MATH 239 Spring 2012: Assignment 8

Due: 9:29 AM, Friday, July 6 2012 in the dropboxes outside MC 4066

Note: There is a note.

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only): /50

Acknowledgments:

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1. {10 marks} Let G be a connected graph where each vertex has degrees 1 or 3. Let $\mathcal{X}$ be the set of vertices that have degree 1. Suppose there exists a set of edges $\mathcal{E}$ such that after removing $\mathcal{E}$ from G, each component of the remaining graph is a tree which contains exactly one vertex from $\mathcal{X}$. Determine $|\mathcal{E}|$ in terms of $|E(G)|$.

2. {15 marks}

   (a) Find a 3-regular graph with a bridge.

   (b) Prove that if every vertex of G has even degree, then G cannot have a bridge.

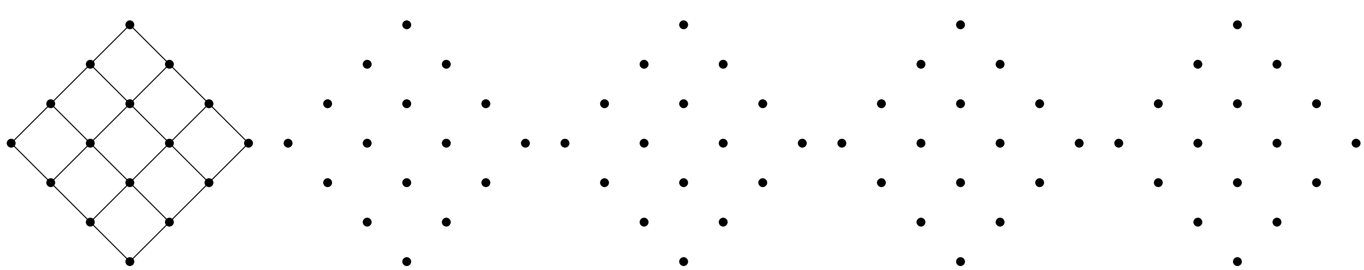
   (c) Prove that if G is a k-regular bipartite graph where $k \geq 2$, then G cannot have a bridge.

3. {15 marks} Let G be a connected graph with $2k$ odd-degree vertices, where $k \geq 1$.

(a) Prove that there exist k walks in G such that each edge of G is used in exactly one walk. (For this question, you may assume that the main theorem about Eulerian circuits is true even for graphs with multiple edges.)

(b) Prove that it is not possible to find $k-1$ walks in G such that each edge is used in exactly one walk.

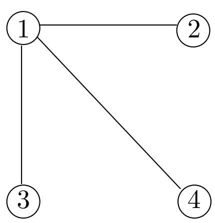
(c) Partition the edges of the leftmost graph below into as few walks as possible.

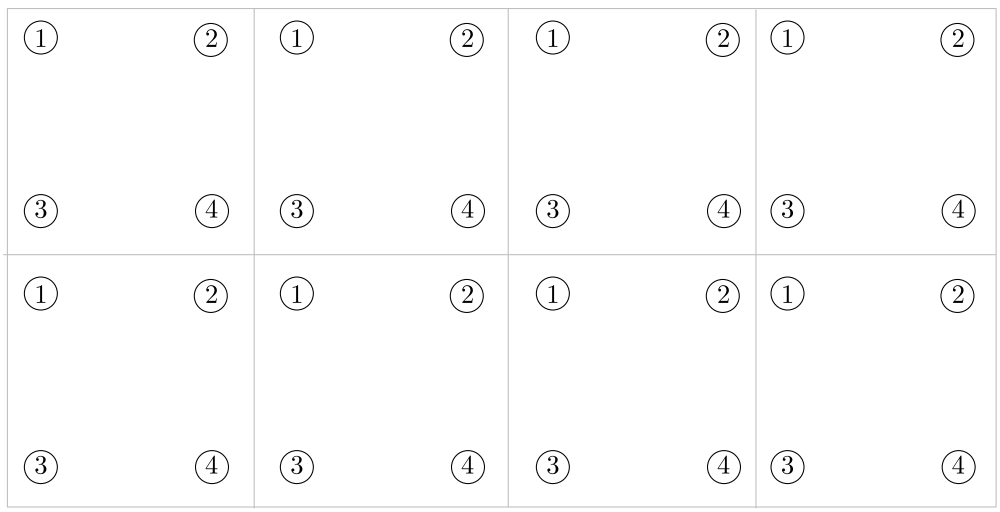


4. {10 marks} Consider the graph G_n where each vertex is a spanning tree of K_n with vertices labelled with $[n]$, and two trees T_1 and T_2 are adjacent if and only if $|E(T_1) \setminus E(T_2)| = 1$ (i.e. there is one edge in T_1 that is not in T_2).

   (a) Draw G_3.

   (b) In G_4, what are the neighbours of the tree on the left?





   (c) Prove that G_n is connected. (Hint: Use induction on $|E(T_1) \setminus E(T_2)|$.)