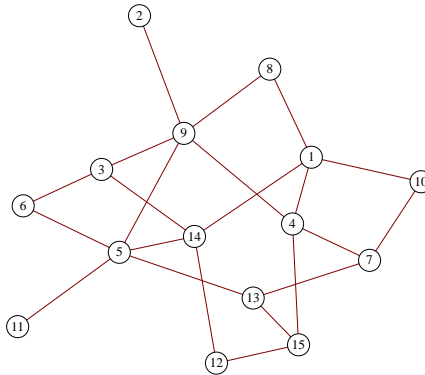


MATH 239 Assignment 8

- This assignment is due on Friday, November 16, 2012, at 10 am in the drop boxes in St. Jerome's (section 1) or outside MC 4067 (the other two sections).
- You may collaborate with other students in the class, provided that you list your collaborators. However, you **MUST** write up your solutions individually. Copying from another student (or any other source) constitutes cheating and is strictly forbidden.

1. For each positive integer n , what is the largest number of bridges in any n -vertex graph? Prove that your answer is correct.
2. Suppose there is a unique path between any pair of vertices in G . Prove that G is a tree.
3. Prove or disprove the following statement: Given a graph G , if T and U are spanning trees of G , then T and U are isomorphic.
4. Let G be the following graph:



- (a) Construct a breadth-first search tree for G . Use vertex 1 as the root, and add vertices to the tree using the smallest possible label first. Denote each tree edge by an arrow from a vertex to its parent, and indicate each non-tree edge by a dashed line.
 - (b) Is G bipartite? Use a breadth-first search tree to justify your answer.
 - (c) Give a shortest path between vertices 1 and 6. Use a breadth-first search tree to prove that your path is as short as possible.
5. Prove that each of the following graphs is planar by exhibiting a planar embedding. (Be sure to label the vertices in your embeddings.)

