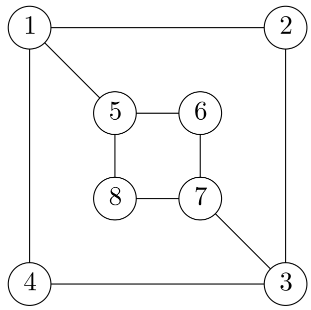
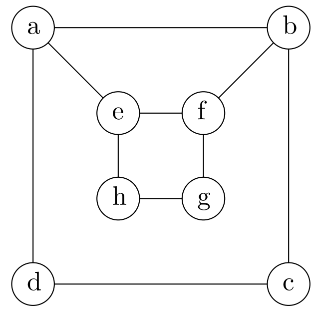
MATH 239 Tutorial 6 Problems

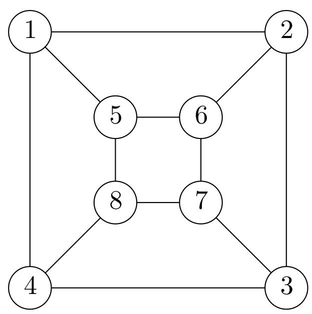
1. Determine if G_1 and G_2 are isomorphic, and if G_3 and G_4 are isomorphic. Prove your claims.



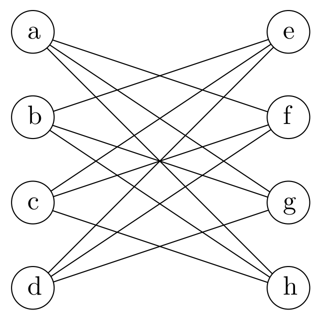
Graph G_1



Graph G_2



Graph G_3



Graph G_4

2. Draw all non-isomorphic graphs with 5 vertices where the degree of each vertex is even.

3. The *degree sequence* of a graph is the list of vertex degrees, usually written in non-increasing order, as $d_1 \geq \cdots \geq d_p$. Determine whether or not each of the following is the degree sequence of a graph. If so, draw the graph. If not, explain why.

   (a) 7, 6, 5, 4, 4, 3, 2, 2, 2, 2

   (b) 3, 3, 3, 2, 2, 1

   (c) 5, 5, 1, 1, 1, 1

4. The definition of the complement of a graph is in the assignment.

   (a) Draw the complement of the cube.

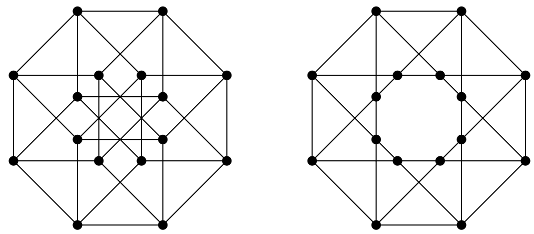
   (b) Find a graph G with 4 vertices that is isomorphic to $\overline{G}$.

   (c) Suppose $|V(G)| = p$. Determine $|E(G)| + |E(\overline{G})|$.

   (d) A subset of vertices S of G is an *independent set* if no two vertices in S are adjacent in G. Prove that the size of a largest independent set in G is equal to the largest n such that K_n is a subgraph of $\overline{G}$.

Additional exercises

1. Are these two graphs isomorphic?



2. For $k \geq 1$, is it true that if G is a k-regular graph, then there exists a subgraph of G that is $(k-1)$-regular?

3. Married couple Mario and Peach invited 3 other couples to the castle on the mountain for a cake party (and it's no lie). During the party, some handshaking took place with the restriction that a person cannot shake hands with themselves nor with their spouse. After all the shakings were done, Peach went around to ask the 7 others in the party how many people they shook hands with, and she received a different answer from everyone. How many hands did Mario shake? How many hands did Peach shake? What happens if Mario and Peach invited n couples to the party?

4. A graph is *self-complementary* if it is isomorphic to its complement.

   (a) Prove that the number of vertices in a self-complementary graph is congruent to 0 or 1 modulo 4.

   (b) (Hard.) Construct a self-complementary graph with n vertices for every $n \equiv 0, 1 \pmod 4$.

   (Hint: For $n \equiv 0 \pmod 4$, generalize the answer from Q4(b) of the tutorial problems by partitioning the vertices into 4 groups. For $n \equiv 1 \pmod 4$, add a vertex to your construction of $n-1$.)