MATH 239 Assignment 3

- This assignment is due on Friday, October 5th, 2012, at 10 am in the drop boxes in St. Jerome's (section 1) or outside MC 4067 (the other two sections).
- You may collaborate with other students in the class, provided that you list your collaborators. However, you MUST write up your solutions individually. Copying from another student (or any other source) constitutes cheating and is strictly forbidden.

1. (a) Let S be a set of configurations, and w be a weight function on S. By definition, the number of elements in S with weight exactly n is just $[x^n]\Phi_S(x)$. Prove that the number of elements in S with weight *at most* n is $[x^n]\frac{\Phi_S(x)}{1-x}$.

   (b) Determine the number of k-tuples $(a_1, ..., a_k)$ of positive integers that satisfy the inequality
   $$a_1 + ... + a_k \leq n.$$

2. (a) Let A_k be the set of all compositions of n with k parts, and let B_{k-1} be the set of binary strings of length $n-1$ which have exactly $k-1$ elements equal to 0. Describe and justify a bijection between A_k and B_{k-1}.

   (b) Using part a), give an alternate proof that there are 2^{n-1} total compositions of n.

3. Show that the generating series for the set of all compositions which have an even number of parts, and each part congruent to 1 modulo 5, is equal to
   $$\frac{1 - 2x^5 + x^{10}}{1 - x^2 - 2x^5 + x^{10}}.$$

4. Determine the exact number of compositions $(a_1, ..., a_k)$ of n which have k parts, and satisfy $a_i \geq i$ for $i = 1, ..., k$.

   (Give a closed-form expression which does not involve any summation.)

5. Let $A = \{101, 00, 1011\}$ and let $B = \{101, 01, 0111\}$. Find the sets AB and BA and determine their generating series. Are the generating series for AB and BA the same? If not, try to explain why they wouldn't be.