w Problem P $P(x) = \begin{cases} 1 & x \in G \\ 0 & x \in B \end{cases}$

Graph coloring: m = # edges
c = # colors

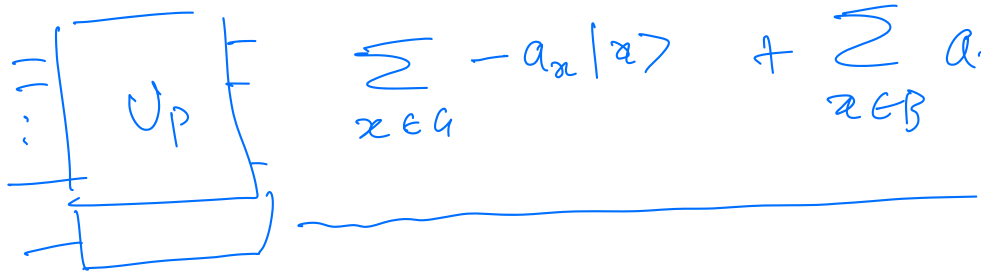
$\Rightarrow \lceil \log c \rceil m = n$ # qubits

Let $U_I|0\rangle$ = starting superpos

Typically $U_I = H^{\otimes n}$

Let $\overbrace{S_G} = \underbrace{U_p \otimes I}$ with $|-\rangle$ ancilla

$\sum a_x|x\rangle$



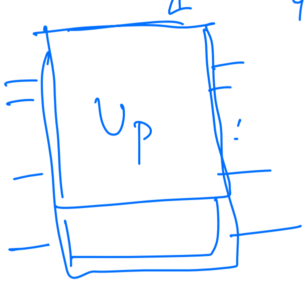
$\sum_{x \in G} -a_x|x\rangle + \sum_{x \in B} a_x|x\rangle$

$|-\rangle$ $|-\rangle$

$S_G = (n+1)$ qubit

$\sum a_x|x\rangle$

$|y\rangle$



$|f(x) \oplus y\rangle = \begin{cases} X|y\rangle & \text{if } f(x)=1 \\ |y\rangle & \text{if } f(x)=0 \end{cases}$

When $|y\rangle = |-\rangle$, $X|-\rangle = -|-\rangle$

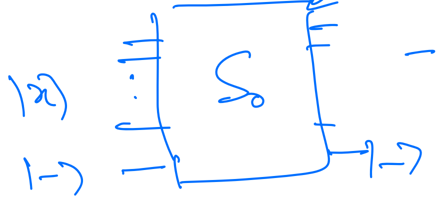
$\sum_{x \in G} a_x|x\rangle|-\rangle + \sum_{x \in B} a_x|x\rangle|-\rangle \xrightarrow{U_p \otimes I} \sum_{x \in G} -a_x|x\rangle|-\rangle + \sum_{x \in B} a_x|x\rangle|-\rangle$

$$D = \overbrace{-\, H^{\otimes n}\, \underset{n+1}{S_0}\, H^{\otimes n}}$$

$$f_0(a) = \begin{cases} 1, & |a\rangle = |0\rangle \\ 0 & o/w \end{cases}$$

$S_0 = (n+1)$

Define U_0



$-|x\rangle$, $x=0$

$|x\rangle$, $x\neq 0$

$$-\left(H^{\otimes n} \otimes I\right) S_0 \left(H^{\otimes n} \otimes I\right)$$

Define $$Q = -\left(H^{\otimes n} \otimes I\right) S_0 \left(H^{\otimes n} \otimes I\right) S_q$$

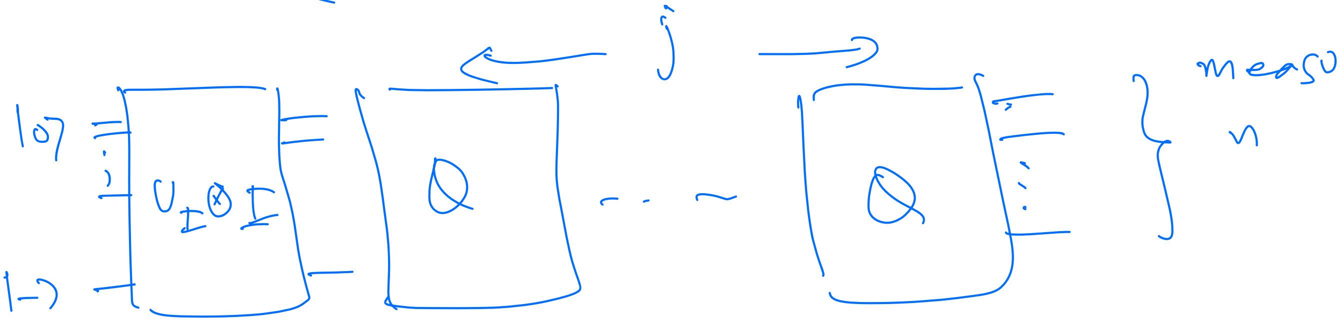
Define $$\bar{f}_0(x) = \begin{cases} 0, & |x\rangle = 0 \\ 1, & |x\rangle \neq 0 \end{cases}$$

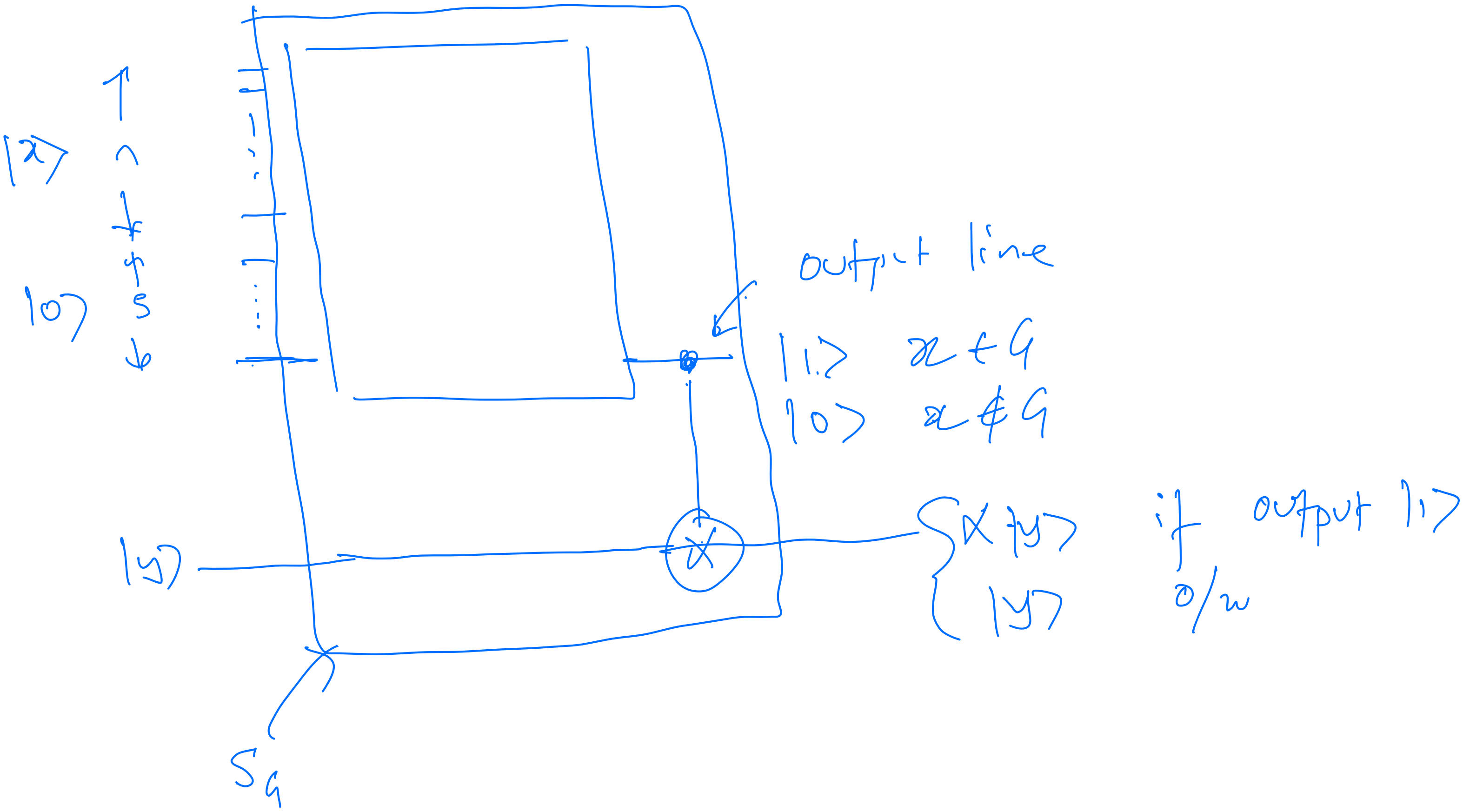
$-|x\rangle$, $x\neq 0$

$|x\rangle$, $x=0$

$$\Rightarrow \quad Q = \underbrace{\left(H^{\otimes n} \otimes I\right) \bar{S}_0 \left(H^{\otimes n} \otimes I\right) S_q}$$

$$Q^j \left(U_I |0\rangle |-\rangle\right) \quad \text{Grover's Algorithm}$$





|x⟩
n + s
|0⟩
|y⟩
S_G
output line
|1⟩ x ∈ G
|0⟩ x ∉ G
|x+y⟩ if output |1⟩
|y⟩ o/w