CSCI 6212 – Design and Analysis of Algorithms (Fall 2023)

Project Assignment 1

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Problem Statement

We are required to analyse the asymptotic time complexity of the following program/code sample:

```
int j = 2
while (j < n) {
        k = 2
        while (k < n) {
                sum += a[k] * b[k]
                k = k * sqrt(k)
        }
        j += j / 2
}
```

Theoretical Analysis

The time Complexity of a loop is considered as O(log n) if the loop variables are divided/multiplied by a constant amount. The Time Complexity of a loop is considered as O(log log n) if the loop variables are reduced/increased exponentially by a constant amount.

- The Outer Loop runs from j=2 to n with increments of j/2 (which can be written as, for each round of loop new value of j would be, $j_{new} = 3/2 * j_{old}$) which approximates to O(log n).
- The Inner Loop runs from k=2 to n with increments of k * SQRT(k) (which can be written as, for each round of loop new value of k would be, $k_n = k_0^c$, where c is some constant) which again approximates to O(log log n).

Hence, we can say the Time Complexity of the above code sample is O((log n * log log n))

Experimental Analysis

- **Program Listing**

  I have executed the code for values of n ranging from 1E1 to 1E49 with multiples of 1E2, which can be seen in the following section of Output Numerical Data.

- **Data Normalization Notes**

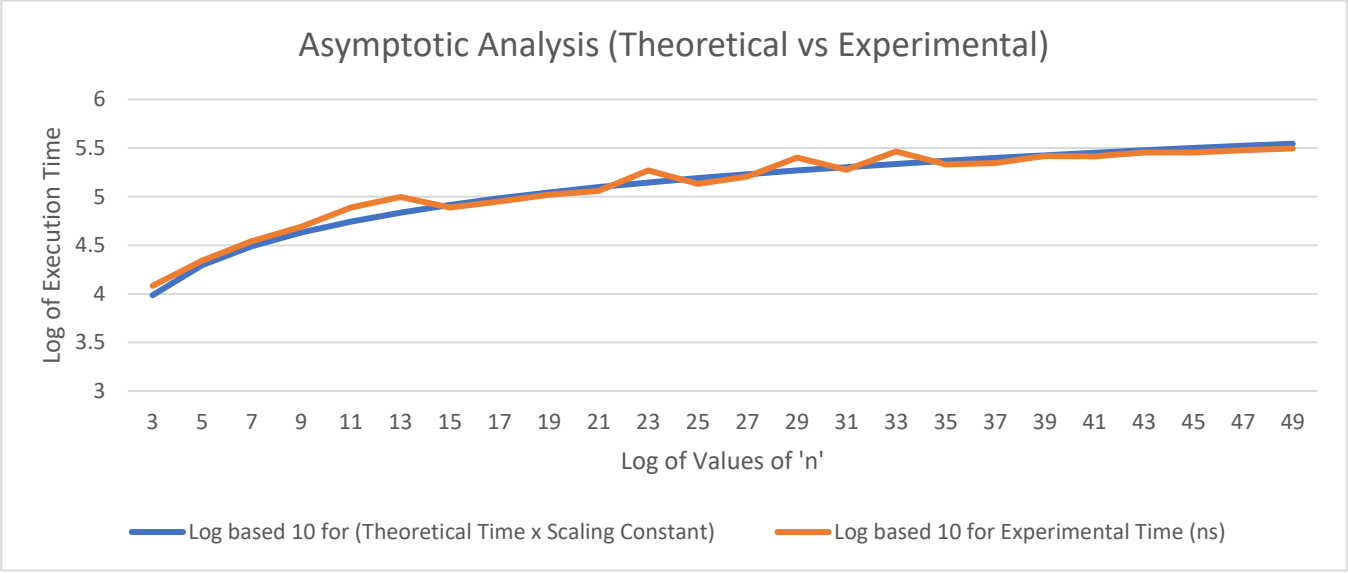
  To Normalize the theoretical time, we take the average of the experimental time and divide that by the average of the theoretical time, that give us the scaling constant which we multiply with the theoretical time. This gets the theoretical and experimental values in the same range. But the values of n are still too large to compare with the execution times. So, we log the values of n as well as we log the values of theoretical and experimental execution Times.

- **Output Numerical Data**

Value of 'n'	Theoretical Time Taken (O(log n * log log n))	Experimental Time Taken (ns)	Scaling Constant	Theoretical Time x Scaling Constant	Log based 10 for 'n'	Log based 10 for New Theoretical Time	Log based 10 for Experimental Time (ns)
1.00E+01	5.75	5400.01	293.12	1686.53	1	3.226994159	3.732394726
1.00E+03	33.06	12100.00	293.12	9689.59	3	3.98630559	4.082785543
1.00E+05	67.33	21999.99	293.12	19737.37	5	4.295289323	4.342422541
1.00E+07	105.56	34800.00	293.12	30941.06	7	4.49053522	4.541579282

1.00E+09	146.56	49200.00	293.12	42958.79	9	4.633052063	4.69196507
1.00E+11	189.70	77400.01	293.12	55606.12	11	4.745122611	4.888741002
1.00E+13	234.60	99600.00	293.12	68767.14	13	4.837380979	4.998259346
1.00E+15	280.98	77200.01	293.12	82362.13	15	4.915727578	4.887617333
1.00E+17	328.64	89600.01	293.12	96332.85	17	4.983774421	4.952308072
1.00E+19	377.43	104699.98	293.12	110634.88	19	5.043892079	5.019946608
1.00E+21	427.24	115100.00	293.12	125233.21	21	5.09771951	5.061075336
1.00E+23	477.95	186200.02	293.12	140099.52	23	5.146436658	5.269979725
1.00E+25	529.51	135700.01	293.12	155210.46	25	5.190920986	5.132579895
1.00E+27	581.82	161899.98	293.12	170546.41	27	5.231842571	5.209246799
1.00E+29	634.85	252300.00	293.12	186090.66	29	5.269724578	5.401917251
1.00E+31	688.55	189899.99	293.12	201828.84	31	5.304983216	5.278524944
1.00E+33	742.86	292400.01	293.12	217748.40	33	5.337954977	5.46597739
1.00E+35	797.75	215499.98	293.12	233838.35	35	5.368915742	5.333447238
1.00E+37	853.19	222200.00	293.12	250088.94	37	5.398094483	5.346744062
1.00E+39	909.14	261699.98	293.12	266491.47	39	5.425683312	5.417803697
1.00E+41	965.59	260400.00	293.12	283038.15	41	5.451844971	5.415640986
1.00E+43	1022.51	284799.99	293.12	299721.93	43	5.476718522	5.454539963
1.00E+45	1079.87	284400.01	293.12	316536.44	45	5.500423717	5.45392961
1.00E+47	1137.66	300899.99	293.12	333475.87	47	5.52306441	5.478422176
1.00E+49	1195.86	313800.00	293.12	350534.88	49	5.544731245	5.496652943

- **Graph**



- **Graph Observations**

  We can visually observe from the graph that the Theoretical Execution Time and Experimental Execution Time follow a similar path for same values of n. Since we assume that the variable assignment and conditional checking and expression evaluation takes constant time for theoretical values, it does not hold true for experimental values. Hence we see the Bumps in the graph for the experimental values.

Conclusion

Here we can conclude that the Theoretical Time Complexity of O((log n * log log n)) is correct which can be validated from the Experimental Results.