MATH 239 Tutorial 8 Problems

1. What is the fewest number of edges that can exist in a graph with n vertices and k components?

2. Let G be a connected graph on $n \geq 3$ vertices where removing any edge from G results in a tree. Determine (with proof) all possible graphs that satisfy this condition.

3. Let T be a tree with n vertices where each vertex has degree either 1 or 4. Determine the number of leaves in T in terms of n.

4. Suppose that G is a connected graph. Prove that an edge e is a bridge if and only if e is in every spanning tree of G.

5. Prove that the edges of a graph G can be partitioned into edge-disjoint cycles if and only if every vertex of G has even degree.

6. Suppose that G is a graph which contains two edge-disjoint spanning trees T_1 and T_2.

   (a) Prove that G does not have any bridge.

   (b) Let $e \in E(T_1) \setminus E(T_2)$. Prove that there exists $e' \in E(T_2) \setminus E(T_1)$ such that $T_1 - e + e'$ is a spanning tree of G.

   (c) Let $X \subseteq V(G)$ be a nonempty subset. What is the maximum number of edges in G that joins two vertices in X? Write this in terms of $|X|$.