sum < goal) continue;
        if (sum == goal) ++res;
        for (int k = j; k < i && arr[k] == 0; k++)
            ++res;
    }
    return res;
}
```

12.3 713. Subarray Product Less Than K

Given an array of integers nums and an integer k, return the number of contiguous subarrays where the product of all the elements in the subarray is strictly less than k.

```
public int numSubarrayProductLessThanK(int[] arr, int k) {
    if (k == 0) return 0;
    int n = arr.length, ans = 0, j = 0, cur = 1;
    for (int i = 0; i < n; i++) {
        cur *= arr[i];
        while (j <= i && cur >= k)
            cur /= arr[j++];
        ans += (i - j + 1); // 当确定了窗口的大小后，就可以统计子数组的个数了，就是窗口的大小。
    }
    return ans;
}
```

12.4 双端队列：数据结构，**O(N)** 解法题目参考

12.5 862. Shortest Subarray with Sum at Least K - Hard

Given an integer array nums and an integer k, return the length of the shortest non-empty subarray of nums with a sum of at least k. If there is no such subarray, return -1.

A subarray is a contiguous part of an array.

```

public int shortestSubarray(int[] nums, int k) {
    int n = nums.length;
    int [] sum = new int[n+1];
    for (int i = 1; i <= n; i++) {
        sum[i] = nums[i-1] + sum[i-1];
    }
    int res = n + 1;
    ArrayDeque<Integer> q = new ArrayDeque<>(); // decreasing sum [] deque
    for (int i = 0; i <= n; i++) {
        while (!q.isEmpty() && sum[i] - sum[q.peekFirst()] >= k) // 左出:
            res = Math.min(res, i - q.pollFirst()); // 取值了 // 取出
        while (!q.isEmpty() && sum[q.peekLast()] >= sum[i]) // 右出
            q.pollLast();
        q.offerLast(i); // 当前元素进队列
    }
    return res <= n ? res : -1;
}

```

12.6 1438. Longest Continuous Subarray With Absolute Diff Less Than or Equal to Limit - Medium 双端队列

Given an array of integers nums and an integer limit, return the size of the longest non-empty subarray such that the absolute difference between any two elements of this subarray is less than or equal to limit.

12.6.1 解题思路与分析

- 如果使用普通列表再排序的话会增加一个 $\log n$ 的时间复杂度，会超时！
- 选对了数据类型后，直接套滑窗模板即可，简单的一。。。
- Intuition:

```

. we can use siliding window but we need to keep track of minimum and maximum in every window
. treemap can be used for this purpose to keep track of min and max both

```

- 这是一种临场应对的偷懒解法，选对数据结构，牺牲一点儿效率，但是可以最大限度地保证结果的正确性

```

// Time complexity: O(N*LogN), insertion in treeMap is logN and for N elements it is O(N*LogN)
// Auxiliary Space: O(N), every key enters the map once and in worst case every key can be present with frequency one
public int longestSubarray(int[] arr, int limit) { // TreeMap to keep Min & Max at O(NlogN)
    TreeMap<Integer, Integer> cnt = new TreeMap<>();
    int l = 0, r = 0;
    while (r < arr.length) {
        cnt.put(arr[r], cnt.getOrDefault(arr[r], 0) + 1);
        if (cnt.lastEntry().getKey() - cnt.firstEntry().getKey() > limit) {
            if (cnt.get(arr[l]) > 1)
                cnt.put(arr[l], cnt.get(arr[l])-1);
            else
                cnt.remove(arr[l]);
            l++;
        }
        r++;
    }
    return r - l; // 这样维护的就是全局最优解了!!!!
}

```

- 但是在平时练习时，还是要不断地寻求最优解，使用双端队列，得 $O(N)$ 线性复杂度
- Intuition:

```

// . we can use siliding window but we need to keep track of minimum and maximum in every window
// . 2 Deques can be used for this purpose to keep track of min and max.
// Time complexity: O(N), N is number of elements in nums
// Auxiliary Space: O(N)
public int longestSubarray(int[] nums, int limit) { // 双端队列: 从右边加入
    ArrayDeque<Integer> min = new ArrayDeque<>(); // 单调递增队列: 左小右大
    ArrayDeque<Integer> max = new ArrayDeque<>(); // 单调递减队列: 左大右小
    int l = 0, r = 0, ans = 0;
    while (r < nums.length) {
        int rval = nums[r];
        // to ensure that minQ have minimum element as head
        while (!min.isEmpty() && min.peekLast() > rval) min.pollLast(); // 维护: 右边, 比当前待入列值大的, 全扔出去
        min.offerLast(rval);
        while (!max.isEmpty() && max.peekFirst() < rval) max.pollFirst(); // 维护: 左边, 比当前待入列值小的, 全扔出去
        max.offerFirst(rval);
        if (max.peekFirst() - min.peekLast() > limit)
            l++;
        else
            ans = Math.max(ans, r - l + 1);
        r++;
    }
    return ans;
}

```

```
//to ensure that maxQ have maximum element as head
    while (!max.isEmpty() && max.peekLast() < rval) max.pollLast(); // 维护：右边，比当前待入列值小的，全扔出去
    min.offerLast(rval); // 入队列，从右边加入
    max.offerLast(rval); //
//In case max-min is greater than the limit slide the left side or window
    if (max.peekFirst() - min.peekFirst() > limit) { // 维护合法窗口范围：会两个队列都把 l 下标的值抓不出去吗？
        if (min.peekFirst() == nums[l]) min.pollFirst(); //
        if (max.peekFirst() == nums[l]) max.pollFirst(); //
        l++;
    }
    r++;
}
return r - l;
}
```

- 另一种把它“刷”过去的应付的写法 (MinSeg, MaxSeg 列在这里，方便自己以后参考时查找)

```
public class MaxSeg {
    List<Integer> tree = new ArrayList<>();
    int n;
    public MaxSeg (int [] arr) {
        n = arr.length;
        tree = new ArrayList<>(2 * n);
        for (int i = 0; i < n; i++)
            tree.add(0);
        for (int i = 0; i < n; i++)
            tree.add(arr[i]); // same effect as below
        for (int i = n-1; i >= 0; i--) // i >= 0
            tree.set(i, Math.max(tree.get(2 * i), tree.get(2 * i+1)));
    }
    public void update(int idx, int v) {
        idx += n;
        tree.set(idx, v);
        while (idx > 1) {
            idx /= 2;
            tree.set(idx, Math.max(tree.get(2 * idx), tree.get(2 * idx+1)));
        }
    }
    public int getMax(int l, int r) {
        l += n;
        r += n;
        int max = Integer.MIN_VALUE;
        while (l < r) {
            if ((l & 1) == 1) {
                max = Math.max(max, tree.get(l));
                l++;
            }
            if ((r & 1) == 1) {
                r--; // order matters !!!
                max = Math.max(max, tree.get(r));
            }
            l >= 1;
            r >= 1;
        }
        return max;
    }
}
public class MinSeg {
    List<Integer> tree = new ArrayList<>();
    int n;
    public MinSeg (int [] arr) {
        n = arr.length;
        tree = new ArrayList<>(2*n);
        for (int i = 0; i < n; i++)
            tree.add(0);
        for (int i = 0; i < n; i++)
            tree.add(arr[i]);
        for (int i = n-1; i >= 0; i--) // i >= 0
            tree.set(i, Math.min(tree.get(2*i), tree.get(2*i+1)));
    }
    public void update(int idx, int v) {
        idx += n;
        tree.set(idx, v);
        while (idx > 1) {
            idx /= 2;
            tree.set(idx, Math.min(tree.get(2*idx), tree.get(2*idx+1)));
        }
    }
    public int getMin(int l, int r) { // [l, r) include left, not included right
        l += n;
        r += n;
        int min = Integer.MAX_VALUE;
        while (l < r) {
```

```

        if ((l & 1) == 1) {
            min = Math.min(min, tree.get(l));
            l++;
        }
        if ((r & 1) == 1) {
            r--; // order matters !!!
            min = Math.min(min, tree.get(r));
        }
        l >= 1;
        r >= 1;
    }
    return min;
}
}

public int longestSubarray(int[] arr, int limit) { // 单调递增队列，左小右大，右进左出
    MaxSeg max = new MaxSeg(arr);
    MinSeg min = new MinSeg(arr);
    int res = 0, tmp = 0, j = 0;
    int n = arr.length;
    for (int i = 1; i <= n; i++) {
        if (max.getMax(j, i) - min.getMin(j, i) <= limit) {
            tmp = i-j;
            res = Math.max(res, tmp);
        } else
            while (max.getMax(j, i) - min.getMin(j, i) > limit && j < i) ++j;
    }
    return res;
}

```

12.7 1687. Delivering Boxes from Storage to Ports - Hard 滑动窗口

You have the task of delivering some boxes from storage to their ports using only one ship. However, this ship has a limit on the number of boxes and the total weight that it can carry.

You are given an array boxes, where boxes[i] = [ports_i, weight_i], and three integers portsCount, maxBoxes, and maxWeight.

ports_i is the port where you need to deliver the *i*th box and weight_i is the weight of the *i*th box. portsCount is the number of ports. maxBoxes and maxWeight are the respective box and weight limits of the ship. The boxes need to be delivered in the order they are given. The ship will follow these steps:

The ship will take some number of boxes from the boxes queue, not violating the maxBoxes and maxWeight constraints. For each loaded box in order, the ship will make a trip to the port the box needs to be delivered to and deliver it. If the ship is already at the correct port, no trip is needed, and the box can immediately be delivered. The ship then makes a return trip to storage to take more boxes from the queue. The ship must end at storage after all the boxes have been delivered.

Return the minimum number of trips the ship needs to make to deliver all boxes to their respective ports.

12.7.1 解题思路与分析: 滑动窗口

- 这里会需要一种决策：需要总运送次数最少，就涉及到同一船是否只装送往同一港口的优化与选择问题，用滑动窗口，也有贪心的解法
- 时间复杂度：O(N)

```

public int boxDelivering(int[][] box, int __, int max, int limit) {
    int n = box.length;
    int[] dp = new int[n+1]; // Minimum trips for first n boxes.
    int wit = 0, cost = 2; // cumulative weight, cumulative cost
    int l = 0; // left ptr
    for (int r = 0; r < n; r++) { // right ptr
        wit += box[r][1];
        if (r > 0 && box[r][0] != box[r-1][0]) cost++;
        /* drop box iff:
           - There are too many box (r-l >= max)
           - The box are too heavy (weight > limit)
           - It is redundant to carry them (dp[l] == dp[l+1]). */
        /* // 送往同一个港口的箱子向右滑动，直到改变消耗的临界点
        while (r - l >= max || wit > limit || (l < r && dp[l] == dp[l+1])) { // 滑动窗口：左窗口右移
            wit -= box[l][1];
            if (box[l+1][0] != box[l][0]) cost--;
            l++;
        }
        dp[r+1] = cost + dp[l]; // 运完下标为 r 的箱子后的最小次数，对应 dp[r+1]
    }
}

```

```

    return dp[n];
}

```

- 一种 dp 的写法，写得天外来仙，可惜不是很好懂

```

public int boxDelivering(int[][] boxes, int portsCount, int maxBoxes, int maxWeight) { // O(N) 滑动窗口
    int n = boxes.length, j = 0, lastj = 0, cnt = 0;
    int[] dp = new int[n+1];
    Arrays.fill(dp, Integer.MAX_VALUE / 3); // 3 to avoid overflow
    dp[0] = 0; // 
    for (int i = 0; i < n; ++i) {
        while (j < n && maxBoxes > 0 && maxWeight >= boxes[j][1]) {
            maxBoxes -= 1;
            maxWeight -= boxes[j][1];
            if (j == 0 || boxes[j][0] != boxes[j-1][0]) { // if the port is different from the previous port
                lastj = j;
                cnt++;
            }
            ++j; // keep expanding the right pointer when we can
            // dp[++j] = 200000; // equivalent
        }
        dp[j] = Math.min(dp[j], dp[i] + cnt + 1); // 这里的 cnt 相当于是一个快进效果，提速
        dp[lastj] = Math.min(dp[lastj], dp[i] + cnt);
        // 这里下面三行：像是回溯一样，绕得好昏呀：感觉是向右滑动，再增加一些空间，向右遍历所有可能性，并保存最优解
        maxBoxes += 1; // now as we move the left pointer i forward (don't put the ith box in this trip),
        maxWeight += boxes[i][1]; // we increase the number of available boxes and available weights
        if (i == n-1 || boxes[i][0] != boxes[i+1][0]) cnt--;
    }
    return dp[n];
}

```

12.8 992. Subarrays with K Different Integers - Hard

Given an integer array nums and an integer k, return the number of good subarrays of nums.

A good array is an array where the number of different integers in that array is exactly k.

For example, [1,2,3,1,2] has 3 different integers: 1, 2, and 3. A subarray is a contiguous part of an array.

12.8.1 解题思路与分析

- 思路依然是滑动窗口，但是这一题是没法直接套用 76 题的模板的，有一些变动（引用）。回忆前面做的滑动窗口的题目，有求过子数组里面最多 K 个不同元素的题（340），所以这个解法的思路是求子数组里面最多 K 个不同元素的子数组的数量 - 子数组里面最多 K - 1 个不同元素的子数组的数量。

```

private int atMostK(int[] arr, int k) {
    int ans = 0, l = 0; // Left boundary of window
    Map<Integer, Integer> cnt = new HashMap<>(); // Map to keep track of number of distinct elements in the current window
    for (int i = 0; i < arr.length; i++) { // i : right
        cnt.put(arr[i], cnt.getOrDefault(arr[i], 0) + 1);
        while (cnt.size() > k) {
            if (cnt.get(arr[l]) > 1) cnt.put(arr[l], cnt.get(arr[l])-1);
            else cnt.remove(arr[l]);
            l++;
        }
        ans += i - l + 1; // Adding the count of subarrays with at most K distinct elements in the current window
    }
    return ans;
}
public int subarraysWithKDistinct(int[] arr, int k) {
    return atMostK(arr, k) - atMostK(arr, k-1);
}

```

12.9 857. Minimum Cost to Hire K Workers - Hard

There are n workers. You are given two integer arrays quality and wage where quality[i] is the quality of the ith worker and wage[i] is the minimum wage expectation for the ith worker.

We want to hire exactly k workers to form a paid group. To hire a group of k workers, we must pay them according to the following rules:

Every worker in the paid group should be paid in the ratio of their quality compared to other workers in the paid group. Every worker in the paid group must be paid at least their minimum wage expectation. Given the integer k,

return the least amount of money needed to form a paid group satisfying the above conditions. Answers within 10-5 of the actual answer will be accepted.

12.9.1 解题思路与分析

- 做题的时候没有想明白，为什么不是性价比最低的 k 个工人，什么情况下性价比最低的 k 个工人不是题目要求的答案？
- 时间复杂度 $O(n \log n)$ ，空间 $O(n)$ 。

```
public double mincostToHireWorkers(int[] quality, int[] wage, int k) {
    int n = quality.length;
    List<int[]> l = new ArrayList<>();
    for (int i = 0; i < n; i++) {
        int[] cur = new int[] {wage[i], quality[i]};
        l.add(cur);
    }
    Collections.sort(l, (a, b) -> Double.compare((double)a[0] / a[1], (double)b[0] / b[1]));
    Queue<Integer> q = new PriorityQueue<>((x, y) -> -Integer.compare(x, y)); // 堆里存所以 t 值小于等于当前枚举的 t 值的工人中, q 值最小的 k 个人的 q 值
    double res = 1e18, sum = 0;
    for (int i = 0; i < n; i++) {
        int[] cur = l.get(i);
        sum += cur[1];
        q.offer(cur[1]);
        if (q.size() > k)
            sum -= q.poll();
        if (q.size() == k)
            res = Math.min(res, sum * (double)cur[0] / (double)cur[1]);
    }
    return res;
}
```

12.10 1425. Constrained Subsequence Sum - Hard

Given an integer array nums and an integer k, return the maximum sum of a non-empty subsequence of that array such that for every two consecutive integers in the subsequence, $\text{nums}[i]$ and $\text{nums}[j]$, where $i < j$, the condition $j - i \leq k$ is satisfied.

A subsequence of an array is obtained by deleting some number of elements (can be zero) from the array, leaving the remaining elements in their original order.

12.11 239. Sliding Window Maximum - Hard

You are given an array of integers nums, there is a sliding window of size k which is moving from the very left of the array to the very right. You can only see the k numbers in the window. Each time the sliding window moves right by one position.

Return the max sliding window.

```
public int[] maxSlidingWindow(int[] arr, int k) {
    int n = arr.length, startWindowIdx = 0;
    ArrayDeque<Integer> q = new ArrayDeque<>(); // 维持一个递减队列
    int[] ans = new int[n - k + 1];
    for (int i = 0; i < n; i++) {
        startWindowIdx = i - k + 1;
        while (!q.isEmpty() && i - q.peekFirst() >= k) q.pollFirst(); // 左出 q: maintain k size window, 去头: 去掉 k windows 之外的元素
        while (!q.isEmpty() && arr[q.peekLast()] <= arr[i]) q.pollLast(); // 右出 q: 去掉递减队列尾部所有不大于当前值的元素, 就留一个最大值也行
        q.offerLast(i); // 进 q: 进后此时 q.size() == k
        if (startWindowIdx >= 0)
            ans[startWindowIdx] = arr[q.peekFirst()]; // 使用递减队列左端最大值
    }
    return ans;
}
```

- 线段树的做法

```
// https://blog.csdn.net/Yaokai_AssultMaster/article/details/79599809
public class MaxSeg {
    List<Integer> tree = new ArrayList<>();
    int n;
    public MaxSeg (int[] arr) {
```

```

n = arr.length;
tree = new ArrayList<>(2*n);
for (int i = 0; i < n; i++)
    tree.add(0);
for (int i = 0; i < n; i++)
    tree.add(arr[i]); // same effect as below
for (int i = n-1; i >= 0; i--) // i >= 0
    tree.set(i, Math.max(tree.get(2*i), tree.get(2*i+1)));
}
public void update(int idx, int v) {
    idx += n;
    tree.set(idx, v);
    while (idx > 1) {
        idx /= 2;
        tree.set(idx, Math.max(tree.get(2*idx), tree.get(2*idx+1)));
    }
}
public int getMax(int l, int r) {
    l += n;
    r += n;
    int max = Integer.MIN_VALUE;
    while (l < r) {
        if ((l & 1) == 1) {
            max = Math.max(max, tree.get(l));
            l++;
        }
        if ((r & 1) == 1) {
            r--; // order matters !!!
            max = Math.max(max, tree.get(r));
        }
        l >= 1;
        r >= 1;
    }
    return max;
}
}
public int[] maxSlidingWindow(int[] arr, int k) {
    int n = arr.length;
    MaxSeg mat = new MaxSeg(arr);
    if (n == k) return new int [] {mat.getMax(0, n)};
    int [] res = new int [n-k+1];
    for (int i = 0; i+k <= n; i++)
        res[i] = mat.getMax(i, i+k);
    return res;
}

```

12.12 76. Minimum Window Substring

Given two strings s and t of lengths m and n respectively, return the minimum window substring of s such that every character in t (including duplicates) is included in the window. If there is no such substring, return the empty string $" "$. The testcases will be generated such that the answer is unique. A substring is a contiguous sequence of characters within the string.

```

private boolean satisfies(Map<Character, Integer> s, Map<Character, Integer> t) {
    if (s.size() < t.size()) return false;
    for (Map.Entry<Character, Integer> en : t.entrySet()) {
        if (!s.containsKey(en.getKey()) || s.containsKey(en.getKey()) && s.get(en.getKey()) < en.getValue()) return false;
    }
    return true;
}
public String minWindow(String s, String t) {
    int m = s.length();
    int n = t.length();
    if (m < n) return "";
    if (m == 1 && n == 1 && s.charAt(0) != t.charAt(0)) return "";
    if (n == 1) {
        boolean contains = false;
        for (char c : s.toCharArray()) {
            if (c == t.charAt(0)) {
                contains = true;
                break;
            }
        }
        return !contains ? "" : t;
    }
    Map<Character, Integer> mt = new HashMap<>();
    for (char c : t.toCharArray())
        mt.put(c, mt.getOrDefault(c, 0) + 1);
    Map<Character, Integer> ms = new HashMap<>();

```

```

int l = 0, r = 0, i = 0, j = 0, pl = 0;
String res = "", tmp = "";
while (i < m) {
    while (i < m && !satisfies(ms, mt)) {
        ms.put(s.charAt(i), ms.getOrDefault(s.charAt(i), 0) + 1);
        ++i;
    }
    if (satisfies(ms, mt)) {
        tmp = s.substring(l, i);
        if (res.equals("") || res.length() > tmp.length()) res = tmp;
    }
    pl = l;
    while (l < i && satisfies(ms, mt)) {
        System.out.println("\n" + l);

        ms.put(s.charAt(l), ms.get(s.charAt(l))- 1);
        if (ms.get(s.charAt(l)) == 0) ms.remove(s.charAt(l));
        ++l;
    }
    if (satisfies(ms, mt) || pl != l) {
        tmp = s.substring(l-1, i);
        if (res.equals("") || res.length() > tmp.length()) res = tmp;
    }
    if (i == m) break;
}
return res;
}

```

12.13 632. Smallest Range Covering Elements from K Lists

You have k lists of sorted integers in non-decreasing order. Find the smallest range that includes at least one number from each of the k lists. We define the range $[a, b]$ is smaller than range $[c, d]$ if $b - a < d - c$ or $a < c$ if $b - a == d - c$.

```

// 时间复杂度: O(nk \log k)O(nk\log k), 其中 nn 是所有列表的平均长度, kk 是列表数量。所有的指针移动的总次数最多是 nk\log k 次, 每次从堆中取出元素和添加元素都需要更新堆,
// 空间复杂度: O(k)O(k), 其中 kk 是列表数量。空间复杂度取决于堆的大小, 堆中维护 kk 个元素。
public int[] smallestRange(List<List<Integer>> nums) {
    int n = nums.size();
    int resMin = 0, resMax = Integer.MAX_VALUE;
    int minRg = resMax - resMin;
    int max = Integer.MIN_VALUE;
    int [] next = new int [n]; // 各子链表中比当前 idx 位数值大的下一个数的下标, 即 idx+1, 初始化全为 0
    Queue<Integer> q = new PriorityQueue<>(new Comparator<Integer>() {
        public int compare(Integer idx1, Integer idx2) {
            return nums.get(idx1).get(next[idx1]) - nums.get(idx2).get(next[idx2]);
        }
    });
    for (int i = 0; i < n; i++) {
        q.offer(i); // 0, 1, 2, ..., n-1 真神奇呀
        max = Math.max(max, nums.get(i).get(0));
    }
    int minIdx = 0, curRg = 0;
    while (true) {
        minIdx = q.poll(); // 取出的是最小值的子链表的序号, 而子链表里的当前最小值所在子链表中的位置存于 next[minIdx] 中
        curRg = max - nums.get(minIdx).get(next[minIdx]);
        if (curRg < minRg) {
            minRg = curRg;
            resMin = nums.get(minIdx).get(next[minIdx]);
            resMax = max;
        }
        next[minIdx]++;
        if (next[minIdx] == nums.get(minIdx).size()) break;
        q.offer(minIdx); // 加回去, 但是 queue 里真正比较的值已经变了, 变强大了。。。 // 更新最小值的替换值
        max = Math.max(max, nums.get(minIdx).get(next[minIdx])); // 更新最大值
    }
    return new int [] {resMin, resMax};
}

// 这里的 BB 序列是什么? 我们可以用一个哈希映射来表示 BB 序列—— B[i]
// B[i] 表示 ii 在哪些列表当中出现过,
// 这里哈希映射的键是一个整数, 表示列表中的某个数值,
// 哈希映射的值是一个数组, 这个数组里的元素代表当前的键出现在哪些列表里。
// 如果列表集合为:
// 0: [-1, 2, 3]
// 1: [1]
// 2: [1, 2]
// 3: [1, 1, 3]
// 那么可以得到这样一个哈希映射
// -1: [0]
// 1: [1, 2, 3, 3]
// 2: [0, 2]

```

```
// 3: [0, 3]
public int[] smallestRange(List<List<Integer>> nums) {
    int n = nums.size();
    Map<Integer, List<Integer>> indices = new HashMap<>();
    int xmin = Integer.MAX_VALUE, xmax = Integer.MIN_VALUE;
    for (int i = 0; i < n; i++) {
        for (int v : nums.get(i)) { // 把大链表中出过的每一个值作键，值为它所存在于子链表序号链表
            List<Integer> list = indices.getOrDefault(v, new ArrayList<>());
            list.add(i);
            indices.put(v, list);
            xmin = Math.min(xmin, v);
            xmax = Math.max(xmax, v); // 这里得到全局的最小最大值
        }
    }
    int [] freq = new int [n];
    int inside = 0; // cnt # of lists included in miniRanges
    int left = xmin, right = xmin - 1;
    int resLeft = xmin, resRight = xmax;
    while (right < xmax) {
        right++;
        if (indices.containsKey(right)) {
            for (int x : indices.get(right)) {
                freq[x]++;
                if (freq[x] == 1) inside++;
            }
        }
        while (inside == n) { // find ONE satisfied solution, try to minimize the range
            if (right - left < resRight - resLeft) {
                resLeft = left;
                resRight = right;
            }
            if (indices.containsKey(left)) { // sliding the left size towards right
                for (int v : indices.get(left)) {
                    freq[v]--;
                    if (freq[v] == 0) --inside;
                }
            }
            left++;
        }
    }
    return new int [] {resLeft, resRight};
}
```

12.14 480 Sliding Window Median

The median is the middle value in an ordered integer list. If the size of the list is even, there is no middle value. So the median is the mean of the two middle values. For examples, if arr = [2,3,4], the median is 3. For examples, if arr = [1,2,3,4], the median is $(2 + 3) / 2 = 2.5$. You are given an integer array nums and an integer k. There is a sliding window of size k which is moving from the very left of the array to the very right. You can only see the k numbers in the window. Each time the sliding window moves right by one position. Return the median array for each window in the original array. Answers within 10-5 of the actual value will be accepted.

```
public double[] medianSlidingWindow(int[] nums, int k) {
    TreeMap<Integer, Integer> ma = new TreeMap<>();
    TreeMap<Integer, Integer> mb = new TreeMap<>();
    for (int i = 0; i < k; i++) {
        if (i % 2 == 0) {
            mb.put(nums[i], mb.getOrDefault(nums[i], 0) + 1);
            int n = mb.firstKey();
            if (mb.get(n) == 1) mb.remove(n);
            else mb.put(n, mb.get(n) - 1);
            ma.put(n, ma.getOrDefault(n, 0) + 1);
        } else {
            ma.put(nums[i], ma.getOrDefault(nums[i], 0) + 1);
            int n = ma.lastKey();
            if (ma.get(n) == 1) ma.remove(n);
            else ma.put(n, ma.get(n) - 1);
            mb.put(n, mb.getOrDefault(n, 0) + 1);
        }
    }
    double [] res = new double[nums.length-k+1];
    if (k % 2 == 1) res[0] = ma.lastKey();
    else res[0] = (double)((long)(ma.lastKey()) + (long)(mb.firstKey())) / 2.0;
    for (int i = 0; i + k < nums.length; i++) {
        ma.put(nums[i+k], ma.getOrDefault(nums[i+k], 0) + 1);
        int n = ma.lastKey();
        if (ma.get(n) == 1) ma.remove(n);
        else ma.put(n, ma.get(n) - 1);
        mb.put(n, mb.getOrDefault(n, 0) + 1);
    }
}
```

```

mb.put(n, mb.getOrDefault(n, 0) + 1);
if (ma.containsKey(nums[i])) {
    if (ma.get(nums[i]) == 1) ma.remove(nums[i]);
    else ma.put(nums[i], ma.get(nums[i]) - 1);
    int v = mb.firstKey();
    if (mb.get(v) == 1) mb.remove(v);
    else mb.put(v, mb.get(v) - 1);
    ma.put(v, ma.getOrDefault(v, 0) + 1);
} else {
    if (mb.get(nums[i]) == 1) mb.remove(nums[i]);
    else mb.put(nums[i], mb.get(nums[i]) - 1);
}
if (k % 2 == 1) res[i+1] = ma.lastKey();
else res[i+1] = (double)((long)(ma.lastKey()) + (long)(mb.firstKey())) / 2.0;
}
return res;
}

```

12.15 826. Most Profit Assigning Work

You have n jobs and m workers. You are given three arrays: difficulty, profit, and worker where:

$\text{difficulty}[i]$ and $\text{profit}[i]$ are the difficulty and the profit of the i th job, and $\text{worker}[j]$ is the ability of j th worker (i.e., the j th worker can only complete a job with difficulty at most $\text{worker}[j]$). Every worker can be assigned at most one job, but one job can be completed multiple times.

For example, if three workers attempt the same job that pays \$1, then the total profit will be \$3. If a worker cannot complete any job, their profit is \$0. Return the maximum profit we can achieve after assigning the workers to the jobs.

```

// 法一: 暴力 TreeMap---O(n^2logn)
public int maxProfitAssignment(int[] difficulty, int[] profit, int[] worker) {
    TreeMap<Integer, Integer> m = new TreeMap<>();
    for (int i = 0; i < difficulty.length; i++)
        m.put(difficulty[i], i);
    int res = 0, idx;
    Integer low;
    for (int i = 0; i < worker.length; i++) {
        low = m.floorKey(worker[i]); // 使用 treemap 排序的特长
        if (low == null) continue;
        idx = m.get(low);
        for (int j = 0; j < difficulty.length; j++)
            if (difficulty[j] <= low && profit[j] >= profit[idx])
                idx = j;
        res += profit[idx];
    }
    return res;
}
// 法二: 优化的 TreeMap---O(nlogn)
// 如果 TreeMap 里面保存的是每个 difficulty[i] 对应的最大的 profit, 则就可以直接找 floorKey 对应的 value 就是对应的要找的 value;
// 那么只需要再遍历一次 TreeMap, 将最大的到目前 key 位置最大的 value 放进去就行了
public int maxProfitAssignment(int[] difficulty, int[] profit, int[] worker) {
    TreeMap<Integer, Integer> m = new TreeMap<>();
    for (int i = 0; i < difficulty.length; i++)
        m.put(difficulty[i], Math.max(m.getOrDefault(difficulty[i], 0), profit[i]));
    int max = 0;
    for (Integer key : m.keySet()) {
        max = Math.max(max, m.get(key));
        m.put(key, max); // 将最大的到目前 key 位置最大的 value 放进去
    }
    int res = 0;
    Integer low;
    for (int i = 0; i < worker.length; i++) {
        low = m.floorKey(worker[i]); // 使用 treemap 排序的特长
        if (low == null) continue;
        res += m.get(low);
    }
    return res;
}
// 法三: Sort + 双指针---O(nlogn)
// 思路就是先将 difficulty[] 和 profit 组成 pair, 然后再将 list 和 worker[] 从小到大排序, 然后遍历 worker, 更新 tempMaxProfit, 得到总的 maxProfit
public int maxProfitAssignment(int[] difficulty, int[] profit, int[] worker) {
    List<int[]> list = new ArrayList<>();
    for (int i = 0; i < difficulty.length; i++)
        list.add(new int[] {difficulty[i], profit[i]});
    Collections.sort(list, (a, b) -> {return a[0] - b[0];});
    Arrays.sort(worker);
    int res = 0, tmpMaxProfit = 0;
    // i, j 同向双指针移动, 更新到目前的 tempMaxProfit
    for (int i = 0, j = 0; i < worker.length; i++) {
        while (j < list.size() && list.get(j)[0] <= worker[i]) {

```

```

        tmpMaxProfit = Math.max(tmpMaxProfit, list.get(j)[1]);
        j++;
    }
    //此时 tmpMaxProfit 是前面所有 difficulty 小于 worker[i] 的最大的 profit
    res += tmpMaxProfit;
}
return res;
}

```

12.16 1838. Frequency of the Most Frequent Element - Medium

The frequency of an element is the number of times it occurs in an array.

You are given an integer array nums and an integer k. In one operation, you can choose an index of nums and increment the element at that index by 1.

Return the maximum possible frequency of an element after performing at most k operations.

由反证可得，在一定操作次数下最高频元素为原数组中的元素，结合贪心算法，应优先选择不大于该元素的最大数字进行递增操作。

因此，可对原数组排序后，结合双指针算法求解。设计左、右两个指针，在移动右指针的同时，维护左指针的位置，使区间内元素全部递增到右指针所在元素值的操作次数符合要求，此时区间的长度就是该元素的频率。

```

public int maxFrequency(int[] nums, int k) {
    int n = nums.length, ans = 1, cnt = 0;
    Arrays.sort(nums);
    for (int l = 0, r = 1; r < n; r++) {
        cnt += (nums[r] - nums[r-1]) * (r - l); // 右指针移动后所需操作次数
        while (cnt > k) // 操作次数超过 k, 移动左指针
            cnt -= nums[r] - nums[l++]; // 左指针向右移动一位
        ans = Math.max(ans, r-l+1); // 区间长度为操作后当前元素的频数
    }
    return ans;
}

```

12.17 1703. Minimum Adjacent Swaps for K Consecutive Ones - Hard

You are given an integer array, nums, and an integer k. nums comprises of only 0's and 1's. In one move, you can choose two adjacent indices and swap their values.

Return the minimum number of moves required so that nums has k consecutive 1's.

方法一：转化 + 中位数

思路与算法

显然我们需要

- 找出数组中 k 个连续的 1；
- 找出数组中 k 个连续的位置。

然后将每个 1 按照位置先后依次放入位置中。如果不按照顺序放入，那么交换的路线就会产生「交叉」，不会比按照顺序要优。

假设 1 的位置从前往后分别为 p_0, \dots, p_{k-1} ，放入的位置分别为 $q, \dots, q+k-1$ ，那么我们需求求出

$$\sum_{i=0}^{k-1} |p_i - (q+i)|$$

的最小值。令 $p'_i = p_i - i$ ，那么

$$\sum_{i=0}^{k-1} |p_i - (q+i)| = \sum_{i=0}^{k-1} |(p'_i + i) - (q+i)| = \sum_{i=0}^{k-1} |p'_i - q|$$

当 q 为 $\{p'_i\}$ 的中位数时，上式取到最小值。

因此我们可以记 f_0, f_1, \dots, f_{m-1} 为数组 $nums$ 中所有 1 的位置，并且令 $g_i = f_i - i$ 。由于 $f_0 < f_1 < \dots < f_{m-1}$ ，因此 $g_0 \leq g_1 \leq \dots \leq g_{m-1}$ 。这样一来，我们可以使用一个大小为 k 的滑动窗口在 $\{g\}$ 上实时找出中位数并计算答案。

设我们当前的滑动窗口中包含 g_i, \dots, g_{i+k-1} ，它们已经有序，那么中位数为 $q = g_{(2i+k-1)/2}$ 。这里的除法为整数除法。记 $mid = (2i+k-1)/2$ ，此时的交换次数为

$$\begin{aligned} \sum_{j=i}^{i+k-1} |g_j - q| &= \sum_{j=i}^{mid-1} (q - g_j) + \sum_{j=mid+1}^{i+k-1} (g_j - q) \\ &= (mid - i)q - \sum_{j=i}^{mid-1} g_j + \sum_{j=mid+1}^{i+k-1} g_j - (i+k-mid-1)q \\ &= (2(mid - i) - k + 1)q + \sum_{j=mid+1}^{i+k-1} g_j - \sum_{j=i}^{mid-1} g_j \end{aligned}$$

其中的后两项求和可以通过前缀和快速得到。

注意到当我们向右移动滑动窗口时，滑动窗口中的 g_i, \dots, g_{i+k-1} 是由对应的 f 值减去 $i, \dots, i+k-1$ 得来，而并不是我们在一开始设定的 $0, \dots, k-1$ 。这里其实并不会影响答案：因为这相当于我们把 f 值减去 $0, \dots, k-1$ 之后再同时减去 i ，那么取 q 为 g_i, \dots, g_{i+k-1} 的中位数计算出的答案仍然是正确的。

```
public int minMoves(int[] arr, int k) {
    if (k == 1) return 0;
    int n = arr.length;
    List<Integer> g = new ArrayList<>();
    List<Integer> sum = new ArrayList<>();
    sum.add(0);
    int cnt = -1, last = 0;
    for (int i = 0; i < n; i++) {
        if (arr[i] == 0) continue;
        ++cnt;
        g.add(i-cnt);
        sum.add(last + i - cnt);
        last += i - cnt;
    }
}
```

```

}
int m = g.size();
int ans = Integer.MAX_VALUE;
for (int i = 0; i+k <= m; i++) {
    int mid = (i + i + k - 1) / 2; // 中位数下标
    int q = g.get(mid);           // 中位数
    ans = Math.min(ans, (2*(mid-i)-k+1) * q + sum.get(i+k) - sum.get(mid+1) - sum.get(mid) + sum.get(i));
}
return ans;
}

```

12.18 220. Contains Duplicate III - Medium

Given an integer array `nums` and two integers `k` and `t`, return true if there are two distinct indices `i` and `j` in the array such that $\text{abs}(\text{nums}[i] - \text{nums}[j]) \leq t$ and $\text{abs}(i - j) \leq k$.

```

public boolean containsNearbyAlmostDuplicate(int[] arr, int k, int t) {
    TreeSet<Long> ts = new TreeSet<>();
    for (int i = 0; i < arr.length; i++) {
        if (i >= k+1) ts.remove((long)arr[i-k-1]);
        Long lower = ts.ceiling((long)arr[i]-t); // E ceiling(E e) , 返回 treeSet 中大于等于 e 的元素中最小的元素, 如果没有大于等于 e 的元素就返回 null
        if (lower != null && lower <= (long)arr[i] + t)
            return true;
        ts.add((long)arr[i]);
    }
    return false;
}
// 维持一个长度为 k 的 window, 每次检查新的值是否与原来窗口中的所有值的差值有小于等于 t 的. 如果用两个 for 循环会超时 O(nk).
// 使用 treeset( backed by binary search tree) 的 subSet 函数, 可以快速搜索. 复杂度为 O(n logk)
public boolean containsNearbyAlmostDuplicate(int[] nums, int k, int t) {
    if (k < 1 || t < 0 || nums == null || nums.length < 2) return false;
    SortedSet<Long> set = new TreeSet<Long>();
    for (int j = 0; j < nums.length; j++) {
        SortedSet<Long> subSet = set.subSet((long)nums[j] - t, (long)nums[j] + t + 1);
        if (!subSet.isEmpty()) return true;
        if (j >= k) set.remove((long)nums[j - k]);
        set.add((long)nums[j]);
    }
    return false;
}

```

Chapter 13

backTracking 回溯

13.1 1723 Find Minimum Time to Finish All Jobs

You are given an integer array jobs, where jobs[i] is the amount of time it takes to complete the ith job.

There are k workers that you can assign jobs to. Each job should be assigned to exactly one worker. The working time of a worker is the sum of the time it takes to complete all jobs assigned to them. Your goal is to devise an optimal assignment such that the maximum working time of any worker is minimized.

Return the minimum possible maximum working time of any assignment.

```
private void dfs(int[] a, int k, int idx) { // todo: 这类题需要总结一下，外加其它高效方法汇总
    if (Arrays.stream(dp).max().getAsInt() >= min) return; // 这里可以豪爽地把 == min 的全扔了
    if (idx < 0) {
        int tmp = Arrays.stream(dp).sum();
        if (tmp != sum) return;
        int cur = Arrays.stream(dp).max().getAsInt();
        // if (cur < min)
        min = cur; // 这里也就可以用再比较，直接取结果
        return;
    }
    // for (int i = idx; i >= 0; i--) { // 为什么会画蛇添足地多加个没用的 loop 呢????!!!
    for (int j = 0; j < k; j++) {
        if (j > 0 && dp[j] == dp[j-1]) continue;
        dp[j] += a[idx];
        dfs(a, k, idx-1);
        dp[j] -= a[idx];
    }
    }
    int n, sum, min = Integer.MAX_VALUE;
    int[] dp;
    public int minimumTimeRequired(int[] jobs, int k) {
        n = jobs.length;
        dp = new int[k];
        Arrays.sort(jobs);
        sum = Arrays.stream(jobs).sum();
        dfs(jobs, k, n-1);
        return min;
    }
}
```

13.2 1986. Minimum Number of Work Sessions to Finish the Tasks - Medium

There are n tasks assigned to you. The task times are represented as an integer array tasks of length n, where the ith task takes tasks[i] hours to finish. A work session is when you work for at most sessionTime consecutive hours and then take a break.

You should finish the given tasks in a way that satisfies the following conditions:

If you start a task in a work session, you must complete it in the same work session. You can start a new task immediately after finishing the previous one. You may complete the tasks in any order. Given tasks and sessionTime, return the minimum number of work sessions needed to finish all the tasks following the conditions above.

The tests are generated such that sessionTime is greater than or equal to the maximum element in tasks[i].

```
private void backtracking(int[] a, int limit, int idx, List<Integer> list) { // 说明对回溯的原理理解得不够透彻
    if (list.size() >= ans) return;
```

```

if (idx < 0) {
    if (sum == list.stream().collect(Collectors.summingInt(Integer::intValue))) // 这个前提条件一定不能忘记
        ans = list.size();
    return;
}
// for (int i = idx; i >= 0; i--) f // 画蛇添足: 第三次!!!
for (int j = 0; j < list.size(); j++) {
    if (list.get(j) + a[idx] > limit) continue;
    if (j > 0 && list.get(j) == list.get(j-1)) continue;
    list.set(j, list.get(j) + a[idx]);
    backtracking(a, limit, idx-1, list);
    list.set(j, list.get(j) - a[idx]);
}
list.add(a[idx]);
backtracking(a, limit, idx-1, list);
list.remove(list.size()-1); // backtracking: 这里是需要回缩的
//
//
}

// boolean [] vis; // 全排列的时候用 vis, 顺序遍历应该不用
int n, ans, sum;
public int minSessions(int[] tasks, int sessionTime) {
    n = tasks.length;
    ans = n;
    Arrays.sort(tasks);
    sum = Arrays.stream(tasks).sum();
    // vis = new boolean[n];
    backtracking(tasks, sessionTime, n-1, new ArrayList<>());
    return ans;
}

```

13.3 996. Number of Squareful Arrays - Hard 对重复数字的处理

An array is squareful if the sum of every pair of adjacent elements is a perfect square.

Given an integer array nums, return the number of permutations of nums that are squareful.

Two permutations perm1 and perm2 are different if there is some index i such that perm1[i] != perm2[i].

```

List<List<Integer>> ll = new ArrayList<>();
boolean [] vis;
int n;
public int numSquarefulPerms(int[] a) {
    n = a.length;
    if (Arrays.stream(a).distinct().count() == 1) {
        if (!isSquare(a[0] + a[1])) return 0;
        return 1;
    }
    vis = new boolean[n];
    dfs(a, 0, new ArrayList<>());
    return ll.size();
}
private void dfs(int [] a, int idx, List<Integer> l) { // tle tle tle
    if (l.size() >= 2 && !isValid(l)) return;
    if (l.size() == n) {
        if (isValid(l) && !ll.contains(l)) ll.add(new ArrayList<>(l));
        return ;
    }
    for (int i = 0; i < n; i++) {
        if (i > 0 && a[i] == a[i-1] && vis[i-1]) continue; // 很重要
        if (!vis[i]) {
            vis[i] = true;
            l.add(a[i]);
            dfs(a, i+1, l);
            l.remove(l.size()-1);
            vis[i] = false;
        }
    }
}
private boolean isSquare(int v) {
    return Math.pow((int) Math.sqrt(v), 2) == v;
}
private boolean isValid(List<Integer> l) {
    for (int i = 0; i <= l.size()-2; i++)
        if (!isSquare(l.get(i) + l.get(i+1))) return false;
    return true;
}

```

13.4 491. Increasing Subsequences

Given an integer array `nums`, return all the different possible increasing subsequences of the given array with at least two elements. You may return the answer in any order.

The given array may contain duplicates, and two equal integers should also be considered a special case of increasing sequence.

```
private void dfs(int[] arr, int idx, List<Integer> l) {
    if (l.size() >= 2)
        res.add(new ArrayList<>(l));
    Set<Integer> vis = new HashSet<>();
    for (int i = idx; i < arr.length; i++) {
        if (vis.contains(arr[i])) continue;
        if (l.size() == 0 || arr[i] >= l.get(l.size() - 1)) {
            vis.add(arr[i]);
            l.add(arr[i]);
            dfs(arr, i + 1, l);
            l.remove(l.size() - 1);
        }
    }
}
List<List<Integer>> res = new ArrayList<>();
public List<List<Integer>> findSubsequences(int[] arr) {
    if (arr == null || arr.length == 0) return res;
    dfs(arr, 0, new ArrayList<Integer>());
    return res;
}
```

Chapter 14

排序与 recursion

常用排序算法							
类别	排序方法	时间复杂度			空间复杂度	稳定性	备注
		平均情况	最好情况	最坏情况			
插入排序	直接插入	$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$	稳定	n 小时较好
	shell排序	$O(n^{1.3})$	$O(n)$	$O(n^2)$	$O(1)$	不稳定	
选择排序	直接选择	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$	不稳定	n 小时较好
	堆排序	$O(n\log_2 n)$	$O(n\log_2 n)$	$O(n\log_2 n)$	$O(1)$	不稳定	n 大时较好
交换排序	冒泡排序	$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$	稳定	n 小时较好
	快速排序	$O(n\log_2 n)$	$O(n\log_2 n)$	$O(n^2)$	$O(n\log_2 n)$	不稳定	n 大时较好
归并排序		$O(n\log_2 n)$	$O(n\log_2 n)$	$O(n\log_2 n)$	$O(1)$	稳定	n 大时较好
基数排序		$O(d(r+n))$	$O(d(n+rd))$	$O(d(r+n))$	$O(rd+n)$	稳定	

注：基数排序的复杂度中， r 代表关键字的基数， d 代表长度， n 代表关键字的个数

排序方法	时间复杂度（平均）	时间复杂度（最坏）	时间复杂度（最好）	空间复杂度	稳定性
插入排序	$O(n^2)$	$O(n^2)$	$O(n)$	$O(1)$	稳定
希尔排序	$O(n^{1.3})$	$O(n^2)$	$O(n)$	$O(1)$	不稳定
选择排序	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$	不稳定
堆排序	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(1)$	不稳定
冒泡排序	$O(n^2)$	$O(n^2)$	$O(n)$	$O(1)$	稳定
快速排序	$O(n \log_2 n)$	$O(n^2)$	$O(n \log_2 n)$	$O(n \log_2 n)$	不稳定
归并排序	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n \log_2 n)$	$O(n)$	稳定
计数排序	$O(n+k)$	$O(n+k)$	$O(n+k)$	$O(n+k)$	稳定
桶排序	$O(n+k)$	$O(n^2)$	$O(n)$	$O(n+k)$	稳定
基数排序	$O(n*k)$	$O(n*k)$	$O(n*k)$	$O(n+k)$	稳定

14.1 1996. The Number of Weak Characters in the Game 桶排序

You are playing a game that contains multiple characters, and each of the characters has two main properties: attack and defense. You are given a 2D integer array properties where $\text{properties}[i] = [\text{attack}_i, \text{defense}_i]$ represents the properties of the i th character in the game. A character is said to be weak if any other character has both attack and defense levels strictly greater than this character's attack and defense levels. More formally, a character i is said to be weak if there exists another character j where $\text{attack}_j > \text{attack}_i$ and $\text{defense}_j > \text{defense}_i$. Return the number of weak characters.

```

public int numberOfWeakCharacters(int[][] properties) {
    int maxAttack = 0; // 找到所有士兵中的最大值
    for (int[] p : properties)
        maxAttack = Math.max(maxAttack, p[0]);
    // 为每一个攻击创建一个桶的位置
    int[] bucket = new int[maxAttack + 2];
    // 在每一个攻击力上找到最大的防御力
    for (int[] p : properties)
        bucket[p[0]] = Math.max(bucket[p[0]], p[1]);
    // 将桶的每一个位置都寻找到大于其攻击力的最大防御数值
    int rightMax = bucket[maxAttack];
    for (int i = maxAttack; i >= 0; i--)
        if (rightMax > bucket[i])
            bucket[i] = rightMax;
        else
            rightMax = bucket[i];
    int ans = 0;
    // 最后遍历 p 寻找所有的弱将
    for (int[] p : properties)
        if (bucket[p[0] + 1] > p[1]) ans++; // 攻击力比当前小兵的攻击力大 1 的桶的位置存放着最大的防御力，与这个防御作比较可以得到当前小兵是否全面弱
    return ans;
}
public int numberOfWeakCharacters(int[][] properties) {
    int cnt = 0;
    int len = properties.length;
    Arrays.sort(properties, (a, b) -> (a[0] != b[0] ? a[0]-b[0] : b[1]-a[1]));
    int max = properties[len-1][1];
    for(int i = len-1;i>=0;i--){
        if(properties[i][1] < max)

```

```

        cnt++;
        max = Math.max(max, properties[i][1]);
    }
    return cnt;
}

```

14.2 632. Smallest Range Covering Elements from K Lists

You have k lists of sorted integers in non-decreasing order. Find the smallest range that includes at least one number from each of the k lists. We define the range $[a, b]$ is smaller than range $[c, d]$ if $b - a < d - c$ or $a < c$ if $b - a == d - c$.

```

// 时间复杂度: O(nk \log k)O(nk\log k), 其中 nn 是所有列表的平均长度, kk 是列表数量。所有的指针移动的总次数最多是 nk\log k 次, 每次从堆中取出元素和添加元素都需要更新堆,
// 空间复杂度: O(k)O(k), 其中 kk 是列表数量。空间复杂度取决于堆的大小, 堆中维护 kk 个元素。
public int[] smallestRange(List<List<Integer>> nums) {
    int n = nums.size();
    int resMin = 0, resMax = Integer.MAX_VALUE;
    int minRg = resMax - resMin;
    int max = Integer.MIN_VALUE;
    int [] next = new int [n]; // 各子链表中比当前 idx 位数值大的下一个数的下标, 即 idx+1, 初始化全为 0
    Queue<Integer> q = new PriorityQueue<>(new Comparator<Integer>() {
        public int compare(Integer idx1, Integer idx2) {
            return nums.get(idx1).get(next[idx1]) - nums.get(idx2).get(next[idx2]);
        }
    });
    for (int i = 0; i < n; i++) {
        q.offer(i); // 0, 1, 2, ..., n-1 真神奇呀
        max = Math.max(max, nums.get(i).get(0));
    }
    int minIdx = 0, curRg = 0;
    while (true) {
        minIdx = q.poll(); // 取出的是最小值的子链表的序号, 而子链表里的当前最小值所在子链表中的位置存于 next[minIdx] 中
        curRg = max - nums.get(minIdx).get(next[minIdx]);
        if (curRg < minRg) {
            minRg = curRg;
            resMin = nums.get(minIdx).get(next[minIdx]);
            resMax = max;
        }
        next[minIdx]++;
        if (next[minIdx] == nums.get(minIdx).size()) break;
        q.offer(minIdx); // 加回去, 但是 queue 里真正比较的值已经变了, 变强大了。。。 // 更新最小值的替换值
        max = Math.max(max, nums.get(minIdx).get(next[minIdx])); // 更新最大值
    }
    return new int [] {resMin, resMax};
}

```

- 另外一种解法的

```

// 这里的 BB 序列是什么? 我们可以用一个哈希映射来表示 BB 序列—— B[i]
// B[i] 表示 ii 在哪些列表当中出现过,
// 这里哈希映射的键是一个整数, 表示列表中的某个数值,
// 哈希映射的值是一个数组, 这个数组里的元素代表当前的键出现在哪些列表里。
// 如果列表集合为:
// 0: [-1, 2, 3]
// 1: [1]
// 2: [1, 2]
// 3: [1, 1, 3]
// 那么可以得到这样一个哈希映射
// -1: [0]
// 1: [1, 2, 3, 3]
// 2: [0, 2]
// 3: [0, 3]
// 时间复杂度: O(nk + |V|)O(nk+|V|), 其中 nn 是所有列表的平均长度, kk 是列表数量, |V|V 是列表中元素的值域, 在本题中 |V| \leq 2*10^5V210
// 5
// 构造哈希映射的时间复杂度为 O(nk)O(nk), 双指针的移动范围为 |V|V, 在此过程中会对哈希映射再进行一次遍历, 时间复杂度为 O(nk)O(nk), 因此总时间复杂度为 O(nk + |V|)O(nk+|V|)
// 空间复杂度: O(nk)O(nk), 即为哈希映射使用的空间。哈希映射的「键」的数量由列表中的元素个数 nk\log k 以及值域 |V|V 中的较小值决定, 「值」为长度不固定的数组, 但是它们的长
public int[] smallestRange(List<List<Integer>> nums) {
    int n = nums.size();
    Map<Integer, List<Integer>> indices = new HashMap<>();
    int xmin = Integer.MAX_VALUE, xmax = Integer.MIN_VALUE;
    for (int i = 0; i < n; i++) {
        for (int v : nums.get(i)) { // 把大链表中出过的每一个值作键, 值为它所存在于子链表序号链表
            List<Integer> list = indices.getOrDefault(v, new ArrayList<>());
            list.add(i);
            indices.put(v, list);
            xmin = Math.min(xmin, v);
            xmax = Math.max(xmax, v); // 这里得到全局的最小最大值
        }
    }
}

```

```

        }
    }

    int [] freq = new int [n];
    int inside = 0; // cnt # of lists included in miniRanges
    int left = xmin, right = xmin - 1;
    int resLeft = xmin, resRight = xmax;
    while (right < xmax) {
        right++;
        if (indices.containsKey(right)) {
            for (int x : indices.get(right)) {
                freq[x]++;
                if (freq[x] == 1) inside++;
            }
        }
        while (inside == n) { // find ONE satisfied solution, try to minimize the range
            if (right - left < resRight - resLeft) {
                resLeft = left;
                resRight = right;
            }
            if (indices.containsKey(left)) // sliding the left size towards right
                for (int v : indices.get(left)) {
                    freq[v]--;
                    if (freq[v] == 0) --inside;
                }
            left++;
        }
    }
    return new int [] {resLeft, resRight};
}

```

14.3 Reverse Pairs

Given an integer array nums, return the number of reverse pairs in the array. A reverse pair is a pair (i, j) where $0 \leq i < j < \text{nums.length}$ and $\text{nums}[i] > 2 * \text{nums}[j]$.

```

private int mergeSortCount(long [] arr, int bgn, int end) {
if (bgn >= end) return 0;
int mid = bgn + (end-bgn)/2;
int cnt = mergeSortCount(arr, bgn, mid) + mergeSortCount(arr, mid+1, end);
for (int i = bgn, j = mid+1; i <= mid; i++) {
    while (j <= end && arr[i] > 2*arr[j]) j++;
    cnt += j - (mid+1);
}
Arrays.sort(arr, bgn, end+1);
return cnt;
}
public int reversePairs(int[] nums) {
    int n = nums.length;
    return mergeSortCount( Arrays.stream(nums).mapToLong(i -> i).toArray(), 0, n-1);
}
// bit 的解法: https://www.cnblogs.com/grandyang/p/6657956.html

```

14.4 306. Additive Number

Additive number is a string whose digits can form additive sequence.

A valid additive sequence should contain at least three numbers. Except for the first two numbers, each subsequent number in the sequence must be the sum of the preceding two.

Given a string containing only digits '0'-'9', write a function to determine if it's an additive number.

Note: Numbers in the additive sequence cannot have leading zeros, so sequence 1, 2, 03 or 1, 02, 3 is invalid.

```

public boolean isAdditiveNumber(String num) {
    int n = num.length();
    if (n < 3) return false;
    for (int i = 1; i <= num.length() >> 1; i++)
        for (int j = 1; j + i < num.length(); j++)
            if (isValid(num, num.substring(0, i), num.substring(i, i + j), i + j)) return true;
    return false;
}
private boolean isValid(String num, String first, String second, int index) {
    if (first.length() > 1 && first.startsWith("0"))
        || second.length() > 1 && second.startsWith("0")) return false;
    if (index == num.length()) return true; // 如果只有两个数是有效的!!!
    long sum = Long.parseLong(first) + Long.parseLong(second);
    if (num.startsWith(sum + "", index)) // 间接检测第三个数

```

```
    if (isValid(num, second, sum + "", index + (sum + "").length())) return true;
return false;
}
```

Chapter 15

dfs 记忆化搜索

15.1 1977. Number of Ways to Separate Numbers - Hard

You wrote down many positive integers in a string called num. However, you realized that you forgot to add commas to separate the different numbers. You remember that the list of integers was non-decreasing and that no integer had leading zeros.

Return the number of possible lists of integers that you could have written down to get the string num. Since the answer may be large, return it modulo $10^9 + 7$.

15.1.1 动态规划与最长公共前缀

全网都找不到一句中文分析

```
private void getLongestCommonPrefixLength() { // Pre compute Longest Common Prefix sequence for each index in the string
    for (int i = n-1; i >= 0; i--)          // 从右向左遍历，计算最长公共前缀序列长度
        for (int j = n-1; j >= 0; j--)
            if (s[i] == s[j]) {
                if (i >= n-1 || j >= n-1) lcp[i][j] = 1;
                else lcp[i][j] = lcp[i+1][j+1] + 1;
            } else lcp[i][j] = 0;
    }
    private boolean compare(int i, int j, int len) { // compare substring of same length for value,
        int commonLength = lcp[i][j];           // 返回以 i 开始长度为 len 的序列是否比以 j 开始长度为 len 的序列（数值）小
        if (commonLength >= len) return true;
        return s[i + commonLength] <= s[j + commonLength]; // <= ? 为什么不可以等于呢?
    }
    long mod = (int)1e9 + 7;
    int [][] lcp;
    char [] s;
    int n;
    public int numberOfCombinations(String t) {
        if (t.charAt(0) == '0') return 0;
        n = t.length();
        this.s = t.toCharArray();
        lcp = new int[n][n];
        int [][] f = new int [n][n];
        int [][] pre = new int [n][n]; // 从右向左的累加和
        getLongestCommonPrefixLength(); // 计算从右向左遍历的最长公共前缀（右边，其实是后缀）
        for (int i = 0; i < n; i++) {
            f[0][i] = 1;
            pre[0][i] = 1;
        }
        for (int j = 1; j < n; j++) { // 跟上面超内存的写法是反着走，这次是从左向右遍历，可是两种方法，为什么就有一个会超内存呢？
            for (int i = 1; i <= j; i++) {
                if (s[i] == '0') {
                    f[i][j] = 0;
                    // continue;
                } else {
                    f[i][j] = pre[i-1][i-1];
                    if (i - (j-i+1) >= 0) // 现在长度为 i-j+1 的数，前面是否存在一个同样长度的数，即前一个数的第一个位下标是否 >= 0
                        f[i][j] -= pre[2*i-j-1][i-1];
                    if (i - (j-i+1) >= 0 && compare(i-(j-i+1), i, j-i+1)) {
                        f[i][j] = (int)((f[i][j] + pre[i-(j-i+1)][i-1]) % mod);
                        if (i - (j-i+1) - 1 >= 0)
                            f[i][j] -= pre[i-(j-i+1)-1][i-1];
                    }
                }
            }
            f[i][j] = (int)((f[i][j] + mod) % mod);
        }
    }
}
```

```

        pre[i][j] = (int)((pre[i-1][j] + f[i][j]) % mod);
    }
}
return pre[n-1][n-1];
}

```

15.1.2 解题思路与分析 **todo:** 其它思路，改天补上

15.2 87. Scramble String - Hard 非常经典：要韧熟于心

We can scramble a string s to get a string t using the following algorithm:

If the length of the string is 1, stop. If the length of the string is > 1 , do the following: Split the string into two non-empty substrings at a random index, i.e., if the string is s , divide it to x and y where $s = x + y$. Randomly decide to swap the two substrings or to keep them in the same order. i.e., after this step, s may become $s = x + y$ or $s = y + x$. Apply step 1 recursively on each of the two substrings x and y . Given two strings $s1$ and $s2$ of the same length, return true if $s2$ is a scrambled string of $s1$, otherwise, return false.

15.2.1 朴素解法 (TLE)

一个朴素的做法根据「扰乱字符串」的生成规则进行判断。

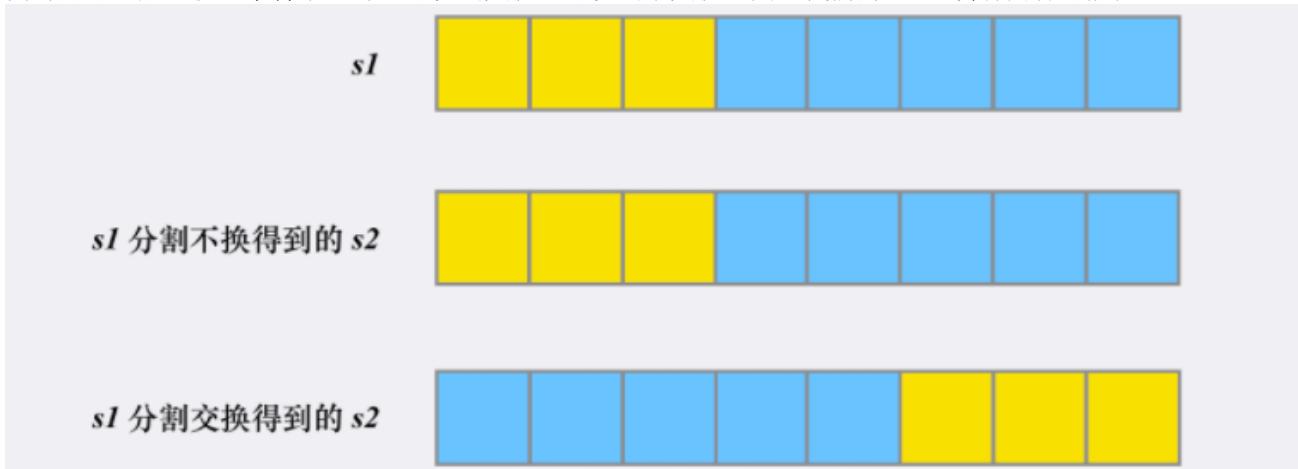
由于题目说了整个生成「扰乱字符串」的过程是通过「递归」来进行。

我们要实现 `isScramble` 函数的作用是判断 s_1 是否可以生成出 s_2 。

这样判断的过程，同样我们可以使用「递归」来做：

假设 s_1s_1 的长度为 n , 第一次分割的分割点为 i , 那么 s_1s_1 会被分成 $[0, i)[0, i)$ 和 $[i, n)[i, n)$ 两部分。

同时由于生成「扰乱字符串」时，可以选交换也可以选不交换。因此我们的 s2s2 会有两种可能性：



因为对于某个确定的分割点， s_1s_1 固定分为两部分，分别为 $[0,i)[0,i)$ & $[i, n)[i,n)$ 。

而 s_2s_2 可能会有两种分割方式，分别 $[0,i)[0,i) \& [i,n)[i,n)$ 和 $[0,n-i)[0,n-i) \& [n-i,n)[n-i,n)$ 。

我们只需要递归调用 `isScramble` 检查 s_1 的 $[0, i)$ & $[i, n)$ 部分能否与「 s_2 的 $[0, i)$ & $[i, n)$ 」或者「 s_2 的 $[0, n-i)$ & $[n-i, n)$ 」匹配即可。

同时，我们将「 s_1s_1 和 s_2s_2 相等」和「 s_1s_1 和 s_2s_2 词频不同」作为「递归」出口。

理解这套做法十分重要，后续的解法都是基于此解法演变过来。

```
private boolean idCheck(String ss, String tt) {
    int [] one = new int [26];
    int [] two = new int [26];
    char [] s = ss.toCharArray();
    char [] t = tt.toCharArray();
    for (int i = 0; i < s.length; i++)
        one[s[i] - 'a']++;
    for (int i = 0; i < t.length; i++)
        two[t[i] - 'a']++;
    for (int i = 0; i < 26; i++)
        if (one[i] != two[i])
            return false;
    return true;
}
public boolean isScramble(String s, String t) { // tle tile tile
    int n = s.length();
    if (n == 1) return s.charAt(0) == t.charAt(0);
    if (n == 2) return s.equals(t);
    if (n == 3) return idCheck(s, t);
    if (n % 2 == 1) return false;
    int m = n / 2;
    String s1 = s.substring(0, m);
    String s2 = s.substring(m, n);
    String t1 = t.substring(0, m);
    String t2 = t.substring(m, n);
    if (idCheck(s1, t1) && idCheck(s2, t2))
        return true;
    if (idCheck(s1, t2) && idCheck(s2, t1))
        return true;
    return false;
}
```

```

if (s.equals(t)) return true;
if (!idCheck(s, t)) return false;
for (int i = 1; i < n; i++) {
    System.out.println("\n i: " + i);
    String ls = s.substring(0, i), rs = s.substring(i);
    String ltone = t.substring(0, i), rtone = t.substring(i);
    String lttwo = t.substring(0, n-i), rttwo = t.substring(n-i);
    if (isScramble(ls, ltone) && isScramble(rs, rtone)
        || isScramble(ls, rttwo) && isScramble(rs, lttwo))
        return true;
}
return false;
}

```

时间复杂度: $O(5^n)$

空间复杂度: 忽略递归与生成子串带来的空间开销, 复杂度为 $O(1)$

15.2.2 记忆化搜索

朴素解法卡在了 286 / 288 个样例。

我们考虑在朴素解法的基础上, 增加「记忆化搜索」功能。

我们可以重新设计我们的「爆搜」逻辑: 假设 s_1 从 i 位置开始, s_2 从 j 位置开始, 后面的长度为 len 的字符串是否能形成「扰乱字符串」(互为翻转)。

那么在单次处理中, 我们可分割的点的范围为 $[1, len]$, 然后和「递归」一下, 将 s_1 分割出来的部分尝试去和 s_2 的对应位置匹配。

同样的, 我们将「入参对应的子串相等」和「入参对应的子串词频不同」作为「递归」出口。

```

private boolean idCheck(String ss, String tt) {
    int [] one = new int [26];
    int [] two = new int [26];
    char [] s = ss.toCharArray();
    char [] t = tt.toCharArray();
    for (int i = 0; i < s.length; i++)
        one[s[i] - 'a']++;
    for (int i = 0; i < t.length; i++)
        two[t[i] - 'a']++;
    for (int i = 0; i < 26; i++)
        if (one[i] != two[i])
            return false;
    return true;
}
private int dfs(int i, int j, int k) { // k: length dp[i][j][len]: 这个 dp 的设计还是比较难想的!
    if (dp[i][j][k] != 0) return dp[i][j][k];
    String a = s.substring(i, i+k), b = t.substring(j, j+k);
    if (a.equals(b)) return dp[i][j][k] = 1;
    if (!idCheck(a, b)) return dp[i][j][k] = -1;
    for (int l = 1; l < k; l++) {
        if (dfs(i, j, l) == 1 && dfs(i+l, j+l, k-l) == 1)
            return dp[i][j][k] = 1;
        if (dfs(i, j+k-l, l) == 1 && dfs(i+l, j, k-l) == 1)
            return dp[i][j][k] = 1;
    }
    return dp[i][j][k] = -1;
}
int [[[[]]] dp;
String s, t;
int n;
public boolean isScramble(String s, String t) {
    this.s = s;
    this.t = t;
    n = s.length();
    if (s.equals(t)) return true;
    if (!idCheck(s, t)) return false;
    dp = new int [n][n][n+1];
    return dfs(0, 0, s.length()) == 1;
}

```

15.2.3 动态规划 (区间 DP)

当然, 这道题也可以用动态规划 Dynamic Programming, 根据以往的经验来说, 根字符串有关的题十有八九可以用 DP 来做, 那么难点就在于如何找出状态转移方程。

其实有了上述「记忆化搜索」方案之后, 我们就已经可以直接忽略原问题, 将其改成「动态规划」了。

根据「`dfs` 方法的几个可变入参」作为「状态定义的几个维度」, 根据「`dfs` 方法的返回值」作为「具体的状态值」。

我们可以得到状态定义 $f[i][j][len]f[i][j][len]$:

$f[i][j][len]f[i][j][len]$ 代表 s_1s_1 从 i 开始, s_2s_2 从 j 开始, 后面长度为 len 的字符是否能形成「扰乱字符串」(互为翻转)。

状态转移方程其实就是翻译我们「记忆化搜索」中的 dfs 主要逻辑部分:

```
// 对应了「s1 的 [0,i) & [i,n]」匹配「s2 的 [0,i) & [i,n]」
if (dfs(i, j, k) && dfs(i + k, j + k, len - k)) {
    cache[i][j][len] = Y;
    return true;
}
// 对应了「s1 的 [0,i) & [i,n]」匹配「s2 的 [n-i,n) & [0,n-i]」
if (dfs(i, j + len - k, k) && dfs(i + k, j, len - k)) {
    cache[i][j][len] = Y;
    return true;
}
```

从状态定义上, 我们就不难发现这是一个「区间 DP」问题, 区间长度大的状态值可以由区间长度小的状态值递推而来。

而且由于本身我们在「记忆化搜索」里面就是从小到大枚举 len , 因此这里也需要先将 len 这层循环提前, 确保我们转移 $f[i][j][len]$ 时所需要的状态都已经被计算好。

这道题看起来是比较复杂的, 如果用 brute force, 每次做切割, 然后递归求解, 是一个非多项式的复杂度, 一般来说这不是面试官想要的答案。

这其实是一道三维动态规划的题目, 我们提出维护量 $res[i][j][n]$, 其中 i 是 s_1 的起始字符, j 是 s_2 的起始字符, 而 n 是当前的字符串长度, $res[i][j][len]$ 表示的是以 i 和 j 分别为 s_1 和 s_2 起点的长度为 len 的字符串是不是互为 scramble。

有了维护量我们接下来看看递推式, 也就是怎么根据历史信息来得到 $res[i][j][len]$ 。判断这个是不是满足, 其实我们首先是把当前 $s_1[i \dots i+len-1]$ 字符串劈一刀分成两部分, 然后分两种情况:

第一种是左边和 $s_2[j \dots j+len-1]$ 左边部分是不是 scramble, 以及右边和 $s_2[j \dots j+len-1]$ 右边部分是不是 scramble;

第二种情况是左边和 $s_2[j \dots j+len-1]$ 右边部分是不是 scramble, 以及右边和 $s_2[j \dots j+len-1]$ 左边部分是不是 scramble。

如果以上两种情况有一种成立, 说明 $s_1[i \dots i+len-1]$ 和 $s_2[j \dots j+len-1]$ 是 scramble 的。而对于判断这些左右部分是不是 scramble 我们是有历史信息的, 因为长度小于 n 的所有情况我们都在前面求解过了(也就是长度是最外层循环)。

上面说的是劈一刀的情况, 对于 $s_1[i \dots i+len-1]$ 我们有 $len-1$ 种劈法, 在这些劈法中只要有一种成立, 那么两个串就是 scramble 的。

总结起来递推式是 $res[i][j][len] = || (res[i][j][k] \&& res[i+k][j+k][len-k] || res[i][j+len-k][k] \&& res[i+k][j][len-k])$ 对于所有 $1 <= k < len$, 也就是对于所有 $len-1$ 种劈法的结果求或运算。因为信息都是计算过的, 对于每种劈法只需要常量操作即可完成, 因此求解递推式是需要 $O(len)$ (因为 $len-1$ 种劈法)。

如此总时间复杂度因为是三维动态规划, 需要三层循环, 加上每一步需要线行时间求解递推式, 所以是 $O(n^4)$ 。虽然已经比较高了, 但是至少不是指数量级的, 动态规划还是有很大本事的, 空间复杂度是 $O(n^3)$ 。

时间复杂度: $O(n^4)$

空间复杂度: $O(n^3)$

```
public boolean isScramble(String s, String t) {
    int n = s.length();
    if (s.equals(t)) return true;
    boolean [[[dp = new boolean[n][n][n+1];
    for (int len = 1; len <= n; len++)
        for (int i = 0; i+len <= n; i++)
            for (int j = 0; j+len <= n; j++) {
                if (len == 1) {
                    dp[i][j][len] = s.charAt(i) == t.charAt(j);
                    continue;
                }
                for (int k = 1; k < len; k++)
                    if (dp[i][j][k] && dp[i+k][j+k][len-k] || dp[i][j+len-k][k] && dp[i+k][j][len-k])
                        dp[i][j][len] = true;
            }
    }
    return dp[0][0][n];
}
```

15.3 2065. Maximum Path Quality of a Graph - Hard 记忆化搜索 + 重复遍历

There is an undirected graph with n nodes numbered from 0 to $n - 1$ (inclusive). You are given a 0-indexed integer array values where $values[i]$ is the value of the i th node. You are also given a 0-indexed 2D integer array edges, where each $edges[j] = [u_j, v_j, time_j]$ indicates that there is an undirected edge between the nodes u_j and v_j , and it takes $time_j$ seconds to travel between the two nodes. Finally, you are given an integer $maxTime$.

A valid path in the graph is any path that starts at node 0, ends at node 0, and takes at most maxTime seconds to complete. You may visit the same node multiple times. The quality of a valid path is the sum of the values of the unique nodes visited in the path (each node's value is added at most once to the sum).

Return the maximum quality of a valid path.

Note: There are at most four edges connected to each node.

15.3.1 解题思路与分析

A straightforward idea is to try all paths from node 0 and calculate the max path quality for paths also end with node 0.

One **optimization** we can add is: once we cannot return back to node 0, we stop. The \min_{time} required from any node to node 0 can be pre-computed using Dijkstra algorithm.

Time complexity: $O(4^{10})$

Note the constraints: $10 \leq time_j, maxTime \leq 100$ and There are at most four edges connected to each node..

It means the max levels of dfs search is 10, and at each level we have maximum of 4 neighbouring nodes to try.

So the time complexity is: $O(4^{10})$.

```

private int[] dijkstra() {
    int[] ans = new int[n];
    Arrays.fill(ans, Integer.MAX_VALUE);
    ans[0] = 0;
    // boolean[] vis = new boolean[n]; // 因为可以重复遍历, 要允许它重复遍历
    // vis[idx] = true; // 因为可以重复遍历, 要允许它重复遍历
    Queue<int[]> q = new PriorityQueue<>((a, b) ->a[1] - b[1]);
    q.offer(new int[] {0, 0});
    while (!q.isEmpty()) {
        int[] cur = q.poll();
        if (cur[1] > ans[cur[0]]) continue;
        for (int[] nei : adj.get(cur[0]))
            // if (nei[0] == cur[0]) continue; // 因为可以重复遍历, 要允许它重复遍历
            if (nei[1] + cur[1] < ans[nei[0]]) {
                ans[nei[0]] = nei[1] + cur[1];
                // if (!vis[nei[0]]) {
                q.offer(new int[] {nei[0], ans[nei[0]]});
                // vis[nei[0]] = true;
            }
    }
    return ans;
}

private void dfs(int idx, int avaTime, int[] t, int[] v, Set<Integer> vis) {
    if (idx == 0) {
        int cur = 0;
        for (Integer node : vis)
            cur += v[node];
        ans = Math.max(ans, cur);
    }
    for (int[] nei : adj.get(idx))
        if (t[nei[0]] + nei[1] <= avaTime) { // 
            boolean added = vis.add(nei[0]);
            dfs(nei[0], avaTime - nei[1], t, v, vis);
            if (added)
                vis.remove(nei[0]);
        }
}
int[] time;
List<List<int[]>> adj = new ArrayList<>();
int n, ans = 0;
public int maximalPathQuality(int[] values, int[][] edges, int maxTime) {
    n = values.length;
    for (int i = 0; i < n; i++)
        adj.add(new ArrayList<>());
    for (int[] e : edges) {
        adj.get(e[0]).add(new int[] {e[1], e[2]});
        adj.get(e[1]).add(new int[] {e[0], e[2]});
    }
    time = dijkstra();
    Set<Integer> si = new HashSet<>();
    si.add(0);
    dfs(0, maxTime, time, values, si);
    return ans;
}

```

15.4 1397. Find All Good Strings - Hard 记忆化搜索

Given the strings s_1 and s_2 of size n and the string $evil$, return the number of good strings.

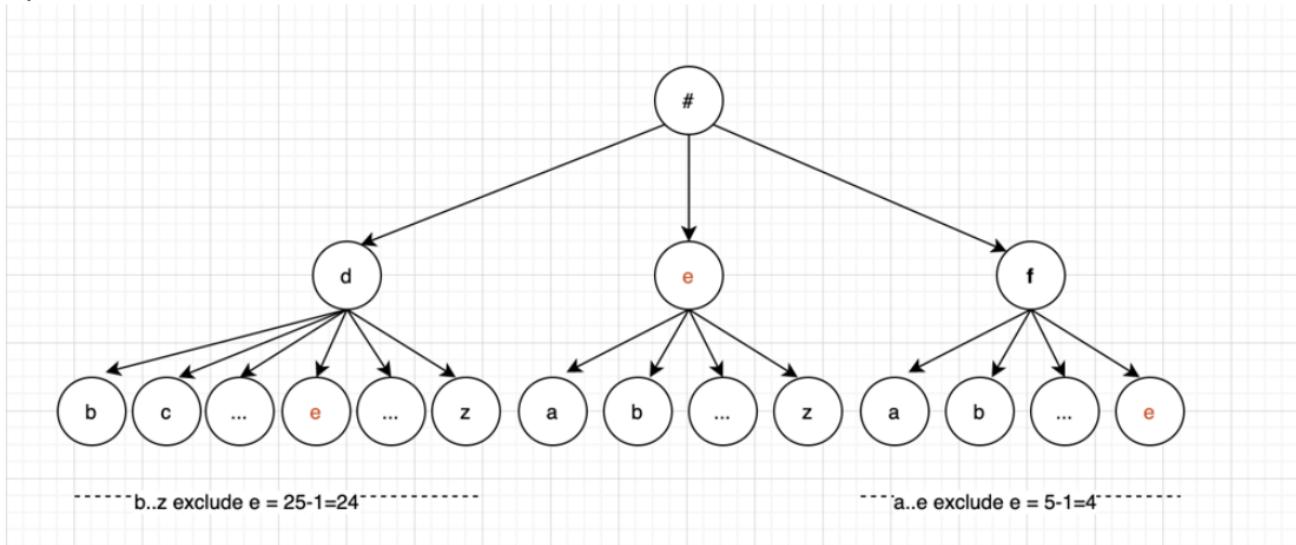
A good string has size n , it is alphabetically greater than or equal to s_1 , it is alphabetically smaller than or equal to s_2 , and it does not contain the string $evil$ as a substring. Since the answer can be a huge number, return this modulo $10^9 + 7$.

15.4.1 解题思路与分析: 记忆化搜索

Input: $n = 2, s_1 = "db", s_2 = "fe", evil = "e"$

Output: $24 + 4 = 28$

Explanation:



- Complexity

Time: $O(n*m*2*2*26)$, where $n \leq 500$, $m \leq 50$ is length of evil

Space: $O(m*n*2*2)$

```

private int[] computeLongestPrefixSuffix (char[] s) { // 这个要再理解一下: 重中之重
    int n = s.length;
    int[] lps = new int[n];
    for (int i = 1, j = 0; i < n; i++) {
        while (j > 0 && s[i] != s[j]) j = lps[j-1]; // 转向它 j 的前一位字符 (在 j-1 下标) 所指向的匹配位置 lps[j-1]
        if (s[i] == s[j]) lps[i] = ++j; // 同时增加两个的下标
    }
    return lps;
}

private int getKey(int i, int j, boolean l, boolean r) { // bits occupied: i 9, j 6, l 1, r 1
    // 9 bits store n (2^9=512), 6 bits for m (2^6=64), 1 bit for b1, 1 bit for b2
    return (i << 8) | (j << 2) | ((l ? 1 : 0) << 1) | (r ? 1 : 0); // 这是一个压缩空间存 key 的聪明技巧
}

private int dfs(int n, int i, int evilMatched, boolean leftBound, boolean rightBound) {
    if (evilMatched == evil.length) return 0; // matched evil string, no good
    if (i == n) return 1; // DIDN'T match evil string, great
    int key = getKey(i, evilMatched, leftBound, rightBound); // state: represented by <= 17 bits integer
    if (dp[key] > 0) return dp[key];
    char from = leftBound ? s[i] : 'a';
    char to = rightBound ? t[i] : 'z';
    int ans = 0;
    for (char c = from; c <= to; c++) {
        int j = evilMatched; // j means the next match between current string (end at char 'c') and `evil` string
        while (j > 0 && e[j] != c) j = lps[j-1]; // 向左回溯寻找 match 字符 c 的上一个位置 ?
        if (c == e[j]) j++;
        ans += dfs(n, i+1, j, leftBound && (c == from), rightBound && (c == to));
        ans %= mod;
    }
    return dp[key] = ans;
}
int mod = (int)1e9 + 7;
char[] s, t, e;
int[] dp, lps;
public int findGoodStrings(int n, String s1, String s2, String evil) {
    dp = new int[1 << 17]; // Need total 17 bits, according to data limits
}

```

```

s = s1.toCharArray();
t = s2.toCharArray();
e = evil.toCharArray();
lps = computeLongestPrefixSuffix(e);
return dfs(n, 0, 0, true, true);
}

```

15.4.2 动态规划: 数位 DP + KMP todo: 改天把这个补上

- <https://leetcode-cn.com/problems/find-all-good-strings/solution/shu-wei-dp-kmp-by-qodjf/>
- <https://www.cnblogs.com/wenruo/p/12616985.html>
- <https://www.codeleading.com/article/42703213478/>
- <https://leetcode-cn.com/problems/find-all-good-strings/solution/shu-wei-dp-kmpqian-zhui-shu-zu-java-by/>
- <https://leetcode-cn.com/problems/find-all-good-strings/solution/kmpshang-de-dpc-by-zhu-mang-4/>

之前做的题大部分是关于数字的数位 dp, 而现在要的就是字符串的数位 dp。

设 $d p [p o s][s t a t s][b o u n d]$ $dp[pos][stats][bound]$ 为数位 dp 的数组, 其中 $p o s$ 表示第 pos 个位置的字符总共有的数量, $s t a t s$ 表示的是匹配 $e v i l$ 的状态, 即能够匹配到 $e v i l$ 数组的位置。 $b o u n d$ 表示此时能够选择字符的范围, 即当前字符选择的时候是否有限制。

对于 $b o u n d$ 我们用四个数字表示

0 00 表示此时的字符选择是没有限制的,	即可以选择的范围为 $a z a \sim z a z$
1 11 表示此时的字符选择是有下限的,	所以选择的范围是 $s 1 [p o s] z s 1 [pos] \sim s 1 [pos] z$
2 22 表示此时的字符选择是有上限的,	所以可以选择的范围是 $a s 2 [p o s] a \sim s 2 [pos] s 2 [pos]$
3 33 表示此时的字符既有上限又有下限。	这种情况只有当 $s 1 [p o s] s 2 [p o s] s 1 [pos] \sim s 2 [pos] s 1 [pos] s 2 [pos]$

而对于 $s t a t s$ 表示匹配 $e v i l$ 字符的状态, 由于 $e v i l$ 的长度最长为 50, 所以可以生成字符串 $e v i l$ 的 next 数组。那么当匹配不成立的时候, 就可以直接进行跳转。

设一个记忆数组 $m e m [e _ p o s][n _ c h a r] mem[e \backslash pos][n \backslash char]$

$char]mem[e$

$pos[nchar] e v i l$ 的位置为 $e _ p o s e \backslash pos$ $n _ c h a r n \backslash char$ $m e m[1, 2] mem[7, 8]$, 因此没必要每次都计算。

对于如何生成 next 的数组, 小伙伴们可以去搜索与 KMP 算法相关的博客查看。

-
-

15.5 2060. Check if an Original String Exists Given Two Encoded Strings - Hard dfs 记忆化搜索

An original string, consisting of lowercase English letters, can be encoded by the following steps:

Arbitrarily split it into a sequence of some number of non-empty substrings. Arbitrarily choose some elements (possibly none) of the sequence, and replace each with its length (as a numeric string). Concatenate the sequence as the encoded string. For example, one way to encode an original string "abcdefghijklmnp" might be:

Split it as a sequence: ["ab", "cdefghijklmn", "o", "p"]. Choose the second and third elements to be replaced by their lengths, respectively. The sequence becomes ["ab", "12", "1", "p"]. Concatenate the elements of the sequence to get the encoded string: "ab121p". Given two encoded strings s_1 and s_2 , consisting of lowercase English letters and digits 1-9 (inclusive), return true if there exists an original string that could be encoded as both s_1 and s_2 . Otherwise, return false.

Note: The test cases are generated such that the number of consecutive digits in s_1 and s_2 does not exceed 3.

¹DEFINITION NOT FOUND.

²DEFINITION NOT FOUND.

15.5.1 解题思路与分析- (这里需要再好好总结一下)

- solution is straight forward we have 2 pointer in each string

1.consider the easy case, they all character, we compare s1.charAt(i) == s2.charAt(j)
 2.digit case, we get a number from s1, we can calculate the number s1 has, (descripton said less than 1000),
 we can pass this value compare with number from s2 name it diff
 3.character case if we still has remaing diff to spend passed from our parents,
 so we can use one dollar a day, one diff one position dfs(i + 1, j, diff - 1)
 4.terminating condition, if both reach the end and diff == 0

```

private boolean dfs(int i, int j, int dif) {
    if (i >= m && j >= n && dif == 0) return true;
    if (memo[i][j][dif+1000] != null) return memo[i][j][dif+1000]; // + 1000
    boolean res = false;
    if (i < m) {
        if (Character.isDigit(s[i])) {
            int cur = 0, cnt = 0;
            while (i+cnt < m && cnt < 3 && Character.isDigit(s[i+cnt])) {
                cur = cur * 10 + (s[i+cnt] - '0');
                cnt++;
                if (dfs(i+cnt, j, dif - cur)) res = true;
            }
        } else {
            if (dif > 0 && dfs(i+1, j, dif-1)) res = true;
            else if (dif == 0 && j < n && s[i] == t[j] && dfs(i+1, j+1, dif)) res = true;
        }
    }
    if (j < n) {
        if (Character.isDigit(t[j])) {
            int cur = 0, cnt = 0;
            while (j+cnt < n && cnt < 3 && Character.isDigit(t[j+cnt])) {
                cur = cur * 10 + (t[j+cnt] - '0');
                cnt++;
                if (dfs(i, j+cnt, dif+cur)) res = true; // when to + cur when to - cur
            }
        } else if (dif < 0 && dfs(i, j+1, dif+1)) res = true; // no limit on i < m any more
    }
    return memo[i][j][dif+1000] = res;
}
Boolean[][][] memo = new Boolean[41][41][2000];
char[] s, t;
int m, n;
public boolean possiblyEquals(String s, String t) {
    m = s.length();
    n = t.length();
    this.s = s.toCharArray();
    this.t = t.toCharArray();
    return dfs(0, 0, 0);
}

```

Chapter 16

排列与组合

16.1 903. Valid Permutations for DI Sequence - Hard

You are given a string s of length n where $s[i]$ is either:

'D' means decreasing, or 'I' means increasing. A permutation perm of $n + 1$ integers of all the integers in the range $[0, n]$ is called a valid permutation if for all valid i:

If $s[i] = 'D'$, then $\text{perm}[i] > \text{perm}[i + 1]$, and If $s[i] = 'I'$, then $\text{perm}[i] < \text{perm}[i + 1]$. Return the number of valid permutations perm. Since the answer may be large, return it modulo $10^9 + 7$.

当是降序时，下一个数字不小于当前最后一个数字，反之是升序时，下一个数字小于当前最后一个数字，所以可以写出状态转移方程如下所示：

```
if (S[i-1] == 'D')    dp[i][j] += dp[i-1][k]    ( j <= k <= i-1 )
else                  dp[i][j] += dp[i-1][k]    ( 0 <= k < j )
```

```
public int numPermsDISequence(String s) {
    int n = s.length(), mod = (int)1e9 + 7, res = 0;
    int [][] dp = new int [n+1][n+1];
    dp[0][0] = 1;
    for (int i = 1; i <= n; i++)
        for (int j = 0; j <= i; j++) // 考虑当前最后一个元素为 j
            if (s.charAt(i-1) == 'D')
                // 可以把序列中所有大于等于 j 的元素都加上 1 得到新序列 (这个时候是不会改变当前序列的大小关系的), 再把 j 添加到序列末尾得到
                for (int k = j; k <= i; k++)
                    dp[i][j] = (dp[i][j] + dp[i-1][k]) % mod;
            else // 考虑当前最后一个元素为 j
                // 把该序列中所有大于等于 j(k < j <= i) 的数字都加上 1, 并在末尾添加一个 j 得到 dp[i][j] 中的一种方案
                for (int k = 0; k < j; k++) // 把序列中所有大于等于 j 的元素都加上 1, 再把 j 放在序列末尾得到
                    dp[i][j] = (dp[i][j] + dp[i-1][k]) % mod;
    for (int i = 0; i <= n; i++)
        res = (res + dp[n][i]) % mod;
    return (int)res;
}
```

16.2 1467. Probability of a Two Boxes Having The Same Number of Distinct Balls - Hard

Given $2n$ balls of k distinct colors. You will be given an integer array balls of size k where $\text{balls}[i]$ is the number of balls of color i .

All the balls will be shuffled uniformly at random, then we will distribute the first n balls to the first box and the remaining n balls to the other box (Please read the explanation of the second example carefully).

Please note that the two boxes are considered different. For example, if we have two balls of colors a and b, and two boxes [] and (), then the distribution [a] (b) is considered different than the distribution [b] (a) (Please read the explanation of the first example carefully).

We want to calculate the probability that the two boxes have the same number of distinct balls.

Example: [2, 1, 1]

for first ball, we can put both ball in first bin and 0 ball in second bin OR we can put 1 ball in first bin, second ball in 2nd bin OR we can put both in second bin

for second ball, we can put ball in first bin and 0 ball in second bin, similarly, we can put 1 ball in second bin. same thing with the third ball

Try all possible permutations recursively. And, in the end check, if there are equal number of distinct balls in both bins or not.

```

public double getProbability(int[] balls) {
    int sum = Arrays.stream(balls).sum();
    double all = allCases(balls, 0, 0, 0, 0, 0, sum);
    double valid = casesWithEqualDistinctBalls(balls, 0, 0, 0, 0, 0, sum);
    return (1.0 * valid / all);
}
// disF = distinct balls in first bin
// disS = distinct balls in second bin
// f = number of balls in first bin
// s = number of balls in second bin
public double allCases(int [] arr, int pos, int f, int s, int disF, int disS, int sum) {
    if (pos == arr.length) {
        // for all cases, we just need to check if both bins have same number of balls or not
        if (f == s) return fact(sum / 2) * fact(sum / 2); // numerator of our permutations
        return 0;
    }
    // we put all balls in second bin
    double ans = 1.0 * allCases(arr, pos+1, f, s+arr[pos], disF, disS+1, sum) / fact(arr[pos]);
    // we put all balls in first bin
    ans += 1.0 * allCases(arr, pos+1, f+arr[pos], s, disF+1, disS, sum) / fact(arr[pos]);
    for (int i = 1; i < arr[pos]; i++) // 把每一种颜色的球放到两个里面盒子里都有
        ans += 1.0 * allCases(arr, pos+1, f+i, s+arr[pos]-i, disF+1, disS+1, sum) / (fact(i) * fact(arr[pos]-i));
    return ans;
}
public double casesWithEqualDistinctBalls(int [] arr, int pos, int f, int s, int disF, int disS, int sum) {
    if (pos == arr.length) {
        if (f == s && disF == disS) return fact(sum / 2) * fact(sum / 2);
        return 0;
    }
    double ans = 1.0 * casesWithEqualDistinctBalls(arr, pos+1, f, s+arr[pos], disF, disS+1, sum) / fact(arr[pos]);
    ans += 1.0 * casesWithEqualDistinctBalls(arr, pos+1, f+arr[pos], s, disF+1, disS, sum) / fact(arr[pos]);
    for (int i = 1; i < arr[pos]; i++)
        ans += 1.0 * casesWithEqualDistinctBalls(arr, pos+1, f+i, s+arr[pos]-i, disF+1, disS+1, sum) / (fact(i) * fact(arr[pos]-i));
    return ans;
}
private double fact(double n) {
    double res = 1;
    for (int i = 2; i <= n; i++)
        res = res * i;
    return res;
}
// Complexity: There can be total of (8 * 6) balls with 8 distinct. Complexity of recursion is f * s * disF * disS = O(48 * 48 * 8 * 8)

```

Chapter 17

Union Find 并查集

17.1 1697. Checking Existence of Edge Length Limited Paths - Hard

An undirected graph of n nodes is defined by edgeList, where edgeList[i] = [ui, vi, disi] denotes an edge between nodes ui and vi with distance disi. Note that there may be multiple edges between two nodes.

Given an array queries, where queries[j] = [pj, qj, limitj], your task is to determine for each queries[j] whether there is a path between pj and qj such that each edge on the path has a distance strictly less than limitj .

Return a boolean array answer, where answer.length == queries.length and the jth value of answer is true if there is a path for queries[j] is true, and false otherwise.

```
private class DisjointSetUnion {
    private int N;
    private int[] parent, rank;
    public DisjointSetUnion( int n) {
        this.N = n;
        this.parent = new int[N];
        this.rank = new int[N];
        for (int i = 0; i < N; i++) {
            this.parent[i] = i;
            this.rank[i] = 1;
        }
    }
    public boolean areConnected(int u, int v) {
        return find(u) == find(v);
    }
    public void union(int u, int v) { // O(Log(N))
        if (u != v) {
            int p = find(u);
            int q = find(v);
            if (p != q) {
                if (rank[p] > rank[q]) {
                    parent[q] = p;
                    rank[p] += rank[q];
                } else {
                    parent[p] = q;
                    rank[q] += rank[p];
                }
            }
        }
    }
    private int find(int v) { // 我记得这里有个同步更新父数组的优化步骤，和 rank 优化冲突吗？要总结一下 // O(Log(N))
        int x = v;
        while (x != parent[x])
            x = parent[x];
        parent[v] = x;
        return x;
    }
}
class Query implements Comparable<Query> {
    public int idx, start, end, weight;
    public Query(int idx, int bgn, int end, int weight) {
        this.idx = idx;
        this.start = bgn;
        this.end = end;
        this.weight = weight;
    }
    @Override public int compareTo(Query query) {
        return this.weight - query.weight;
    }
}
```

```

// 我们可以分别对 edges 和 queries 进行一次升序排序。本题和 1170. 比较字符串最小字母出现频次 类似，都可以采取 * 离线排序优化 * 的方式来解。
// 接下来，遍历 queries。遍历 queries 的同时将权值小于 limitj 的边进行合并。
// 接下来，我们只需要判断 pj 和 qj 是否已经在同一个联通域即可。
// 因此如果 pj 和 qj 在同一个联通域，那么其联通的路径上的所有边必定都小于 limitj，其原因就是前面加粗的那句话。
// 注意到排序打乱了 queries 的索引，因此我们需要记录一下其原始索引。
public boolean[] distanceLimitedPathsExist(int n, int[][][] edgeList, int[][][] queries) {
    DisjointSetUnion set = new DisjointSetUnion(n);
    Arrays.sort(edgeList, (a, b) -> a[2]-b[2]); // arrange edges in ascending order of weights
    int m = queries.length, i;
    int E = edgeList.length;
    Query [] qarr = new Query[m];
    for (i = 0; i < m; i++) {
        qarr[i] = new Query(i, queries[i][0], queries[i][1], queries[i][2]);
    }
    Arrays.sort(qarr); // arrange queries in ascending order of threshold weights
    boolean [] ans = new boolean [m];
    int idx = 0;
    for (i = 0; i < m; i++) {
        while (idx < E && edgeList[idx][2] < qarr[i].weight) {
            set.union(edgeList[idx][0], edgeList[idx][1]);
            ++idx;
        }
        ans[qarr[i].idx] = set.areConnected(qarr[i].start, qarr[i].end);
    }
    return ans;
}
// 令 m, q edges 和 queries 的长度。
// 时间复杂度: $O(m \log m + q \log q)$
// 空间复杂度: $O(n + q)$
// Runtime is bound by sorting: $O(E \log E + N \log N + N + E)$;

```

17.2 2003. Smallest Missing Genetic Value in Each Subtree - Hard

There is a family tree rooted at 0 consisting of n nodes numbered 0 to n - 1. You are given a 0-indexed integer array parents, where parents[i] is the parent for node i. Since node 0 is the root, parents¹ == -1.

There are 105 genetic values, each represented by an integer in the inclusive range [1, 105]. You are given a 0-indexed integer array nums, where nums[i] is a distinct genetic value for node i.

Return an array ans of length n where ans[i] is the smallest genetic value that is missing from the subtree rooted at node i.

The subtree rooted at a node x contains node x and all of its descendant nodes.

```

private void dfs(int i, Set<Integer> visited, int [] arr) { // 图的遍历：自顶向下 ok，自底向上还不太熟悉，需要练习，还有一个没有消化好的类似题，找出来 binary in
    if (!visited.contains(arr[i])) {
        Set<Integer> children = tree.getOrDefault(i, new HashSet<Integer>());
        for (int v : children)
            dfs(v, visited, arr);
        visited.add(arr[i]);
    }
}
Map<Integer, Set<Integer>> tree = new HashMap<>();
int [] ans;
int n;
public int[] smallestMissingValueSubtree(int[] parents, int[] nums) {
    n = parents.length;
    ans = new int [n];
    Arrays.fill(ans, 1);
    int oneIdx = -1;
    for (int i = 0; i < n; i++)
        if (nums[i] == 1) {
            oneIdx = i;
            break;
        }
    if (oneIdx == -1) return ans;
    for (int i = 1; i < n; i++) {
        tree.computeIfAbsent(parents[i], k -> new HashSet<Integer>());
        tree.get(parents[i]).add(i);
        // tree.computeIfAbsent(i, k -> new HashSet<Integer>()); // 这里要想一下：为什么双向图他只加一个方向？
        // tree.get(i).add(parents[i]);
    }
    Set<Integer> visited = new HashSet<>(); // 这个直接转化为想要的结果，很便捷
    int parentIter = oneIdx;
    int miss = 1;
    while (parentIter >= 0) { // 从值为 1 的节点向根遍历（自底向上），没有任何重复计算，只走完这一条自底向顶的路径就可以了
        dfs(parentIter, visited, nums);
        while (visited.contains(miss)) ++miss;
        ans[parentIter] = miss;
        parentIter = parents[parentIter];
    }
}

```

```
    return ans;
}
```

17.3 803. Bricks Falling When Hit - Hard 反向: 变 de-Union vs 为 Re-Union

You are given an $m \times n$ binary grid, where each 1 represents a brick and 0 represents an empty space. A brick is stable if:

It is directly connected to the top of the grid, or At least one other brick in its four adjacent cells is stable. You are also given an array hits, which is a sequence of erasures we want to apply. Each time we want to erase the brick at the location $\text{hits}[i] = (\text{row}_i, \text{col}_i)$. The brick on that location (if it exists) will disappear. Some other bricks may no longer be stable because of that erasure and will fall. Once a brick falls, it is immediately erased from the grid (i.e., it does not land on other stable bricks).

Return an array result, where each $\text{result}[i]$ is the number of bricks that will fall after the i th erasure is applied.

Note that an erasure may refer to a location with no brick, and if it does, no bricks drop.

看了discussion之后发现别人提出了很好的想法。

与其将砖头一块块按顺序敲碎，不如把砖头按相反的顺序一块块的加回来。

这样的好处是很好的解决了没敲碎一块砖头之后需要更新grid的操作!! (因为后面要敲的很可能在之前已经跟着落掉了)

而且这种方法也很好的解决了 **de-union** 的问题。把砖头加回来只需要 **union**。

```
private class UnionFind {
    int[] id; // parent
    int[] cnt; // size
    public UnionFind (int n) {
        id = new int[n];
        cnt = new int[n];
        for (int i = 0; i < n; i++) {
            id[i] = i;
            cnt[i] = 1;
        }
    }
    public int find(int i) {
        while (id[i] != i) {
            id[i] = id[id[i]];
            i = id[i];
        }
        return i;
    }
    public void union(int i, int j) {
        int rootI = find(i);
        int rootJ = find(j);
        if (rootI != rootJ) {
            id[rootI] = rootJ;
            cnt[rootJ] += cnt[rootI];
        }
    }
}
private void unionAround(int x, int y, int[][] arr, UnionFind uf) {
    for (int[] d : dirs) {
        int i = x + d[0];
        int j = y + d[1];
        if (i < 0 || i >= m || j < 0 || j >= n) continue;
        if (arr[i][j] == 1) uf.union(x*n+y+1, i*n+j+1); // +
    }
    if (x == 0) uf.union(x*n+y+1, 0); // 第一排的直接与顶相连 // trick: to help calculate cnts connecting top easier/faster
}
int[][] dirs = {{1, 0}, {-1, 0}, {0, 1}, {0, -1}};
int m, n;
public int[] hitBricks(int[][][] grid, int[][][] hits) {
    m = grid.length;
    n = grid[0].length;
    for (int[] hit : hits) // 首先把所有要打的砖块标记为 2.
        if (grid[hit[0]][hit[1]] == 1) // 如果有砖头
            grid[hit[0]][hit[1]] = 2;
    UnionFind uf = new UnionFind(m*n+1); // 这里的 + 1 主要是多一个 0 来表示顶，所有的第一排的砖在 unionfind 的时候都会直接与这个 0 相连。
    for (int i = 0; i < m; i++) // 然后对打掉后的数组中的砖块进行四个方向的 union
        for (int j = 0; j < n; j++)
```

```

    if (grid[i][j] == 1)
        unionAround(i, j, grid, uf);
int cnt = uf.cnt[uf.find(0)];           // 这个 count 就是打完后一定会剩下的砖块数量.
int [] ans = new int [hits.length];
for (int i = hits.length-1; i >= 0; i--) {
    int [] hit = hits[i];
    if (grid[hit[0]][hit[1]] == 2) { // 对于需要复原的这个砖块做四个方向 union, 主要是为了得到有多少砖必须通过这块砖才能连接到顶部。
        unionAround(hit[0], hit[1], grid, uf);
        grid[hit[0]][hit[1]] = 1;    // 由于是从后向前, 做完要把这块砖重新标记回来: 这些砖是有可能被 hits 前序砖敲掉后掉落下来的, 不复原影响前序结果
    }
    int newCnt = uf.cnt[uf.find(0)];
    ans[i] = (newCnt - cnt > 0 ? newCnt - cnt - 1 : 0);
    cnt = newCnt;
}
return ans;
}

```

17.4 721. Accounts Merge

Given a list of accounts where each element `accounts[i]` is a list of strings, where the first element `accounts[i]1` is a name, and the rest of the elements are emails representing emails of the account.

Now, we would like to merge these accounts. Two accounts definitely belong to the same person if there is some common email to both accounts. Note that even if two accounts have the same name, they may belong to different people as people could have the same name. A person can have any number of accounts initially, but all of their accounts definitely have the same name.

After merging the accounts, return the accounts in the following format: the first element of each account is the name, and the rest of the elements are emails in sorted order. The accounts themselves can be returned in any order.

- Similar to the most voted solution, my solution uses the index of the owner as the key for the union map, to avoid possible issue caused by different owners have the same name, thus uses one less loop.

```

private int findParent(int [] arr, int x) {
    if (arr[x] == x) return x;
    arr[x] = findParent(arr, arr[x]);
    return arr[x];
}
public List<List<String>> accountsMerge(List<List<String>> accounts) {
    Map<String, Integer> owner = new HashMap<>();
    Map<Integer, TreeSet<String>> union = new HashMap<>(); // match idx & Set<String emails>
    int n = accounts.size(), p = 0;
    int [] par = new int [n];
    for (int i = 0; i < n; i++) par[i] = i;
    List<String> ls = new ArrayList<>();
    for (int i = 0; i < n; i++) { // find the ownerIdx for each email address
        ls = accounts.get(i);
        for (int j = 1; j < ls.size(); j++) {
            String email = ls.get(j);
            if (owner.containsKey(email)) {
                p = findParent(par, owner.get(email));
                par[p] = i; // union accounts that belong to the same user here by updating parent relation
            }
            owner.put(email, i);
        }
    }
    // union all emails belong to the same owner
    for (String emal : owner.keySet()) {
        int ownerIdx = findParent(par, owner.get(emal));
        TreeSet<String> set = union.getOrDefault(ownerIdx, new TreeSet<>());
        set.add(emal);
        union.put(ownerIdx, set);
    }
    // Generate return result
    List<List<String>> res = new ArrayList<>();
    for (int ownerIdx : union.keySet()) {
        ls = new ArrayList<>();
        ls.add(accounts.get(ownerIdx).get(0)); // get the owner name
        ls.addAll(unon.get(ownerIdx));
        res.add(ls);
    }
    return res;
}

```

17.5 1202. Smallest String With Swaps

You are given a string s , and an array of pairs of indices in the string pairs where $\text{pairs}[i] = [a, b]$ indicates 2 indices(0-indexed) of the string.

You can swap the characters at any pair of indices in the given pairs any number of times.

Return the lexicographically smallest string that s can be changed to after using the swaps.

```

int [] par;
int [] rank;
int n;
public int find(int v) {
    if (v != par[v])
        par[v] = find(par[v]);
    return par[v];
}
public boolean union(int i, int j) {
    int ri = find(i);
    int rj = find(j);
    if (ri == rj) return false;
    if (rank[ri] < rank[rj]) par[ri] = rj;
    else if (rank[ri] > rank[rj]) par[rj] = ri;
    else {
        par[rj] = ri;
        rank[ri]++;
    }
    return true;
}
public String smallestStringWithSwaps(String s, List<List<Integer>> pairs) {
    int n = s.length();
    par = new int [n];
    rank = new int [n];
    List<Queue<Character>> list = new ArrayList<>(n);
    for (int i = 0; i < n; i++) {
        par[i] = i;
        list.add(new PriorityQueue<>());
    }
    Arrays.fill(rank, 1);
    pairs.forEach(p -> union(p.get(0), p.get(1))); // Perform union for each pair.
    // Add each character to the priority queue associated with its component.
    IntStream.range(0, n).forEach(index -> list.get(find(index)).add(s.charAt(index)));
    // Build the result, by removing chars from the corresponding priority queue.
    StringBuilder buffer = new StringBuilder(n);
    IntStream.range(0, n).forEachOrdered(index -> buffer.append(list.get(find(index)).remove()));
    return buffer.toString();
}
// O(NlogN). Worst-case, all indices are part of the same component. So we will essentially be popping off from the same priority queue.
// Space Complexity: O(N).

```

17.6 1998. GCD Sort of an Array - Hard

You are given an integer array nums , and you can perform the following operation any number of times on nums :

Swap the positions of two elements $\text{nums}[i]$ and $\text{nums}[j]$ if $\text{gcd}(\text{nums}[i], \text{nums}[j]) > 1$ where $\text{gcd}(\text{nums}[i], \text{nums}[j])$ is the greatest common divisor of $\text{nums}[i]$ and $\text{nums}[j]$. Return true if it is possible to sort nums in non-decreasing order using the above swap method, or false otherwise.

```

private int find (int v) {
    if (!parent.containsKey(v)) {
        parent.put(v, v);
        return v;
    }
    if (parent.get(v) != v)
        parent.put(v, find(parent.get(v)));
    return parent.get(v);
}
private void union(int x, int y) {
    int rx = find(x);
    int ry = find(y);
    if (rx != ry) parent.put(rx, ry);
}
/**
 * general idea (not accepted)
 * we can simply union pairs of numbers which has gcd > 1 in quadratic time and then check of groups that
 * are formed by union of pairs can be individually sorted.
 * improved (accepted)
 * In above approach problem is we are union-ing pairs in quadratic time. To improve upon it. We union a number
 * which is present in 'nums' with its smallest prime factor. thus if two numbers has same smallest prime factor

```

```

    their gcd is guaranteed to be > 1.
*/
Map<Integer, Integer> parent = new HashMap<>();
public boolean gcdSort(int[] arr) {
    int n = arr.length;
    parent = new HashMap<Integer, Integer>();
    int [] sorted = arr.clone();
    Arrays.sort(sorted);
    int max = Arrays.stream(arr).max().getAsInt();
    Set<Integer> numSet = new HashSet<>();
    numSet.addAll(Arrays.stream(arr).boxed().collect(Collectors.toList()));
    int p = 2; // Sieve algorithm
    boolean [] primes = new boolean [max + 1];
    Arrays.fill(primes, true);
    while (p < max) {
        if (primes[p]) {
            for (int i = p; i <= max; i += p) { // 我合并的是数组的索引，他优化成合并所有拥有公约数为 p 的数组中未合并的值
                if (numSet.contains(i)) union(p, i);
                primes[i] = false;
            }
        }
        p++;
    }
    for (int i = 0; i < n; i++)
        if (arr[i] != sorted[i] && find(sorted[i]) != find(arr[i])) return false;
    return true;
}

```

17.7 1724 给定一个 n 个顶点的无向带权图，要求在线回答若干询问，每次询问是个三元组 (p, q, x) ，是问是否存在 p 到 q 的每条边都小于 x 的路径。

先以 Kruskal 算法求最小生成森林，显然对于任何 p 和 q ，它们之间所有路径中最大边最小的那条路径的最大边一定是最小生成树的某条边（如果存在路径的话）。建树完成之后，对每个连通块，以任意顶点为根，将该连通块做成一棵有根树。对于每次询问，我们只需求 p 和 q 所有到它们的最近公共祖先所经过的边的最大边权就行了。可以考虑用倍增思想（以下的内容可以参考https://blog.csdn.net/qq_46105170/article/details/116217633）和 g ，其中 $f[i][k]$ 指的是从 i 节点向上跳 2^k 步能过走到的顶点是谁（如果跳出界了则规定值为 $1-11$ ）， $g[i][k]$ 指的是从 i 节点向上跳 2^k 的过程中经过的边的最大权值（在不会跳出界的情况下），然后从每个树根做 BFS，初始化 $f[.][0] = 1$ 和 $g[.][0] = \infty$ 。由于在询问的时候需要知道最近公共祖先，所以还需要一个数组 d 记录每个顶点的深度，在 BFS 的时候可以同时求出。接下来，用倍增的思想计算 f 和 g ：

```

f[i][k] = f[f[i][k-1]][k-1]
g[i][k] = max{g[i][k-1], g[f[i][k-1]][k-1]}

```

至此，所有的预处理就完成了。

接下来询问的时候，如果 p 与 q 不连通则直接返回 `false`。否则看一下两个顶点的深度，不妨设 p 更深，则将 p 向上跳若干步，使得 p 与 q 一样深，同时用经过的边权更新答案。此时如果 $p = q$ ，则答案已经求出，与 x 比较即可；否则，将 p 与 q 继续向上跳，一路跳到它们的最近公共祖先的孩子的那层位置，一路更新答案，最后再用最近公共祖先与它们的连边更新答案，最后将答案与 x 比较。代码如下：

```

public class DistanceLimitedPathsExist {
    class UnionFind {
        private int[] p;
        public UnionFind(int size) {
            p = new int[size];
            for (int i = 0; i < size; i++)
                p[i] = i;
        }
        public int find(int x) {
            if (p[x] != x)
                p[x] = find(p[x]);
            return p[x];
        }
        public void union(int x, int y) {
            int px = find(x), py = find(y);
            if (px != py)
                p[px] = py;
        }
    }
    private UnionFind uf;
    // 这里是链式前向星建图
}

```

17.7. 1724 给定一个 NNN 个顶点的无向带权图, 要求在线回答若干询问, 每次询问是个三元组 (P, Q, X) (P, Q, X) 是问是否存在

```
private int[] h, e, ne, w;
private int idx;
private void add(int a, int b, int c) {
    e[idx] = b;
    ne[idx] = h[a];
    w[idx] = c;
    h[a] = idx++;
}
// 这里是倍增法所需的数组和变量
private int[][] f, g;
private int[] depth;
// n 是顶点数, log 是 n 对 2 的对数, log + 1 也是 f 的第二维应该开的长度
private int n, log;
public DistanceLimitedPathsExist(int n, int[][] edgeList) {
    this.n = n;
    depth = new int[n];
    h = new int[n];
    Arrays.fill(h, -1);
    // 无向图, 边要开两倍
    e = new int[n << 1];
    ne = new int[n << 1];
    w = new int[n << 1];
    // 这一段是 Kruskal 算法建最小生成森林
    Arrays.sort(edgeList, (e1, e2) -> Integer.compare(e1[2], e2[2]));
    uf = new UnionFind(n);
    for (int[] e : edgeList) {
        int a = e[0], b = e[1], len = e[2];
        if (uf.find(a) != uf.find(b)) {
            uf.union(a, b);
            add(a, b, len);
            add(b, a, len);
        }
    }
    log = (int) (Math.log(n) / Math.log(2));
    f = new int[n][log + 1];
    g = new int[n][log + 1];
    for (int[] row : f) {
        Arrays.fill(row, -1);
    }
    boolean[] vis = new boolean[n];
    for (int i = 0; i < n; i++) {
        if (!vis[i]) {
            bfs(i, vis);
        }
    }
    init();
}
// 递推一遍 f 和 g 数组
private void init() {
    for (int i = 1; i < log + 1; i++) {
        for (int j = 0; j < n; j++) {
            if (f[j][i - 1] != -1) {
                f[j][i] = f[f[j][i - 1]][i - 1];
                g[j][i] = Math.max(g[j][i - 1], g[f[j][i - 1]][i - 1]);
            }
        }
    }
}
// BFS 一遍 x 所在连通块, 并初始化 f 和 g 数组, 并求出 depth 数组
private void bfs(int x, boolean[] vis) {
    Queue<Integer> q = new ArrayDeque<>();
    q.offer(x);
    vis[x] = true;
    while (!q.isEmpty()) {
        int u = q.poll();
        for (int i = h[u]; i != -1; i = ne[i]) {
            int v = e[i];
            if (vis[v]) continue;
            vis[v] = true;
            f[v][0] = u;
            g[v][0] = w[i];
            q.offer(v);
            depth[v] = depth[u] + 1;
        }
    }
}
public boolean query(int p, int q, int limit) {
    if (uf.find(p) != uf.find(q))
        return false;
    if (depth[p] < depth[q]) {
        int tmp = p;
        p = q;
        q = tmp;
    }
    // 先走到同一深度
}
```

```

int diff = depth[p] - depth[q];
int pow = 0, max = 0;
while (diff > 0) {
    if ((diff & 1) == 1) {
        max = Math.max(max, g[p][pow]);
        p = f[p][pow];
    }
    pow++;
    diff >>= 1;
}
// 已经走到同一点了，那深度更深的那个点就是最近公共祖先，max 就是经过的边的最大值
if (p == q) return max < limit;
// 否则跳到最近公共祖先下面一层，沿途更新答案
for (int i = log; i >= 0; i--) {
    if (f[p][i] != f[q][i]) {
        max = Math.max(max, g[p][i]);
        max = Math.max(max, g[q][i]);
        p = f[p][i];
        q = f[q][i];
    }
}
// 最后别忘了用最后一步更新答案
max = Math.max(max, g[p][0]);
max = Math.max(max, g[q][0]);
return max < limit;
}

// 初始化时间复杂度 O(m log m + n log n) O(m log m+n log n) O(m log m+n log n) O(m log m+n log n)

```

17.8 1172. 祖孙询问

给定一棵包含 n 个节点的有根无向树，节点编号互不相同，但不一定是 1 n 。有 m 个查询，每个查询给出了一对节点的编号 x 和 y ，询问 x 与 y 的祖孙关系。

输入格式：输入第一行包括一个整数表示节点个数；接下来 n 行每行一对整数 a 和 b ，表示 a 和 b 之间有一条无向边。如果 b 是 1，那么 a 就是树的根；第 $n+2$ 行是一个整数 m 表示询问个数；接下来 m 行，每行两个不同的正整数 x 和 y ，表示一个询问。

输出格式：对于每一个询问，若 x 是 y 的祖先则输出 1，否则输出 0。

数据范围：1 $n, m \leq 10^4$, $m \leq 10^{11}$, $m \leq 10^4$

1 $v \leq 10^4$, $v \leq 10^4$, $v \neq 1$, v 是顶点编号

可以用倍增的思想来求。预处理两个数组，一个是 $d[i]$ ，指的是顶点 i 的深度，树根深度是 1，其余顶点的深度就是其与树根的路径边数；另一个是 $f[i][k]$ ，是从顶点 i 向树根方向（以下均称“向上”）跳 k 步走到的顶点。由于顶点编号都是大于 0 的，我们可以人为规定一个 0 号节点作为哨兵，并且 $f[r][k] = 0 \forall k, f[r][k] = 0$ ， $d[0] = 0$ ，其中 r 是树根编号。这样如果 k 太大导致跳出树根的话，就会得到跳到了哨兵的结论。那么对于 f ，有 $f[i][0] = p$ ， $f[i][1] = f[f[i][0]]$ ， $f[i][2] = f[f[i][1]]$ ， \dots ， $f[i][k] = f[f[i][k-1]]$ 。

即分两次跳，一次跳 2^k 步等价于两次跳 2^{k-1} 步。初始化 f 和 d 数组的过程，可以用一次从树根的 BFS 来做到。

在询问的时候，比如询问 a 和 b 的公共祖先，不妨设 a 的深度更深，那么先让 a 向上跳到与 b 深度相同，可以先计算一下 $d[a] - d[b]$ ，然后将这个数字做二进制分解，就可以由 f 数组算出从 a 向上走到与 b 深度相同的时候是哪个顶点；接着再从 a 和 b 一起向上走，直到走到它们真正的最近公共祖先的下一层为止（即跳的步数是当前深度差减 1），此时 $f[a][0] f[a][1] \dots f[a][k]$ 即为最近公共祖先。代码如下：

```

const int N = 40010, M = N * 2;
int n, m;
int h[N], e[M], ne[M], idx;
int depth[N], fa[N][16];
int q[N];
void add(int a, int b) {
    e[idx] = b, ne[idx] = h[a], h[a] = idx++;
}
// 从树根开始 bfs，预处理出 d 数组和 f 数组
void bfs(int root) {
    memset(depth, 0x3f, sizeof depth);
    depth[0] = 0, depth[root] = 1;
    int hh = 0, tt = 0;
    q[tt++] = root;
    while (hh < tt) {
        int t = q[hh++];
        for (int i = h[t]; ~i; i = ne[i]) {
            int j = e[i];
            if (depth[j] > depth[t] + 1) {

```

```

    depth[j] = depth[t] + 1;
    q[tt++] = j;
    // 预处理 fa 数组
    fa[j][0] = t;
    for (int k = 1; k <= 15; k++)
        fa[j][k] = fa[fa[j][k - 1]][k - 1];
}
}
}

int lca(int a, int b) {
    // 强制让 a 的深度大于等于 b 的深度
    if (depth[a] < depth[b]) swap(a, b);
    // 从大到小枚举 k, 将 a 向上跳到与 b 同层
    for (int k = 15; k >= 0; k--)
        if (depth[fa[a][k]] >= depth[b])
            a = fa[a][k];
    // 如果 a 和 b 重合了, 那么说明 b 就是最近公共祖先
    if (a == b) return b;
    // 否则将 a 和 b 同时向上跳, 直到跳到最近公共祖先下一层为止
    for (int k = 15; k >= 0; k--)
        if (fa[a][k] != fa[b][k])
            a = fa[a][k], b = fa[b][k];
    return fa[a][0];
}

int main() {
    scanf("%d", &n);
    memset(h, -1, sizeof h);
    int root = 0;
    for (int i = 0; i < n; i++) {
        int a, b;
        scanf("%d%d", &a, &b);
        if (b == -1) root = a;
        else add(a, b), add(b, a);
    }
    bfs(root);
    scanf("%d", &m);
    while (m--) {
        int a, b;
        scanf("%d%d", &a, &b);
        int p = lca(a, b);
        if (p == a) puts("1");
        else if (p == b) puts("2");
        else puts("0");
    }
    return 0;
}

```

17.9 某个网友列的相关题目：并查集参考：数据结构-并查集（Disjoint-Set）

- 261. 以图判树（全部连通 + 边数 =V-1）
- 305. 岛屿数量 II（并查集）
- 323. 无向图中连通分量的数目（并查集）
- 684. 兀余连接（并查集）
- 685. 兀余连接 II（并查集）
- 721. 账户合并（并查集）（字符串合并）
- 737. 句子相似性 II（并查集）
- 886. 可能的二分法（着色 DFS/BFS/拓展并查集）
- 947. 移除最多的同行或同列石头（并查集）
- 990. 等式方程的可满足性（并查集）
- 959. 由斜杠划分区域（并查集）
- 1061. 按字典序排列最小的等效字符串（并查集）
- 1101. 彼此熟识的最早时间（排序 + 并查集）

- 1202. 交换字符串中的元素（并查集）
- 1319. 连通网络的操作次数（BFS/DFS/并查集）
- 5510. 保证图可完全遍历（并查集）
- 程序员面试金典 - 面试题 17.07. 婴儿名字（并查集）

Chapter 18

Graph

18.1 2045. Second Minimum Time to Reach Destination - Hard

A city is represented as a bi-directional connected graph with n vertices where each vertex is labeled from 1 to n (inclusive). The edges in the graph are represented as a 2D integer array edges , where each $\text{edges}[i] = [u_i, v_i]$ denotes a bi-directional edge between vertex u_i and vertex v_i . Every vertex pair is connected by at most one edge, and no vertex has an edge to itself. The time taken to traverse any edge is time minutes.

Each vertex has a traffic signal which changes its color from green to red and vice versa every change minutes. All signals change at the same time. You can enter a vertex at any time, but can leave a vertex only when the signal is green. You cannot wait at a vertex if the signal is green.

The second minimum value is defined as the smallest value strictly larger than the minimum value.

For example the second minimum value of $[2, 3, 4]$ is 3, and the second minimum value of $[2, 2, 4]$ is 4. Given n , edges , time , and change , return the second minimum time it will take to go from vertex 1 to vertex n .

Notes:

You can go through any vertex any number of times, including 1 and n . You can assume that when the journey starts, all signals have just turned green.

Observation:

We are tasked to find 2nd minimum time to reach city N .

We can use Dijkstra's algorithm to find minimum time taken to the city N . However to find 2nd min time we need a little modification on vanilla Dijkstra. i.e. visit cities again beyond their optimal times once found

The Question now becomes ?

How many times do we need to visit any city at maximum ? - answer is 2 times :) . We don't need to care about 3rd min times and beyond for any of the cities. thats it!

Algorithm:

1. Use Dijkstra to find min elapsed time by visiting cities in order and store this time
2. When visiting the city again, check if this elapsed time is 2nd min time to previously recorded value
 - 2.1 If yes, continue evaluation
 - 2.2 If no, ignore this state and continue to next state in the heap
3. Also check if we already have 2 min times for the city already, if yes, make sure never evaluate for this city again.

This will optimize your solution

```
public int secondMinimum(int n, int[][] edges, int time, int change) {  
    Map<Integer, List<Integer>> adj = new HashMap<>();  
    for (int [] e : edges) {  
        adj.computeIfAbsent(e[0], z -> new ArrayList<>()).add(e[1]);  
        adj.computeIfAbsent(e[1], z -> new ArrayList<>()).add(e[0]);  
    }  
    Queue<int []> q = new PriorityQueue<>((a, b) ->(a[1] - b[1]));  
    q.offer(new int []{1, 0});  
    Map<Integer, Integer> cache = new HashMap<>(); // use cache to record min time per city  
    // modification: we want to visit each city maximum two times with different times,  
    // this will help in early termination when we visit the city again (3rd time or more)  
    Set<Integer> exhausted = new HashSet<>();  
    while (!q.isEmpty()) {  
        int [] top = q.poll();  
        int cur = top[0], t = top[1];  
        // Base Termination : we have found our 2nd min time for city n  
        if (cur == n && t > cache.getOrDefault(cur, Integer.MAX_VALUE))  
            return t;  
        if (!cache.containsKey(cur)) // we visited this city for first time, so elapsed time is min for this city  
            cache.put(cur, t);  
    }  
}
```

```

// early termination, if we are trying to visit the city 3rd time or more ,
// or the elapsed time will not help in finding the solution
else if (cache.get(cur) == t || exhausted.contains(cur)) continue;
else // this means we are visiting the city with 2nd optimal time , we dont need to visit the city ever again
    exhausted.add(cur);
// we visit the city on elapsedTime, we need to check if on basis of change time, whether this time falls in cycle (green or red)
// if odd cycle (red), we must wait for this cycle to end
int factor = t / change;
if (factor % 2 == 1)
    t = (factor + 1) * change;
for (int nb : adj.getOrDefault(cur, new ArrayList<>())) { // visit the neighbours
    int visTime = t + time;
    if (!exhausted.contains(nb))
        q.offer(new int[] {nb, visTime});
}
}
return -1;
}

```

- 另一个也是写得直接了当的

```

public int secondMinimum(int n, int[][] edges, int time, int change) {
    Map<Integer, Set<Integer>> map = new HashMap<>();
    for (int[] e : edges) {
        map.computeIfAbsent(e[0], z -> new HashSet<>()).add(e[1]);
        map.computeIfAbsent(e[1], z -> new HashSet<>()).add(e[0]);
    }
    Queue<int[]> q = new PriorityQueue<>((a, b)->(a[1]-b[1]));
    Map<Integer, Set<Integer>> vis = new HashMap<>();
    q.offer(new int[] {1, 0});
    int min = -1;
    while (!q.isEmpty()) {
        int[] top = q.poll();
        int cur = top[0], t = top[1];
        if (cur == n) {
            if (min == -1 || min == t) min = t;
            else return t;
        }
        if (t % (2 * change) >= change)
            t += 2 * change - t % (2 * change);
        // 源码中传入 key 和 value, 根据 key 获取看是否存在 value, 如果 value==null, 然后调用 put 方法把传入的 key 和 value put 进 map, 返回根据 key 获取的老
        // 如果传入 key 对应的 value 已经存在, 就返回存在的 value, 不进行替换。如果不存在, 就添加 key 和 value, 返回 null
        vis.putIfAbsent(cur, new HashSet<>());
        if (!vis.get(cur).add(t) || vis.get(cur).size() >= 3) continue;
        if (map.containsKey(cur))
            for (int next : map.get(cur))
                q.offer(new int[] {next, t + time});
    }
    return -1;
}

```

昨天晚上的比赛，状态相对早上要差了很多，感觉到晚上的头脑已经不是很清醒。比赛的时候没有感觉，白白折腾了一个小时。结束后去看别人题解的时候，弱弱的内心是崩溃的：应该说这些考点平时自己也都练习过，没有过偏的地方，可为什么自己做题的时候，一个个考点穿上马甲我就再也认不出它们来了呢？把可以重复遍历当作是多余的，不想去重复遍历；又被 n 迷惑住，全然忘记其它 node 也会被重复遍历到；明明知道 pq 里还有一堆杂物，比赛时却浑然想不到我是可以去裁枝优化的!!!! 昨天晚上的比赛应该是最近几周参加比赛以来做得相对较好的一次，可相比于早上最后一道题完全没有时间做、几乎没有下笔思路，这个最后一题留有整整一个小时，考点也都练习到过，却探肩而过、失之交臂、被自己活活折腾死。。。感触很多。。。。。或许这也是一个磨砺心志、相定信念的过程吧!!!! 练习练习练习、总结总结总结，让它们深入骨髓，一看便知似曾相识？

18.2 1334. Floyd 算法 - Find the City With the Smallest Number of Neighbors at a Threshold Distance - Medium

There are n cities numbered from 0 to $n-1$. Given the array edges where $\text{edges}[i] = [\text{from}_i, \text{to}_i, \text{weight}_i]$ represents a bidirectional and weighted edge between cities from_i and to_i , and given the integer distanceThreshold .

Return the city with the smallest number of cities that are reachable through some path and whose distance is at most distanceThreshold . If there are multiple such cities, return the city with the greatest number.

Notice that the distance of a path connecting cities i and j is equal to the sum of the edges' weights along that path.

```

public int findTheCity(int n, int[][] edges, int distanceThreshold) {
    // 1. 创建邻接矩阵
    int[][] graph = new int[n][n]; // 相比于我只会用 HashMap 来建邻接关系，邻接链表与数组都可能，看哪个用起来方便
    for (int i = 0; i < n; i++)

```

```

Arrays.fill(graph[i], Integer.MAX_VALUE); // pre filled n equivalent to Integer.MAX_VALUE
for (int [] eg : edges) {
    graph[eg[0]][eg[1]] = eg[2];
    graph[eg[1]][eg[0]] = eg[2];
}
// 2. floyd 算法
for (int k = 0; k < n; k++)           // 中间结点
    for (int i = 0; i < n; i++)        // 开始结点
        for (int j = 0; j < n; j++) { // 结尾结点
            if (i == j || graph[i][k] == Integer.MAX_VALUE || graph[k][j] == Integer.MAX_VALUE) continue;
            graph[i][j] = Math.min(graph[i][j], graph[i][k] + graph[k][j]);
        }
// 3. 每个城市距离不大于 distanceThreshold 的邻居城市的数目
int [] mark = new int [n]; //记录小于 distanceThreshold 的邻居城市个数
for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        if (graph[i][j] <= distanceThreshold)
            mark[i]++;
// 4. 找数目少，编号最大的
int min = n;
int ans = 0;
for (int i = 0; i < n; i++)
    if (min >= mark[i]) {
        min = mark[i];
        ans = i;
    }
return ans;
}

```

- 另一种解法

```

// 之前用原创想法也写了很多图的题，但缺乏归纳总结，原创想法更多的是解决了题目，但解法与效率、与优化算法间的距离还需要很多比较归纳与总结，才能把图这一块吃透
// https://leetcode.jp/leetcode-1334-find-the-city-with-the-smallest-number-of-neighbors-at-a-threshold-distance-%E8%A7%A3%E9%A2%98%E6%80%9D%E8%B7%AF%E5%
// map: 图结构
// city: 当前城市
// dis: 当前所剩距离
// v: 已经被记录为邻居的节点
// maxDis: 走到某个节点时，剩余距离的最大值
// 返回值为当前城市的邻居数。
private int dfs(int [][] arr, int city, int dis, boolean [] vis, int [] maxDis) {
    int res = 0;
    for (int i = 0; i < arr[0].length; i++) { // 循环当前城市的所有相邻城市
        int distance = arr[city][i]; // 与相邻城市的距离，如果为 0，说明与该城市不相连
        int diffDis = dis - distance; // 到达相邻城市后，与阈值相比的剩余距离。
        if (distance > 0 && diffDis >= maxDis[i]) { // 与该城市相连并且剩余距离大于等于访问数组中的值
            maxDis[i] = diffDis; // 更新访问数组中的剩余距离
            if (!vis[i]) {
                vis[i] = true;
                res++;
            }
            res += dfs(arr, i, diffDis, vis, maxDis); // 递归 dfs 与该城市相连的其他城市：图中我似乎还很没有 dfs 以及递归的概念
        }
    }
    return res;
}
public int findTheCity(int n, int[][] edges, int distanceThreshold) {
    int [][] map = new int [n][n];
    for (int [] eg : edges) {
        map[eg[0]][eg[1]] = eg[2];
        map[eg[1]][eg[0]] = eg[2];
    }
    int min = n;
    int res = 0;
    for (int i = 0; i < n; i++) {
        boolean [] vis = new boolean [n];
        vis[i] = true;
        int cnt = dfs(map, i, distanceThreshold, vis, new int [n]);
        if (cnt <= min) {
            min = cnt;
            res = i;
        }
    }
    return res;
}

```

18.3 1129. Shortest Path with Alternating Colors - Medium

Consider a directed graph, with nodes labelled 0, 1, ..., n-1. In this graph, each edge is either red or blue, and there could be self-edges or parallel edges.

Each $[i, j]$ in red_{edges} denotes a red directed edge from node i to node j . Similarly, each $[i, j]$ in blue_{edges} denotes a blue directed edge from node i to node j .

Return an array answer of length n , where each $\text{answer}[X]$ is the length of the shortest path from node 0 to node X such that the edge colors alternate along the path (or -1 if such a path doesn't exist).

```
// 找最短路径应该用 queue 来做，入队列的时候需要标记红边或是蓝边以便找交替路径
public int[] shortestAlternatingPaths(int n, int[][] red_edges, int[][] blue_edges) {
    HashMap<Integer, List<Integer>> [] maps = new HashMap [2]; // 0 : red; 1: blue
    for (int i = 0; i < 2; i++)
        maps[i] = new HashMap<>();
    for (int i = 0; i < red_edges.length; i++)
        maps[0].computeIfAbsent(red_edges[i][0], k->new ArrayList<>()).add(red_edges[i][1]);
    for (int i = 0; i < blue_edges.length; i++)
        maps[1].computeIfAbsent(blue_edges[i][0], k->new ArrayList<>()).add(blue_edges[i][1]);
    int [] ans = new int[n];
    Arrays.fill(ans, -1);
    Queue<int []> q = new LinkedList<>();
    q.offer(new int [] {0, 0}); // red edge
    q.offer(new int [] {0, 1}); // blue edge
    boolean [][] inQueue = new boolean [n][2]; // 0: red, 1: blue
    inQueue[0][0] = true;
    inQueue[0][1] = true;
    int cnt = 0, color = 0;
    while (!q.isEmpty()) {
        for (int size = q.size(); size > 0; size--) {
            int [] cur = q.poll();
            System.out.println(Arrays.toString(cur));
            color = cur[1];
            if (ans[cur[0]] == -1) ans[cur[0]] = cnt;
            List<Integer> nextNodes = maps[1-color].get(cur[0]);
            if (nextNodes == null) continue;
            for (Integer next : nextNodes)
                if (!inQueue[next][1-color]) {
                    q.offer(new int [] {next, 1-color});
                    inQueue[next][1-color] = true;
                }
        }
        ++cnt;
    }
    return ans;
}
```

- 不是总喜欢省掉大括号吗，试试省掉下面的。。。。。

```
public int[] shortestAlternatingPaths(int n, int[][] red_edges, int[][] blue_edges) {
    int [][] red = new int[n][2]; // 红 0 蓝 1
    int [][] blue = new int[n][2];
    for (int i = 1; i < n; i++) {
        red[i][0] = i;
        red[i][1] = 0xffffffff; // 初始化红边权值
    }
    red [0][0] = 0;
    red [0][1] = 0;
    for (int i = 1; i < n; i++) {
        blue[i][0] = i;
        blue[i][1] = 0xffffffff;
    }
    blue [0][0] = 0;
    blue [0][1] = 0;
    dfs(red, blue, 0, 0, red_edges, blue_edges);
    dfs(red, blue, 1, 0, red_edges, blue_edges);
    int [] ans = new int[n];
    for(int i = 0; i < n; i++){
        ans[i] = Math.min(red[i][1], blue[i][1]);
        if (ans[i] == 0xffffffff) // 没有改变说明不存在
            ans[i] = -1;
    }
    return ans;
}
public void dfs(int [][] red, int [][] blue, int color, int node, int[][] red_edges, int[][] blue_edges){
    if (color == 0) { // 这个括号可以省吗???
        for (int [] blue_to : blue.edges) // 以 node 为 from to 为终的边
            if (node == blue_to[0] && red[node][1]+1 < blue[blue_to[1]][1]) { // 0 到 from 点加 1 是否小于 0 到 to 的距离
                blue[blue_to[1]][1] = red[node][1]+1; // 作距离的更新
                dfs(red, blue, 1-color, blue_to[1], red_edges, blue_edges);
            }
    } else for (int [] red_to : red.edges) // 以 node 为 from to 为终的边
        if (node == red_to[0] && blue[node][1]+1 < red[red_to[1]][1]) { // 0 到 from 点加 1 是否小于 0 到 to 的距离
            red[red_to[1]][1] = blue[node][1]+1;
            dfs(red, blue, 1-color, red_to[1], red_edges, blue_edges);
    }
}
```

```

    }
}
```

18.4 882. Reachable Nodes In Subdivided Graph - Hard

You are given an undirected graph (the "original graph") with n nodes labeled from 0 to $n - 1$. You decide to subdivide each edge in the graph into a chain of nodes, with the number of new nodes varying between each edge.

The graph is given as a 2D array of edges where $\text{edges}[i] = [u_i, v_i, c_{nti}]$ indicates that there is an edge between nodes u_i and v_i in the original graph, and c_{nti} is the total number of new nodes that you will subdivide the edge into. Note that $c_{nti} == 0$ means you will not subdivide the edge.

To subdivide the edge $[u_i, v_i]$, replace it with $(c_{nti} + 1)$ new edges and c_{nti} new nodes. The new nodes are $x_1, x_2, \dots, x_{c_{nti}}$, and the new edges are $[u_i, x_1], [x_1, x_2], [x_2, x_3], \dots, [x_{c_{nti}-1}, x_{c_{nti}}], [x_{c_{nti}}, v_i]$.

In this new graph, you want to know how many nodes are reachable from the node 0, where a node is reachable if the distance is maxMoves or less.

Given the original graph and maxMoves , return the number of nodes that are reachable from node 0 in the new graph.

再进一步来分析，其实上对于每个结点来说（不论有没有编号），若我们能算出该结点离起始结点的最短距离，且该距离小于等于 M 的话，那这个结点就一定可以到达。这样来说，其实本质就是求单源点的最短距离，此时就要祭出神器迪杰斯特拉算法 Dijkstra Algorithm 了，LeetCode 中使用了该算法的题目还有 Network Delay Time 和 The Maze II。该算法的一般形式是用一个最小堆来保存到源点的最小距离，这里我们直接统计到源点的最小距离不是很方便，可以使用一个小 trick，即用一个最大堆来统计当前结点所剩的最大步数，因为剩的步数越多，说明距离源点距离越小。由于 Dijkstra 算法是以起点为中心，向外层层扩展，直到扩展到终点为止。根据这特性，用 BFS 来实现时再好不过了，首先来建立邻接链表，这里可以使用一个 $N \times N$ 的二维数组 graph ，其中 $\text{graph}[i][j]$ 表示从大结点 i 往大结点 j 方向会经过的小结点个数，建立邻接链表的时候对于每个 edge，要把两个方向都赋值，前面解释过了这里要当作有向图来做。然后使用一个最大堆，里面放剩余步数和结点编号组成的数对儿，把剩余步数放前面就可以默认按步数从大到小排序了，初始化时把 $\{M, 0\}$ 存入最大堆。还需要一个一维数组 visited 来记录某个结点是否访问过。

```

public int reachableNodes(int[][][] edges, int maxMoves, int n) {
    int [][] graph = new int [n][n];
    for (int i = 0; i < n; i++)
        Arrays.fill(graph[i], -1);
    for (int [] v : edges) {
        graph[v[0]][v[1]] = v[2];
        graph[v[1]][v[0]] = v[2];
    }
    Queue<int []> q = new PriorityQueue<>((a, b) -> (b[0] - a[0]));
    boolean [] vis = new boolean [n];
    q.offer(new int [] {maxMoves, 0});
    int res = 0;
    while (!q.isEmpty()) {
        int [] cur = q.poll();
        int cnt = cur[0], u = cur[1];
        if (vis[u]) continue;
        vis[u] = true;
        ++res;
        for (int i = 0; i < n; i++) {
            if (graph[u][i] == -1) continue;
            if (cnt > graph[u][i] && !vis[i])
                q.offer(new int [] {cnt - graph[u][i]-1, i});
            graph[i][u] -= Math.min(cnt, graph[u][i]);
            res += Math.min(cnt, graph[u][i]);
        }
    }
    return res;
}
```

- 我们也可以使用 `HashMap` 来建立邻接链表，最后的运行速度果然要比二维数组形式的邻接链表要快一些，其他的地方都不变，参见代码如下：

```

public int reachableNodes(int[][][] edges, int maxMoves, int n) {
    int res = 0;
    Map<Integer, Map<Integer, Integer>> graph = new HashMap<>();
    for (int [] v : edges) {
        graph.computeIfAbsent(v[0], k->new HashMap<>()).put(v[1], v[2]);
        graph.computeIfAbsent(v[1], k->new HashMap<>()).put(v[0], v[2]);
    }
    Queue<int []> q = new PriorityQueue<>((a, b) -> (b[0] - a[0]));
    boolean [] vis = new boolean [n];

```

```

q.offer(new int [] {maxMoves, 0});
while (!q.isEmpty()) {
    int [] cur = q.poll();
    int cnt = cur[0], u = cur[1];
    if (vis[u]) continue;
    vis[u] = true;
    ++res;
    for (int i = 0; i < n; i++) {
        if (!graph.containsKey(u) || !graph.get(u).containsKey(i) || graph.get(u).get(i) == -1) continue;
        if (cnt > graph.get(u).get(i) && !vis[i])
            q.offer(new int [] {cnt - graph.get(u).get(i)-1, i});
        graph.get(i).put(u, graph.get(u).get(i) - Math.min(cnt, graph.get(u).get(i)));
        res += Math.min(cnt, graph.get(u).get(i));
    }
}
return res;
}

```

18.5 1782. Count Pairs Of Nodes - Hard

You are given an undirected graph defined by an integer n , the number of nodes, and a 2D integer array edges , the edges in the graph, where $\text{edges}[i] = [u_i, v_i]$ indicates that there is an undirected edge between u_i and v_i . You are also given an integer array queries .

Let $\text{incident}(a, b)$ be defined as the number of edges that are connected to either node a or b .

The answer to the j th query is the number of pairs of nodes (a, b) that satisfy both of the following conditions:

$a < b$ $\text{incident}(a, b) > \text{queries}[j]$ Return an array answers such that $\text{answers.length} == \text{queries.length}$ and $\text{answers}[j]$ is the answer of the j th query.

Note that there can be multiple edges between the same two nodes.

```

// https://leetcode.com/problems/count-pairs-of-nodes/discuss/1096740/C%2B%2BJavaPython3-Two-Problems-O(q-*- (n-%2B-e))
public int[] countPairs(int n, int[][] edges, int[] queries) { // 别人家的思路好清晰
    int [] cnt = new int [n+1], sortedCnt = new int [n+1], ans = new int [queries.length];
    Map<Integer, Integer> [] m = new HashMap[n+1];
    for (var e : edges) {
        sortedCnt[e[0]] = cnt[e[0]] = cnt[e[0]] + 1;
        sortedCnt[e[1]] = cnt[e[1]] = cnt[e[1]] + 1;
        int min = Math.min(e[0], e[1]), max = Math.max(e[0], e[1]);
        m[min] = m[min] == null ? new HashMap<>() : m[min];
        m[min].put(max, m[min].getOrDefault(max, 0) + 1); // 仍然是当作有向图、单向图来做
    }
    Arrays.sort(sortedCnt);
    int res = 0, cur = 0;
    for (int k = 0; k < queries.length; k++) {
        for (int i = 1, j = n; i < j;)
            if (queries[k] < sortedCnt[i] + sortedCnt[j])
                ans[k] += (j--) - i;
            else ++i;
        for (int i = 1; i <= n; i++)
            if (m[i] != null)
                for (var en : m[i].entrySet()) {
                    int j = en.getKey(), sharedCnt = en.getValue();
                    if (queries[k] < cnt[i] + cnt[j] && cnt[i] + cnt[j] - sharedCnt <= queries[k])
                        ans[k]--;
                }
    }
    return ans;
}

```

18.6 332. Reconstruct Itinerary - Medium

You are given a list of airline tickets where $\text{tickets}[i] = [\text{from}_i, \text{to}_i]$ represent the departure and the arrival airports of one flight. Reconstruct the itinerary in order and return it.

All of the tickets belong to a man who departs from "JFK", thus, the itinerary must begin with "JFK". If there are multiple valid itineraries, you should return the itinerary that has the smallest lexical order when read as a single string.

For example, the itinerary ["JFK", "LGA"] has a smaller lexical order than ["JFK", "LGB"]. You may assume all tickets form at least one valid itinerary. You must use all the tickets once and only once.

```

private void dfs(String cur, List<String> list) {
    Queue<String> next = m.get(cur);
    while (next != null && next.size() > 0)

```

```

        dfs(next.poll(), list);
        list.add(cur);
    }
    Map<String, PriorityQueue<String>> m = new HashMap<>(); // TreeSet does not handle repeats, use queue
    public List<String> findItinerary(List<List<String>> tickets) {
        for (int i = 0; i < tickets.size(); i++)
            m.computeIfAbsent(tickets.get(i).get(0), k -> new PriorityQueue<>()).offer(tickets.get(i).get(1));
        List<String> res = new ArrayList<>();
        dfs("JFK", res);
        Collections.reverse(res);
        return res;
    }
}

```

18.7 1591. Strange Printer II - Hard

There is a strange printer with the following two special requirements:

On each turn, the printer will print a solid rectangular pattern of a single color on the grid. This will cover up the existing colors in the rectangle. Once the printer has used a color for the above operation, the same color cannot be used again. You are given a $m \times n$ matrix targetGrid, where targetGrid[row][col] is the color in the position (row, col) of the grid.

Return true if it is possible to print the matrix targetGrid, otherwise, return false.

这道题可以认为是在研究：是否有一种颜色序列，按照这个序列进行染色，最终矩阵就会呈现输入的状态。

矩形上的某一个像素点，可能会先后经历多次染色。比如先染红，再染绿，再染黄，最后染蓝，最后呈现出的就是蓝色。我们知道这个像素现在是蓝色；

而它在红色/绿色/黄色矩形范围内，说明这个像素曾经红过/绿过/黄过。

此时我们可以提炼出信息：假定先染的优先于后染的，那么红色优于蓝色，绿色优于蓝色，黄色优于蓝色。
(红绿黄之间的顺序未定)。

题中指出，颜色最多有 6060 种，我们可以建立一个有向图，图中的结点就是这 6060 个颜色 1~60160。

按照刚才的方法找出所有的有向边，进行拓扑排序即可判断出结果。

```

public boolean isPrintable(int[][] targetGrid) { // 觉得这个题思路奇特
    int m = targetGrid.length, n = targetGrid[0].length, max = Math.max(m, n);
    for (int i = 0; i < m; i++)
        max = Math.max(max, Arrays.stream(targetGrid[i]).max().getAsInt());
    int[] top = new int[max+1], bot = new int[max+1], left = new int[max+1], right = new int[max+1];
    Arrays.fill(top, max+1);
    Arrays.fill(left, max+1);
    Arrays.fill(bot, -1);
    Arrays.fill(right, -1);
    for (int i = 0; i < m; i++) // 建立每个颜色的矩阵边界：界定每种颜色的上下左右边界，每种颜色最早（上左）最晚（下右）出现的位置
        for (int j = 0; j < n; j++) {
            int color = targetGrid[i][j];
            top[color] = Math.min(top[color], i);
            bot[color] = Math.max(bot[color], i);
            left[color] = Math.min(left[color], j);
            right[color] = Math.max(right[color], j);
        }
    // 根据矩阵建立有向图，遍历 targetGrid,
    // 当前位置颜色 X 在某个矩阵 A 中但是不为矩阵 A 的颜色时，建立从 A 到 X 的边
    // X 可以存在于多个矩阵中
    // 变量：是否存在边-防止重复建立边；入度，便于后期判断是否拓扑排序；邻接表，从 i 出发到达的点
    boolean[][] eg = new boolean[max+1][max+1];
    int[] ins = new int[max+1];
    List<List<Integer>> adj = new ArrayList<>();
    for (int i = 0; i <= max; i++)
        adj.add(new ArrayList<>());
    int cur = 0; // 建立图
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++) {
            cur = targetGrid[i][j]; // 当前格子的最终打印颜色
            for (int color = 1; color <= max; color++) // 遍历所有颜色：暴搜当前格子颜色是否可能后打印
                if (i >= top[color] && i <= bot[color] && j >= left[color] && j <= right[color])
                    if (cur != color && !eg[color][cur]) {
                        adj.get(color).add(cur); // color 先染后，cur 后染色
                        ins[cur]++;
                        eg[color][cur] = true;
                    }
            }
        }
    // 寻找入度为 0 的颜色点，减小该点连结的点的入度，直到所有点的入度都为 0
    List<Integer> l = new ArrayList<>();
    while (true) {
        int i;
        for (i = 1; i <= max; i++)
            if (ins[i] == 0) {

```

```

        l.add(i);
        for (int v : adj.get(i)) ins[v]--;
        ins[i] = -1;
        break;
    }
    if (i == max+1) break;
}
return l.size() == max;
}

```

- topological sort

```

private boolean topologicalSort(Set<Integer> visited, Set<Integer> visiting, int c, Map<Integer, Set<Integer>> graph) {
    if (visited.contains(c)) return true;
    if (visiting.contains(c)) return false;
    visiting.add(c);
    for (Integer nei : graph.getOrDefault(c, Collections.emptySet()))
        if (!topologicalSort(visited, visiting, nei, graph)) return false;
    visiting.remove(c);
    visited.add(c);
    return true;
}
// 1. For each color, find left top corner and right bottom corner
// 2. All colors interfering with given color means they should go after given
// 3. Create dependencies (1 -> [2,3])
// 4. TopSort. Loops mean it is not possible
public boolean isPrintable(int[][] targetGrid) {
    Map<Integer, Set<Integer>> graph = new HashMap<>();
    Set<Integer> colors = new HashSet<>();
    int m = targetGrid.length, n = targetGrid[0].length;
    for (int i = 0; i < m; i++)
        for (int j = 0; j < n; j++)
            colors.add(targetGrid[i][j]);
    for (Integer c : colors) {
        int fi = -1, fj = Integer.MAX_VALUE, li = -1, lj = -1; // f: first, f row, f col, l: last, l row, l col
        for (int i = 0; i < m; i++)
            for (int j = 0; j < n; j++)
                if (targetGrid[i][j] == c) {
                    if (fi == -1) fi = i;
                    fj = Math.min(fj, j);
                    li = i;
                    lj = Math.max(lj, j);
                }
        for (int i = fi; i <= li; i++)
            for (int j = fj; j <= lj; j++)
                if (targetGrid[i][j] != c)
                    graph.computeIfAbsent(c, z -> new HashSet<>()).add(targetGrid[i][j]);
    }
    Set<Integer> visited = new HashSet<>();
    Set<Integer> visiting = new HashSet<>();
    for (Integer c : colors)
        if (!topologicalSort(visited, visiting, c, graph)) return false;
    return true;
}

```

Chapter 19

扫描线

19.1 2054. Two Best Non-Overlapping Events - Medium

You are given a 0-indexed 2D integer array of events where $\text{events}[i] = [\text{start Time}_i, \text{end Time}_i, \text{value}_i]$. The i th event starts at start Time_i and ends at end Time_i , and if you attend this event, you will receive a value of value_i . You can choose at most two non-overlapping events to attend such that the sum of their values is maximized.

Return this maximum sum.

Note that the start time and end time is inclusive: that is, you cannot attend two events where one of them starts and the other ends at the same time. More specifically, if you attend an event with end time t , the next event must start at or after $t + 1$.

19.1.1 解题思路与分析

- 因为最多只有两个事件，联想到股票问题的只有两次交易，很容易想到从头到尾扫一遍，从尾到头扫一遍
- 也是一道扫描线问题，借助 TreeMap, PriorityQueue 等数据结构，或 binary search 算法帮助保证正确性以及提速
- 再熟悉一下 NavigableMap 类中的 floorKey() 和 ceilingKey() 的 API:

类型	方法	描述
K	ceilingKey(K key)	返回大于或等于给定键的最小键，如果没有这样的键，则 null
K	floorKey(K key)	返回小于或等于给定键的最大键，如果没有这样的键，则 null

```
public int maxTwoEvents(int[][] events) {
    int n = events.length, maxSeenVal = 0;      // for any timestamp - find max of values to left and max of values to right
    Arrays.sort(events, (a,b) -> a[1] - b[1]); // Sort by end time - to know what events happened previously
    TreeMap<Integer, Integer> leftMaxSeen = new TreeMap<>();
    for (int [] event : events) {
        int eventVal = event[2];
        if (eventVal >= maxSeenVal) {
            maxSeenVal = eventVal;
            leftMaxSeen.put(event[1], maxSeenVal); // save maxSeenVal event value seen so far at END timestamp
        }
    }
    // Sort by start time but in descending order - as we parse events and store max value of future events to current event start timestamp
    Arrays.sort(events, (a,b) -> b[0] - a[0]);
    int ans = 0, maxSeenRight = 0;
    for (int [] event : events) {
        int eventVal = event[2];
        if (eventVal >= maxSeenRight) {
            maxSeenRight = eventVal;
            Integer maxOnLeftKey = leftMaxSeen.floorKey(event[0] - 1);
            if (maxOnLeftKey != null)
                ans = Math.max(ans, leftMaxSeen.get(maxOnLeftKey) + maxSeenRight);
        }
    }
    return Math.max(maxSeenVal, ans); // 有一种单个事件得最大值的情况不能漏掉
}
```

- 扫描线算法

```
public int maxTwoEvents(int[][] events) {
    int result = 0, maxOfCompletedEvents = 0;
```

```

Arrays.sort(events, (x,y) -> x[0] - y[0]); // Sort by Start time
PriorityQueue<int[]> inProgressQueue = new PriorityQueue<>((x,y)->x[1]-y[1]); // sorted by end time
for (int[] currentEvent : events) {
    while (!inProgressQueue.isEmpty() && inProgressQueue.peek()[1] < currentEvent[0])
        maxOfCompletedEvents = Math.max(maxOfCompletedEvents, inProgressQueue.poll()[2]);
    result = Math.max(result, maxOfCompletedEvents + currentEvent[2]);
    inProgressQueue.offer(currentEvent);
}
return result;
}

```

- 二分查找: 改天补上

19.2 2055. Plates Between Candles - Medium

There is a long table with a line of plates and candles arranged on top of it. You are given a 0-indexed string s consisting of characters '*' and '|' only, where a '*' represents a plate and a '|' represents a candle.

You are also given a 0-indexed 2D integer array queries where $\text{queries}[i] = [\text{left}_i, \text{right}_i]$ denotes the substring $s[\text{left}_i \dots \text{right}_i]$ (inclusive). For each query, you need to find the number of plates between candles that are in the substring. A plate is considered between candles if there is at least one candle to its left and at least one candle to its right in the substring.

For example, $s = "||**|***|**"$, and a query $[3, 8]$ denotes the substring $"*||**|"$. The number of plates between candles in this substring is 2, as each of the two plates has at least one candle in the substring to its left and right. Return an integer array answer where $\text{answer}[i]$ is the answer to the i th query.

19.2.1 解题思路与分析

- 先把每个调用的最左、以及最右的蜡烛的位置找出来，假如作必要的前置处理的话，可以做到 $O(1)$ 时间
- 而要数这最左与最右蜡烛之间的盘子个数的话，如果我们前置数清楚所有位置蜡烛个数，我们也可以做到 $O(1)$ 时间
- 所以，使用三个数组，一个记录各个位置蜡烛总个数，另两个分别纪录左右端点

```

public int[] platesBetweenCandles(String t, int[][] queries) {
    int n = t.length();
    int [] sum = new int [n+1];
    int [] pre = new int [n+1], suf = new int [n+1];
    char [] s = t.toCharArray();
    for (int i = 0; i < n; i++) {
        sum[i+1] = sum[i] + (s[i] == '|' ? 1 : 0);
        pre[i+1] = s[i] == '|' ? i : pre[i]; // pre[i] matches i-1
    }
    for (int i = n-1; i >= 0; i--)
        suf[i] = s[i] == '|' ? i : suf[i+1]; // suf[i] matches i
    int [] ans = new int [queries.length];
    for (int i = 0; i < queries.length; i++) {
        int l = suf[queries[i][0]], r = pre[queries[i][1]+1]; // 注意：右蜡烛边界
        if (l < r)
            ans[i] = r - l - (sum[r] - sum[l]);
    }
    return ans;
}

```

19.3 218. The Skyline Problem - Hard

A city's skyline is the outer contour of the silhouette formed by all the buildings in that city when viewed from a distance. Given the locations and heights of all the buildings, return the skyline formed by these buildings collectively.

The geometric information of each building is given in the array buildings where $\text{buildings}[i] = [\text{left}_i, \text{right}_i, \text{height}_i]$: left_i is the x coordinate of the left edge of the i th building. right_i is the x coordinate of the right edge of the i th building. height_i is the height of the i th building. You may assume all buildings are perfect rectangles grounded on an absolutely flat surface at height 0.

The skyline should be represented as a list of "key points" sorted by their x-coordinate in the form $[[x_1, y_1], [x_2, y_2], \dots]$. Each key point is the left endpoint of some horizontal segment in the skyline except the last point in the list, which always has a y-coordinate 0 and is used to mark the skyline's termination where the rightmost building ends. Any ground between the leftmost and rightmost buildings should be part of the skyline's contour.

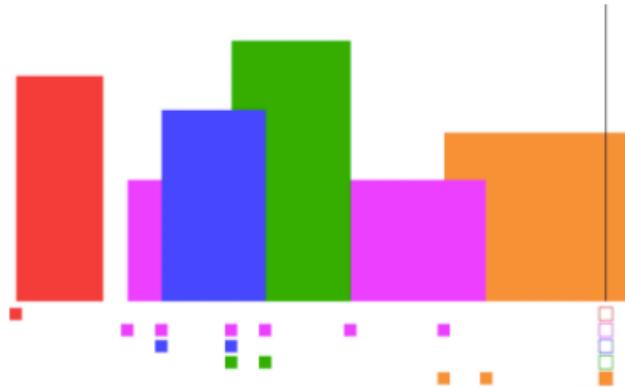
Note: There must be no consecutive horizontal lines of equal height in the output skyline. For instance, `[..., [2 3], [4 5], [7 5], [11 5], [12 7], ...]` is not acceptable; the three lines of height 5 should be merged into one in the final output as such: `[..., [2 3], [4 5], [12 7], ...]`

使用扫描线，从左至右扫过。如果遇到左端点，将高度入堆，如果遇到右端点，则将高度从堆中删除。使用 last 变量记录上一个转折点。

扫描线法

使用扫描线，从左至右扫过。如果遇到左端点，将高度入堆，如果遇到右端点，则将高度从堆中删除。使用 last 变量记录上一个转折点。

可以参考下面的图，扫描线下方的方格就是堆。



```

public List<List<Integer>> getSkyline(int[][][] buildings) { // 从左向右扫一遍过去
    List<List<Integer>> ans = new ArrayList<>();
    Map<Integer, List<Integer>> map = new TreeMap<>();
    for (int [] v : buildings) {
        map.computeIfAbsent(v[0], z -> new ArrayList<>()).add(-v[2]); // 插入左节点: 高度为负
        map.computeIfAbsent(v[1], z -> new ArrayList<>()).add(v[2]); // 插入右节点: 高度为正
    }
    Map<Integer, Integer> heights = new TreeMap<>((a, b) -> b-a); // 保留当前位置的所有高度 重定义排序: 从大到小
    int [] last = {0, 0}; // 保留上一个位置的横坐标及高度
    for (Integer key : map.keySet()) {
        List<Integer> ys = map.get(key);
        Collections.sort(ys);
        for (int y : ys) {
            if (y < 0) // 左端点, 高度入队
                heights.put(-y, heights.getOrDefault(-y, 0) + 1);
            else { // 右端点移除高度
                heights.put(y, heights.getOrDefault(y, 1) - 1);
                if (heights.get(y) == 0) heights.remove(y);
            }
        }
        Integer maxHeight = 0; // 获取 heights 的最大值: 就是第一个值
        if (!heights.isEmpty())
            maxHeight = heights.keySet().iterator().next();
        if (last[1] != maxHeight) { // 如果当前最大高度不同于上一个高度, 说明其为转折点
            last[0] = key; // 更新 last, 并加入结果集
            last[1] = maxHeight;
            ans.add(Arrays.asList(key, maxHeight));
        }
    }
    return ans;
}

```

19.4 715. Range Module - Hard

A Range Module is a module that tracks ranges of numbers. Design a data structure to track the ranges represented as half-open intervals and query about them.

A half-open interval $[left, right)$ denotes all the real numbers x where $left \leq x < right$.

Implement the RangeModule class:

RangeModule() Initializes the object of the data structure.
void addRange(int left, int right) Adds the half-open interval $[left, right)$, tracking every real number in that interval. Adding an interval that partially overlaps with currently tracked numbers should add any numbers in the interval $[left, right)$ that are not already tracked.
boolean queryRange(int left, int right) Returns true if every real number in the interval $[left, right)$ is currently being tracked,

and false otherwise. void removeRange(int left, int right) Stops tracking every real number currently being tracked in the half-open interval [left, right).

```

class Range {
    int left, right;
    public Range(int left, int right) {
        this.left = left;
        this.right = right;
    }
}
TreeSet<Range> ts;
public RangeModule() {
    ts = new TreeSet<>((a, b)->(a.left != b.left ? a.left - b.left : a.right - b.right));
}
public void addRange(int left, int right) {
    int nl = left, nr = right;
    Range high = new Range(right, Integer.MAX_VALUE);
    while (true) {
        Range r = ts.lower(high);
        if (r == null || r.right < left) break;
        if (r.right > right) nr = r.right;
        if (r.left < left) nl = r.left;
        ts.remove(r);
    }
    ts.add(new Range(nl, nr));
}
public boolean queryRange(int left, int right) {
    Range target = ts.floor(new Range(left, Integer.MAX_VALUE));
    return target != null && target.left <= left && target.right >= right;
}
public void removeRange(int left, int right) {
    Range high = new Range(right, right);
    while (true) {
        Range r = ts.lower(high);
        if (r == null || r.right <= left) break;
        if (r.right > right)
            ts.add(new Range(right, r.right));
        if (r.left < left)
            ts.add(new Range(r.left, left));
        ts.remove(r);
    }
}

```

19.5 352. Data Stream as Disjoint Intervals - Hard

Given a data stream input of non-negative integers a_1, a_2, \dots, a_n , summarize the numbers seen so far as a list of disjoint intervals.

Implement the SummaryRanges class:

SummaryRanges() Initializes the object with an empty stream. void addNum(int val) Adds the integer val to the stream. int[][] getIntervals() Returns a summary of the integers in the stream currently as a list of disjoint intervals [starti, endi].

```

class Range implements Comparable<Range> {
    int bgn, end;
    public Range(int bgn, int end) {
        this.bgn = bgn;
        this.end = end;
    }
    @Override public int compareTo(Range other) {
        return this.bgn - other.bgn;
    }
}
TreeSet<Range> ts;
public SummaryRanges() {
    ts = new TreeSet<Range>();
}
public void addNum(int val) {
    Range cur = new Range(val, val);
    Range bef = ts.floor(cur);
    Range aft = ts.ceiling(cur);
    if (bef != null && bef.end + 1 >= val) {
        cur.bgn = bef.bgn;
        cur.end = Math.max(val, bef.end);
        ts.remove(bef);
    }
    if (aft != null && aft.bgn == val + 1) {
        cur.end = aft.end;
    }
}

```

```

        ts.remove(aft);
    }
    ts.add(cur);
}
public int[][] getIntervals() {
    int [][] ans = new int [ts.size()][2];
    int i = 0;
    for (Range cur : ts) {
        ans[i][0] = cur.bgn;
        ans[i][1] = cur.end;
        i++;
    }
    return ans;
}

```

19.6 1419. Minimum Number of Frogs Croaking - Medium

Given the string `croakOfFrogs`, which represents a combination of the string "croak" from different frogs, that is, multiple frogs can croak at the same time, so multiple "croak" are mixed. Return the minimum number of different frogs to finish all the croak in the given string.

A valid "croak" means a frog is printing 5 letters 'c' , 'r' , 'o' , 'a' , 'k' sequentially. The frogs have to print all five letters to finish a croak. If the given string is not a combination of valid "croak" return -1.

```

public int minNumberOfFrogs(String s) { // 写得有点儿乱
    int n = s.length();
    int cc = 0, cr = 0, co = 0, ca = 0, cnt = 0, max = 0;
    for (int i = 0; i < n; i++) {
        char c = s.charAt(i);
        if (c == 'c') {
            ++cc;
            ++cnt;
            max = Math.max(max, cnt);
        } else if (c == 'r') {
            if (cc == 0) return -1;
            --cc;
            ++cr;
        } else if (c == 'o') {
            if (cr == 0) return -1;
            --cr;
            ++co;
        } else if (c == 'a') {
            if (co == 0) return -1;
            --co;
            ++ca;
        } else if (c == 'k') {
            if (ca == 0) return -1;
            --ca;
            --cnt;
        }
    }
    if (cc + cr + co + ca > 0) return -1;
    return max;
}

```

We can track how many frogs are 'singing' each letter in `cnt`:

- Increase number of frogs singing this letter, and decrease number singing previous letter.
- When a frog sings 'c', we increase the number of (simultaneous) frogs .
- When a frog sings 'k', we decrease the number of (simultaneous) frogs .
- If some frog is singing a letter, but no frog sang the previous letter, we return -1 .

	c	r	c	o	a	r	o	a	k	k
0 c										
1 r										
2 o										
3 a										
4 k										
frogs	1	1	2	2	2	2	2	1	0	



Track and return the maximum number of frogs ever singing together.

Catch: if some frog hasn't finished croaking, we need to return -1 .

```
public int minNumberOfFrogs(String s) {
    int n = s.length();
    int cnt [] = new int [5];
    int cur = 0, max = 0;
    for (int i = 0; i < n; i++) {
        char c = s.charAt(i);
        int idx = "croak".indexOf(c);
        ++cnt[idx];
        if (idx == 0)
            max = Math.max(max, ++cur);
        else if (--cnt[idx-1] < 0)
            return -1;
        else if (idx == 4)
            --cur;
    }
    return cur == 0 ? max : -1; // 如果最后所有的青蛙都叫完了的话
}
```

• 解法三:

这个跟上面的解法差不多，优化了查询位置的时间。直接对比每一步的结果。如果当前字符位的个数比上一位多，那么说明当前位没有办法被匹配成功。

```
public int minNumberOfFrogs(String croakOfFrogs) {
    char[] ch = croakOfFrogs.toCharArray();
    int curr = 0;
    int res = 0;
    int c = 0, r = 0, o = 0, a = 0, k = 0;
    for(int i = 0; i < ch.length; i++) {
        if(ch[i] == 'c') {
            c++;
            curr++;
        } else if(ch[i] == 'r') {
            r++;
            curr++;
        } else if(ch[i] == 'o')
            o++;
        } else if(ch[i] == 'a')
            a++;
        } else if(ch[i] == 'k')
            k++;
        if(c > r && r > o && o > a && a > k)
            return -1;
        res = Math.max(res, curr);
    }
    return res;
}
```

```
else if(ch[i] == 'a')
    a++;
else {
    k++;
    curr--;
}
res = Math.max(res, curr);
if (c < r || r < o || o < a || a < k) // 要满足所有的条件
    return -1;
if ((c == r) && (r == o) && (o == a) && (a == k))
    return res;
return -1;
}
```

Chapter 20

PreSum 差分数组

20.1 1674. Minimum Moves to Make Array Complementary - Medium 差分数组

You are given an integer array nums of even length n and an integer limit. In one move, you can replace any integer from nums with another integer between 1 and limit, inclusive.

The array nums is complementary if for all indices i (0-indexed), $\text{nums}[i] + \text{nums}[n - 1 - i]$ equals the same number. For example, the array [1,2,3,4] is complementary because for all indices i , $\text{nums}[i] + \text{nums}[n - 1 - i] = 5$.

Return the minimum number of moves required to make nums complementary.

20.1.1 解题思路与分析 Sweep Line / Prefix Sum

Let $a = \min(\text{nums}[i], \text{nums}[n-i-1])$, $b = \max(\text{nums}[i], \text{nums}[n-i-1])$

The key to this problem is how many moves do we need to make $a + b == T$.

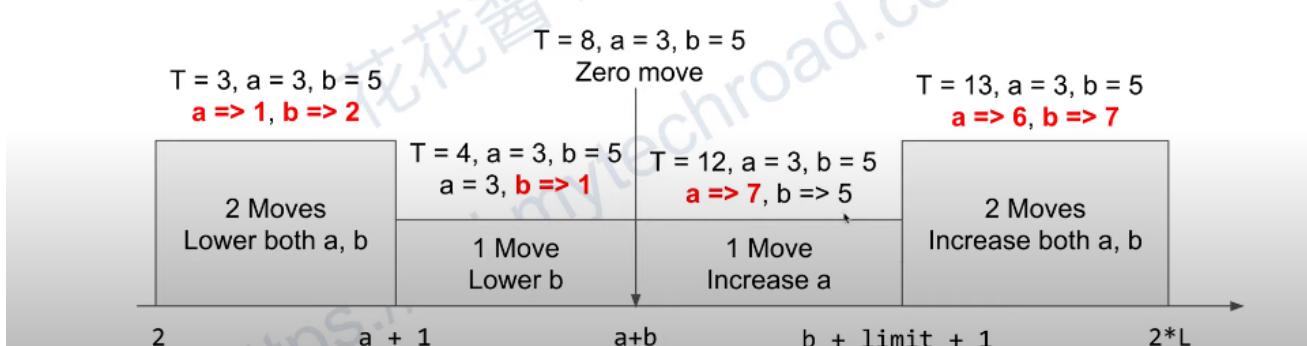
```
if 2 <= T < a + 1, two moves, lower both a and b.  
if a + 1 <= T < a + b, one move, lower b  
if a + b == T, zero move  
if a + b + 1 <= T < b + limit + 1, one move, increase a  
if b + limit + 1 <= T <= 2*limit, two moves, increase both a and b.
```

- 关键是找到五个关键点，并利用扫描线、差分数组来暴搜所有数对和，并最全局最优解

To better understand this problem, we can start with the brute force method:
Try all possible pair sum T from 2 to $\text{limit} * 2$, and check how many moves we need.

Let $a = \min(\text{nums}[i], \text{nums}[n - i - 1])$, $b = \max(\text{nums}[i], \text{nums}[n - i - 1])$
How many moves do we need to sum up to T ? It depends on (a, b, T, Limit)

Time complexity: $O(\text{limit} * n)$ TLE 72/110 passed, Space complexity: $O(1)$



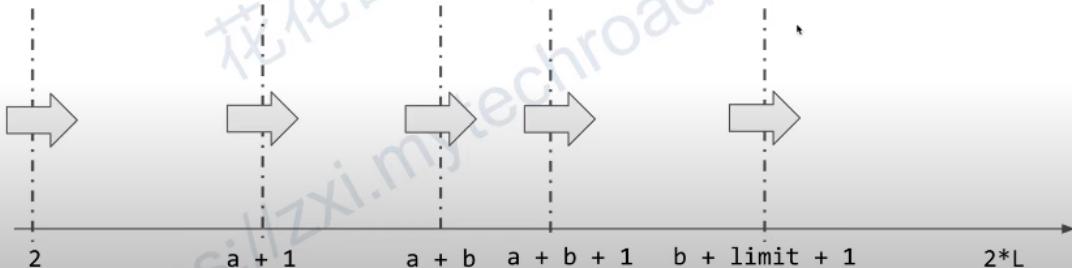
Optimization: If we move T in the x-axis, whenever T cross one boundary, we need to perform some action.

We can record the delta for each pair at each boundary.

Then we only need to do a prefix sum to determine how many moves we need at the current pair sum T in O(1). Similar to LeetCode 731. My Calendar II.

Time complexity: O(n + limit), Space complexity: O(limit)

+2 (lower a, lower b) -1 (lower a) -1 (no-op) +1 (increase a) +1 (increase a, increase b)



- 用一个实例来看。这里强调一下：不可以使用二分搜索，因为这里的答案并不唯一。

a, b	2 / +2	a+1 / -1	a+b / -1	a+b+1 / +1	a+b+L+1 / + 1	delta
1, 3	2 / +2	2 / -1	4 / -1	5 / +1	9 / +1	{2:1,4:-1,5:1,9:1}
2, 4	2 / +2	3 / -1	6 / -1	7 / +1	11 / +1	{2:3,3:-1,4:-1,5:1,6:-1,7:1,9:1,11:1}

T	delta	Prefix sum / ans
2	3	3
3	-1	2
4	-1	1
5	1	2
6	-1	1
7	1	2
9	1	3
11	1	4

[1, 2, 2, 3]

[3, 2, 4, 3]

Since there are at most O(n) unique boundaries, we can sort them to obtain O(nlogn) time complexity and O(n) space complexity. If limit >> n.

```

public int minMoves(int[] nums, int limit) {
    int n = nums.length;
    int [] delta = new int [2 * limit + 2]; // 差分数组
    for (int i = 0; i < n/2; i++) {
        int a = Math.min(nums[i], nums[n-1-i]);
        int b = Math.max(nums[i], nums[n-1-i]); // 把各区间所需要的操作简化到五行代码中去了
        delta[2] += 2; // dec a, decreasing b [2, a] 区间的和 所需要的变换次数都是 2 次
        --delta[a+1]; // dec a [a + 1, a+b-1] 区间中的和 所需要变的次数都是 1 次
        --delta[a+b]; // no operations [a+b+1, b+limit] 区间中的和 只需要变动 1 次;
        ++delta[a+b+1];// inc a
        ++delta[b+limit+1]; // inc a, inc b
    }
    int ans = n, sum = 0;
    for (int t = 2; t < 2 * limit + 2; t++) {
        sum += delta[t]; // 差分数组的前缀和等于 (共需要的操作次数)
        ans = Math.min(ans, sum);
    }
    return ans;
}

```

20.2 1074. Number of Submatrices That Sum to Target - Hard 二维数组转化为一维数组：数组中 target Sum 的 2D 版、presum 思想的 2d 版

Given a matrix and a target, return the number of non-empty submatrices that sum to target.

A submatrix x_1, y_1, x_2, y_2 is the set of all cells $\text{matrix}[x][y]$ with $x_1 \leq x \leq x_2$ and $y_1 \leq y \leq y_2$.

Two submatrices (x_1, y_1, x_2, y_2) and (x'_1, y'_1, x'_2, y'_2) are different if they have some coordinate that is different: for example, if $x_1 \neq x'_1$.

本题也需要使用到 presum 思路来解题, 只不过这个前缀和 presum 的计算对象是二维数组。对于任意一个点, $\text{presum}[i][j]$ 代表了从 $\text{Matrix}^{1,1}$ 到 $\text{Matrix}[i][j]$ 之间的和。

有了前缀和之后, 我们可以将二维数组拆解为多个一维数组, 再用一维数组的思路去解题。

拆分数组时可以以列拆分, 也可以以行拆, 无论如何都可以达到遍历所有子矩阵的效果。本题以列拆分为例, 对于任意两列 col1 和 col2 , 我们可以得到所有行的前缀和

这样, 二维数组就转化为了一维数组。接下来只要遍历所有列的组合即可。

```
public int numSubmatrixSumTarget(int[][] matrix, int target) {
    int m = matrix.length, n = matrix[0].length, rowSum = 0;
    int [][] sum = new int [m][n]; // sums of row sum only
    for (int i = 0; i < m; i++) {
        rowSum = 0;
        for (int j = 0; j < n; j++) {
            rowSum += matrix[i][j];
            sum[i][j] = rowSum + (i == 0 ? 0 : sum[i-1][j]);
        }
    }
    int cnt = 0, cur = 0;
    for (int j = 0; j < n; j++) // col1
        for (int k = j; k < n; k++) // col2
            Map<Integer, Integer> map = new HashMap<>(); // 用于记录前缀和个数
            for (int i = 0; i < m; i++) {
                cur = sum[i][k] - (j == 0 ? 0 : sum[i][j-1]);
                if (cur == target) ++cnt;
                cnt += map.getOrDefault(cur - target, 0);
                map.put(cur, map.getOrDefault(cur, 0) + 1);
            }
    }
    return cnt;
}
```

- 对 corner case 的处理相对简洁的写法

```
public int numSubmatrixSumTarget(int [][] matrix, int target) {
    int res = 0, m = matrix.length, n = matrix[0].length;
    int [][] sum = new int [m+1][n+1];
    for (int i = 1; i <= m; ++i)
        for (int j = 1; j <= n; ++j) // 对全数组纵横求和
            sum[i][j] = sum[i][j - 1] + sum[i - 1][j] - sum[i - 1][j - 1] + matrix[i - 1][j - 1];
    for (int i = 1; i <= m; ++i)
        for (int j = 1; j <= n; ++j)
            for (int p = 1; p <= i; ++p)
                for (int q = 1; q <= j; ++q) {
                    int t = sum[i][j] - sum[i][q - 1] - sum[p - 1][j] + sum[p - 1][q - 1];
                    if (t == target) ++res;
                }
    return res;
}
```

```
public int numSubmatrixSumTarget(int[][] matrix, int target) {
    int m = matrix.length, n = matrix[0].length;
    int [][] sum = new int [m][n+1]; // sums of row sum only
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++)
            sum[i][j+1] = sum[i][j] + matrix[i][j]; // row sums only
    }
    int cnt = 0, cur = 0;
    for (int j = 0; j < n; j++) // col1
        for (int k = j; k < n; k++) // col2
            cur = 0;
            Map<Integer, Integer> map = new HashMap<>(); // 用于记录前缀和个数
            map.put(0, 1);
            for (int i = 0; i < m; i++) {
                cur += sum[i][k+1] - sum[i][j];
                cnt += map.getOrDefault(cur - target, 0);
                map.put(cur, map.getOrDefault(cur, 0) + 1);
            }
    }
    return cnt;
}
```

首先对每一行，计算一个 presum

然后对任意两个列， $[j, k]$ ，计算 j 和 k 之间的所有数的和，因为对于每一行，我们已经计算了 presum，所以，对于 $[j, k]$ 之间的树，可以通过 $\text{presum}[i][k] - (\text{presum}[i][j-1] \text{ if } j == 0 \text{ else } 0)$ 得到某一行在 $[j, k]$ 之间的值。这样，对于任意两列 $[j, k]$ 之间的数，我们可以形成一个临时的一维数组，表示从第 0 行到最后一行的数，那么，问题就转换成在这个一维数组里面，找一个连续子数组，使得这些数的和是 target，那么，后面这个问题就转换成 leetcode 560。

<http://www.noteanddata.com/leetcode-560-Subarray-Sum-Equals-K-javascript-solution-note.html>

所以，这个是 presum 思想的 2d 版，类似的题目还有 leetcode 304 Range Sum Query 2D - Immutable

```
public int numSubmatrixSumTarget(int [][] matrix, int target) {
    int res = 0, m = matrix.length, n = matrix[0].length;
    for (int i = 0; i < m; ++i)
        for (int j = 1; j < n; ++j)
            matrix[i][j] += matrix[i][j - 1];
    for (int i = 0; i < n; ++i)
        for (int j = i; j < n; ++j) {
            Map<Integer, Integer> cntMap = new HashMap<>();
            cntMap.put(0, 1);
            int cur = 0;
            for (int k = 0; k < m; ++k) {
                cur += matrix[k][j] - (j > 0 ? matrix[k][j - 1] : 0);
                res += cntMap.getOrDefault(cur - target, 0);
                cntMap.put(cur, cntMap.getOrDefault(cur, 0) + 1);
            }
        }
    return res;
}
```

```
private int cntTarget(int [] arr, int target) { // 比较一下与上面解法、写法的不同!!!
    int cnt = 0, sum = 0;
    Map<Integer, Integer> cntMap = new HashMap<>();
    cntMap.put(0, 1);
    for (int i = 0; i < arr.length; i++) {
        sum += arr[i];
        cnt += cntMap.getOrDefault(sum - target, 0);
        cntMap.compute(sum, (k, v) -> {
            if (v == null) v = 0;
            return v+1;
        });
    }
    return cnt;
}
public int numSubmatrixSumTarget(int[][] matrix, int target) {
    int m = matrix.length, n = matrix[0].length, cur = 0;
    int [][] sum = new int [m][n]; // sums of row sum only
    for (int i = 0; i < m; i++) {
        cur = 0;
        for (int j = 0; j < n; j++) {
            cur += matrix[i][j];
            sum[i][j] = cur;
        }
    }
    int cnt = 0;
    for (int j = 0; j < n; j++) {
        for (int k = j; k < n; k++) {
            int [] tmp = new int [m];
            for (int i = 0; i < m; i++)
                tmp[i] = sum[i][k] - (j == 0 ? 0 : sum[i][j-1]);
            int oneCnt = cntTarget(tmp, target);
            cnt += oneCnt;
        }
    }
    return cnt;
}
```

20.3 798. Smallest Rotation with Highest Score

Hard

308

19

Add to List

Share You are given an array `nums`. You can rotate it by a non-negative integer `k` so that the array becomes `[nums[k], nums[k + 1], ... nums[nums.length - 1], nums1, nums2, ..., nums[k-1]]`. Afterward, any entries that are less than or equal to their index are worth one point.

For example, if we have `nums = [2,4,1,3,0]`, and we rotate by `k = 2`, it becomes `[1,3,0,2,4]`. This is worth 3 points because $1 > 0$ [no points], $3 > 1$ [no points], $0 \leq 2$ [one point], $2 \leq 3$ [one point], $4 \leq 4$ [one point]. Return the rotation index `k` that corresponds to the highest score we can achieve if we rotated `nums` by it. If there are multiple answers, return the smallest such index `k`.

20.3.1 解题思路与分析

答案的思路也十分巧妙，并没有采用 brute force 那种直接求每一个 `K` 值的得分，而是反其道而行之，对于每个数字，探究其跟 `K` 值之间的联系。首先我们要讨论一下边界情况，那么就是当 `A[i] = 0` 或 `N` 的情况，首先如果 `A[i] = 0` 的话，那么 0 这个数字在任何位置都会小于等于坐标值，所以在任何位置都会得分的，那么其实可以忽略之，因为其不会对最大值产生任何影响，同理，如果 `A[i] = N` 的时候，由于长度为 `N` 的数组的坐标值范围是 `[0, N-1]`，所以数字 `N` 在任何位置都不得分，同样也不会对最大值产生任何影响，可以忽略之。那么我们关心的数字的范围其实是 `[1, N-1]`。在这个范围内的数字在旋转数组的过程中，从位置 0 变到 `N-1` 位置的时候，一定会得分，因为此范围的数字最大就是 `N-1`。这个一定得的分我们在最后统一加上，基于上面的发现，我们再来分析下题目中的例子 `[2, 3, 1, 4, 0]`，其中红色数字表示不得分的位置：

A:	2	3	1	4	0	(K = 0)
A:	3	1	4	0	2	(K = 1)
A:	1	4	0	2	3	(K = 2)
A:	4	0	2	3	1	(K = 3)
A:	0	2	3	1	4	(K = 4)
idx:	0	1	2	3	4	

对于某个数字 `A[i]`，我们想知道其什么时候能旋转到坐标位置为 `A[i]` 的地方，这样就可以得分了。比如上面博主标记了红色的数字 3，最开始时的位置为 1，此时是不得分的，我们想知道其什么时候能到位置 3，答案是当 `K=3` 的时候，其刚好旋转到位置 3，`K` 再增加的时候，其又开始不得分了。所以这个最后能得分的临界位置是通过 $(i - A[i] + N) \% N$ 得到，那么此时如果 `K` 再增加 1 的话，`A[i]` 就开始不得分了（如果我们 suppose 每个位置都可以得分，那么不得分的地方就可以当作是失分了），所以我们可以在这个刚好开始不得分的地方标记一下，通过-1 进行标记，这个位置就是 $(i - A[i] + 1 + N) \% N$ 。我们用一个长度为 `N` 的 `change` 数组，对于每个数字，我们都找到其刚好不得分的地方，进行-1 操作，那么此时 `change[i]` 就表示数组中的数字在 `i` 位置会不得分的个数，如果我们仔细观察上面红色的数字，可以发现，由于是左移，坐标在不断减小，所以原先失分的地方，在 `K+1` 的时候还是失分，除非你从开头位置跑到末尾去了，那会得分，所以我们要累加 `change` 数组，并且 `K` 每增加 1 的时候，要加上额外的 1，最后 `change` 数组中最大数字的位置就是要求的 `K` 值了。

Key point

Don't calculate the score for $K=0$, we don't need it at all.

(I see almost all other solutions did)

The key point is to find out how score changes when $K++$

Time complexity:

"A will have length at most 20000."

I think it means you should find a $O(N)$ solution.

How does score change when $K++$?

- **Get point**

Each time when we rotate, we make index 0 to index $N-1$, then we get one more point.

We know that for sure, so I don't need to record it.

- **Lose point**

$(i - A[i] + N) \% N$ is the value of K making $A[i]$'s index just equal to $A[i]$.

For example, If $A[6] = 1$, then $K = (6 - A[6]) \% 6 = 5$ making $A[6]$ to index 1 of new array.

So when $K=5$, we get this point for $A[6]$

Then if K is bigger when $K = (i - A[i] + 1) \% N$, we start to lose this point, making our score $-= 1$

All I have done is record the value of K for all $A[i]$ where we will lose points.

- **$A[i]=0$**

Rotation makes no change for it, because we always have $0 \leq \text{index}$.

However, it is covered in "get point" and "lose point".

Explanation of codes

1. Search the index where score decrease and record this change to a list change .

2. A simple for loop to calculate the score for every K value.

```
score[K] = score[K-1] + change[K]
```

In my code I accumulated changes so I get the changed score for every K value compared to $K=0$

3. Find the index of best score.

时间复杂度: $O(n)$

空间复杂度: $O(n)$

```
public int bestRotation(int[] a) {
    int n = a.length, ans = 0;
    int [] change = new int [n];
    for (int i = 0; i < n; ++i) change[(i - a[i] + n + 1) % n] -= 1;
    for (int i = 1; i < n; ++i) {
        change[i] += change[i-1] + 1;
        ans = change[i] > change[ans] ? i : ans;
    }
    return ans;
}
```

20.3.2 Java $O(N\lg N)$ optimized Brute Force with Fenwick Tree $O(N\log N)$

我们假设 A 的大小为 n , 如果采用暴力法逐个测试, 则时间复杂度为 $O(n^2)$, 应该过不了大数据测试。

我采取的方法是: 首先计算使得每个元素 $A[i]$ 要符合条件, 需要 rotate 的 K 的集合, 用线段表示; 然后再扫描一遍, 求出这些线段中重合最多的点, 那么这个点对应的 rotate 次数就是题目所要计算的 K 。对应 $A[i]$ 来讲, 如果 $A[i] \leq i$, 那么它向左移动到 j 也可能维持 $A[i] \leq j$, 所以我们计算出此时它向左移动的合法区间 $[0, i - A[i]]$ 。那么 $A[i]$ 向右移动的合法区间是多少呢? 我们知道它向右移动最多移动到 $n - 1$, 即移动 $n - 1 - i$ 步; 而最少需要移动 $\max(1, A[i] - i)$ 步, 其中 1 表示 $A[i] \leq i$ 的情况。那么如果这个区间合法, 就可以同样构成了一个合法的移动区间 $[i + 1, n - \max(1, A[i] - i)]$ 。

得到多个线段构成的合法移动区间之后，我们的任务就是求出这些区间的最大重合点。首先对 segment 中的各个点进行排序，然后采用扫描线的方法计算最大最大重合处。为了便于区分某个点是起点还是终点，我们定义一个 pair<int, bool> 来表示点，并且让起点的 bool 值为 false，终点的 bool 值为 true，这样就可以在扫描到某个点之后，先处理起点，再处理终点。

由于每个 A[i] 最多对应 2 个合法移动区间，所以 segments 大小也是 O(n) 量级的。这样可以得知，本算法的时间复杂度是 O(nlogn)，空间复杂度是 O(n)。

<https://www.codeleading.com/article/62063257456/>

- 不是我的代码风格，需要再学习一下

```

public int bestRotation(int[] A) {
    int LEN = A.length;
    int score, ans, ansInd, k; score = k = ans = ansInd = 0;
    for(int i = 0; i < LEN; i++) A[i] -= i;
    int[] bit = new int[3 * LEN + 1];
    for(int i = k; i < LEN; i++) bitAdd(bit, 3 * LEN + 1, vToIndex(A[i], LEN), 1);
    while (k < LEN) {
        score = bitPreSum(bit, vToIndex(-k, LEN));
        if(score > ans) {
            ansInd = k;
            ans = score;
        }
        bitAdd(bit, 3 * LEN + 1, vToIndex(A[k], LEN), -1);
        bitAdd(bit, 3 * LEN + 1, vToIndex(A[k] - LEN, LEN), 1);
        k++;
    }
    return ansInd;
}
void bitAdd(int[] bit, int bLEN, int ind, int delta) {
    for(; ind < bLEN; ind = (ind | (ind + 1))) {
        bit[ind] += delta;
    }
}
int bitPreSum(int[] bit, int ind) {
    int ans = 0;
    for(; ind >= 0; ind = (ind & (ind + 1)) - 1) {
        ans += bit[ind];
    }
    return ans;
}
int vToIndex(int v, int LEN) {
    return v + 2 * LEN;
}

```

Chapter 21

Greedy

21.1 1585. Check If String Is Transformable With Substring Sort Operations - Hard

Given two strings s and t, you want to transform string s into string t using the following operation any number of times:

Choose a non-empty substring in s and sort it in-place so the characters are in ascending order. For example, applying the operation on the underlined substring in "14234" results in "12344".

Return true if it is possible to transform string s into string t. Otherwise, return false.

A substring is a contiguous sequence of characters within a string.

```
public boolean isTransformable(String s, String t) {
    ArrayDeque<Integer> [] q = new ArrayDeque[10];
    for (int i = 0; i < 10; i++)
        q[i] = new ArrayDeque<Integer>();
    for (int i = 0; i < s.length(); i++)
        q[s.charAt(i)-'0'].offerLast(i);
    for (char c : t.toCharArray()) {
        int d = c - '0';
        if (q[d].isEmpty()) return false;
        for (int i = 0; i < d; i++)
            if (!q[i].isEmpty() && q[i].peekFirst() < q[d].peekFirst())
                return false;
        q[d].pollFirst();
    }
    return true;
}
```

21.2 1330. Reverse Subarray To Maximize Array Value - Hard

You are given an integer array nums. The value of this array is defined as the sum of $|nums[i]-nums[i+1]|$ for all $0 \leq i < \text{nums.length}-1$.

You are allowed to select any subarray of the given array and reverse it. You can perform this operation only once.

Find maximum possible value of the final array.

解题思路

给出一个整数数组，定义这个数组的和为相邻元素之差的和。现在允许将数组中的某个子数组进行整体翻转，然后计算对应的和。返回数组可得到的最大和。

这个题分析一下可知。有如下几种情况。

如果最优的最大和是不翻转的情况，那么就等于是翻转整个数组。

如果最优的情况下，翻转子串的边界是数组边界，那么用 $O(n)$ 的时间遍历两次，分别找出是右边界和左边界的情况就可以了。

如果最优的情况下，翻转子串的边界在数组内部，这是最复杂的情况，那么该如何考虑呢？假设我们已经计算出来数组在不翻转的情况下和 S ，那么翻转之后其实只是改变了子串和整体数组的交界处的情况，其它地方均没有改变。因此假设边界左端点的值是 b ，接着的前一个是 a ，右端点是 c ，接着的后一个是 d 。那么变换之后的结果其实就是 $S + \text{abs}(d-b) + \text{abs}(c-a) - \text{abs}(b-a) - \text{abs}(d-c) = S + x$ 。只要 x 大于0，就意味着增量为正，变换会导致结果优化。我们再假设 $a <= b, c <= d$ ，那么增量就是 $2 * (c - b)$ 。而分析可知我们要让 c 尽可能大， b 尽可能小。因此我们用两个 $O(n)$ 的时间。第一次找 b ，怎么找呢？就是每两个相邻的，选大的那一个，然后在所有对中选对应 b 最小的那一对。第二次找 c 就是每两个相邻的，选小的那个，然后在所有的对中选最大的 c 的那一对。

这里假设了 $abcd$ 的关系，其实也可以假设其他情况，最后都是一样的。

时空复杂度

代码如下，时间上， $O(n)$ ，空间上， $O(1)$ 。

```

public int maxValueAfterReverse(int[] a) {
    int n = a.length, base = 0, ans = 0, sum = 0;
    if (n == 1) return 0;
    for (int i = 0; i < n-1; i++)
        sum += abs(a[i] - a[i+1]);
    // 交换的区间左端点是数组的左边界
    for (int i = 0; i < n-1; i++)
        ans = max(ans, sum + abs(a[i+1]-a[0]) - abs(a[i+1] - a[i]));
    // 交换的区间右端点是数组的右边界
    for (int i = n-1; i > 0; i--)
        ans = max(ans, sum + abs(a[n-1] - a[i-1]) - abs(a[i] - a[i-1]));
    // 交换区间左右端点在数组内部
    int l = 1; // 1. 找出左端点
    for (int i = 2; i < n; i++) {
        int ia = min(a[i], a[i-1]), ib = max(a[i], a[i-1]);
        if (max(a[i], a[i-1]) < max(a[l], a[l-1])) l = i;
    }
    int r = 0; // 2. 找出右端点
    for (int i = 1; i < n-1; i++)
        if (min(a[i], a[i+1]) > min(a[r], a[r+1])) r = i;
    ans = max(ans, sum + 2 * (min(a[r], a[r+1]) - max(a[l], a[l-1])));
    return ans;
}

```

Chapter 22

others

22.1 Predict the Winner

You are given an integer array `nums`. Two players are playing a game with this array: player 1 and player 2. Player 1 and player 2 take turns, with player 1 starting first. Both players start the game with a score of 0. At each turn, the player takes one of the numbers from either end of the array (i.e., `nums1` or `nums[nums.length - 1]`) which reduces the size of the array by 1. The player adds the chosen number to their score. The game ends when there are no more elements in the array. Return true if Player 1 can win the game. If the scores of both players are equal, then player 1 is still the winner, and you should also return true. You may assume that both players are playing optimally.

```
private int helper( int [] arr, int i, int j) {  
    if (i == j) return arr[i];  
    else return Math.max(arr[i] - helper(arr, i+1, j), arr[j] - helper(arr, i, j-1));  
}  
public boolean PredictTheWinner(int[] nums) {  
    int n = nums.length;  
    if (n == 1) return true;  
    return helper(nums, 0, n-1) >= 0;  
}
```

22.2 Rectangle Area II

We are given a list of (axis-aligned) rectangles. Each `rectangle[i] = [xi1, yi1, xi2, yi2]`, where (xi_1, yi_1) are the coordinates of the bottom-left corner, and (xi_2, yi_2) are the coordinates of the top-right corner of the i th rectangle. Find the total area covered by all rectangles in the plane. Since the answer may be too large, return it modulo $10^9 + 7$.

```
private void addRectange(List<int[]> l, int [] arr, int idx) {  
    if (idx >= l.size()) {  
        l.add(arr);  
        return;  
    }  
    int [] r = l.get(idx);  
    // 新增矩形 处在 左 右 上 下 四侧完全不重叠的区域  
    if (arr[2] <= r[0] || arr[0] >= r[2] || arr[1] >= r[3] || arr[3] <= r[1]) {  
        addRectange(l, arr, idx+1);  
        return;  
    }  
    if (arr[0] < r[0]) // 新增矩形 左侧 有交叠  
        addRectange(l, new int[]{arr[0], arr[1], r[0], arr[3]}, idx+1);  
    if (arr[2] > r[2]) // 新增矩形 右侧 也可以有 交叠  
        addRectange(l, new int[]{r[2], arr[1], arr[2], arr[3]}, idx+1);  
    if (arr[1] < r[1]) // 新增矩形 下侧 也可以有 交叠: 注意左侧、右侧前面已经加进去了, 现在只加中间部分, 不要重复计算  
        addRectange(l, new int[]{Math.max(arr[0], r[0]), arr[1], Math.min(arr[2], r[2]), r[1]}, idx+1);  
    if (arr[3] > r[3]) // 新增矩形 上侧 也可以有 交叠: 注意左侧、右侧、下侧 前面已经加进去了, 现在只加中间部分, 不要重复计算  
        addRectange(l, new int[]{Math.max(arr[0], r[0]), r[3], Math.min(arr[2], r[2]), arr[3]}, idx+1);  
    }  
public int rectangleArea(int[][] rectangles) {  
    int mod = (int) Math.pow(10, 9)+7;  
    long res = 0;  
    List<int[]> rlist = new ArrayList<>();  
    for (int [] val : rectangles)  
        addRectange(rlist, val, 0);  
    for (int [] arr : rlist)  
        res = (res + ((long)(arr[2]-arr[0]))*(long)(arr[3]-arr[1]))) % mod;
```

```

    return (int)res % mod;
}

```

22.3 Construct Binary Tree from Preorder and Postorder Traversal

Given two integer arrays, preorder and postorder where preorder is the preorder traversal of a binary tree of distinct values and postorder is the postorder traversal of the same tree, reconstruct and return the binary tree. If there exist multiple answers, you can return any of them.

```

public TreeNode constructFromPrePost(int[] preorder, int[] postorder) {
    int n = preorder.length;
    TreeNode r = new TreeNode(preorder[0]);
    if (n == 1) return r;
    Stack<TreeNode> s = new Stack<>();
    s.push(r);
    int idx = 0;
    for (int i = 1; i < n; i++) {
        TreeNode cur = new TreeNode(preorder[i]);
        if (s.peek().left == null) s.peek().left = cur;
        else s.peek().right = cur;
        s.push(cur);
        while (idx < n && postorder[idx] == s.peek().val) {
            s.pop();
            ++idx;
        }
    }
    return r;
}

```

22.4 Path Sum III

Given the root of a binary tree and an integer targetSum, return the number of paths where the sum of the values along the path equals targetSum. The path does not need to start or end at the root or a leaf, but it must go downwards (i.e., traveling only from parent nodes to child nodes).

```

private int solve(TreeNode r, int t, int value) {
    if (r == null) return 0;
    if (value + r.val == t)
        return 1 + solve(r.left, 0, 0) + solve(r.right, 0, 0);
    return solve(r.left, t, value + r.val) + solve(r.right, t, value + r.val);
}
public int pathSum(TreeNode root, int targetSum) {
    if (root == null) return 0;
    return solve(root, targetSum, 0) + pathSum(root.left, targetSum) + pathSum(root.right, targetSum);
}

```

22.5 Critical Connections in a Network

- There are n servers numbered from 0 to $n - 1$ connected by undirected server-to-server connections forming a network where $\text{connections}[i] = [a_i, b_i]$ represents a connection between servers a_i and b_i . Any server can reach other servers directly or indirectly through the network.
- A critical connection is a connection that, if removed, will make some servers unable to reach some other server.
- Return all critical connections in the network in any order.

```

static class Eg {
    int u, v, next;
    // int w;
    boolean cut;
    // int num;
}
public Eg[] egs;
public int cnt;
public int [] fir; // 边的出发点
int [] low;
int [] dfn;
int recdfn;

```

```

void tarjanAddEg(int u, int v, int w) {
    egs[cnt] = new Eg();
    egs[cnt].u = u;
    egs[cnt].v = v;
    // egs[cnt].w = w;
    egs[cnt].cut = false;
    // egs[cnt].num = 0;
    egs[cnt].next = fir[u]; // ?
    fir[u] = cnt++; // ?
}
private void initTarjan(int nodeSize, int edgeSize) {
    cnt = 0;
    egs = new Eg [edgeSize];
    low = new int [nodeSize];
    dfn = new int [nodeSize];
    fir = new int [edgeSize];
    Arrays.fill(fir, -1);
}
private void tarjan(int u, int fa) { // fa: father
    low[u] = ++recdnf;
    dfn[u] = recdnf;
    int have = 0;
    for (int i = fir[u]; i != -1; i = egs[i].next) {
        int v = egs[i].v;
        if (have == 0 && v == fa) { // 走过你来时的路
            have++;
            continue;
        }
        if (dfn[v] == 0) { // dfs 过程中还未经过该点
            tarjan(v, u);
            low[u] = Math.min(low[u], low[v]);
            if (dfn[u] < low[v]) { // 连通世外桃源与外界的路
                // 当 dfn[x] < low[y] 的时候:
                // --- 我们发现从 yy 节点出发, 在不经过 (x,y)(x,y) 的前提下, 不管走哪一条边, 我们都无法抵达 xx 节点, 或者比 xx 节点更早出现的节点
                // --- 此时我们发现 yy 所在的子树似乎形成了一个封闭圈, 那么 (x,y)(x,y) 自然也就是桥了.
                egs[i].cut = true;
                egs[i^1].cut = true; // ???
            }
        } else {
            low[u] = Math.min(low[u], dfn[v]); // 取已访问的节点的 dfs 序的最小值
        }
    }
}
private boolean findEdgeCut(int l, int r) {
    Arrays.fill(low, 0);
    Arrays.fill(dfn, 0);
    recdnf = 0;
    tarjan(l, l);
    for (int i = l; i <= r; i++) {
        if (dfn[i] == 0) return false;
    }
    return true;
}
public List<List<Integer>> criticalConnections(int n, List<List<Integer>> connections) {
    initTarjan(n, connections.size()*2);
    for (List<Integer> eg : connections) {
        tarjanAddEg(eg.get(0), eg.get(1), 1);
        tarjanAddEg(eg.get(1), eg.get(0), 1);
    }
    // boolean ans = findEdgeCut(0, n-1);
    Arrays.fill(low, 0);
    Arrays.fill(dfn, 0);
    recdnf = 0;
    tarjan(0, 0);
    List<List<Integer>> res = new ArrayList<>();
    int l = connections.size();
    for (int i = 0; i < l * 2; i += 2) { // i += 2 skipped egs[i^1] ?
        Eg eg = egs[i];
        if (eg != null && eg.cut) {
            List<Integer> t = new ArrayList<>();
            t.add(eg.u);
            t.add(eg.v);
            res.add(t);
        }
    }
    return res;
}

```

22.6 891. Sum of Subsequence Widths - Hard 考 sorting 和对 subsequence 的理解

The width of a sequence is the difference between the maximum and minimum elements in the sequence.

Given an array of integers nums, return the sum of the widths of all the non-empty subsequences of nums. Since the answer may be very large, return it modulo 109 + 7.

A subsequence is a sequence that can be derived from an array by deleting some or no elements without changing the order of the remaining elements. For example, [3,6,2,7] is a subsequence of the array [0,3,1,6,2,2,7].

22.6.1 解题思路与分析

- 这道题的最优解法相当的 tricky，基本有点脑筋急转弯的感觉了。在解题之前，我们首先要知道的是一个长度为 n 的数组，共有多少个子序列，如果算上空集的话，共有 2^n 个。

那么在给数组排序之后，对于其中任意一个数字 A[i]，其前面共有 i 个数是小于等于 A[i] 的，这 i 个数字共有 2^i 个子序列，它们加上 A[i] 都可以组成一个新的非空子序列，并且 A[i] 是这里面最大的数字，那么在宽度计算的时候，就要加上 A[i] $\times (2^i)$ ，

同理，A[i] 后面还有 n-1-i 个数字是大于等于它的，后面可以形成 $2^{(n-1-i)}$ 个子序列，每个加上 A[i] 就都是一个新的非空子序列，同时 A[i] 是这些子序列中最小的一个，那么结果中就要减去 A[i] $\times (2^{(n-1-i)})$ 。对于每个数字都这么计算一下，就是最终要求的所有子序列的宽度之和了。

可能你会怀疑虽然加上了 A[i] 前面 2^i 个子序列的最大值，那些子序列的最小值减去了么？其实是减去了的，虽然不是在遍历 A[i] 的时候减去，在遍历之前的数字时已经将所有该数字是子序列最小值的情况减去了，同理，A[i] 后面的那些 $2^{(n-1-i)}$ 个子序列的最大值也是在遍历到的时候才加上的，所以不会漏掉任何一个数字。

在写代码的时候有几点需要注意的地方，首先，结果 res 要定义为 long 型，因为虽然每次会对 1e9+7 取余，但是不能保证不会在取余之前就已经整型溢出，所以要定义为长整型。

其次，不能直接算 2^i 和 $2^{(n-1-i)}$ ，很容易溢出，即便是长整型，也有可能溢出。那么解决方案就是，在累加 i 的同时，每次都乘以一个 2，那么遍历到 i 的时候，也就乘到 2^i 了，防止溢出的诀窍就是每次乘以 2 之后就立马对 1e9+7 取余，这样就避免了指数溢出，同时又不影响结果。

最后，由于这种机制下的 2^i 和 $2^{(n-1-i)}$ 不方便同时计算，这里又用了一个 trick，就是将 A[i] $\times (2^{(n-1-i)})$ 转换为了 A[n-1-i] $\times 2^i$

```
sum(A[i] * 2^(n-1-i)) = A[0]*2^(n-1) + A[1]*2^(n-2) + A[2]*2^(n-3) + ... + A[n-1]*2^0
sum(A[n-1 - i] * 2^i) = A[n-1]*2^0 + A[n-2]*2^1 + ... + A[1]*2^(n-2) + A[0]*2^(n-1)
```

```
public int sumSubseqWidths(int[] a) {
    long mod = (int)1e9 + 7, c = 1;
    long ans = 0;
    Arrays.sort(a);
    for (int i = 0; i < a.length; i++) {
        ans = (ans + (long)a[i] * c - a[a.length-1-i] * c) % mod;
        c = (c << 1) % mod;
    }
    return (int)ans;
}
```
