MATH 239 Spring 2012: Assignment 7
Due: 9:29 AM, Friday, June 22 2012 in the dropboxes outside MC 4066

Note: There is no note.

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only):   /50

Acknowledgments:

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1. {16 marks} For the following statements, determine whether they are true or false, and give justifications through a proof or counterexample.

   (a) If there is a u, v-walk of odd length, then there is a u, v-path of odd length.

   (b) If G is a bipartite graph, then any closed walk has even length.

(c) If there is a cycle containing u and v and another cycle containing v and w, then there is a cycle containing u and w.

(d) Any 3-regular graph must have a Hamilton cycle.

2. {8 marks} Prove that if G is a graph where every vertex has degree at least k, then G contains a path of length at least k.

3. {16 marks} The Kneser graph $G_{n,k}$ is the graph where the vertices are all k-subsets of $[n]$, and two vertices are adjacent if and only if their corresponding sets are disjoint.

(a) Draw $G_{4,1}$ and $G_{5,2}$.

(b) How many vertices and edges are there in $G_{n,k}$?

(c) Prove that if $n \geq 3k - 1$, then $G_{n,k}$ is connected.

(d) Prove that if $n \geq 3k$, then $G_{n,k}$ is not bipartite. As {Extra credit: 4 marks}, prove this statement for $n \geq 2k + 1$.

4. {10 marks} Let a_n be the number of cycles of length 4 in an n-cube. So $a_1 = 0, a_1 = 1, a_2 = 6$. For $n \geq 1$, determine an explicit formula for a_n.

   (Hint: There are at least 2 ways to solve this question. One way is to use the recursive construction of the n-cube to generate a recurrence relation, and solve it. Another way is to enumerate certain pairs of binary strings.)