MATH 239 Spring 2012: Assignment 10

Due: 9:29 AM, Friday, July 20 2012 in the dropboxes outside MC 4066

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only): /50

Acknowledgments:

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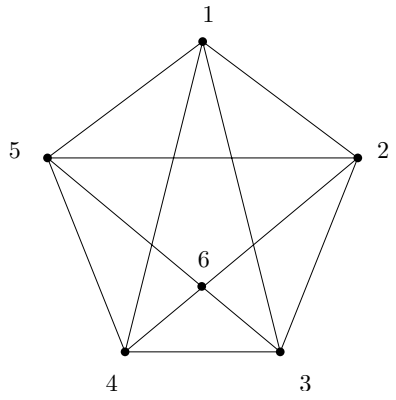
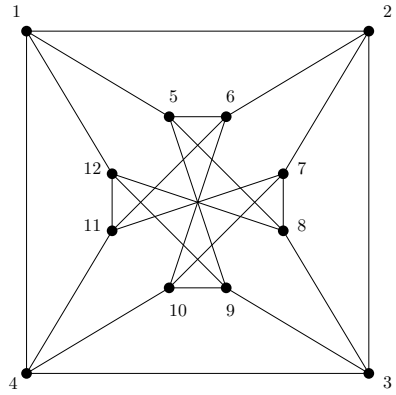
1. {7 marks} Let G be a simple planar graph with at least 4 vertices where every vertex has degree at least 3.

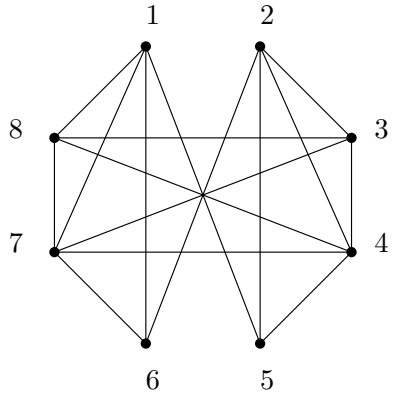
   (a) Prove that G has at least 4 vertices of degree at most 5. (For {Extra credit: 2 marks}, prove the case without the assumption that "every vertex has degree at least 3.")

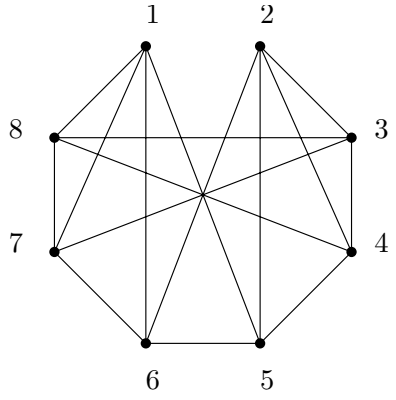
   (b) Give a planar embedding of a graph with 8 vertices which has exactly 4 vertices of degree at most 5.

2. {16 marks} For each of the following graphs, determine whether it is planar or not. Prove your assertion.









3. {12 marks} For each of the following description of a graph, draw a planar embedding for it and briefly explain why the colouring requirements are met.

   (a) A planar 3-regular graph that is not 3-colourable.

   (b) A planar 3-regular graph that is 3-colourable but not 2-colourable.

   (c) A planar 3-regular graph that is 2-colourable.

4. {10 marks} Let G be a planar graph whose shortest cycle has length at least 6 (if it has any cycle at all).

   (a) Prove that G contains a vertex of degree at most 2.

   (b) Prove that G is 3-colourable.

5. {5 marks} Let G be a simple planar graph with at least 2 vertices, and let G^* be the dual of a planar embedding of G. Prove that if G is isomorphic to G^*, then G is not bipartite.

6. {Extra credit: 5 marks} Let G be a planar graph. Prove that there is a bipartite subgraph of G containing at least $\frac{2}{3}|E(G)|$ edges.