MATH 239 Winter 2013
Assignment 5

Due Friday, February 15, 10am.

1. Let
$$A(x) = \frac{2x^2 - 16x + 27}{x^3 - 3x^2 + 4}.$$
Find an expression for $[x^n]A(x)$ as a function of n where $A(x)$ is to be considered a formal power series.

2. Consider the homogeneous recurrence relation
$$c_n - 13c_{n-2} - 12c_{n-3} = 0 \text{ for } n \geq 3.$$
   (a) Write down a formula for c_n in terms of n, c_0, c_1, and c_2.
   (b) Write down the solution satisfying $c_0 = 0, c_1 = 0, c_2 = 0$.
   (c) Write down the solution satisfying $c_0 = 35, c_1 = 0, c_2 = 0$.
   (d) Write down the solution to the non-homogeneous recurrence relation
$$b_n - 13b_{n-2} - 12b_{n-3} = 24 \text{ for } n \geq 3.$$
   satisfying $b_0 = 34, b_1 = -1, b_2 = -1$.

3. Consider the homogeneous recurrence relation
$$c_n + 4c_{n-1} + 3c_{n-2} = 0 \text{ for } n \geq 2.$$
   (a) Write down a formula for c_n in terms of n, c_0, and c_1.
   (b) For the solution found in (a), define the formal power series $C(x)$ by
$$C(x) = \sum_{n \geq 0} c_n x^n.$$
   Find polynomials $P(x), Q(x)$ with $\deg Q = 2$ such that
$$C(x) = \frac{P(x)}{Q(x)}.$$
   Express the coefficients of $P(x)$ in terms of c_0, c_1.

4. Consider the two sequences $\{a_n\}, \{b_n\}$ given by
$$\begin{aligned} a_n &= n3^n - 6 \\ b_n &= n3^n - 6 + 4n \end{aligned}$$
   (a) Derive a third order homogeneous recurrence relation that a_n satisfies.
   (b) Write down a third order non-homogeneous reoccurrence relation that b_n satisfies whose left-hand side is the same as that found in (a).