MATH 239 Assignment 5

- This assignment is due on Friday, October 19, 2012, at 10 am in the drop boxes in St. Jerome's (section 1) or outside MC 4067 (the other two sections).
- You may collaborate with other students in the class, provided that you list your collaborators. However, you MUST write up your solutions individually. Copying from another student (or any other source) constitutes cheating and is strictly forbidden.
- The first problem is optional and may be solved for bonus marks.

1. **(Bonus problem)** Find the generating series (with respect to length) for the set of binary strings that do not contain the substring 110011.

2. Prove Lemma 3.1.1: If $f(x)$ is a polynomial of degree less than r, then there is a polynomial $P(x)$ with degree less than r such that

$$[x^n]\frac{f(x)}{(1-\theta x)^r} = P(n)\theta^n.$$

3. (a) Find values of a and b so that

$$\frac{x+8}{(x-3)(2x+5)} = \frac{a}{x-3} + \frac{b}{2x+5}.$$

   (b) Find a closed-form expression for

$$[x^n]\frac{x+8}{(x-3)(2x+5)}.$$

4. Suppose $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, $a_3 = 4$, and

$$a_n = 8a_{n-2} - 16a_{n-4}$$

   for all integers $n \geq 4$. Determine a_n explicitly for all non-negative integers n.

5. Let n be a fixed positive integer. Suppose $b_i = (-1)^i i$ for $i = 0, 1, \ldots, n-1$ and

$$b_i = -\sum_{k=1}^{n} \binom{n}{k} b_{i-k}.$$

   for all integers $i \geq n$. Determine b_i explicitly for all non-negative integers i.

6. Suppose $c_0 = 0$, $c_1 = -1$, and

$$c_i = -2c_{i-1} - c_{i-2} + 4i - 4$$

   for all integers $i \geq 2$. Determine c_i explicitly for all non-negative integers i.