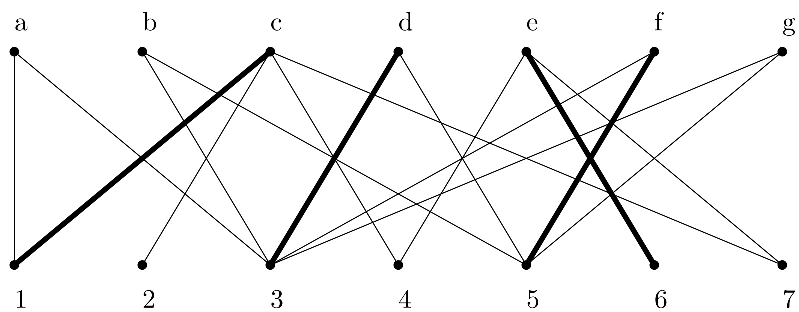
MATH 239 Spring 2012: Assignment 11

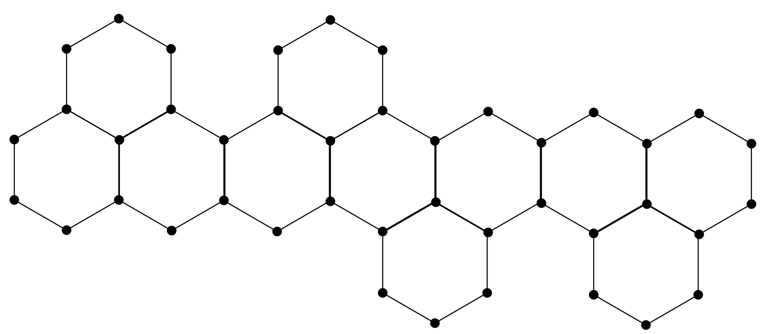
Due: Never. (Never say never.)

Note: Do not hand in this assignment. Solutions will be posted by July 27th.

1. For the following bipartite graph with bipartition $A = \{a, b, c, d, e, f, g\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$, perform the maximum matching algorithm using XY-construction. At the end of the algorithm, produce a maximum matching, a minimum cover, and the sets X and Y from the algorithm. Prove that there is no matching that saturates every vertex in A by giving a set $D \subseteq A$ such that $|N(D)| < |D|$.



2. Find a maximum matching of the following graph. Prove that your matching is maximum using a vertex cover.



3. An *independent set* of a graph G is a subset of the vertices $S \subseteq V(G)$ such that no two vertices in S are adjacent. Prove that C is a vertex cover of G if and only if $V(G) \setminus C$ is an independent set. If x is the size of a maximum independent set and y is the size of a minimum vertex cover, determine $x + y$.

4. Suppose that a connected graph G has exactly one maximum matching. Prove that G has a perfect matching.

5. Prove that the edges of a k-regular bipartite graph can be partitioned into k perfect matchings.

6. Let G be a bipartite graph with bipartition (A, B) where $|A| = |B| = 2n$. Suppose for each $X \subseteq A$ where $|X| \leq n$, $|N(X)| \geq |X|$, and for each $Y \subseteq B$ where $|Y| \leq n$, $|N(Y)| \geq |Y|$ (i.e. Hall's condition holds for subsets of A and B of size at most n). Prove that G has a perfect matching.

7. Two people play a game on a graph G by alternately selecting distinct vertices $v_1, v_2, \ldots$ forming a path. The last player able to select a vertex wins. Prove that the second player has a winning strategy if G has a perfect matching, and the first player has a winning strategy if G has no perfect matching.