1. (a) Let k be a given positive integer. Find the generating function for the number of compositions of n having k parts, where each part is an odd positive integer.

   (b) **[6 marks]** Find the generating function for the number of compositions of n with an odd number of parts, where each part is congruent to $2 \pmod 4$.

2. (a) [**3 marks**] Write down a decomposition that uniquely creates the set of binary strings in which every block of 0's has odd length and every block of 1's has length greater than 2.

   (b) [**3 marks**] Write down a decomposition that uniquely creates the set of binary strings that do not contain the substring 111000.

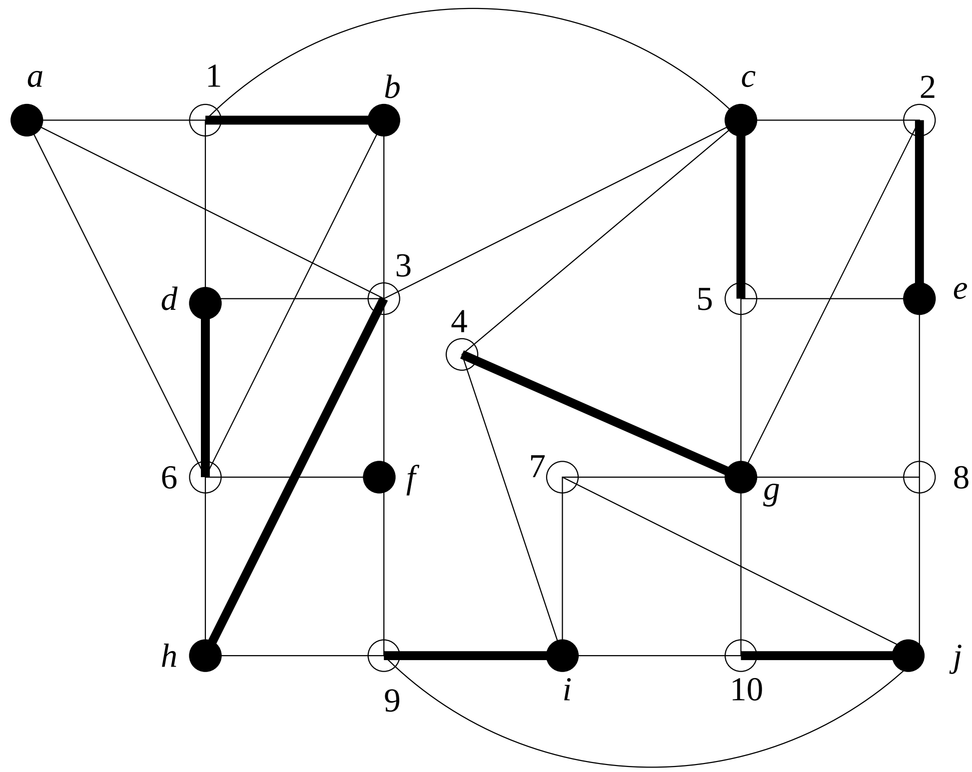
3. **[8 marks]** As usual, let the weight of a binary string be its length. Determine the generating function for the set of binary strings in which a block of 1's is never followed by a block of 0's of the same length.

4. (a) [**2 marks**] Define a *bridge* in a graph.

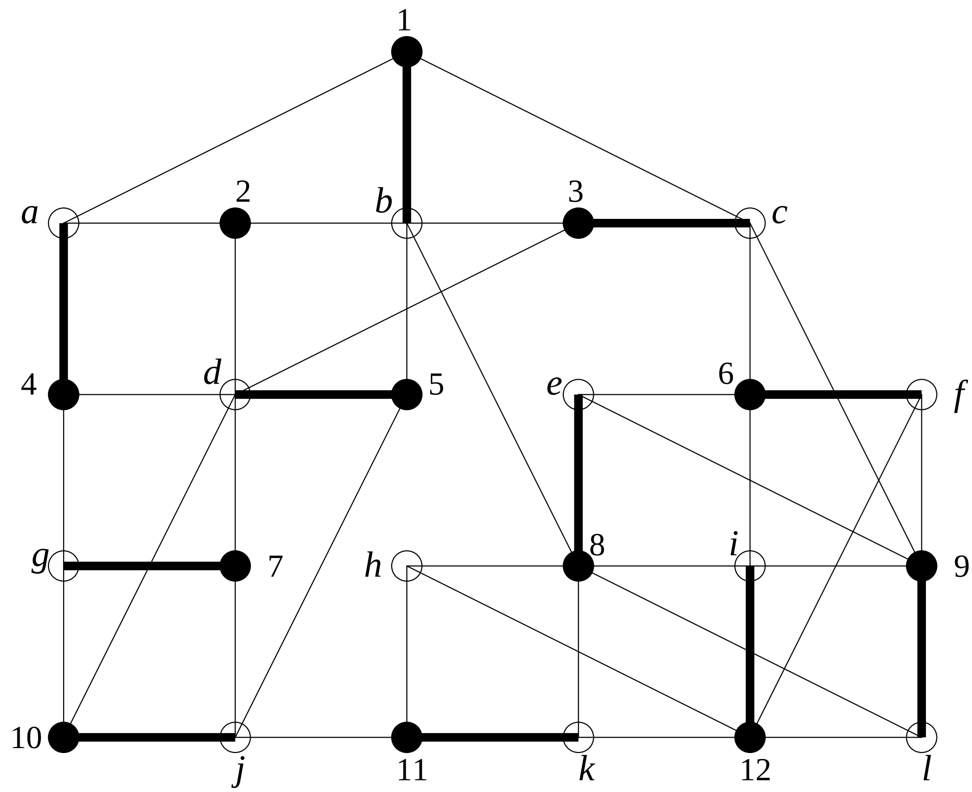
   (b) [**5 marks**] Show that if $e = \{x, y\}$ is a bridge of a connected graph G, then the graph we get by deleting e from G has exactly two components.

   (c) [**5 marks**] A graph is a *forest* if it has no cycles. Prove that a forest with v vertices and e edges has exactly $v - e$ components.

5. (a) **[6 marks]** Let G be the graph shown below. Let $A = \{a, b, \ldots, j\}$ and let $B = \{1, 2, \ldots, 10\}$. The thick edges form a matching M. Use the algorithm from the lectures to construct a matching with one more edge than M.



(b) [**6 marks**] Let H be the graph shown below. Let $A = \{1, 2, \ldots, 12\}$ and let $B = \{a, b, \ldots, l\}$. The thick edges form a matching M. Use the algorithm from the lectures to find a cover C such that $|C| = |M|$.



6. Let G be a planar embedding of a simple graph with v vertices, e edges and f faces.

   (a) [**3 marks**] State Euler's formula.

   (b) [**8 marks**] If G is connected, every vertex has degree 3, and all faces have degree five or six, show that G has exactly 12 faces of degree five.

7. **[10 marks]** Prove that a simple planar graph is 6-colourable.

8. (a) [**3 marks**] State Kuratowski's theorem.

(b) [**8 marks**] Draw a planar embedding of the graph below, or prove that it is not planar.