MATH 239 Tutorial 10 Problems

1. Prove that the 4-cube is not planar. (Find two different proofs of this statement.)

2. Let G be a 3-regular planar bipartite graph with 14 vertices. How many faces does G have? Assuming that G has faces of degrees 4 or 6, how many faces of degree 4 does G have? Draw one such graph.

3. Suppose G has a planar embedding where every face boundary is an even cycle. Prove that G is bipartite. (Hint: Pick an odd cycle that has the fewest number of faces inside of it.)

4. (a) Let G be a planar embedding where every face has degree 3. Suppose G is 3-colourable. Prove that the dual of G is 3-edge-colourable (meaning there is a 3-colouring of the edges such that all edges joining the same vertex receive different colours).

   (b) Find a 3-colouring of the octahedron. The dual of the octahedron is the cube. Find a 3-edge-colouring of the cube.

5. Suppose G is a graph (not necessarily planar) such that the edges of G can be partitioned into two bipartite subgraphs (i.e. $E(G) = A \cup B$ where $A \cap B = \emptyset$, and both $(V(G), A)$ and $(V(G), B)$ are bipartite graphs). Prove that G is 4-colourable.