MATH 239 Assignment 10

This assignment is for practice only, and is not to be handed in.

1. Use the bipartite matching algorithm to find a maximum matching and a minimum cover in the graph in Figure 1.

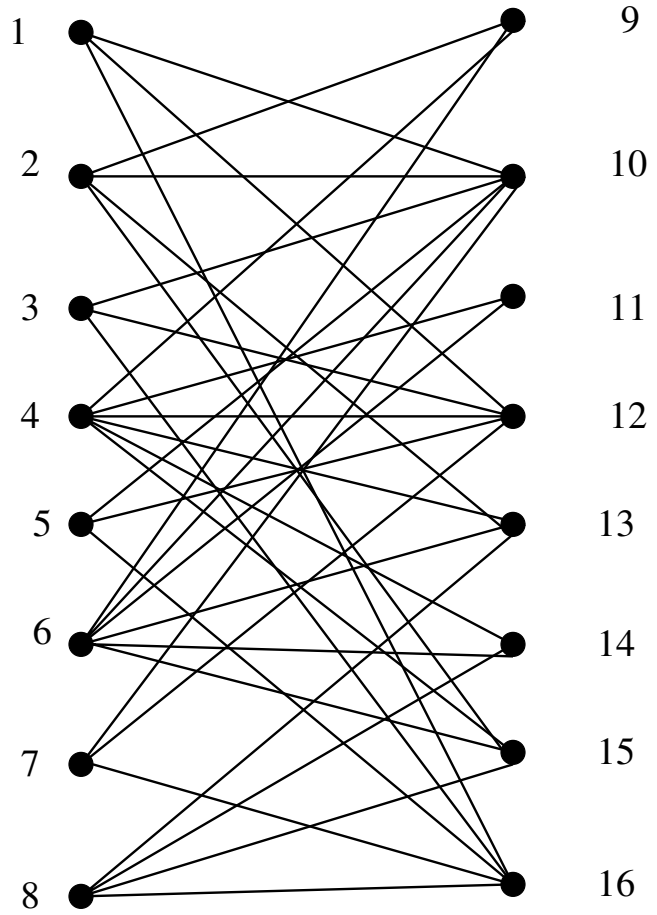

Figure 1:

2. Find a subset D of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ such that $|N(D)| < D$.

3. Let k be a positive integer and suppose G is a bipartite graph in which every vertex has degree precisely k. Prove that G has k perfect matchings, no two having an edge in common.

4. For each positive integer $n \geq 24$, find an example of a bipartite graph with n vertices on each side, with minimum degree at least three, and with no matching of size larger than $n/4$.

5. Let G be a graph with $2n$ vertices such that every vertex has degree at least n. Prove that G has a perfect matching.

6. Give an example of a 3-regular graph that does not have a perfect matching. (Note that such a graph cannot be bipartite.)

7. Let G be a bipartite graph with vertex classes A and B, where $|A| = |B| = 2n$. Suppose that $|N(X)| \geq |X|$ for all subsets $X \subset A$ with $|X| \leq n$, and $|N(X)| \geq |X|$ for all subsets $X \subset B$ with $|X| \leq n$. Prove that G has a perfect matching.