MATH 239 Assignment 4

- This assignment is due on Friday, October 12, 2012, at 10am in the drop boxes in St. Jerome's (section 1) or outside MC 4067 (the other two sections).
- You may collaborate with other students in the class, provided that you list your collaborators. However, you MUST write up your solutions individually. Copying from another student (or any other source) constitutes cheating and is strictly forbidden.

1. Let S be the set of binary strings that do not contain the substring 0111.
   (a) Give an unambiguous decomposition for S, and explain why it is unambiguous.
   (b) Find the generating series for S with respect to length. Indicate wherever you use results such as the Product Lemma. Express your answer as a rational function (i.e. in the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials).

2. Let S' be the set of binary strings that do not contain the substring 11, in which every block of 0's is of even length.
   (a) Give an unambiguous decomposition for S', and explain why it is unambiguous.
   (b) Find the generating series for S' with respect to length. Indicate wherever you use results such as the Product Lemma. Express your answer as a rational function.

3. Let S'' be the set of binary strings that are empty or end with a 0, in which each block of 1's has odd length, and each block of 0's that is preceded by (at least one) 1 has length exactly two.
   (a) Give an unambiguous decomposition for S'', and explain why it is unambiguous.
   (b) Find the generating series for S'' with respect to length. Indicate wherever you use results such as the Product Lemma. Express your answer as a rational function.

4. Let S' and S'' be as in Questions 2 and 3. Let $S'(n)$ denote the set of strings in S' of length n, and similarly $S''(n)$ denote the set of strings in S'' of length n. Prove that for every $n \geq 0$ there is a bijection between $S'(n)$ and $S''(n)$. (Hint: you do not necessarily have to find the bijection explicitly.)

5. Let S be the set of all binary strings that are either empty or begin with a block of 1's and end with a block of 0's.

(a) Give a recursive decomposition for S, that is an unambiguous expression. Explain why your decomposition is unambiguous.

(b) Use the decomposition in (a) to find the generating series for S with respect to length.

(c) Find the number of strings in S of length n (as an explicit closed-form expression in terms of n).

6. For each of the following sets A, either prove that A^* is unambiguous, or give an example to show that A^* is ambiguous. (Hint: one way to prove an expression is unambiguous is by induction on length of strings.)

   (a) $A = \{1, 100, 110011\}$

   (b) $A = \{110, 001, 0001\}$.