MATH 239 Spring 2012: Assignment 4

Due: 9:29 AM, Friday, June 1 2012 in the dropboxes outside MC 4066

Note: In this assignment, the weight of a binary string is its length.

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only): /50

Acknowledgments:

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1. {16 marks} For each of the following sets of binary strings, determine an unambiguous expression which generates every string in that set. (No justification required.)

   (a) The set of binary strings where the length of each block is divisible by 3.

   (b) The set of binary strings which do not contain 01111 as a substring.

   (c) The set of binary strings where each block of 1's must be followed by a block of 0's of length at least 3.

   (d) The set of binary strings which do contain 1111000 as a substring.

2. {12 marks} Inside Bertrand's special box, there is an unlimited supply of blue and red balls. You draw one ball at a time, and Bertrand will offer you \$1 for each ball you draw, as long as you do not draw 4 of the same-coloured balls in a row, at which point you lose everything. (For this question, represent your answers as coefficients of rational expressions.)

   (a) How many ways can you win $\$n$ from Bertrand?

   (b) How many ways can you win $\$n$, but get greedy and lose everything on the next draw?

3. {Extra credit: 3 marks} Describe the set of binary strings which is generated by the following expression:

$$(1(0\{1\}^*0)^*1\{0\}^*)^*$$

4. {12 marks} Let S be the set of all binary strings where consecutive blocks have different parities. For example, things in S include 00011011111100000110, 1111111, 0011111, ε. Prove that the generating series for S is

$$\Phi_S(x) = \frac{1 + 2x + x^3 - x^4}{1 - 2x^2 - x^3 + x^4}.$$

5. {5 marks} For some positive integer m, let $s_1, \ldots, s_k$ be distinct binary strings of length m. Prove that $S = \{s_1, \ldots, s_k\}^*$ is an unambiguous expression.

6. {5 marks} Prove that for any choice of positive integers m and n where $m \neq n$, there exist binary strings s and t of lengths m and n respectively where $\{s, t\}^*$ is an ambiguous expression.