MATH 239 Assignment 6

- This assignment is due on Friday, March 1.

1. For $n \geq r \geq 1$, define the graph $G_{n,r}$ as follows: The vertices of $G_{n,r}$ are r-element subsets of $\{1, ..., n\}$. Two vertices U and V are adjacent if and only if $U \cap V \neq \emptyset$.

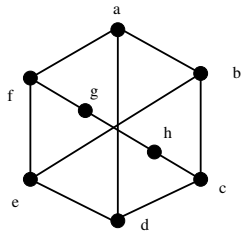
   (a) Draw $G_{4,1}$ and $G_{4,2}$.

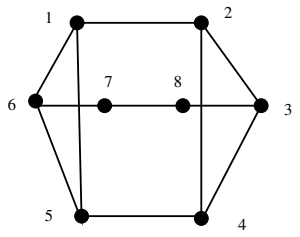
   (b) Determine how many vertices $G_{n,r}$ has.

   (c) Prove that $G_{n,r}$ is k-regular for each $n \geq r \geq 1$, and determine k in terms of n and r.

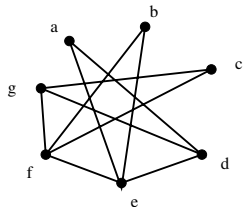
   (d) Determine how many edges $G_{n,r}$ has.

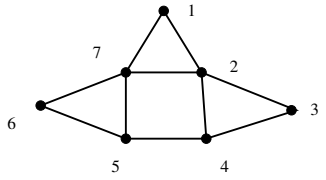
2. For the two graphs shown below, determine whether they are isomorphic. If they are isomorphic, find an explicit isomorphism and verify that it is an isomorphism. If they are not isomorphic, give a proof that they are not.





3. For the two graphs shown below, determine whether they are isomorphic. If they are isomorphic, find an explicit isomorphism and verify that it is an isomorphism. If they are not isomorphic, give a proof that they are not.





4. For a graph G, we define the complement graph of G, denoted $\overline{G}$, with $V(\overline{G}) = V(G)$, and $\{u, v\} \in E(\overline{G})$ if and only if $\{u, v\} \notin E(G)$.

   (a) Let G be the graph with vertex set $\{1, 2, 3, 4, 5\}$ and edge set $\{\{i, j\} : j \equiv i + 1 \mod 5\}$. Draw G and $\overline{G}$.

   (b) For a graph G, the *degree sequence* of G is a list $d_1, \ldots, d_p$ of the degrees of all the vertices of G, written in nondecreasing order. For example, the degree sequence of the graph G in part (a) is $2, 2, 2, 2, 2$. Prove that $d_1, \ldots, d_p$ is the degree sequence of a graph if and only if $p - d_p - 1, \ldots, p - d_1 - 1$ is the degree sequence of a graph.

   (c) Prove that $4, 4, 4, 4, 8, 8, 8, 8, 8, 8$ is not the degree sequence of a graph.

5. Let G be a graph with at least two vertices. Prove that G has two vertices of the same degree.