MA311 (Scientific computing)-IITG

1-11-18

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1. Use the Forward-Difference method to approximate the solution to the following parabolic partial differential equations.

a. $\dfrac{\partial u}{\partial t} - \frac{4}{\pi^2}\dfrac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 4,\ 0 < t;$

$$u(0,t) = u(4,t) = 0, \quad 0 < t,$$

$$u(x,0) = \sin\tfrac{\pi}{4}x\left(1 + 2\cos\tfrac{\pi}{4}x\right), \quad 0 \le x \le 4.$$

Use $\Delta x = 0.2$ and $\Delta t = 0.04$, and compare your results at $t = 0.4$ to the actual solution $u(x,t) = e^{-t}\sin\frac{\pi}{2}x + e^{-t/4}\sin\frac{\pi}{4}x$.

2. Repeat question number 1 with Backward difference scheme and Crank-Nicolson Scheme.

Note: Tabulate x_i, approximated solution at (x_i, t) and the corresponding errors at $t = 0.4$. Also you need to plot the approximated solution at $t = 0.4$, and compare it with the exact solution. Do a comparison of solutions obtained from three different schemes in a single plot.