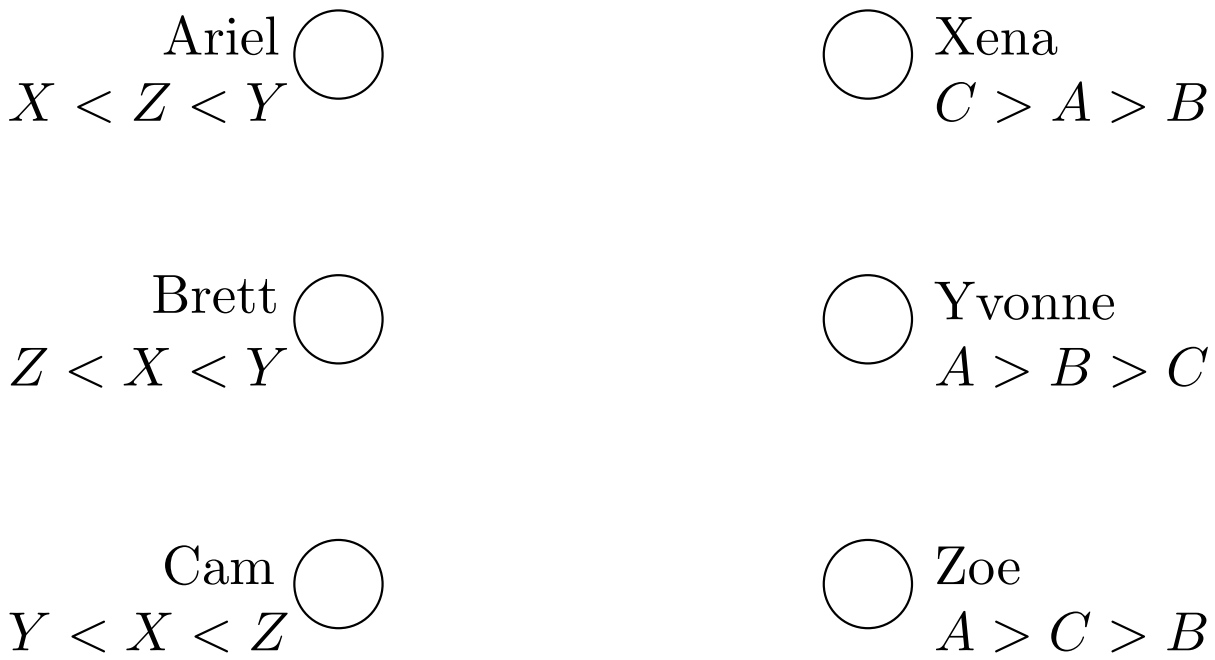
Stable marriage

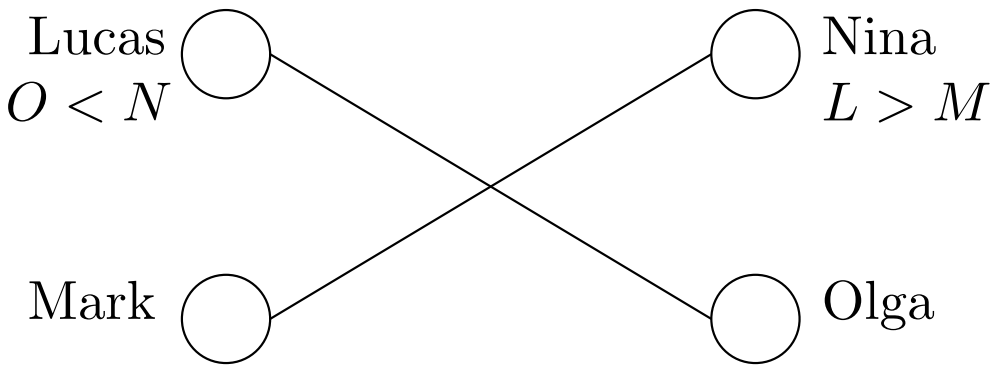
Given n men and n women, each rank the n members of the opposite gender.



Goal: Pair up the men and women so that there is no "jealousy."

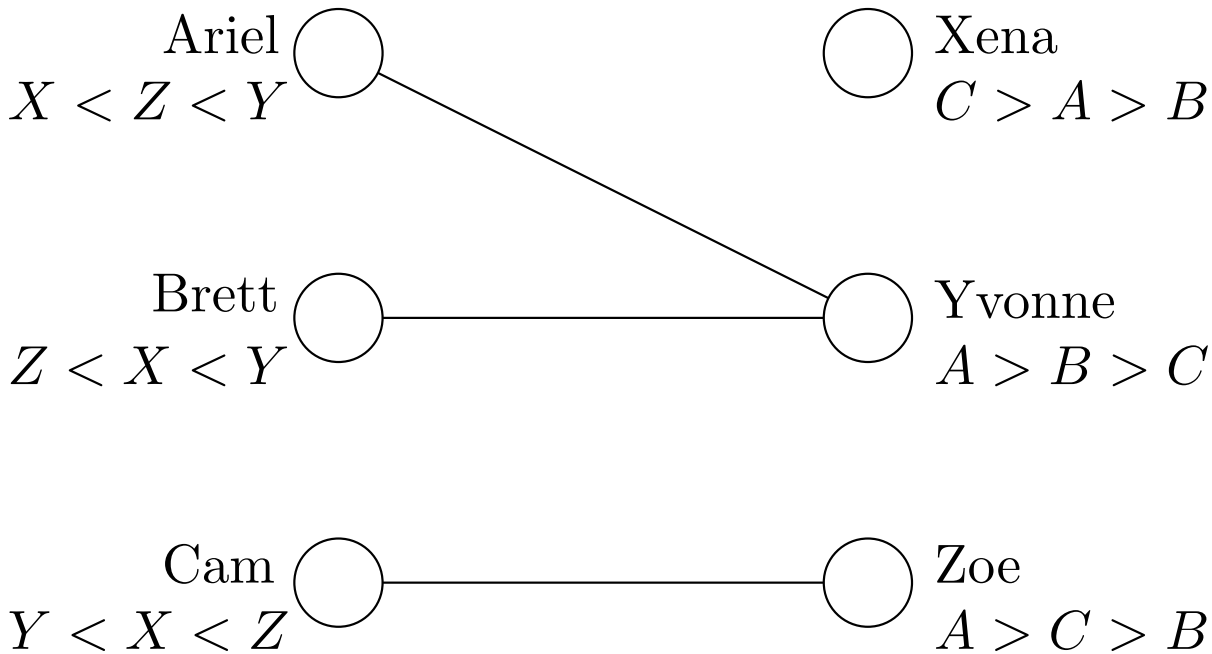
Stable marriage

Jealousy occurs when a pair both prefer each other over their current partner.



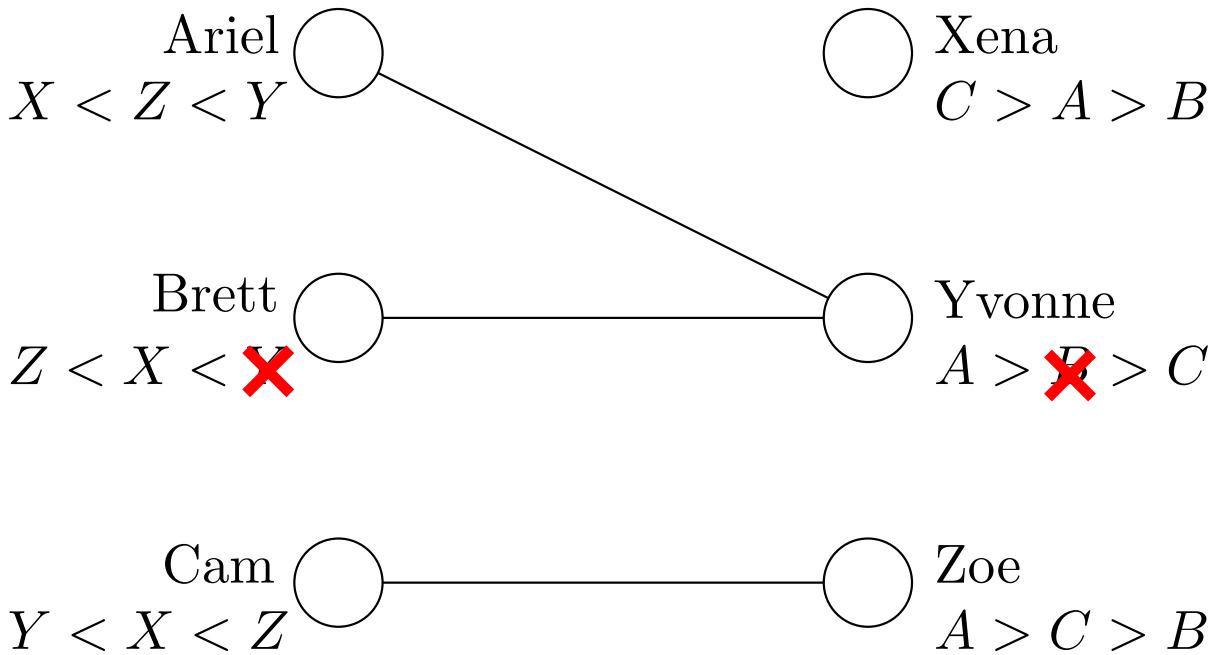
Stable marriage: Algorithm

Proposal: Each man proposes to the highest-ranking woman who has not rejected him.



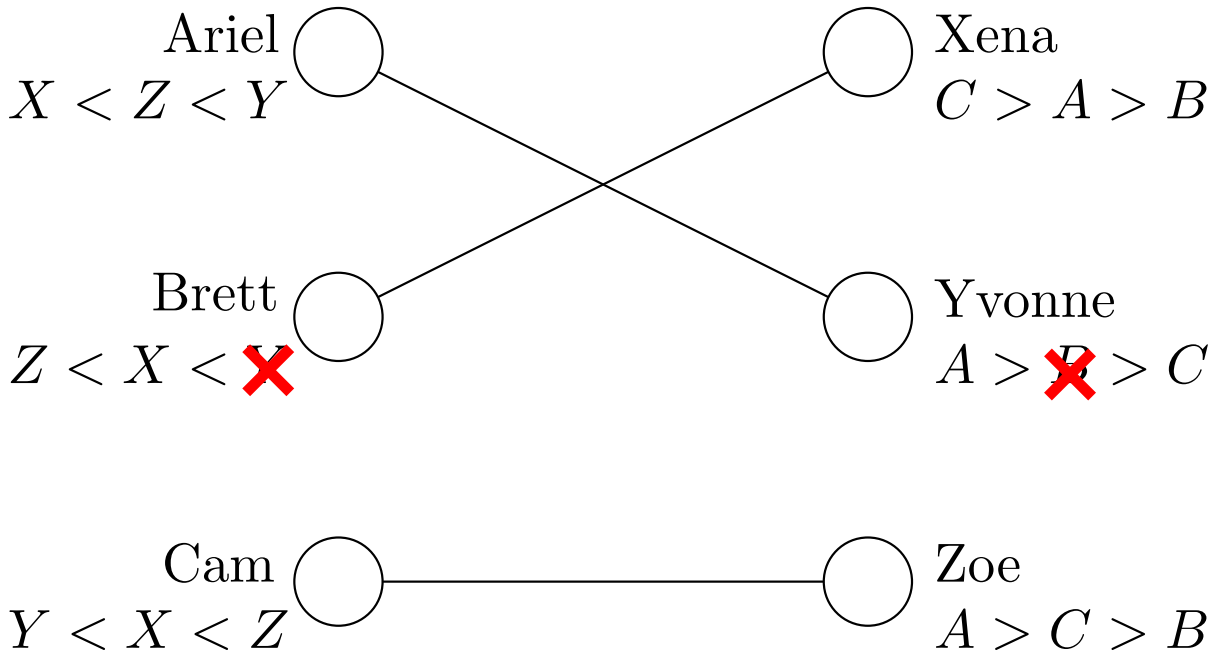
Stable marriage: Algorithm

Rejection: Each woman says "maybe" to the highest-ranking proposal received, and rejects the rest of the proposals.



Stable marriage: Algorithm

Repeat the proposal/rejection process until a perfect matching is obtained.



Stable marriage: Large example

Ariel
$T < Z < X < U < V < Y < W$

Brett
$Z < X < U < V < Y < T < W$

Cam
$Z < U < T < W < Y < X < V$

David
$Z < T < U < Y < X < W < V$

Eric
$W < V < U < X < Z < T < Y$

Francis
$Z < T < V < Y < U < X < W$

Greg
$W < X < V < U < Z < Y < T$

Tina
$C > B > A > D > F > G > E$

Ursula
$C > D > F > G > A > B > E$

Vivian
$B > C > E > D > F > A > G$

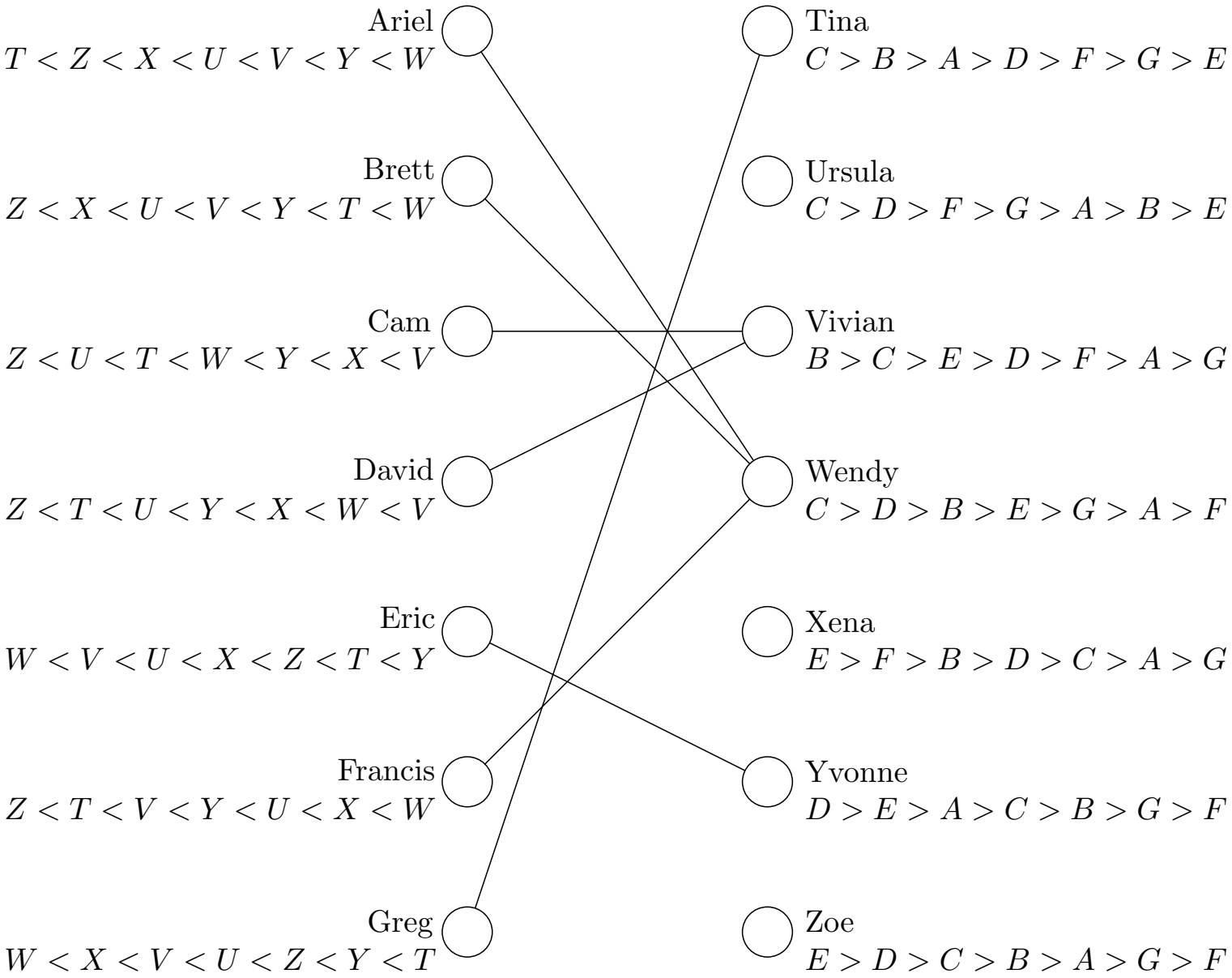
Wendy
$C > D > B > E > G > A > F$

Xena
$E > F > B > D > C > A > G$

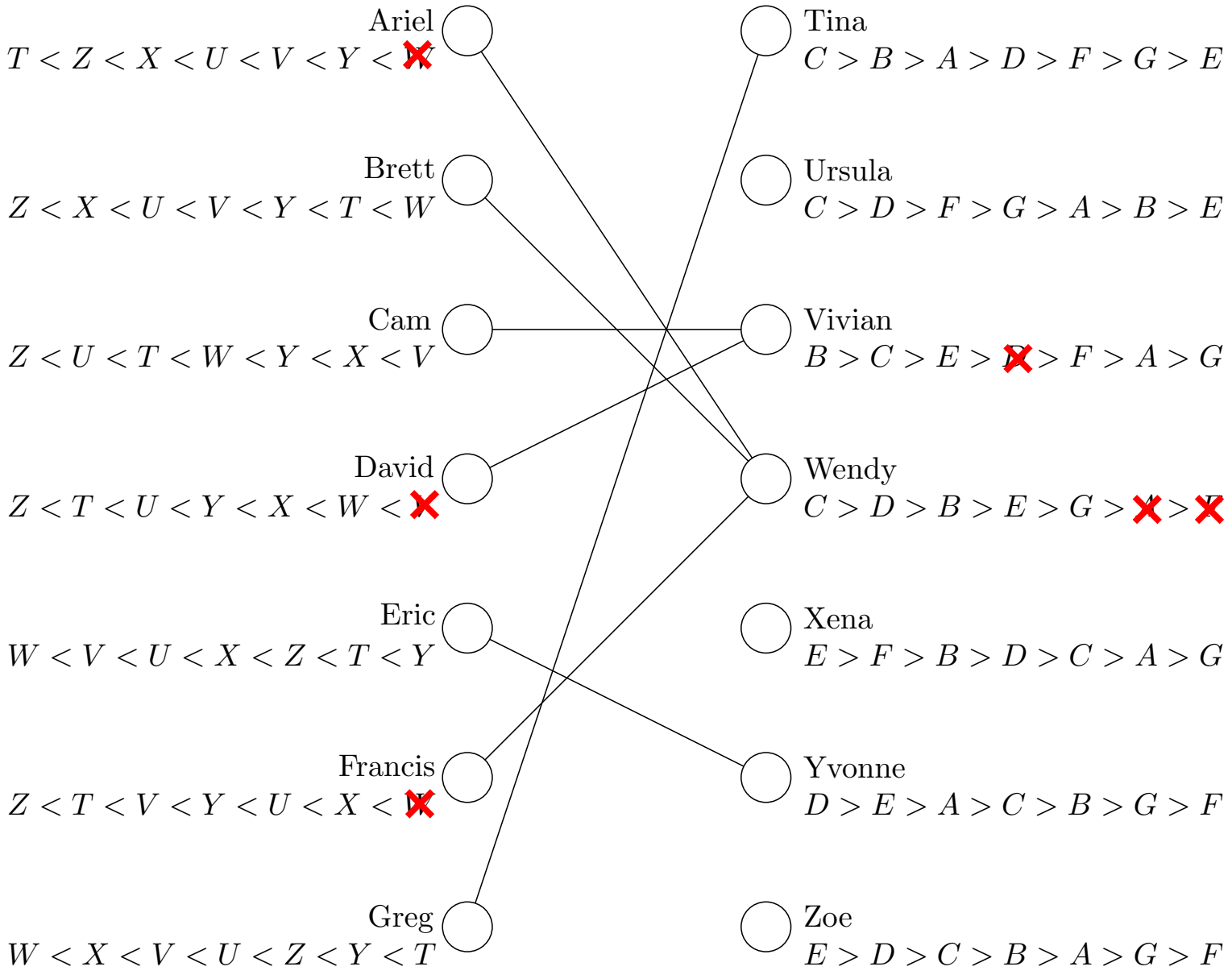
Yvonne
$D > E > A > C > B > G > F$

Zoe
$E > D > C > B > A > G > F$

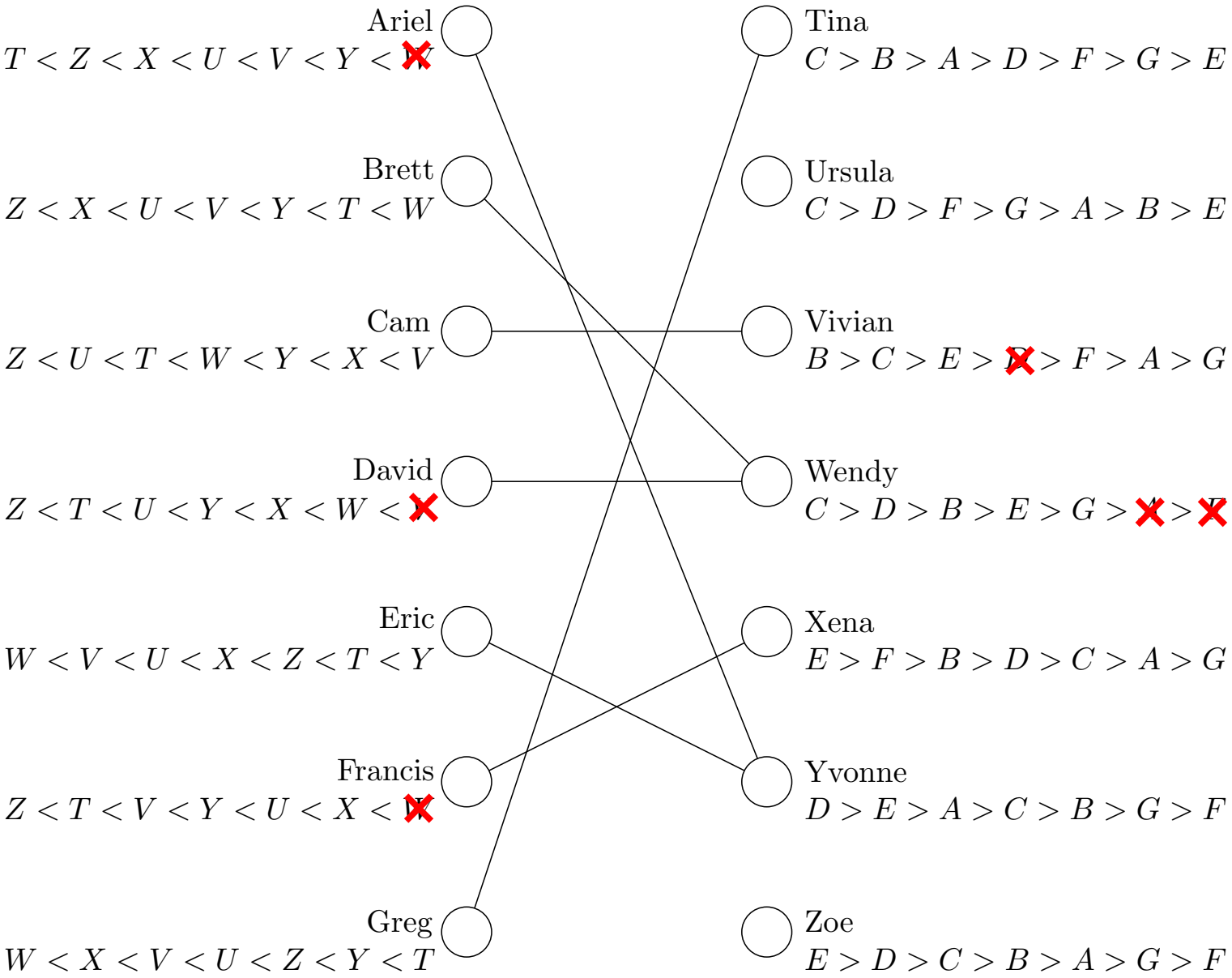
Stable marriage: Large example



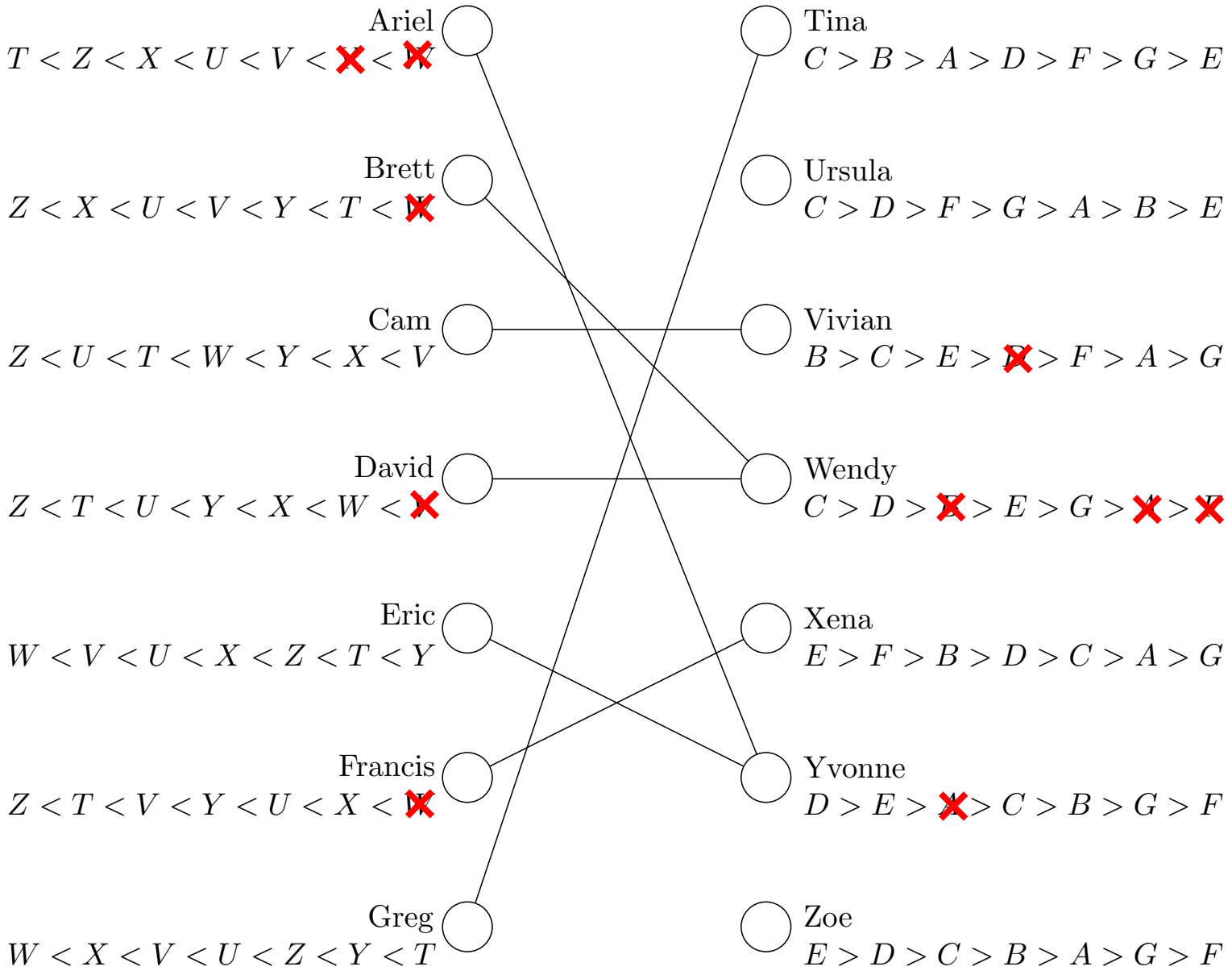
Stable marriage: Large example



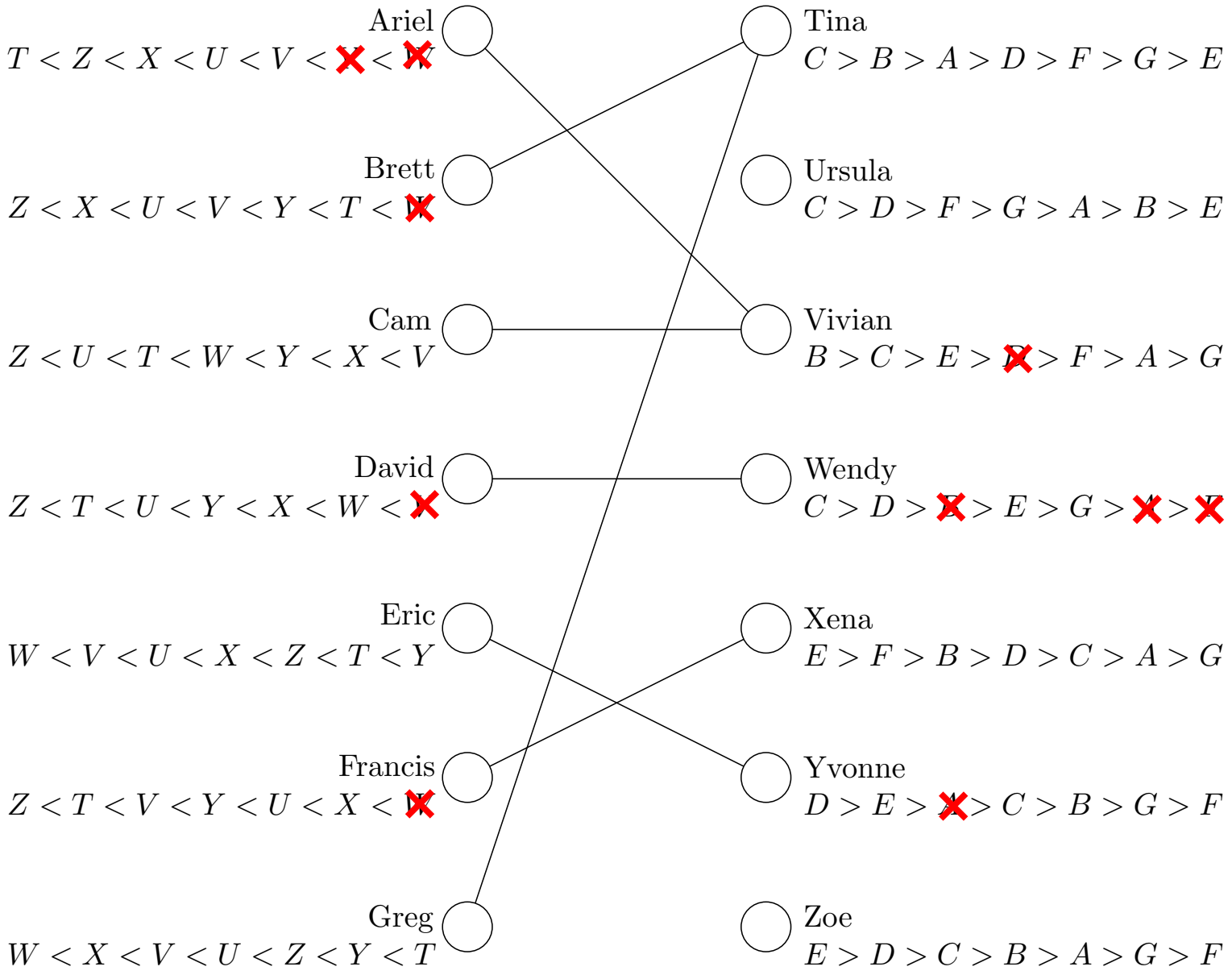
Stable marriage: Large example



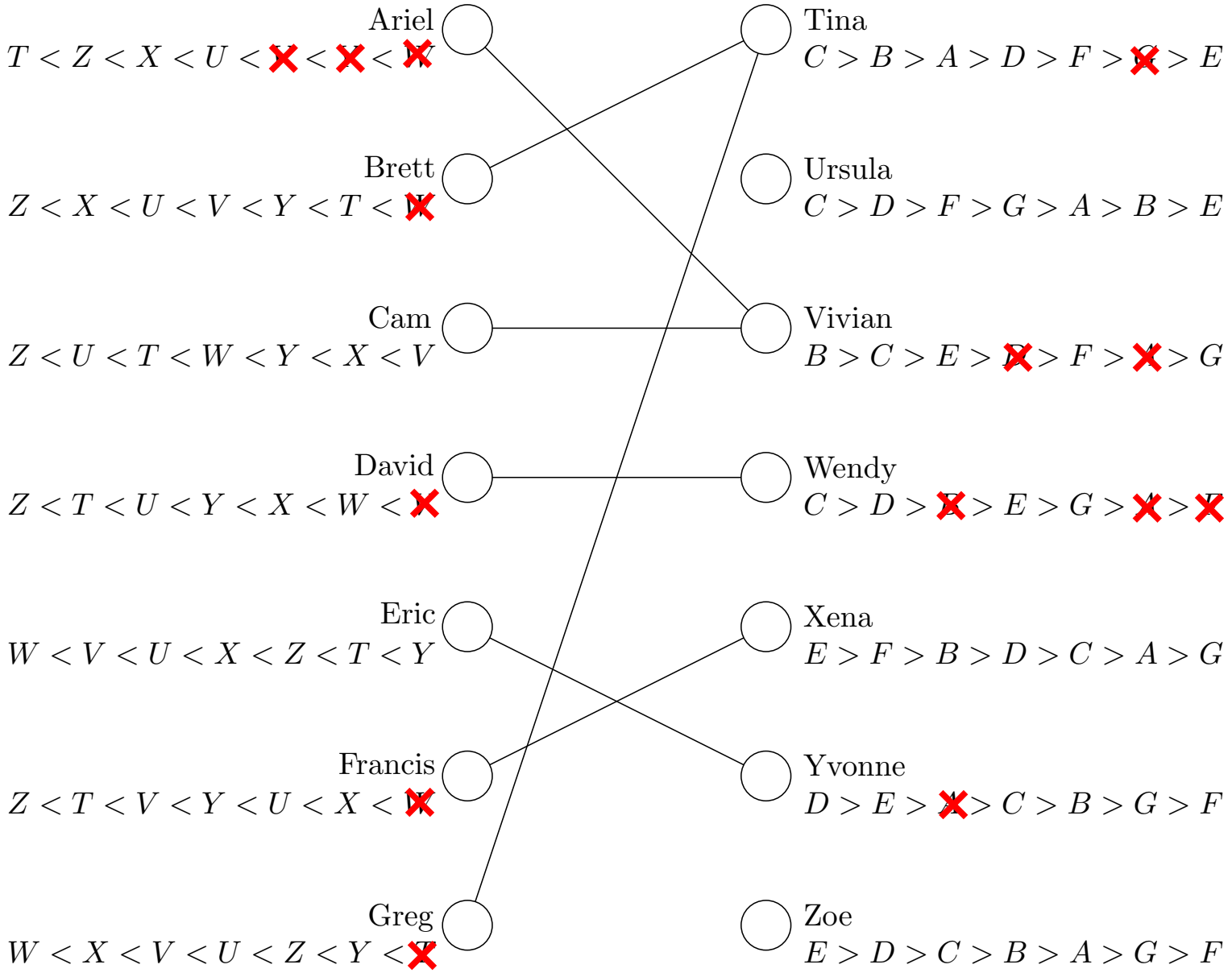
Stable marriage: Large example



Stable marriage: Large example



Stable marriage: Large example



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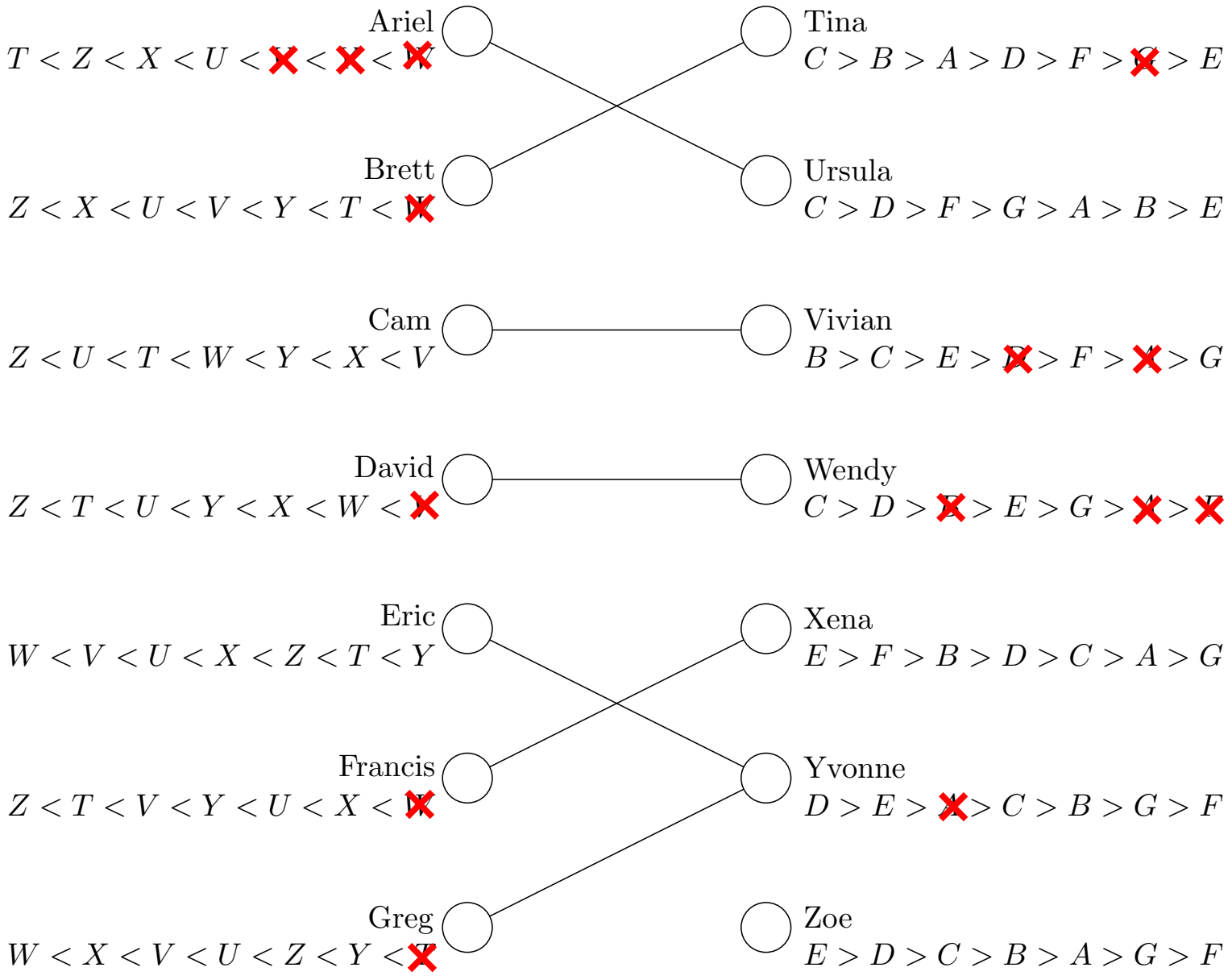

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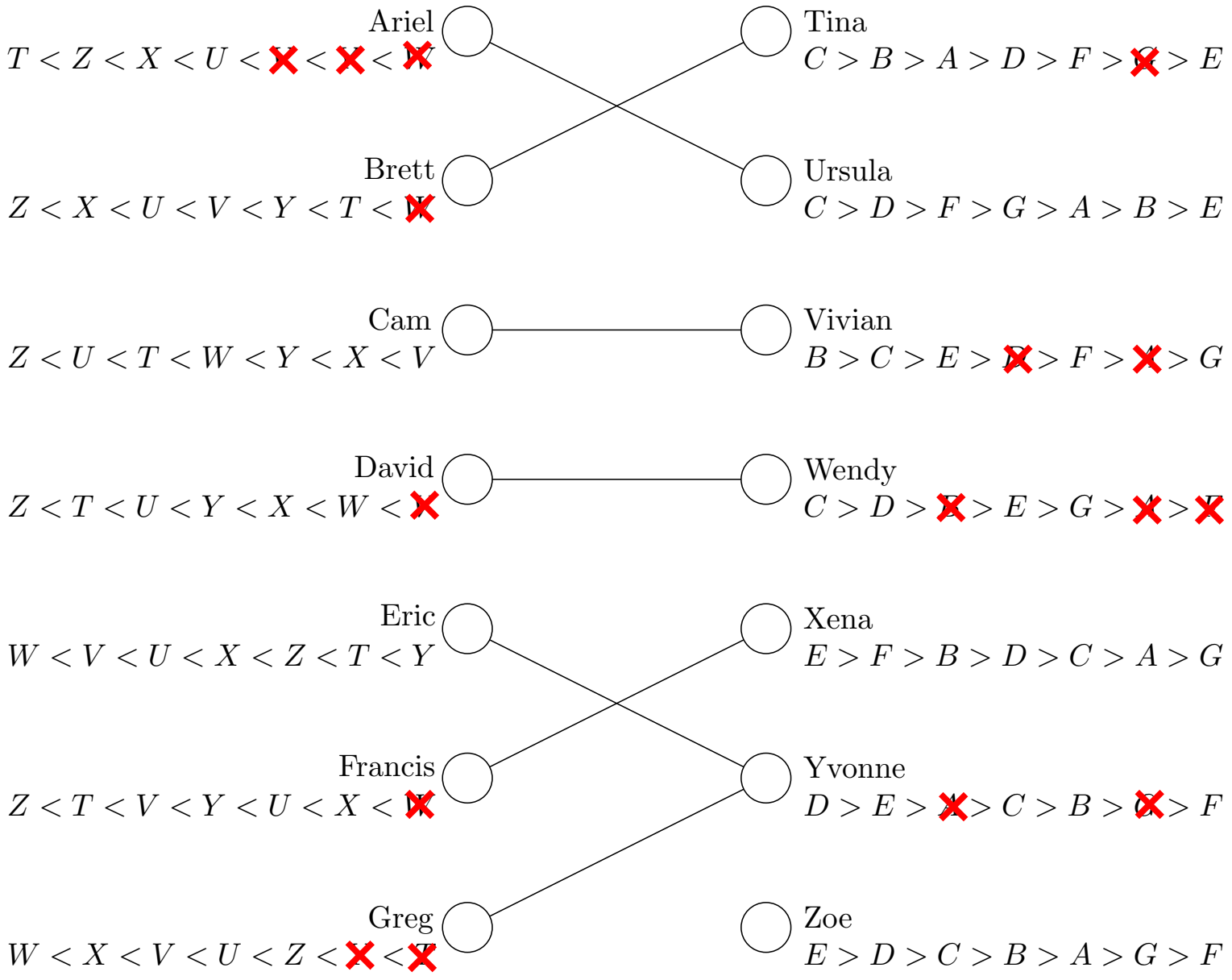

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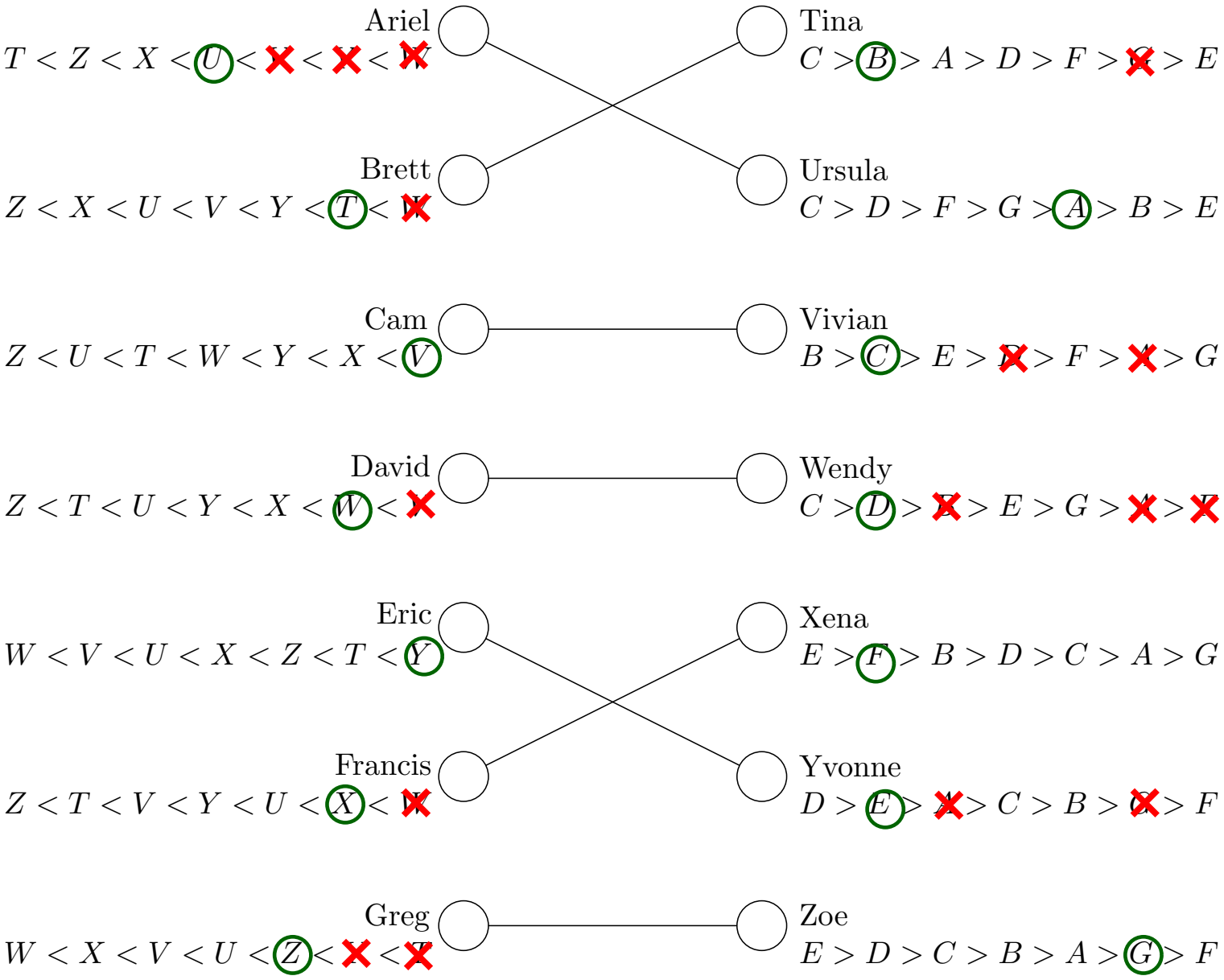
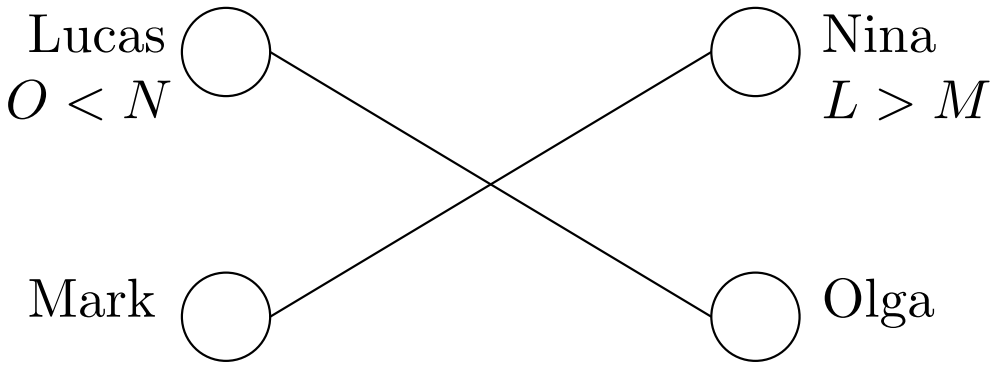


Stable marriage: Why does it work?

Why doesn't jealousy happen?



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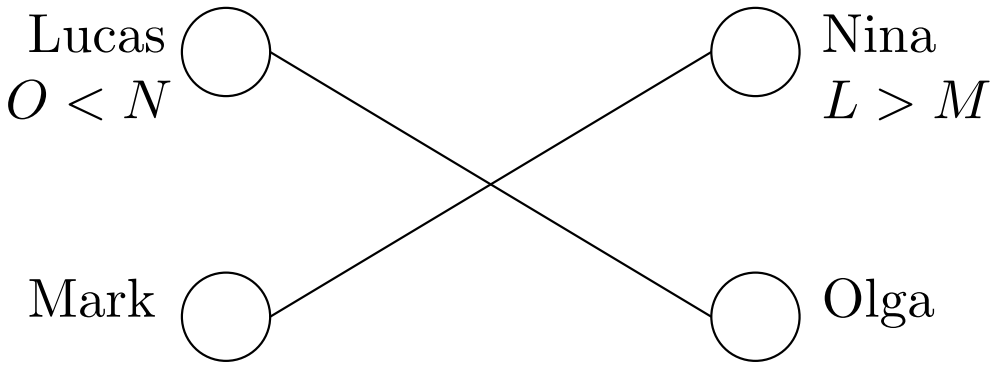
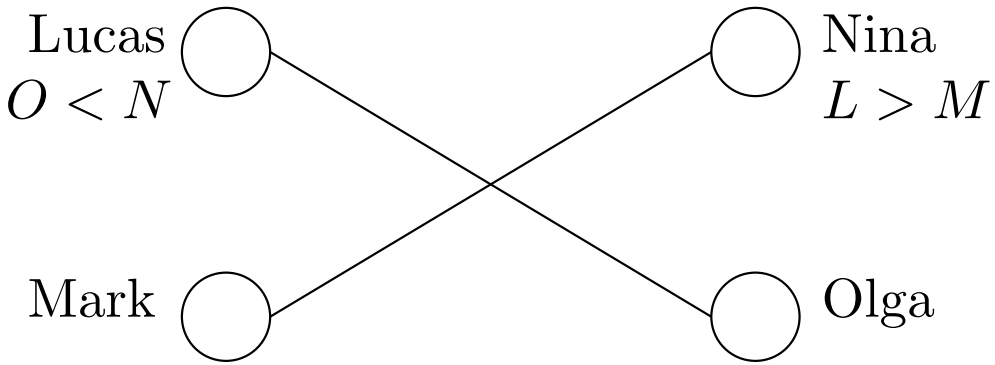


Key: For each woman, the ranks of the rejected proposals are lower than the "maybe", and ultimately lower than the accepted proposal.

Stable marriage: Why does it work?

Why doesn't jealousy happen?



Key: For each woman, the ranks of the rejected proposals are lower than the "maybe", and ultimately lower than the accepted proposal.
Lucas proposes to Nina before Olga. So Lucas must have been rejected by Nina. However, Nina accepted Mark, yet Nina ranks Mark lower than Lucas. This is not possible.

Stable marriage: Who propose?

Q: Who is happier? The men or the women?

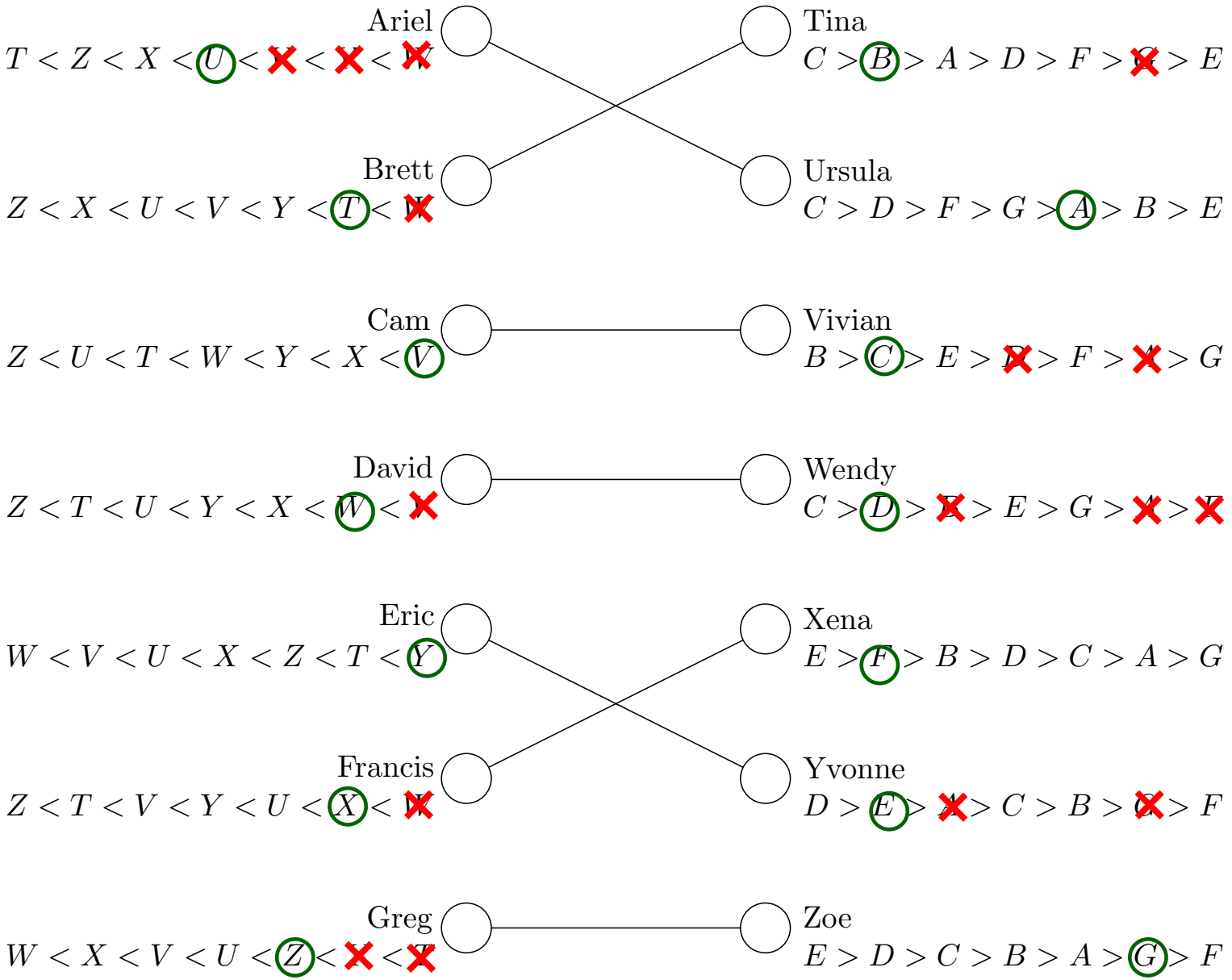
Stable marriage: Who propose?

Q: Who is happier? The men or the women?
A: The men.

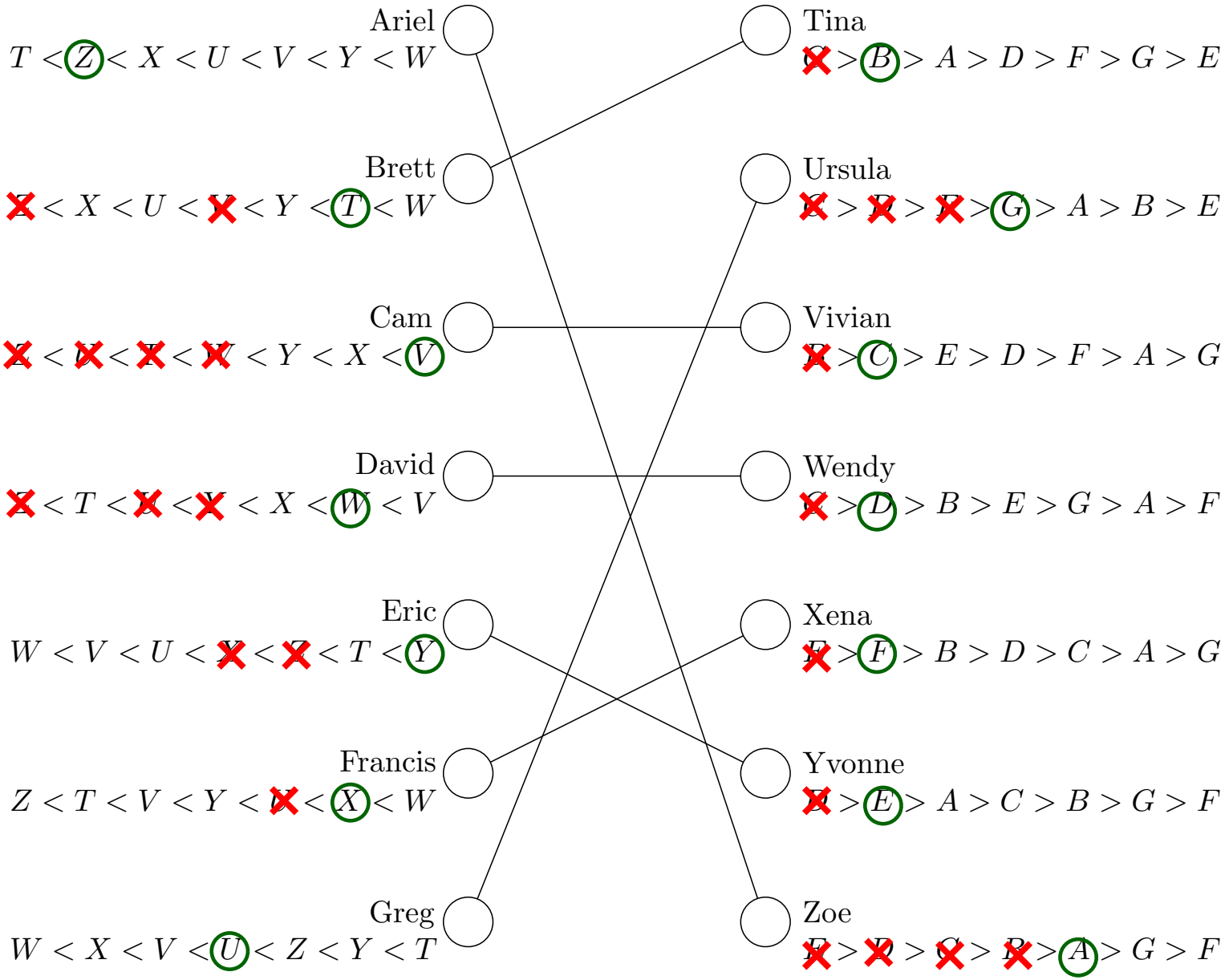
Stable marriage: Who propose?

Q: Who is happier? The men or the women?
A: The men.
If we instead run the algorithm by having the women propose, then every woman winds up at least as happy as in the original algorithm, and every man winds up at least as unhappy.

Stable marriage: Men propose



Stable marriage: Women propose



Stable marriage: Who propose?

Q: Why don't you get the co-op job that you want?

Stable marriage: Who propose?

Q: Why don't you get the co-op job that you want?
A: It's the employers that make the proposals.