MA311 (Scientific computing)-IITG

04-10-18

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1. Use Euler's method to approximate the solutions for each of the following initial-value problems.

   a. $y' = y/t - (y/t)^2, \quad 1 \le t \le 2, \quad y(1) = 1,$ with $h = 0.1$
   b. $y' = 1 + y/t + (y/t)^2, \quad 1 \le t \le 3, \quad y(1) = 0,$ with $h = 0.2$
   c. $y' = -(y+1)(y+3), \quad 0 \le t \le 2, \quad y(0) = -2,$ with $h = 0.2$
   d. $y' = -5y + 5t^2 + 2t, \quad 0 \le t \le 1, \quad y(0) = \frac{1}{3},$ with $h = 0.1$

   The exact solutions to the corresponding IVP are obtained as;

   a. $y(t) = \dfrac{t}{1 + \ln t}$ **b.** $y(t) = t\tan(\ln t)$
   c. $y(t) = -3 + \dfrac{2}{1 + e^{-2t}}$ **d.** $y(t) = t^2 + \dfrac{1}{3}e^{-5t}$

2. Use the result of question 1 and linear interpolation to approximate the following values of $y(t)$. Compare the approximations obtained to the actual values obtained using the exact solutions given above.

   a. $y(1.25)$ and $y(1.93)$ **b.** $y(2.1)$ and $y(2.75)$
   c. $y(1.3)$ and $y(1.93)$ **d.** $y(0.54)$ and $y(0.94)$

3. For each case in question 1 compute the approximate order of convergence and draw the h vs error plot in log-log scale. Also find the optimum value of h after which the error blows up.