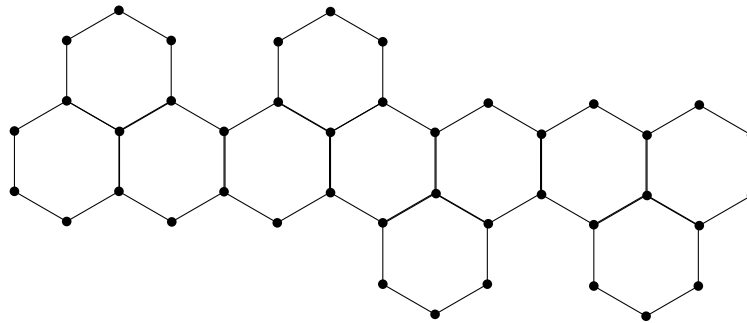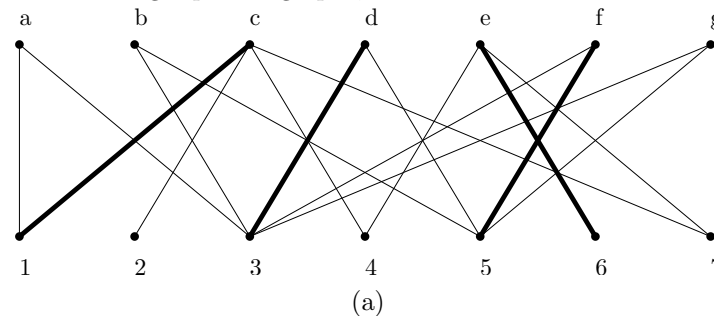**Note:** Do not hand in this assignment. Solutions will be posted on Monday April 8th.
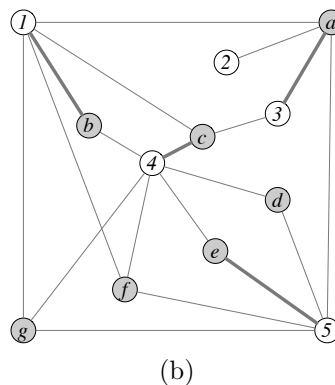
Exercise 1. Find a maximum matching M and a minimum cover C in the following graph (using whatever mean you wish). Then find a simple argument proving that M is indeed a maximum matching and that C is indeed a minimum cover.

Exercise 2. Consider the following bipartite graphs,

(a)

and

(b)

For (a) the bipartition is given by $A = \{a, b, c, d, e, f, g\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. For (b) the bipartition is given by $A = \{a, b, c, d, e, f, g\}$ and $B = \{1, 2, 3, 4, 5\}$.

Run the maximum matching algorithm starting from the matching indicated by the bold edges for both (a) and (b). At each iteration,

(i) determine the sets $X \subseteq A$ and $Y \subseteq B$, and

(ii) either,

• find a larger matching, or

• prove the matching is maximum by exhibiting a vertex cover.

Explain each iteration of the procedure. In particular, explain how you find a larger matching, explain how you find a vertex cover.