MATH 239 - Assignment 4

DUE: 10am Friday Feb. 8th in the drop boxes opposite the Math Tutorial Centre MC 4067.

**Exercise 1** (20pts)**.**
The C&O Zoo has a fire sale and the following animals are purchasable at the following unit prices:

- lion (7 gold coins),
- kangaroo (5 gold coins),
- moose (4 gold coins),
- rabbit (1 gold coin).

Customers are not allowed to buy more than four animals of the same type, with the exception of rabbits (customers can buy as many rabbits as they wish).

(a) Find a formal power series $\Phi(x)$ such that $[x^n]\Phi(x)$ is the number of ways you can spend exactly $n$ gold coins buying animals from the zoo.

(b) Since lions find rabbits very tasty the zoo has a new policy: no customer is allowed to purchase both rabbits and lions. Under this new policy find a formal power series $\Phi'(x)$ such that $[x^n]\Phi'(x)$ is the number of ways you can spend exactly $n$ gold coins buying animals from the zoo.

**Exercise 2** (10 pts)**.**
Your friend Tony Montana, proposes the following game: flip a coin $n$ times, if tail appears 5 times in a row you loose, otherwise you win. Define a power series $\Phi(x)$, such that $[x^n]\Phi(x)$ is the number of winning sequences of $n$ coin flips.

**Exercise 3** (20pts)**.**
For each of the following sets of binary strings, determine an unambiguous expression which generates every string in that set.

(a) The set of binary strings where the length of each block is divisible by 3.

(b) The set of binary strings which do not contain 01111 as a substring.

(c) The set of binary strings where each block of 1's must be followed by a block of 0's of length at least 3.

(d) The set of binary strings which do contain 1111000 as a substring.

**Exercise 4** (10pts)**.**

(a) Let $s_1, \ldots, s_k$ be distinct binary strings all of the same length $m \geq 1$. Prove that $\{s_1, \ldots, s_k\}^*$ is an unambiguous expression.

(b) Prove that for any choice of positive integers $m$ and $n$ where $m \neq n$, there exist binary strings $s$ and $t$ of lengths $m$ and $n$ respectively where $\{s, t\}^*$ is an ambiguous expression.

**Exercise 5** (10pts)**.** A binary string is a palindrome if it reads the same forwards and backwards. Examples of palindromes include 01100110, 11011, 1. Let $\mathcal{P}$ be the set of all binary strings that are palindromes. Determine a recursive definition of $\mathcal{P}$, and use it to find the generating series for $\mathcal{P}$.