MATH 239 Spring 2012: Assignment 5

Due: 9:29 AM, Friday, June 8 2012 in the dropboxes outside MC 4066

Note: Again, the weight of a string is its weight. For systems of equations, once you set up the equations properly, you don't need to show the work on how you solve them. You also do not need to show how you obtain the roots of a polynomial.

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only): /50

Acknowledgments:

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1. {8 marks} A binary string is a palindrome if it reads the same forwards and backwards. Examples of palindromes include 01100110, 11011, 1. Let P be the set of all binary strings that are palindromes. Determine a recursive definition of P, and use it to find the generating series for P.

2. {12 marks} Let S be the set of binary strings which do not contain 01010 as a substring. Let T be the set of binary strings that have exactly one copy of 01010 at the right end. Two (incomplete) recursive relations between the two sets are

$$\begin{aligned}\{\varepsilon\} \cup S\{0,1\} &= S \cup T \\ S\{01010\} &= \dots\dots\dots\end{aligned}$$

(a) Complete the second equation in terms of T and justify this equality.

(b) Determine the generating series for S.

3. {20 marks}

   (a) Let $\{a_n\}$ be the sequence which satsifies

   $$a_n - 2a_{n-1} - 7a_{n-2} - 4a_{n-3} = 0$$

   for $n \geq 3$ with initial conditions $a_0 = 3, a_1 = 7, a_2 = 8$. Determine an explicit formula for a_n.

(b) Let $\{b_n\}$ be the sequence which satisfies

$$b_n - 2b_{n-1} - 7b_{n-2} - 4b_{n-3} = 3 \cdot (-2)^{n-1}$$

for $n \geq 3$ with initial conditions $b_0 = 1, b_1 = 9, b_2 = 4$. Determine an explicit formula for b_n. (Note: This recurrence is similar to the one in part (a).)

4. {5 marks} Let $\{a_n\}$ be the sequence where $a_n = (n^2 - 1)(-2)^n + 3^{n+1}$. Determine a homogeneous recurrence that a_n satsifies, together with sufficient initial conditions.

5. {5 marks} In assignment 4, we have an unambiguous decomposition for the set of binary strings that begin with 1 and whose values as binary representations are multiples of 3:

$$(1(0\{1\}^*0)^*1\{0\}^*)^*$$

In particular, this set includes no string of length 1 and 1 string of length 2. For $n \geq 1$, determine the number of such binary strings of length n as an explicit formula. (Your answer should make sense in some way.)