Surname: ____________________

First Name: ____________________

Signature ____________________

Id.#: ____________________

MATH 239 MIDTERM EXAMINATION

Midterm Examination

4:30-6:30 pm, Thursday, July 3, 2003

Please check your lecture section:

001	Bauer	9:30	☐
002	Schellenberg	1:30	☐

INSTRUCTIONS:

1. Write your name and Id.# in the blanks above.
2. This exam has 9 pages. Please ensure you have them all.
3. Calculators permitted, no additional materials allowed.
4. Write your answers in the space provided. If you require additional space, use the back of the previous page, and indicate clearly where your solution continues.

Question	Value	Mark Awarded	Question	Value	Mark Awarded
1	12		5	10	
2	10		6	12	
3	8		7	8	
4	10		Total	70	

CANNOT BE REMOVED FROM OFFICE!!!

1. **Each of the following parts is worth 3 marks. No proofs are required; only the answer will be graded.**

   (a) Give a decomposition that uniquely creates all binary strings in which every block of 1's has odd length and every block of 0's has even length.

   (b) Give a decomposition that uniquely creates all binary strings that have an even number of blocks.

   (c) Give a decomposition that uniquely creates all binary strings that do not contain the substring 010.

   (d) Let A be the set of binary strings determined by the recursive definition

$$A = \{0, 01, 10\} \cup AA \cup \{0\}A\{1\} \cup \{1\}A\{0\}.$$

   (Note: this definition does not generate the strings of A uniquely.) Describe the strings of A in words. (You need not prove your claim.)

[Marks] 2. Let S be the set of all compositions having $2k$ parts, where each odd part is an odd integer and each even part is a multiple of 3.

[5] (a) Show that the generating function for S is

$$\Phi_S(x) = \left(\frac{x^4}{(1-x^2)(1-x^3)}\right)^k.$$

[5] (b) Express $[x^n]\Phi_S(x)$ as a sum of binomial coefficients.

[**3 Marks**] 3.(a) Determine the generating function for the set $\{1, 101, 10101\}^*$.

[5] (b) Determine the generating function for the set $\{1, 10, 10101\}^*$.

[**4 Marks**] 4.(a) Determine a decomposition that uniquely generates the set of binary strings S in which no block of 0's has the same length as the block of 1's that immediately precedes it. (For example, 0010011 is in S, but 01110001 is not.)

[6] (b) Determine the generating function $\Phi_S(x)$ where the weight of a string is its length.

[**5 Marks**] 5.(a) Find an explicit formula for c_n, where c_n is given by the recurrence equation

$$c_n = 3c_{n-1} - 4c_{n-3}, \quad \text{for } n \geq 3,$$

with initial conditions $c_0 = 7$, $c_1 = -3$, and $c_2 = 5$.

[5] (b) Find a linear homogeneous recurrence relation and initial conditions for the sequence defined by

$$a_n = (5 + 2n)3^n - 2(-2)^n \quad \text{for } n \geq 0.$$

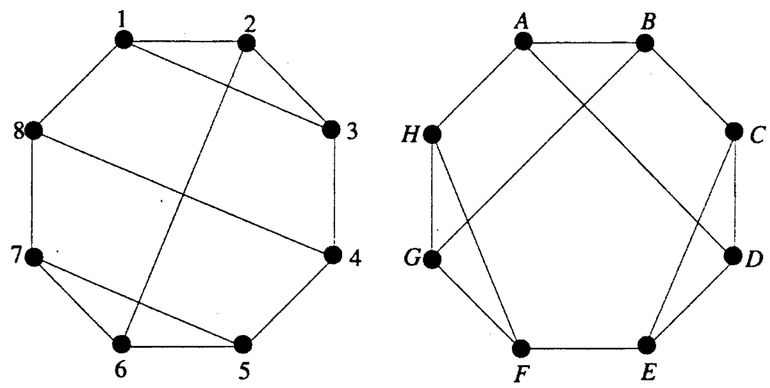
6. **Each of the following parts is worth 3 marks. No proofs are required; only the answer will be graded.**

   (a) Determine, without proof, the number of edges in a graph G with no cycles having p vertices and c components.

   (b) Is there a 5-regular graph on 11 vertices?
   Give an example or explain why no such graph exists.

   (c) What is the minimum number of vertices in a tree with 1 vertex of degree 3 and one vertex of degree 5? Draw a tree with 1 vertex of degree 3 and one vertex of degree 5 having 10 vertices, or explain why no such tree exists.

(d) Give an isomorphism of the following two graphs or prove one does not exist.



[**8 Marks**] 7. If every vertex of a graph G with p vertices has degree at least $p/2$, prove that G is connected.