Hints to Solutions and Proofs of Some Supplementary Problems of Chapter 1 : **Vectors and Scalars** of *Schaum's Outlines of Vector Analysis, Second Edition*

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1 Vectors and Scalars

1.1 Question 1.34

The first important thing to do in mathematics is to draw figures and imagine it.
Draw a figure of ABCDEF Regular Hexagon with forces represented as vectors **AB**,**AC**,**AD**,**AE** and **AF**.

Now, observe by looking at the figure that :

AB=**ED**

AF=**CD**

BC=**FE**

Let the Resultant force between **AB** and **AC** is R_1, **AF** and **AE** is R_2, R_2 and **AD** is R_3, R_3 and R_1 is R_4 respectively.

Now,
R_4=R_1+R_3

Using above all obervations, Simplifying R_4

R_4=**AB**+**AC**+R_2+**AD**

R_4=**AB**+**AC**+**AF**+**AE**+**AD**

Also,
AC=**AB**+**BC**

AE=**AF**+**FE**

Considering all the observations and using it in previous expression of R_4

R_4=**AB**+**AC**+**AF**+**AE**+**AD**

R_4=**AB**+**AB**+**BC**+**AF**+**AF**+**FE**+**AD**

Since, **AB**+**BC**+**CD**=**AD**,
and

AB=**ED**

AF=**CD**

BC=**FE**

R_4=3**AD**

Hence, the resultant of the forces is 3**AD**.

1.2 Question 1.39

Given :

A=(x+4y)**a**+(2x+y+1)**b**

B=(y-2x+2)**a**+(2x-3y-1)**b**

Now, 3**A**=2**B**

3**A**=2**B**

3**A**-2**B**=0

⇒ (3x+12y)**a**+(6x+3y+3)**b**-(2y-4x+4)**a**+(4x-6y-2)**b**=0

⇒ (7x+10y-4)**a**+(2x+9y+5)**b**=0

Since, **a** and **b** are **Non-Collinear**,

7x+10y=4,

2x+9y=-5

By solving above equations, we get x=2 and y=-1.

1.3 Question 1.43

It is given that **a**,**b** and **c** are non-coplanar vectors.

Also,

r_1 = 2**a** - 3**b**+**c** ,

r_2 = 3**a** - 5**b** + 2**c** ,

r_3 = 4**a** - 5**b** + **c** ,

We need to check whether r_1,r_2 and r_3 are linearly independent or linearly dependent.

Let L = xr_1 + yr_2 + cr_3 be a Linear Combination of r_1,r_2 and r_3, where x,y and z are any scalars.

L = xr_1 + yr_2 + cr_3

L = x(2a - 3b + 2c) + y(3a - 5b + 2c) + z(4a - 5b + c)

L = (2x + 3y + 4z)a + (-3x - 5y - 5z)b + (x + 2y + z)c

Since, **a**,**b** and **c** are non-coplanar vectors,

L = 0 and

2x + 3y + 4z = 0

-3x - 5y - 5z = 0

x + 2y + z = 0

By solving above equations, we get y = 2z and x = -5z.

Taking z = 1, yields y = 2 and x = -5.

Hence, L = 0
$\Rightarrow$ xr_1 + yr_2 + cr_3 = 0

$\Rightarrow 5r_1 = 2r_2 + r_3$

Therefore, r_1,r_2 and r_3 are linearly dependent.

1.4 Question 1.44

Part (a)

It is given that if O is any point within triangle ABC and P,Q, and R are midpoints of the sides AB,BC and CA respectively.

We need to prove that :
OA+OB+OC=OP+OQ+OR

First of all, we need to draw a triangle ABC with sides representing vectors.

By looking at the figure, we find :

AP=PB

BQ=QC

CR=RA

$$\mathbf{OA} = \begin{cases} 1. & \mathbf{OR+RA} \\ 2. & \mathbf{OP\text{-}AP} \end{cases}$$

$$\mathbf{OB} = \begin{cases} 3. & \mathbf{OP+PB} \\ 4. & \mathbf{OQ\text{-}BQ} \end{cases}$$

$$\mathbf{OC} = \begin{cases} 5. & \mathbf{OQ+QC} \\ 6. & \mathbf{OR\text{-}CR} \end{cases}$$

Adding all above 6 equations,

2(**OA+OB+OC**)= (**OR+RA+OP+PB+OQ+QC**) + (**OP-AP+OQ-BR+OR-CR**)

$\Rightarrow$ 2(**OP+OQ+OR**) + (**RA-AP+PB-CR+QC-BR**)

$\Rightarrow$ **OA+OB+OC=OP+OQ+OR**

Hence Proved.

Part (b)

Yes. You can do on your own.
Hint: Consider Point O outside the triangle ABC and follow Part (a).

1.5 Question 1.46

Consider a triangle ABC with vertices **AC**,**CB** and **BC** and with mid point P of **BA** joining Q, the mid point of **CB**.

We need to prove that $\mathbf{PQ} \| \mathbf{AC}$ and PQ=$\frac{1}{2}$AC

First draw the figure with given conditions. By looking at the figure :

$\mathbf{BP} = \mathbf{PA} = \frac{1}{2}\mathbf{BA}$

$\mathbf{CQ} = \mathbf{QB} = \frac{1}{2}\mathbf{CB}$

Also,

$\mathbf{PQ} = \mathbf{QB} + \mathbf{BP}$

$\Rightarrow \frac{1}{2}\mathbf{CB} + \frac{1}{2}\mathbf{BA}$

$\Rightarrow \frac{1}{2}(\mathbf{CB} + \mathbf{BA})$

$\Rightarrow \mathrm{PQ} = \frac{1}{2}\mathrm{AC}$

$\Rightarrow$ $\mathbf{PQ} \| \mathbf{AC}$ and PQ=$\frac{1}{2}$AC

Hence Proved.