6th South African Regional
ACM Collegiate Programming Competition

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Problem F – Green balloon
Rendering Fractal Images

Fractal images are a special class of images that posses some interesting properties. One of the defining characteristics of fractal images is that they are self-similar.

Self-similarity can be illustrated by looking at the coastline of a continent. If one considers what the coastline looks like on a map showing a stretch of the coastline several hundreds of kilometers in length, then one gets a sense of the "roughness" of the coastline. If you were to zoom in (by a factor of 2 or so), then one is likely to see similar features, resulting in a comparable impression of the "roughness" of the coastline. With a real coastline, the illusion of self-similarity is destroyed if you continue to zoom in on the map, as the coastline gradually becomes smoother.

There exists a class of mathematically-constructed images that exhibit self-similarity on all scales of magnification. One of the best-known fractal images was discovered by Benoit Mandelbrot, and is thus called the Mandelbrot set. An approximation of the points belonging to the Mandelbrot set can be rendered to produce beautiful pictures. Your task will be to render some of these images in ASCII text.

The Mandelbrot set lives in the complex number plane, so all capitalised variables below will denote complex numbers.

Let Z_n represent element n of a Mandelbrot sequence. Then we define

$$Z_{n+1} = Z_n^{\ 2} + C,$$

where $Z_0 = 0 + 0i$ (i represents the square root of -1), and C represents an arbitrary constant value on the complex plane.

The Mandelbrot set is then defined as the set of all starting points C (complex numbers) such that $|Z_n| < 2.0$ for all n. (Note that |Z| denotes the magnitude of the complex number Z, in other words, the square root of the sum of the square of the real component and the square of the imaginary component).

The set of points C belonging to the Mandelbrot set are said to *converge*. All points outside the set cause Z_n to diverge as n approaches infinity.

For example, let $C = 0 + 0i$. Note that $Z_n = 0 + 0i$ for all values of n, thus $(0 + 0i)$ belongs to the Mandelbrot set.

On the other hand, the starting value $C = 1 + 1i$ does not lead to a convergent Z_n sequence. Here's the first few values:

$$Z_0 = 0 + 0i$$
$$Z_1 = (0 + 0i)^2 + (1 + 1i) = 1 + 1i$$
$$Z_2 = (1 + 1i)^2 + (1 + 1i) = 1 + 3i$$

Note that $|Z_2| = 3.1623$, so that after two iterations, we can already see that the sequence starting at $C = 1 + 1i$ does not belong to the Mandelbrot set (because $|Z_2| > 2$).

Note that it is possible to identify some points C *outside* the Mandelbrot set quickly, because $|Z_n|$ will exceeded 2 after only a few iterations.

An approximation of the Mandelbrot set can be rendered for a region of the complex plane by calculating the first k elements of the sequence Z_n for each of the points in the region (by choosing those points as C values). If $|Z_n| < 2$ after k iterations, that point can be labeled as belonging to the Mandelbrot set. A discrete picture can be formed by evaluating the Mandelbrot sequence for each point on a grid defined in such a region.

Input

You will be provided with the dimensions of the desired output image, as well as four real values denoting the region of the complex plane that you must render. Your input will thus consist of records of the following form:

<rows> <columns> <left> <top> <right> <bottom>

The *rows* and *columns* fields denote the number of rows and columns that your output image must occupy.

The *left*, *top*, *right* and *bottom* fields denote the boundaries of the region of the complex plane that you must generate the Mandelbrot set for.

Your input may contain any number of records formatted as described above; you must render the image for each record specified.

Output

Your output will be an ASCII image of the Mandelbrot set for the specified region of the complex plane. If a point is found to be in the Mandelbrot set after 2048 iterations of the generating equation (thus the value $|Z_{2047}| < 2$), you must output a space character. For all points not belonging to the Mandelbrot set you must print a '*' character.

Sample Input

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31 60 -2 1 1 -1
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Sample Output

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