Mathematical Applications in BINGO: The Number Board Game

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ABSTRACT:

The project implements a single player game named "BINGO". This game is a number board game between the player and the CPU with certain rules to be satisfied in order to win the game. The description of the game is such that the player decides a linear function. This linear function is used to form a matrix with unique elements in it. The value of the variable used in the function is the value in the default matrix. The values of random variable are taken from default matrix and the value of output random variable is substituted in the respective position after applying modular arithmetic operation on it. This makes sure that the numbers are just scrambled in the default matrix. When the matrix is ready for the player, then the player can begin the game. The CPU also generates a matrix for itself. The game is played with these matrices which will be explained in the following topics. We developed this game using Python IDE. The mathematical concepts that we have used in this project is discussed in this report.

I. INTRODUCTION

The number board game involves of various forms like playing with few piece of blocks on the board. There are many varieties of number board game this may differ from having a particular pattern to play the game or maybe just some number required to play the game. Each game differ from the set of rules. But all these board games are related to each other by the way you play the game and the approach to the game. Each tactic used to solve a puzzle involves the application of the mathematics. The involvement of mathematics is the common link for most of the number board game as the games are dealt with numbers and every logic being solved involves the application of mathematics in it.

Bingo is one such number game designed based upon the application of discrete mathematics. The board itself represents the application of Matrix and the formation of matrix involves other parts of mathematics dealing with linear function, greatest common divisor and to solve the board game there is always probability involved in it.

The Design of the game is presented using a programming language called python and has been implemented.

II. HOW TO PLAY BINGO-INSTRUCTIONS

The "BINGO" game is played between 2 or more players. Each player has to draw a table with 'n' number of rows and 'n' number of columns (where 'n' is any odd real number starting from 3). The most common table formation is with 5 rows and 5 columns. Each player will be filling the table of 25 boxes (5 X5) with 25 numbers starting from 1 to 25, in a random manner.

5*5 matrix

The numbers can be filled in any form, so that each box contains one number. The following figure shows an example

1	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

BINGO NUMBER BOARD

The Traditional way of the game was such that both the players start with a pen and paper. Then they both draw a random scrambled table of numbers without each other knowing what are the positions of that particular table they have made. Then one of them starts telling the numbers one after other and the game continues until someone says BINGO. The rules of deciding a player won or not are described below.

After each player completes forming this box, the game starts. First a player say player 'X' reads out a number. Each player would strike out that particular number on the table. Let us consider player 'X' told number 5, that particular number would be striken off,

1	18	11	13	X
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

Next player 'Y" reads out another non striken number and both players strikes that particular number on their respective table. The game continues this way, till someone wins

A. What Determines Winning and Losing

A line is considered to be striken completely if player has all the numbers of that line striken off. The line can be a vertical one or horizontal or a diagonal one, but it has to be straight.

1	18	11	13	X
16	2	19	7	X
8	17	20	4	X
9	3	10	12	X
15	21	22	23	X

Once the player strikes off any of 5 combinations of the lines then the player wins. 5 combinations represent the 5 letters in the word BINGO. So when either of the player 'X' or player 'Y' strikes off the word BINGO first wins the game. The turns taken by each player will be one after other, there aren't any advantages such that the player can repeat their turn after getting a set completed. The number of sets stricken remains anonymous until the end of the game.

X	X	X	X	X
X	X	X	X	X
X	17	X	4	24
9	X	X	X	25
X	21	X	23	X

The above matrix shows the winning matrix. Here in this matrix the number of combinations of winning strikes are 5. Hence, the win. The winning set can be shown by the first 2 columns of the matrix. Then the 3rd set is the diagonal element from the top left to the bottom right, the 4th set is another diagonal element from the top right to the bottom left. The final set can be displayed by the middle row. So, overall there are 5 combinations.

III. Application of Matrices in BINGO

The BINGO game is a number board game. So, there must be some board to design the game. The numbers are generally evaluated in the form of list. But the numbers are available in the form of the list. These list includes the numbers starting from 1 to 25 which are supposed to arrange in a number board with the number of rows to be 5 and the number of columns as 5. So, the suitable format for this kind of arrangement is Matrix form. In Matrix the elements can be arranged in a symmetrical format and it gives the desired number board format. Like for example, let's arrange the set of numbers starting from 1 to 25 in the matrix format starting from top left corner to the bottom right corner.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

This is the number board available for the game to be played. The order of the matrix determines the number of rows and the number of columns present in it. So, in this case the number of rows and columns are 5 so the order of the matrix can be defined as 5*5 matrix, where it represents the number of rows and the number of columns respectively.

There are various applications of matrix, like linear transformation, vectors in 3 dimensional can be computed using the matrix. Here, the operations performed on the matrix are that the elements of the matrix are used as input for the linear functions.

The other operations which will be performed on the matrix along with the linear functions are GCD, modular arithmetic. The game itself works on the principle of probability such that the probability of choosing an element over other elements. Even the application of sets such that the in order to win the game there are certain sets which are supposed to filled by cancelling the elements.

Matrix can be of any order. The order of the matrix is evaluated based upon the number of the rows and the column. So, it can also be said that a single element can be represented as a matrix of order 1*1. The other matrix representation are like of order 2*3, which shows that the number of rows is 2 and the number of columns are 3. The total number of elements in the matrix can be represented by the order i.e. 2*3 order matrix has 6 elements. The matrix of order 5*5 which is designed for the game Bingo, the number of elements is 25. This is where we get to design the 25 elements in the matrix.

The matrix can also be used to map a certain element to some other element in some other matrix. This can be used as a dictionary. Like an element representing some value from some other matrix, the position of that element can be used to get the value in the corresponding matrix and this acts like a dictionary.

A square matrix is defined as the matrix which has the number of rows and the number of columns same. That is a matrix of order 3*3 is a square matrix. Every square matrix has a quantity known as determinant. Determinant is generally used to represent the coefficient in the linear systems. The inverse of the matrix is also defined based upon the determinant value. The inverse of the matrix is possible for a square matrix only when the determinant is non-zero. The inverse matrix and the matrix when multiplied together using the row and column operation the resultant is an identity matrix. The identity matrix is the matrix where the diagonal elements are 1 and the remaining all elements are zero.

IV. APPLICATION OF MODULAR ARITHEMETIC

Modular arithmetic is a system of numbers where the numbers wrap around such that the whole number system consists of only those numbers. It is defined by an integer. Modular arithmetic is mathematically expressed with congruence relation on integers. The operations that can done are addition, subtraction and multiplication.

Here we use modulo of 25. Hence the number system defined is within the limits of this number. The set can be defined as

Numbers= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 ,13 , 14, 15, 16 , 17, 18, 19, 20, 21, 22, 23, 24, 25}

Here we consider 25 = 0, as 25 mod 25 is 0. It can be denoted both as 0 and 25, we consider 25.

The examples defining the modular arithmetic are:

$26 \bmod 25 \equiv 1$

$-3 \bmod 25 \equiv 22$

This shows that any number greater than the modular value is wrapped up back to the number which is defined in the set. The application of the modular arithmetic in BINGO is that, whenever a certain number is given as input by the player and if the given input number is beyond the number system defined then this is used to bring the given input number back to the number system.

The implementation of modular arithmetic is done using the modulus operator. The Modulus operator is generally defined to get the remainder of a certain input number. So, the remainder left behind is the desired number.

V. APPLICATION OF SET THEORY

The sets are nothing but a collection of elements. The elements or objects are mostly distinct. The collection of numbers is an example for a set. E.g., {1, 2, 4, 3, 5}. We have used this concept in this project. The arrangement of numbers in the BINGO table can be interpreted in the form of sets and can be used for coding purpose. This can be explained clearly with the following figure.

1	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

A. Horizontal Rows:

The five horizontal rows containing the numbers are interpreted as 5 sets

Set1- {1, 18, 11, 13, 5},

Set2- {16, 2, 19, 7, 6},

Set3-{8, 17, 20, 4, 24},

Set4-{9, 3, 10, 12, 25},

Set5-{15, 21, 22, 23, 14}

B. Vertical Columns:

Likewise the vertical columns are 5 such sets.

Set6-{1, 16, 8, 9, 15},

Set7-{18, 2, 17, 3, 21},

Set8-{11, 19, 20, 10, 22},

Set9-{13, 7, 4, 12, 23},

Set10-{5, 6, 24, 25, 14}.

The 2 remaining diagonal sets are

Set11-{1, 2, 20, 12, 14},

Set12-{5, 7, 20, 3, 15}.

If the user selects a random number, the CPU would check the sets in which that particular number is striken off and the CPU selects the non striken number from the set that has many numbers striken off. Consider the situation as shown in the following figure.

X	18	11	X	X
16	2	19	7	6
8	17	20	4	24
9	3	X	12	25
X	21	22	23	X

The sets 1, 4 and 5 has stricken numbers in them. Each set has at least one stricken number in them .of these the set 1 contains 3 stricken numbers. Only the numbers 18 and 11 are not stricken. So the CPU would select the numbers 18 or 11 so that it can increase its chance of getting one full stricken set. Storing these values in the form of sets would help to know what all the stricken numbers are and what could be done in the next play so that a fully stricken set can be attained. The decision of selecting number 11 or 18 is decided by considering the neighboring sets as well. (i.e.) if number 11 is selected ,it would be advantageous than selecting number 18 because the number 11 already has a stricken number and selecting 11 would make it 2 thereby getting closer to attaining a full stricken set.

The set theory can be explained as the branch of mathematics that studies the sets, which in turn is the collection of objects. Set theory forms the basis of discrete mathematics. A set is specified by its numbers. The numbers of the set are written within the curly braces. The members of set can also be defined using statements. E.g. X is a member of A. There are two ways of defining sets.

(i) Intentional definition e.g. X is a member of A and

(ii) Extensional definition, which includes using curly braces.

The study of sets has many sub domains. Combinational set theory, descriptive set theory, fuzzy set theory, inner model set theory, large cardinals, determinacy, forcing, cardinal invariants, set-theoretic topology are various fields of research under set theory.

Set theory is an important tool for reasoning and formalizing about computing and the objects of computation. The logics used in computer programming and the set theory are indivisible. The fundamental ideas of computer science can be transferred to a practical one with the help of concepts like set theory. It serves as a tool to unify the disparate ideas and the notations of programming. Set theory is basically identifying objects, grouping them according to the property that is being shared between them. The union and intersection are some of the properties used in common. The intersection of two sets is nothing but the set that has elements that are in common for both sets .e.g. A={1,2,3,4,5} and B={3,4,5,6}, 'A' intersection 'B' ={3,4,5}. Another important property is the union .the union of two sets includes all the elements from the sets. E.g. A={1,2,3,4,5} and B={6,7,8} . A union B = {1, 2, 3, 4, 5, 6}. These two properties are extensively in every logical operations.

The Bingo uses any of the 5 sets to win. So, this introduces the concept of combinatorics. The number of ways where 5 sets are to be selected from a total of 12 sets. So, the number of ways in which it can be done is 5/12. This is same for both the players. So the game is even on both sides that both the players have equal probability of winning unlike few games where the player to begin first has the advantage or vice versa.

VI. APPLICATION OF LINEAR FUNCTIONS

The linear functions are also called as matrix generating function. The function used is a linear function with a single variable 'x' and two constants b and a. It is of the form of an equation y= (ax+b)%25. This function is used as a substitution function, used to substitute the default matrix with newly generated matrix where each element is mapped to a numerical equivalent. The constants 'b' and 'a' are selected based upon certain conditions such that the resultant y is in the range of {1, 2... 25}. The range is defined based upon the matrix used. As the matrix is of order 5*5, the number of elements in the matrix are 25.

The constant 'a' is selected such that the GCD (a, 25) is 1. This can also be said that the constant 25 and 'a' are coprime. The reason that 'a' has to be a coprime of 25 is that every time 'a' multiplies an element in the default matrix it maps it to a new element in the range, but if it multiplies it with some factor of 25, then every multiple of the factor generates the same number.

So, the number ‘a’ selected such that ‘a’ & 25 are coprime.

The Default matrix is chosen because it can used as common beginning for both the players. Both the players have the equal chance of beginning the game and both form the scrambled matrix using the default matrix. The Default matrix could the just numbers in increasing order starting from top left corner to the bottom right corner could be from left to right or top to bottom. The default matrix used in here is the top to bottom, starting from top left to the bottom right of the matrix defined for the game.

Now for example, there is a 5*5 matrix with a default matrix arrangement. The matrix generating function can be used to scramble the default matrix elements. The default matrix can be represented as made from following elements in order:

default matrix = [1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]

Default matrix=

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Now, let’s generate a function with a=2 (GCD (2, 25) =1) and b= 5. As the number of elements in the 5*5 matrix is 25, so a modulus 25 is applied on the function.

The generated matrix after using the matrix function is calculated as follows,

a=2, b=5

y= (2x+5) %25

For x=1 to 25

y= (2(1) +5) %25= 7%25=7
y= (2(2) +5) %25=9%25=9
y= (2(3) +5) %25=11%25=11
y= (2(4) +5) %25=13%25=13
y= (2(5) +5) %25=15%25=15
y= (2(6) +5) %25= 17%25=17
y= (2(7) +5) %25= 19%25=19
y= (2(8) +5) %25= 21%25= 21
y= (2(9) +5) %25= 23%25= 23
y= (2(10) +5) %25= 25%25 = 25
y= (2(11) +5) %25= 27%25= 2
y= (2(12) +5) %25= 29%25= 4
y= (2(13) +5) %25=31%25= 6
y= (2(14) +5) %25=33%25= 8
y= (2(15) +5) %25=35%25= 10
y= (2(16) +5) %25=37%25= 12
y= (2(17) +5) %25=39%25= 14
y= (2(18) +5) %25=41%25= 16
y= (2(19) +5) %25=43%25= 18
y= (2(20) +5) %25=45%25= 20
y= (2(21) +5) %25=47%25= 22
y= (2(22) +5) %25=49%25= 24
y= (2(23) +5) %25=51%25= 1
y= (2(24) +5) %25=53%25= 3
y= (2(25) +5) %25=55%25= 5

So, the newly generated matrix will be of form,

matrix = [7 , 9 , 11, 13, 15, 17, 19, 21, 23, 25, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 1, 3, 5] =

7	9	11	13	15
17	19	21	23	25
2	4	6	8	10
12	14	16	18	20
22	24	1	3	5

This function basically scrambled the elements in the default matrix and generated a random matrix just using two variables. This function is invertible. The default matrix can be obtained by using the inverse of the function used. The inverse of the matrix can be calculated as,

y=ax+b

$x=a^{-1}(y-b)$

Here a^{-1} is selected such that $aa^{-1}=1$ %9. Hence, it shows that the function maps each unique element to another unique element which shows that it is both ONE-TO-ONE and ONTO function. A function which is both one to one and onto is invertible and hence the reverse mapping is possible.

ONE to ONE function is defined as relation where one element in one set maps to one element in another set. This is like point to point mapping. This includes the same number of elements in the domain and the range. The one to one type of functions have a certain set of input called as domain mapping to the certain set of outputs called as range. These elements in domain and range are equal in number, along with a property that the each element in the domain map to some individual element to range. ONTO function are defined as the property of the function where every element in range are mapped to some or other element in the domain. That is there is no orphaned element in the range such that it is not mapped to some element in the domain. Together these properties both one to one and the onto of functions prove that the function is invertible. The inverse of the function defines that the element can be mapped back from the range to the domain. Here the range becomes the domain and the domain becomes the range. The original input can be traced back from the output.
Here if the inverse is evaluated then the resultant value will be the defined default matrix. The matrix which was used to form the scrambled matrix. So, the elements in default matrix become the elements of domain and the scrambled matrix elements become the range. When done in reverse, the elements of the scrambled matric become the domain and the elements of the default matrix become the range.

VII. APPLICATIONS OF PROBABILITY

The probability is another concept that has been used for selecting the numbers for striking. Consider the case in which two sets has equal number of stricken numbers. The CPU has to select one number from one set and that selection has to be advantageous for winning the game. The probability of a number being advantageous is determined and selected by the CPU for striking. For e.g. the following figure has two sets of numbers with 2 numbers stricken in each set. Technically selecting a number from either of the set is the same but logically selecting number 25 from set 5 would be advantageous as it would simultaneously add a stricken number in the horizontal set.

13	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

A. Test Case I

Consider the following table with the stricken numbers Highlighted with red color.

12	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

In this case, selecting the number 1 has a higher probability of making the game advantageous the CPU. This is because it is present at the junction of a row and a column and it is accessible to both the row and column. The row has already 2 stricken numbers and the column also has two stricken numbers, selecting the number 1 would add a stricken number to both row and column, whereas selecting other numbers like 18, 11, 16 or 8 would add one stricken number either to the row or column and the CPU would need another round to add one more stricken number. In this way the probability of number is calculated according to its position and the stricken numbers present around it.

B. Test Case II

Consider the following test case

11	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

In this case, the numbers 22 and 25 have a higher probability of making the game advantageous to the CPU. Of these numbers 25 has higher probability than 22. Selecting number 25 would add a stricken number to column 4 and row 5. Also it would make those rows and columns 2 numbers less for a completely stricken number.

C. Test Case III

Consider the case at the later stage of the game.

11	18	11	13	5
16	2	19	7	6
8	17	20	4	24
9	3	10	12	25
15	21	22	23	14

Here, this represents the game in later stage. The probable case would be selecting the number 20 or the number 25.
Both the number results in the formation of a complete set. We can assume that both the numbers are equally probable. Selecting any number in this case it won't matter if you select either the number 20 or the number 25. But now we better look at the better choice because this is later stage of the game where even player 2 would be at the verge of winning the game, the player 2 might need just one more number to complete a set and finish the bingo and at the same time even if the number benefits us by completing a set and not helping in finishing the total number of set then it would be waste that the number didn't help you instead helped the opponent. So, let's look at the matrix again. Here the number 20 shows that once it is strike then we will get 2 diagonal elements at the same time. Which would help to complete 2 sets at a time instead of just one set using the number 25. Hence it is more likely to win in case of 20 than that of 25. So, the probability of selecting the number 20 is higher than selecting the number 20.
Probability is the measure of how likely an event would occur, given the circumstances. The higher the probability of the event, the higher the event would occur. It deals with the outcomes of the experiment by considering the circumstances. The set of all outcomes of an experiment is called the sample space and it contains all the possible results that could be the outcome of that particular experiment. An event is a subset of a sample space. The probability of an event can be calculated in many ways and the outcome can be judged before the event occurs. If the events are unlikely then the way in which the probability is calculated varies. This type is called as the conditional probability. Also probability shares many principles with set theory. Finding the probability of an event would help us to decide the occurrence of the other events in future. The probability of events are calculated by taking all the circumstances into consideration. The probability is calculated by dividing the occurrence of the event by sample space.
The permutation concept can also be introduced in the BINGO. The number of matrix which can be made using 25 different elements in 25 different position is 25!

VIII. IMPLEMENTATION OF BINGO

The game is implemented using a programming language. The programming language used is Python on the IDLE interface. Python is a scripting language which is helpful in all the mathematical applications. Even the numbers in sets can be dealt with on of the important feature of python known as list. The list is used to store all the elements in it and the operations can be performed accordingly.

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