MATH 239 Spring 2012: Assignment 6

Due: 9:29 AM, Friday, June 15 2012 in the dropboxes outside MC 4066

Note: All graphs are assumed to be simple and finite.

Last Name:

First Name:

I.D. Number:

Section:

Mark (For the marker only): /50

Acknowledgments:

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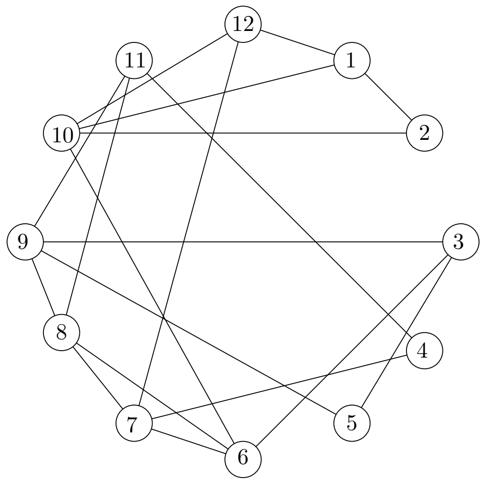
1. {8 marks} In assignment 3, you have discovered that if b_n is the total number of parts over all compositions of n, then b_n satisfies the recurrence

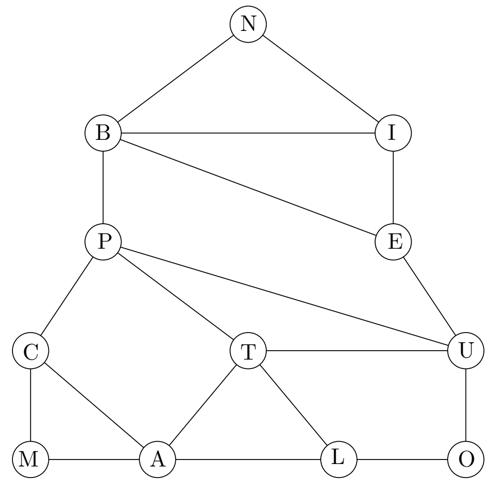
$$b_n = 2b_{n-1} + 2^{n-2}$$

with initial condition $b_1 = 1$. Determine an explicit formula for b_n when $n \geq 1$.

2. {5 marks} The following two graphs G and H are isomorphic. Find an isomorphism.

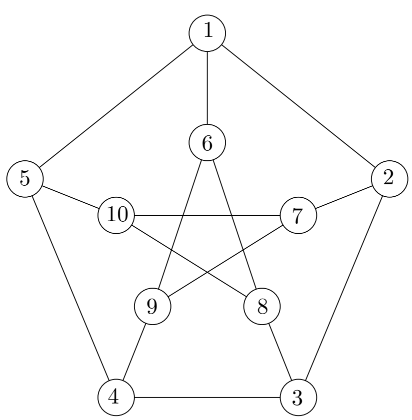


Graph G

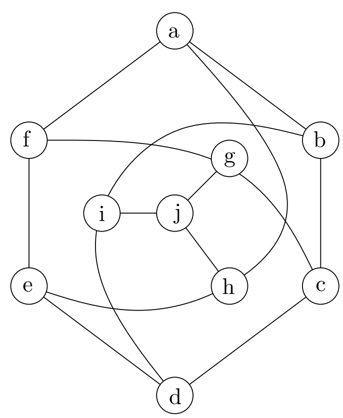


Graph H

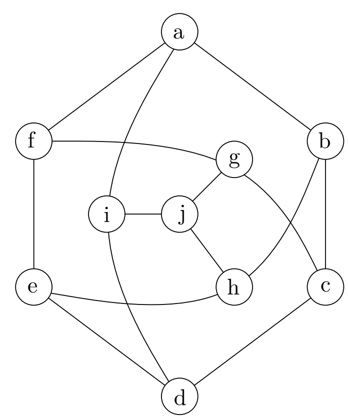
3. {8 marks} Graph G is the Petersen graph. Among the graphs H_1 and H_2, one of them is isomorphic to the Petersen graph, and the other is not. For the one that is isomorphic, give an isomorphism. For the one that is not isomorphic, explain why it is not isomorphic to G.



Graph G



Graph H_1



Graph H_2

4. {6 marks} Draw all non-isomorphic graphs on 6 vertices and 5 edges which do not contain a cycle.

5. {4 marks} Prove that if k is odd, then any k-regular graph has even number of vertices.

6. {7 marks} At Jochen's party of combinatorialists (of more than 2 guests, of course), a series of handshakes were generated among the guests. We are not a narcissistic bunch, so no shaking onto oneself is allowed. Prove that at least two of the guests shook hands with the same number of people.

7. {12 marks} Let G be a graph. The complement of G, denoted $\overline{G}$, is the graph where $V(\overline{G}) = V(G)$, and $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$.

   (a) Draw a graph G on 5 vertices such that G is isomorphic to $\overline{G}$.

   (b) Suppose G has p vertices where $p \equiv 2 \pmod 4$. Prove that G is not isomorphic to $\overline{G}$.

   (c) Prove that if G is regular, then $\overline{G}$ is also regular.

8. {Extra credit: 4 marks} Prove that given any set of 6 people, there exist either 3 mutual friends or 3 mutual strangers. We adopt the unusual social convention that a pair of people is either friends or strangers. (A proof involving lengthy case analysis will not be accepted.)