L’APPEL AU VIDE: A THOUGHT EXPERIMENT

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Approval Sheet

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Dedication

Dedicated to the **FALLEN** .

Abstract

Contents

List of Tables

List of Figures

Chapter 1

Introduction

Falling through the black hole, albeit a stellar or a supermassive one, static or rotating, in short, in all kinds and colours, have been a subject of fascination for theoretical physicists. However, Given the limitations of the current technology, the immense distances involved, the nearest Black hole *Gaia BH1* being $1,560$ *light years* away (European Space Agency, 2023), and the extreme conditions in the vicinity of the Black hole, had made it more than unlikely for humankind to go through with such an adventure physically in the foreseeable future.

This fact has not however stopped the scientific community from researching as well as interpreting the *mathematics of General Relativity (GR)* in order to make such journeys understandable and imaginable if not comprehensible scenarios, along with their fair share of myths and misconceptions. Such journeys from advanced technically written research papers with their more than fair share of mathematics, to easily digestible content for the general public while still retaining the same level of mathematical rigor, have been done by various science journalists, youtube channels, as well as simulations on the web.

In all that, We aim to try and do something different. Instead of falling head first into the black hole and documenting the journey for either fun or education purposes for the general masses, we aim to provide a different view into this, mainly on the matters of *Gravitational Redshift*, *Time Dilation* and how they will impact an exchange of information between two observers, one being closest to the event horizon, while the other far from the gravitational influence of the said event horizon.

1.1 Mission Statement

Our mission is simple. Just like the equations of General Relativity.

The mission is to figure out the safest event horizon, safest being the one whose tidal forces won't rip you apart on close enough approach, calculate different physical parameters for the observer near it, like the relation of it's proper to coordinate distance, proper acceleration, proper and coordinate time, redshift parameter etc. Afterwards, calculate the distance of the second observer such that it feels same gravitational acceleration due to the black hole as a person on the surface of earth feels due to the gravitational acceleration due to moon. Then to define the mode of communication between them, i.e. the wavelength of light used, etc and calculate signal travel times for both sides. Finally, provide the information on the relative time delay between the two observers, as well as how long the experience will be for both sides.

1.2 Mission's Significance

Black Hole are the most simplest phenomena in the universe, defined only by the three parameters, mass, charge and angular momentum, and yet the universe around this beasts is the most complicated and least understood, to the point that all of our understanding of the current universe, *laws of physics*, and *mathematics* blows up as one reaches the singularity, the heart of darkness. Due to this, Black hole not only provide us the most extreme places to experiment and test *General Relativity*, our current best theory of gravitation, as well as hope for some key clue about quantum gravity. Due to this, keeping the strangeness and awesomeness of them aside, Black holes have been a subject of fascination for scientists, journalists, and even the general masses for half a century or so. New research is being done in the field as the times goes by.

In all this, the simple but straightforward significance of our mission can be seen as a way of trying to bridge the gap between the two worlds, the one we live in, and the one we dream of. We aim to show, how something as simple as having a conversation or exchange of some flashes can be effected by the gravitational effects of the black hole, how the same experience will take mere second for one observer, while a lifetime for the other. The goal is to present a way as well as a tool that can be used to show the time frame for such exchanges for general masses, signifying the importance of research into black holes as well as promoting and even sparking curiosity in the readers.

1.3 Mission Objectives

To enlist the core objectives of our mission, we have the following:

- Calculate the mass of the black hole (M) such that the gravitational gradient at it's schwarzschild radius r_s is Δg_e for the person of same height on earth.
- Calculate the distance of observer B (D), such that acceleration due to gravity of the black hole at (D) is same as the acceleration due to gravity of the moon on the surface of earth.
- Calculate the distance between two observers (d).
- Calculate the Redshift parameter z and use that to calculate emitted wavelength

$\lambda_e(observer)$ for both observers such that they both observed certain same wavelength λ_o.

- Calculate the relative time delay between the two observers for a defined wavelength of light.

1.4 Research Questions

The main questions asked in this mission are:

How the presence of a black hole affects the communication between two observers in temporal terms? How change in a person height, effect the required lower mass limit of the black hole? Exactly how much longer it take to compose same message far from black hole relative to near it's event horizon? How long will the whole experience be for both sides? What can these insights help us with?

1.5 Assumptions

As mentioned earlier in 1.2, All (stationary) black holes can be completely defined by three parameters, i.e. *mass M, angular momentum J and charge Q.* As mass is fundamental for every black holes, the other two parameter provide us with four possible categories of black holes. Among them, for our mission, we will be using the simplest case, a case where both angular momentum and charge are zero. In essence the assumptions we will be having are all following:

Stationary The spacetime geometry surrounding the black hole does not change over time, (metric components are independent of time coordinates).

Static The black hole has zero charge.

Non-rotating The angular momentum or spin parameter for the black hole is zero.

Shape The black hole is symmetrical and spherical.

Surrounding The black hole is in vacuum and there are no other gravitational influences nor any other source of matter or radiation (no accretion disk, jets.).

Inertial Frame Both observers are at rest relative to each other.

Free Fall Chamber Our free fall chamber works.

Based on these assumptions, the spacetime geometry around our black hole can be described by using **Schwarzschild Metric**, the line element for which is given by 1.1,

$$ds^2 = (1 - \frac{r_s}{r})c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi) \tag{1.1}$$

where, r_s is the *schwarzschild radius*, which is equal to $\frac{2GM}{c^2}$. The equation 1.1, can be written in metric form as,

$$g_{\mu\nu} = \begin{bmatrix} (1 - \frac{r_s}{r}) & 0 & 0 & 0 \\ 0 & \frac{-1}{(1 - \frac{r_s}{r})} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{bmatrix} \tag{1}$$

1.6 Limitations

Tidal acceleration (aka gravitational gradient (Δg)) and proper acceleration (α) are the two main limiting factors in our mission. Humans, our observers are humans too, cannot withstand high acceleration and extreme tidal forces around the black holes can rip apart anything, aka spaghettification. Interestingly, lower mass black holes (stellar black holes), have low proper acceleration near their event horizon compared to supermassive black holes (several billions times stronger), but they also have extreme tidal forces, meaning everything will be ripped apart before coming even a $1000 \times r_s$ radial distance to them. On the other hand, supermassive black hole can have almost negligible (even earth like) tidal forces at their event horizon because tidal acceleration is inversely proportional to the distance (r^3) and supermassive black holes have larger r_s.

Insight of that, we decided to use tidal acceleration as our limiting factor, and use that to calculate the mass of the black hole. In order to deal with high g-forces (proper acceleration), we have our observers orbit the black hole, so they will be in constant free fall, i.e. a state of weightlessness and won't have to experience the proper acceleration. This in turn created a limit factor, the distance of the orbit from the center of the black hole. To be as near as possible to the even horizon, we chose **Innermost Stable Circular Orbit**, (r_{isco}), which is 3 times r_s for the Schwarzschild Black hole.

To keep model simple, so far the only real variable in our model is the height of the observer 'A' (h).

Chapter 2

Review of Related Literature

For this chapter, we will briefly be going over the literature used in this work, and related work of this kind. The aforementioned literature does include books, articles, papers, internet publications and work related to or like ours include content from youtube videos to podcasts and review articles on the internet. As it is not in the realm of possibilities and sensibility to cover all of them, we will be mainly mentioning these most relevant and pertinent ones, which are as follows:

Gravitation and Cosmology book

Profound Physics Website by *Ville Hirvonen* about physics concepts including General Relativity

Physics Forums Insights An article titled *Learn time Dilation and Redshift for a Static Black Hole* by *stevebd1*

ScienceClic English A youtube channel, about scientific concepts, including videos about falling into black holes as well as simulations of black hole

PBS SpaceTime A youtube channel, currently hosted by *Dr. Matt O'Dowd* about astrophysics, including playlist about black holes.

Code Pen A Front end code online code editor, including many simulations of black hole like *Black Hole (WEBGL shader)*.

Circular Bit An online game using black hole physics and simulations.

For review purposes, instead of having dedicated sections for each of these sources we will simply have sections, namely, *Mathematical Part*, *Conceptual Parts* and *Coding Part*.

2.1 Mathematical Part

The most fundamental equation in our work, *Schwarzschild Metric* (eq: 1.1) was originally given by **Karl Schwarzschild** in 1916 paper titled *'On the Gravitational field of a Mass point according Einstien's Theory'* (Schwarzschild, 1999) (1999 english translation reference). Since then, this metric has been used quite a good deal in research related to general relativity. In fact, it is safe to say Schwarzschild geometry is one of the most well studied and understood among all GR geometrics.

For the same conditions as Schwarzschild (event horizon at *2M*) in different coordinate geometrics like *Edditngton-Finkelstein coordinates* as well as for in depth explanations, we refer to (Hartle, 2003) and (Weinberg, 1972). Here, we found in-depth mathematical as well as scientific base for our work along with world lines for the observers falling into the black hole. We did not however used any of the advanced tensor mathematics or differential calculus for our work.

Other main equations like for mass (eq 3.19) and tidal forces, were basic newtonian physics that we mostly dervied for ourselved but can easily be found in college physics books. The equations of proper and coordinate time and distance were derived from (eq: 1.1) (see derivations.) and were also tallied over different internet sources like (Hirvonen, 2023) and (stevebd1, 2017).

2.2 Conceptual Parts

What happens near a black hole or inside it's event horizon is still an open mystery. In fact, till recently, the very existence of entities like black hole was under research, and to top that the matter of whether or singularities exist is still a hot question (Kerr, 2023). Keeping all this in sight, the prospect of how an exchange of information among two observers each at a different gravitational well and hence experiencing rate of flow of time differently was no easy feat.

The work of (Hartle, 2003) and (Weinberg, 1972) provided the insight into the flow of time for the observers falling into a black hole, as well as time at certain distance (r) from the center of the black hole. Along with equations and mathematical framework providing the quantative understanding of the scenario, the provided explanations and thought experiments were also very insightful.

More than simple time dilation and length contraction or gravitational redshift is experienced at close proximity of the black hole. The matter of gravitational lensing, the view based on the geodesics of light coming in to the black hole from outside universe, the horizon and the look of the view, all of that was also explained in that texts as well as some rather nice youtube videos like one of *ScienceClic English* (English, 2021), provides stunning visuals as well as the explanations for the fall into the black hole at some distance.

Beside that the work of *Andrew Hamilton* (Hamilton, 2010) gives a nice insight into the dynamics of the black hole, (though sadly the *Black Hole Flight Simulator*

(BHFS) is not publically available). For Schwarzschild geometrics, his simulations like (Hamilton, 1998) are really awesome and helpful.

2.3 Coding Part

The last part of our

Chapter 3

Research Methodology

Our study is mainly theoretical and creative in nature, using mathematical equations and scientific principles to model our scenario, calculating most of the parameters using real life value wherever possible. Due to the complex dynamic of the research in question, the model of our mission, the methods of our research aren't very straight forward. Our study area or stage is set at the simplest of the black holes defined by the mathematics of *General Relativity*, **'Schwarzschild Black Hole'**, defined only by their mass. The main players are two human beings, observer and the goal is simple, defining a mode of communication between them. Based on that, This chapter3, is in correspondence to this is divided into four parts, namely, *Black hole, Observer One, Observer Two and The signal*, where each sections contains detailed informations about the parameters and equations for the respective part of our scenario.

3.1 Black Hole

We assume a *spherically symmetrical* black hole, with zero charge ($Q = 0$), i.e. a *static* as well as no angular momentum ($J = 0$), i.e.*non-rotating*. The black hole is also stationary, which means that the spacetime geometry surrounding the black hole does not change as time goes by. According to *No hair Theorem*, such a black hole can be completely defined by only it's **Mass** (M). We also assume that the space surrounding our black hole is completely empty, i.e. no matter or any radiation nor any other gravitational influences are present in it's sphere of influences. This assumption implies that the black hole in question has no accretion disk of any kind, presence of which can complicate the overall scenario as well as make the calculations more complex.

The spacetime geometry around such a black hole in a vacuum space, is described by the most simplest as well as the most studied solution of Einstein field equations, *Schwarzschild metric* as given in1.1,

$$ds^2 = (1 - \frac{r_s}{r})c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2(d\theta^2 + \sin^2\theta d\phi) \tag{1.1}$$

where, $r_s = 2GM/c^2$ and known as *Schwarzschild radius*.

3.1.1 Mass of the Black hole

There are three main things that can be potentially lethal for a person in the neighbourhood of a black hole,

1. Intense radiation due to accretion (radiation burns)
2. Extreme gravitational acceleration (crushing)
3. Strong Tidal Forces (spaghettification)

(note: there are many other phenomena involved that can also lead to death, but for now, we will consider this major ones.)

We eliminated the first one by assuming a black hole with no matter or radiation nearby, an isolated black hole in vacuum space. This leaves us with *extreme gravitational* *acceleration* (aka proper acceleration) and *strong tidal forces* (aka tidal acceleration or gravitational gradient), both of which are phenomenal inevitabilities.

Interestingly, both *proper acceleration* (α), given by equation 3.1 and *gravitational gradient* (Δg), given by equation 3.2, involves the term *mass* (M).

$$\alpha = \frac{GM}{r^2}\frac{1}{\sqrt{1-\frac{r_s}{r}}} \tag{3.1}$$

$$\Delta g = \frac{2GM}{r^3}dr \tag{3.2}$$

In case of proper acceleration, the equation cannot be solved for M directly or explicitly, simplifying up to equation 3.3, after quite a bit of algebra, but still contain r and r_s, where, $r = \Delta r + r_s$ and $r_s = 2GM/c^2$, meaning, M is still a factor on both sides, leaving only numerical approaches to solve it, which is quite cumbersome. Furthermore, as $r \longrightarrow r_s$, the term α goes to ∞, meaning we cannot solve this equation for $r \approx r_s$.

$$M = \frac{g^2}{G^2}r(r+r_s)^3 \tag{3.3}$$

Another interesting fact, is that proper acceleration for one of the most massive black hole discovered so far, TON, with mass. All of this leaves us with *gravitational gradient* (Δg)

Mass through Gravitational Gradient

Gravitational gradient is simply the difference in gravitational acceleration across the small distance (dr), (see equation: 3.2). It is also the reason of tides on Earth, hence is commonly related to as well as referred as tidal forces (or acceleration), However, gravitational gradient is much more of a general term. It can be derived by differentiating gravitational acceleration (proper acceleration) with respect to distance (dr), giving the equation, 3.2.

Letting dr as h, height of the observer near the black hole, assuming the observer is parallel to the radial direction of the black hole for maximum gradient, and r to be at Schwarzschild radius, i.e. $(r = r_s)$, the equation 3.2 can be rearranged for mass as in equation 3.4

$$M = \frac{c^3}{2G}\sqrt{\frac{h}{\Delta g}} \tag{3.4}$$

The equation 3.4 gives the mass of a black hole that has Δg gradient at it's event horizon for an observer with height 'h'. Further having $\Delta g = \Delta g_e$, where, Δg_e is the gravitational gradient for a person with height 'h' on the surface of earth, (as we know, such low Δg are bearable for humans), provided by equation 3.5

$$\Delta g_e = \frac{2GM_e}{(r_e + h)^3} \tag{3.5}$$

where, 'M_e' is the mass of the earth and 'r_e' is the radius of earth.

Substituting, Δg_e in to equation 3.4, we can further simplify it into,

$$M = \frac{c^3}{2G}\sqrt{\frac{h(h + r_e)^3}{2GM_e}} \tag{3.6}$$

where equation 3.6 is the final equation that we will be using for calculating the *mass* of our black hole, such that mass is only a function of 'h', as every other term is a constant.

3.1.2 Schwarzschild Radius

Now, equipped with the mass of black hole, the calculation for the *Schwarzschild Radius* is quite simple. Based on schwarzschild metric 1.1, the Schwarzschild radius r_s

for a black hole with mass M is given by the equation, 3.7

$$r_s = \frac{2GM}{c^2} \tag{3.7}$$

where, G is the *Gravitational constant* and c is the *speed of light*. In simple, r_s for a black hole with mass M is simply $const$ times the M, where M is in solar units ($M_\odot$) and one solar mass is $solar_m ass_i n_k gs$.

In interpretational terms, Schwarzschild radius refers to boundary of an **Event Horizon**, which in case of non-rotating black hole, is a sphere, and defined as *the boundary of spacetime inside which escape velocity equals to the speed of light*. This implies once inside this region, nothing can escape.

As our mission to set up a mode of communication between two observers, we will stay out of event horizon. However, as the effects like *gravitational time dilation and gravitational redshift* become more prominent as one approaches the event horizon, we will have observer one as close to the event horizon as we can.

3.2 Observer One

Next important piece of our play is the *Observer One* (*referred to as O1 from here on*). In respect of terms such as distance from the center of the black hole, O1 is closer to it as compared to *Observer Two*. This means that O1 will be the one experiencing effects like **gravitational time dilation, gravitational redshift**. This in turns implies that clock of O1 will run significantly slower as compared to someone at infinity. O1's proper and coordinate distance as well time will vary enough to be taken into consideration.

3.2.1 Proper and Coordinate Distance

In *Schwarzschild Metric*, the relation of proper to coordinate distance is given by the equation 3.8

$$\Delta r' = \Delta r \frac{1}{\sqrt{1 - \frac{r_s}{r}}} \tag{3.8}$$

where, $\Delta r'$ is the *proper distance*, Δr is the *coordinate distance* and r is the distance between center of the black hole to the O1, aka, *radial distance* given by 3.9, . Furthermore, equation 3.8 clearly shows that proper distance is always greater than the coordinate distance by the factor of $1/\sqrt{1 - rs/r}$ and as r approaches infinity, the term approaches 1, meaning at great distance from schwarzschild radius, coordinate and proper distances are almost same.

$$r = \Delta r + r_s \tag{3.9}$$

For our purposes, instead of arbitrarily assuming a coordinate distance and calculating the proper distance, which is not much use later on anyways, we rearranged the equation 3.8 in order to calculate *coordinate distance*, giving us the equation 3.10. This way the distance that O1 measures themselves, i.e. their proper distance will always remain same regardless of change in mass hence, change in r_s. This also assures that radial distance also varies in accordance, in compliance of the equation 3.9.

$$\Delta r = \frac{-r_s + \sqrt{r_s^2 + 4\Delta r'^2}}{2} \tag{3.10}$$

3.2.2 Proper and Coordinate Time

As proper and coordinate distances are related by the inverse of the factor $1/\sqrt{1 - rs/r}$ in schwarzschild system, the proper and coordinate time is related as a simple multiple given by the equation 3.11,

$$\tau = t\sqrt{1 - \frac{r_s}{r}} \tag{3.11}$$

where, τ is the *proper time* and *coordinate time* and it is clear that proper time is always less than the coordinate time, i.e. O1's clock runs slower.

3.2.3 Proper acceleration and extreme g-forces

As stated in subsubsection 3.1.1, tidal forces or gravitational gradient experienced by the O1 would be even less than what they would feel on the surface of earth (as they would have to be at the event horizon to feel the same gravitational gradient they would on earth and since $r > r_s$, and Δg decreases with r, it will be less), we need not to concern ourselves with it or it's effects.

On the other hand, the proper acceleration, given by equation 3.1 caused by the extreme curvature of the spacetime too close to the event horizon, would still be quite lethal. The only viable ways to survive such extreme acceleration without being crushed to death is to be in the state of *free fall* (In accordance to *Equvilance Principle*, objects in free fall experience zero acceleration or weightlessness). And any object can be in the state of free fall around a massive body in two ways.

Radial Free Fall

The simplest scenario is to letting the object fall towards the massive body by releasing it at distance (d) away from the center of the black hole, such that it falls under the sole influence of the gravity of the massive body, decreasing d as times goes by. The obvious problem with using this to counter g-forces through this kind of free fall is that mainly, that O1 will cross the event horizon after some time (t). In addition to that, following are the list of reasons why we won't be using this.

- In this scenario, r changes as time goes, meaning almost every single parameters will change. Like, time dilation factor for O1 will vary, so will the time to send and recieve signals, making our model cumbersome.
- O1 can no longer be consider in an inertial frame with respect to the observer two, which is one of the required assumption.
- Variable redshift parameter will make any sensible and constant communication obsolete.
- Regardless of how far we start the fall, for most masses range, the time frame for the fall from the prespective of O1 will be in the range of few minutes.
- O1 would also not notice nor know when they crossed the event horizon.

Orbit around

Another way for O1 to free fall is instead of falling radially towards the black hole, fall around the black hole, i.e. orbit around the black hole at some distance d. In this scenario, even though d remains constant, making almost all the parameters unchange throughout, O1's frame again is no longer at rest, as even with uniform velocity, velocity direction component changes throughout, making the frame of reference non-inertial.

However, the biggest hurdle in orbit scenario is that in order for observer two to remain in line of sight of O1, they will have to move faster than the speed of light which is a violation of theory of relativity. Along with that, in order for O1 to have the lowest or inner most stable circular orbit, d would have to be 3 times r_s, which is the Innermost stable circular orbit possible in schwarzschild geometry as well as move half the speed of light regardless of the mass of the black hole in question. The question of how to factor in the velocity component in the calculations is not even worth asking at this point because, it would not be possible for O1 to rotate around the black hole and communicate with O2 with ease (as much ease as one can expect around a black hole anyways).

Free Fall Chamber

Imagine a long cylindrical rod with a length 'L', that is hollow inside with enough cross sectional area that an average human being can fall through the rod without coming in contact with rod walls. The said rod is in the vacuum of the space and rotating around it's center of mass, (the point $L/2$), such that it takes the rod time 'T' to complete one rotation. If the rotation and a person falls linearly inside of it from one end to the other in time t_f covering the distance L, i.e. the length of the rod.

At rest

On the basis of subsubsection 3.2.3 and 3.2.3, as O1 can neither radially fall towards the black hole or orbit it at a constant distance d, our only option remains is that O1 is at rest with respect to the black hole at distance r and somehow can survive the g-forces without getting crushed to death. We chose this scenario because in this case the problem is subjective in nature and has nothing to do with the underlying physics of our experiment.

In order to remain at rest at close distance to the event horizon, O1 will be applying thrust through some craft, equal in magnitude to the acceleration due to spacetime curvature in opposite direction and will experience full extent of g-forces, which in most cases will be millions to billions of times the earth gravitational acceleration g_e, calculated by equation 3.1. Of course, realistically, it would kill O1 most certainly but for our aims and purposes, we will consider that O1 survive somehow.

3.3 Observer Two

As mentioned in subsection 3.2.1, proper quantities and coordinate ones (like time, distance etc) are approximately equal so in case of Observer two (referred to as *O2* from here on) are basically the same. In general examples and explanations of such phenomena and thought experiment, O2 is considered to be at infinity, where infinity implies far from the influences of the black hole's gravity. As for our purposes, we need a bit more quantitative information about O2's distance D from the center of the black hole.

3.3.1 Distance

We can always assume a random high value of D that is far enough from the black hole, such that the gravitational force of the black hole on O2 is negligible, but for our model, we use the *newtonian approximation* of gravitational acceleration equation (given in 3.12), as we know O2 is far far away from the black hole. we rearrange the equation 3.12 to calculate the distance D and take the value of g equals to the gravitational acceleration felt by an object on the surface of the earth due to moon, i.e. $g = g_{me}$. This gives us the equation 3.13.

$$g = \frac{GM}{D^2} \tag{3.12}$$

$$D = \sqrt{\frac{GM}{g_{me}}} \tag{3.13}$$

The reason of not using a constant value of D for the experiment is because then for the different masses of the black hole, g experienced by the O2 would vary and can have effects that we might end up neglecting as we will be considering g as negligible for O2.

Now, that we have both O1's distance from the center of the black hole as given by eq 3.9 and O2's distance given by eq 3.13, we can calculate the distance between O1 and O2 by simple algebra.

$$d = D - r \tag{3.14}$$

where, d is the distance between O1 and O2, assuming both are at rest and on same radial line from the black hole. As, the time and everything else for the O2 will be normal, this is the only thing we really need to know about O2.

3.4 Signals

Now, as we know all the important things we need to know about our black hole (section 3.1) and our observers (sections 3.2 and 3.3), we are ready to discuss the main player of our experiment, *signals.*. Both of our observers will communicate via electromagnetic signals, which will be travelling at the speed of light (c). To keep things simple and interesting, they will be using the **Morse code**, where the *dots* and *dashes* will be distinguished by durations of the flash. The important consideration about the signals are given in following subsections.

3.4.1 Gravitational Redshift

One of the most prominent effects of a black hole is *gravitational redshift*, which is the effect of the gravitational potential on the wavelength of light. According to *GR*, as a light beam travels opposite to the gravitational potential of a massive body, it loses energy and it's wavelength increases, i.e. the wavelength shifts towards longer wavelengths and less energetic parts of the EM spectrum, or towards red, just like the doppler effect. In our scenario, light travelling from O1 to O2 would be redshifted.

For gravitational redshift, the redshift parameter in terms of the mass of the black hole is given by equation 3.14. This factor z, then provides a way to relate the measured wavelength of light as compared to the emitted one given by the equation 3.15

$$z = \frac{1}{\sqrt{1 - \frac{r_s}{r}}} - 1 \tag{3.14}$$

$$\lambda_o = \lambda_e(1 + z) \tag{3.15}$$

where, λ_o is the *observed wavelength* and λ_e is the *emitted wavelength*. Further, we can use equations 3.14 and 3.15 to simplify the relation of λ_o and λ_e in terms of r_s and r, stated in equation 3.16

$$\lambda_o = \lambda_e \frac{1}{\sqrt{1 - \frac{r_s}{r}}} \tag{3.16}$$

From equation 3.16, it is obvious that observed wavelength is always greater than that of emitted, i.e. $\lambda_o > \lambda_e$. Beside that, at $r = r_s$, the observed wavelength goes to infinity. This is why event horizons are called the surfaces of infinite redshift.

Blueshift

In case of light travelling in the opposite direction, i.e. towards the black hole from afar, the converse of everything in subsection 3.4.1 also applies. This means, in case of light coming towards the black hole, i.e. from O2 to O1 will have same amount of increase in energy, hence decrease in wavelength as the light ray travelling in the opposite direction loses. Hence, for O1, observed light's wavelength is given by,

$$\lambda'_o = \lambda'_e \sqrt{1 - \frac{r_s}{r}} \tag{3.17}$$

where, λ'_o is the *observed wavelength* by O1 and λ'_e is the *emitted wavelength* by O2.

For our experiment, we assume a constant observed wavelength for O1 and O2, i.e. $\lambda_o = \lambda'_o$ and calculated the required emitting wavelengths for both observers given by equations 3.18 and 3.19.

$$\lambda_e = \lambda_o \sqrt{1 - \frac{r_s}{r}} \tag{3.18}$$

$$\lambda'_e = \lambda_o \frac{1}{\sqrt{1 - \frac{r_s}{r}}} \tag{3.19}$$

where, λ_e is the wavelength of light emitted by O1 and λ'_e is the wavelength of light emitted by O2, and λ_o is the observed wavelength by both observers.

3.4.2 Message Time

If we denote duration of a *dot* flash as $t_{(dot)}$ and duration of a *dash* flash as $t_{(dash)}$, such that $t_{(dot)} < t_{(dash)}$, and keep the time between two flashes same as $t_{(dot)}$ and between two words same as $t_{(dash)}$, we can calculate the total time spend on composing a single message as given in equation 3.20

$$t_{(msg)} = nt_{(dot)} + mt_{(dash)} \tag{3.20}$$

where n is the number of all the dots in the single message as well as the spaces and m is the number of all the dashes in the single message as well as the spaces between words. To be duly noted, this is the time measured by the observers themselves, proper time.

Relative message time

Let's assume O1 transmit a single dot flash towards O2 having a duration of $t_{(dot)}$. Now, according to subsection 3.2.2, the dot will have a duration of $t_{(dot)}/\sqrt{1 - r_s/r}$ i.e. longer than the actual duration and in opposite case, it will be less than the actual duration of flash send by O2. This can be very bothersome and not very ideal for a communication system.

In order to deal with that, instead of sending flashes of simple $t_{(dot)}, t_{(dash)}$, both observers will adjust the duration in order to compensate for time dilation effects. Based on equation 3.11, O1 will have flashes duration in term of inverse of the factor $\sqrt{1 - rs/r}$, while O2 will have their duration as multiple. This will take care of time disrepancy due to gravitational time dilation.

$$
\begin{aligned}
t_{(A)} &= t_{(dur)} \frac{1}{\sqrt{1 - \frac{r_s}{r}}} \\
t_{(B)} &= t_{(dur)} \sqrt{1 - \frac{r_s}{r}}
\end{aligned}
\tag{3.21}
$$

where, $t_{(dur)}$ is either $t_{(dot)}$ or $t_{(dash)}$ generalised durations, and equations 3.21 provide the durations for respective observer. Furthermore, for equation 3.20, the use of dot and dash will depend on the observer, to provide the total time in their local frame of reference. It is to be noted that these will be durations they will keep the flashes. Observed flashes, regardless of observer, will be simple $t_{(dot)}$ and $t_{(dash)}$.

3.4.3 Signal Travel Time

The time it will take for signal to travel from O1 to O2 considering relativistic effects is given by

$$\Delta t = \frac{d}{c} + \frac{r_s}{c} \ln(\frac{D - r_s}{r - r_s}) \tag{3.22}$$

Chapter 4

Data Analysis and Interpretation

Chapter 5

Summary, Findings, Discussion, Conclusions and Recommendations

Appendix A: Derivations

.0.1 Mass of the Black hole

According to *Newton's 2nd law*,

$$F = ma \tag{1.1}$$

In case, of acceleration due to gravity (g), this can be written as,

$$F_g = mg \tag{1.2}$$

And in accordance with *Newton's Law of Gravitation*,

$$F_g = \frac{GMm}{r^2} \tag{1.3}$$

From equation (1.2) and (1.3), we get g, Gravitational acceleration as,

$$g = \frac{GM}{r^2} \tag{1.4}$$

Tidal acceleration (a_t) or Gravitational gradient (Δg) is defined as the rate of change of gravitational acceleration across small distance Δr, given by the derivative of g from equation (1.4), as

$$\frac{dg}{dr} = \frac{d}{dr}(\frac{GM}{r^2}) \tag{1.5}$$
$$\frac{dg}{dr} = GM\frac{d}{dr}(r^{-2}) \tag{1.6}$$
$$\frac{dg}{dr} = GM(2r^{-3}) \tag{1.7}$$
$$\frac{dg}{dr} = \frac{2GM}{r^3} \tag{1.8}$$

For a finite change in r as Δr, for a finite change in g, Δg,

$$\Delta g = \frac{dg}{dr}\Delta r \tag{1.9}$$

from equation (1.9) and (1.5), and replacing $Deltar$ with h to represent height of a person, we get the same equation as in (**??**),

$$\Delta g = \frac{2GMh}{r^3} \tag{1.10}$$

Till here, the derivation can be found in text books. Afterwards, we are gonna solve it for our purposes.

We would like to derive an equation that calculate the mass of the BH M, that has a certain Δg at the surface of it's event horizon, i.e. at r_s.

As, $r_s = 2GM/c^2$ and putting $r = r_s$ in equation (1.10), we get,

$$\Delta g = 2GMh(\frac{c^2}{2GM})^3 \tag{1.11}$$

$$\Delta g = 2GMh\frac{c^8}{8G^3M^3} \tag{1.12}$$

$$\Delta g = h\frac{c^8}{4G^2M^2} \tag{1.13}$$

$$M^2 = \frac{hc^8}{4G^2\Delta g} \tag{1.14}$$

$$M = \sqrt{\frac{hc^8}{4G^2\Delta g}} \tag{1.15}$$

$$M = \frac{c^3}{2G}\sqrt{\frac{h}{\Delta g}} \tag{1.16}$$

where, (1.11) is the desired equation (**??**).

Proper and Coordinate Distances

The *Schwarzchild Metric* is given by,

$$ds^2 = -c^2d\tau^2 = -(1-\frac{r_s}{r})c^2dt^2 + (1-\frac{r_s}{r})^{-1}dr^2 + r^2(d\theta^2 - sin^2\theta d\phi^2) \tag{2.1}$$

where (t, r, θ, ϕ) are *Schwarzchild coordinates* that describe the curvature around a non-rotating, uncharged spherical body (black hole). for simplification of the equation to get the relation for proper and coordinate distance, we will assume a spacetime event that is fixed in time, $dt = 0$, and no angular motion, i.e. a pure radial movement, hence, $d\theta = d\phi = 0$, which simplifies equation (2.1) to,

$$ds^2 = (1-\frac{r_s}{r})^{-1}dr^2 \tag{2.2}$$

then for a proper distance ($\Delta r'$) for coordinate distance (Δr), we have,

$$
\begin{aligned}
\Delta r' &= \sqrt{ds^2} \\
\Delta r' &= \sqrt{(1 - \frac{r_s}{r})^{-1} \Delta r^2} \\
\Delta r' &= \Delta r \sqrt{(1 - \frac{r_s}{r})^{-1}} \\
\Delta r' &= \Delta r \frac{1}{\sqrt{1 - \frac{r_s}{r}}}
\end{aligned}
\tag{2.3}
$$

where , substituting $\sqrt{1 - (rs/r)}$ as q will gives us equation (**??**).

Now, Further more,

$$
\begin{aligned}
\Delta r &= \Delta r' \sqrt{1 - \frac{r_s}{r}} \\
\Delta r &= \Delta r' \sqrt{\frac{r - r_s}{r}} \\
\Delta r &= \Delta r' \sqrt{\frac{r_s + \Delta r - r_s}{r_s + \Delta r}} \text{ from eq (??)} \\
\Delta r &= \Delta r' \sqrt{\frac{\Delta r}{r_s + \Delta r}} \\
\Delta r^2 &= \Delta r'^2 \frac{\Delta r}{r_s + \Delta r} \\
0 &= \Delta r^2 (r_s + \Delta r) - \Delta r'^2 \Delta r \\
0 &= \Delta r^3 + r_s \Delta r^2 - \Delta r'^2 \Delta r \\
0 &= \Delta r (\Delta r^2 + r_s \Delta r - \Delta r'^2)
\end{aligned}
\tag{2.4}
$$

from equation (2.4), we get $\Delta r = 0$ and the quadratic equation,

$$
\Delta r^2 + r_s \Delta r - \Delta r'^2 = 0 \tag{2.5}
$$

comparing with the general quadratic equation, $ax^2 + bx + c = 0$, we get,

$$a = 1$$

$$b = r_s$$

$$c = -\Delta r'$$

using quadratic formula (QF),

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{QF}$$

putting values we get,

$$\Delta r = \frac{-r_s \pm \sqrt{r_s^2 + 4(\Delta r')r_s}}{2} \tag{2.6}$$

Proper and coordinate Time

For a timelike spacetime event, we will assume an event that is stationary in space, i.e. $ds = d\theta = d\phi = 0$, which reduces the *Schwarzchild Metric* to,

$$-c^2 d\tau^2 = -(1 - \frac{r_s}{r})c^2 dt^2 \tag{2.7}$$

simplifying it by cancelling out $-c^2$ on both sides, and taking the square root and neglecting the derivative, leaves the following,

$$\tau = t \cdot \sqrt{1 - \frac{r_s}{r}} \tag{2.8}$$

or, using equation (**??**), we get the same equation as (**??**)

$$\tau = tq \tag{2.9}$$

Appendix B: Formulas

$$M = \frac{c^3}{2G}\sqrt{\frac{h^2 r_e^3}{2GM_e}} \tag{3.6}$$

$$r_s = \frac{2GM}{c^2} \tag{3.7}$$

$$\Delta r = \frac{-r_s + \sqrt{r_s^2 + 4\Delta r'^2}}{2} \tag{3.10}$$

$$r = \Delta r + r_s \tag{3.9}$$

$$\tau = t\sqrt{1 - \frac{r_s}{r}} \tag{3.11}$$

$$D = \sqrt{\frac{GM}{g_e}} \tag{3.13}$$

$$d = D - r \tag{3.14}$$

$$z = \frac{1}{\sqrt{1 - \frac{r_s}{r}}} - 1 \tag{3.14}$$

$$\lambda_e = \lambda_o\sqrt{1 - \frac{r_s}{r}} \tag{3.18}$$

$$\lambda'_e = \lambda_o\frac{1}{\sqrt{1 - \frac{r_s}{r}}} \tag{3.19}$$

$$t_{(A)} = t_{(dur)}\frac{1}{\sqrt{1 - \frac{r_s}{r}}}$$

$$t_{(B)} = t_{(dur)}\sqrt{1 - \frac{r_s}{r}} \tag{3.21}$$

$$t_{(msg)} = nt_{(dot)} + mt_{(dash)} \tag{3.20}$$

$$\Delta t = \frac{d}{c} + \frac{r_s}{c}\ln(\frac{D - r_s}{r - r_s}) \tag{3.22}$$

Symbol	**Name**	**Units**	**Formula**
M	Mass of the Black Hole	$M_\odot$	
r_s	Schwarzschild Radius	m	
$\Delta r'$	Proper Distance of *O1*	m	
$\Delta r'$	Coordinate Distance of *O1*	m	
r	Radial distance of *O1*	m	
t	Coordinate time of *O1*	sec	
τ	Proper time of *O1*	sec	
D	Radial Distance of *O2*	m	
d	Distance between *O1* and *O2*	m	
z	Redshift factor	[unitless]	
λ_o	Observed wavelength by *O1* and *O2*	m	
λ_e			

Table 1: Table of Symbols, their names, units and formulas

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