MATH 239 Assignment 9

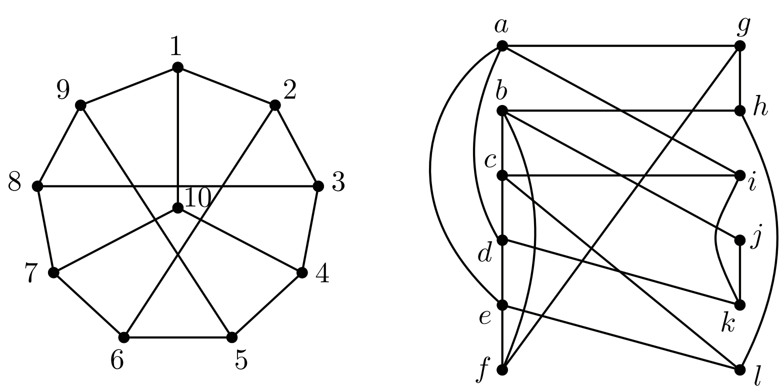
- This assignment is due on Friday, November 23rd, 2012, at 10 am in the drop boxes in St. Jerome's (section 1) or outside MC 4067 (the other two sections).
- You may collaborate with other students in the class, provided that you list your collaborators. However, you MUST write up your solutions individually. Copying from another student (or any other source) constitutes cheating and is strictly forbidden.

1. Suppose G is a 4-regular connected graph with a planar embedding such that every face has degree 3 or 4, and further that any 2 adjacent faces have different degrees.
   (a) Prove that G has no bridges and hence that every edge in G is on the boundary of 2 distinct faces.
   (b) Determine precisely the number of vertices, edges, faces of degree 3, and faces of degree 4 in G.
   (c) Draw a planar embedding of a graph having these properties.

2. Suppose G is a connected 3-regular planar graph which has a planar embedding such that every face has degree either 5 or 6. Prove that G has precisely 12 faces of degree 5.

3. Recall from assignment 6 the definition of graph complement: If G is a graph, the complement graph of G, denoted $\overline{G}$, is a graph with $V(\overline{G}) = V(G)$, and $\{u, v\} \in E(\overline{G})$ if and only if $\{u, v\} \notin E(G)$. Suppose G and $\overline{G}$ are both connected and have $p \geq 11$ vertices. Prove that at least one of G or $\overline{G}$ is not planar.

4. For each of G and H below, either give a planar embedding of the graph, or use Kuratowski's Theorem to prove that none exist.



G H

5. (a) Suppose G is a connected planar graph having girth at least 6. Prove that G has at least one vertex with degree at most 2.
   (b) Prove that all connected planar graphs with girth at least 6 are 3-colourable.