

MATH 239 Tutorial 7 Problems

1. Let G be a graph with p vertices and every vertex of G has degree at least $(p-1)/2$. Prove that G is connected.
2. Let G_n be the graph where the vertices are all binary strings of length n , and two vertices are adjacent if the two strings differ in exactly 2 positions.
 - (a) Draw G_2 and G_3 .
 - (b) How many edges are in G_n ?
 - (c) For what values of n is G_n connected?
 - (d) For what values of n is G_n bipartite?
3. Prove that if every vertex of a graph G has degree at least 3, then G contains a cycle of even length.
4. How many Hamilton cycles are there in K_n where the vertices are labelled with $1, 2, \dots, n$? We consider two Hamilton cycles to be the same if they use the same set of edges.

Additional exercises

1. Let $k \geq 1$. If G is a k -regular bipartite graph with a bipartition (A, B) of the vertices, then $|A| = |B|$.
2. Determine (with proof) a bipartite graph with the fewest number of edges such that it is NOT the subgraph of any n -cube.
3. Prove that for $n \geq 2$, the n -cube has a Hamilton cycle.
4. Suppose that P and Q are two paths of maximum length in a connected graph G . Prove that there is at least one vertex that is in both P and Q .
5. Let G_n be the graph whose vertices are all permutations of $[n]$, and two vertices are adjacent if and only if one permutation can be obtained from another by swapping two entries. For example, in G_4 , (1234) is adjacent to (1324) and (1432) , but not (3142) .
 - (a) Draw G_2 and G_3 .
 - (b) How many vertices and edges are in G_n ?
 - (c) Prove that G_n is bipartite.
 - (d) Prove that G_n is connected.