0-π Qubit

From Basics of Quantum Mechanics to 0-π Qubit

Defne Dilbaz

Outline

1. Objective
2. Background Information
3. Circuit Models
4. Future Steps

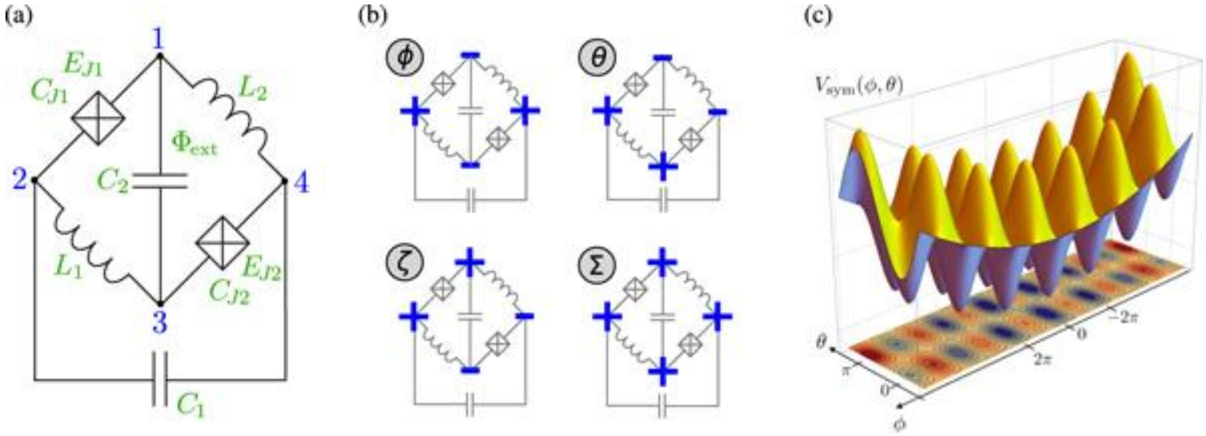
QUANTUM?

“Quantum fluctuation messes with the Planck scale, which then triggers the Deutsch proposition. Can we agree on that?”
- Tony Stark

Purdy, M. (2018, March 21). For the MCU to Thrive, Tony Stark Must Die. Retrieved from https://www.cbr.com/mcu-tony-stark-must-die/
Janaraghi, P. (2019, May 03). Avengers: Endgame Best Quotes - 'This is the fight of our lives.' Retrieved from https://www.moviequotesandmore.com/avengers-endgame-best-quotes/

Objective

Analyzing 0-π qubit structure, running simulations, and suggesting an improvement on current architecture for increased feasibility



Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.

Background Information

Classical vs Quantum Computation

Classical	Quantum
Data is stored as 0 or 1	Data is stored as a superposition $\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$
Data representation depends on voltage measurement	Data representation depends on the eigenvectors and eigenvalues of the qubit Hamiltonian
Classical circuit models	Superconducting circuits, ion-traps, photonic devices, etc.

A. Nielsen, Michael & Chuang, Isaac. (2004). Quantum Computation and Quantum Information. 10.1063/1.1428442.

Background Information

Quantum Mechanics Vocabulary

- **Hamiltonian: an analogy to the energy of a system which consists of kinetic and potential terms**

- **2nd Postulate of Quantum Mechanics: The time evolution of a closed quantum system is described by the Schrödinger's Equation: [1]**

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

Given initial condition |ψ>, the Schrödinger's Equation determines all |ψ(t)> for any given future time (t).[2]

[1]: A. Nielsen, Michael & Chuang, Isaac. (2004). Quantum Computation and Quantum Information. 10.1063/1.1428442.
[2]: J. Griffiths, David & Schroeter, Darrell. (2018). Introduction to Quantum Mechanics. 10.1017/9781316995433.

Background Information

Algebra for Quantum Mechanics

Hamiltonian:

$$H = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{n \times n \text{ matrix}}$$

example: $H = -\frac{1}{2}(\epsilon \hat{\sigma}_z + \Delta \hat{\sigma}_x)$

$$\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$H = -\frac{1}{2}\begin{bmatrix} \epsilon & \Delta \\ \Delta & -\epsilon \end{bmatrix}$$

States:

$|\psi\rangle = a|0\rangle + b|1\rangle$ where $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$|a|^2 =$ probability of $|0\rangle$,

$|b|^2 =$ probability of $|1\rangle$ $\Rightarrow |a|^2 + |b|^2 = 1$

Background Information

Algebra for Quantum Mechanics

Schrödinger's Equation: $i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$, where $\hbar =$ Planck's constant

$$\frac{d|\psi\rangle}{dt} = \frac{H|\psi\rangle}{i\hbar} = -\frac{iH|\psi\rangle}{\hbar} \Rightarrow \frac{d|\psi\rangle}{|\psi\rangle} = \frac{-iH}{\hbar} dt$$

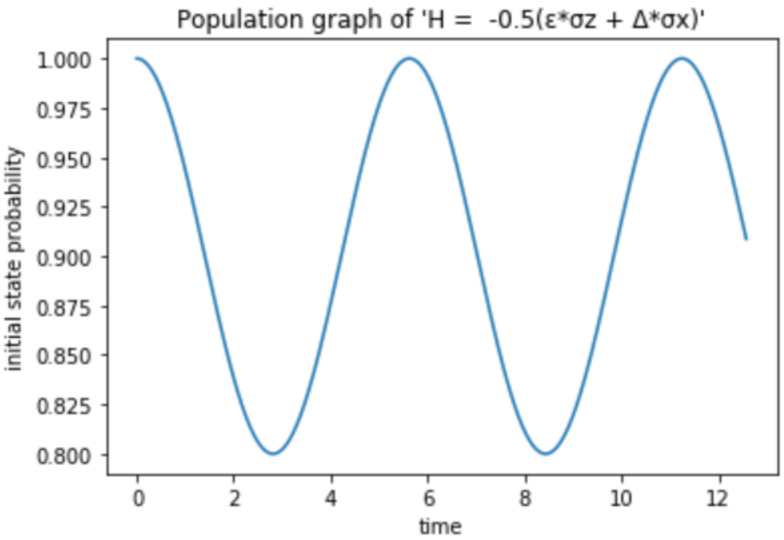
$$\int_{\psi_0}^{\psi_t} \frac{d|\psi\rangle}{|\psi\rangle} = \int_0^t \frac{-iH}{\hbar} dt \Rightarrow \ln|\psi_t| - \ln|\psi_0| = \frac{-iH}{\hbar} t$$

$$\ln\left|\frac{\psi_t}{\psi_0}\right| = \frac{-iH}{\hbar} t \Rightarrow \frac{\psi_t}{\psi_0} = e^{\frac{-iH}{\hbar} t}$$

$$\boxed{|\psi(t)\rangle = e^{-\frac{iH}{\hbar} t} |\psi(0)\rangle}$$

Background Information

Unitary Time Evolution Simulation



- **We have the following Hamiltonian*:**

$$\hat{H} = -\frac{1}{2}(\epsilon\hat{\sigma}_z + \Delta\hat{\sigma}_x).$$

- **Start with basis |0>**
- **Evolve the system for time [0, 4π]**
- **Calculate the probability of obtaining state |0> - the original start state**

Kyaw, T. (n.d.). A single two-level system (qubit): Landau-Zener problem Thi Ha (Tech.).

Background Information

Adiabatic Evolution

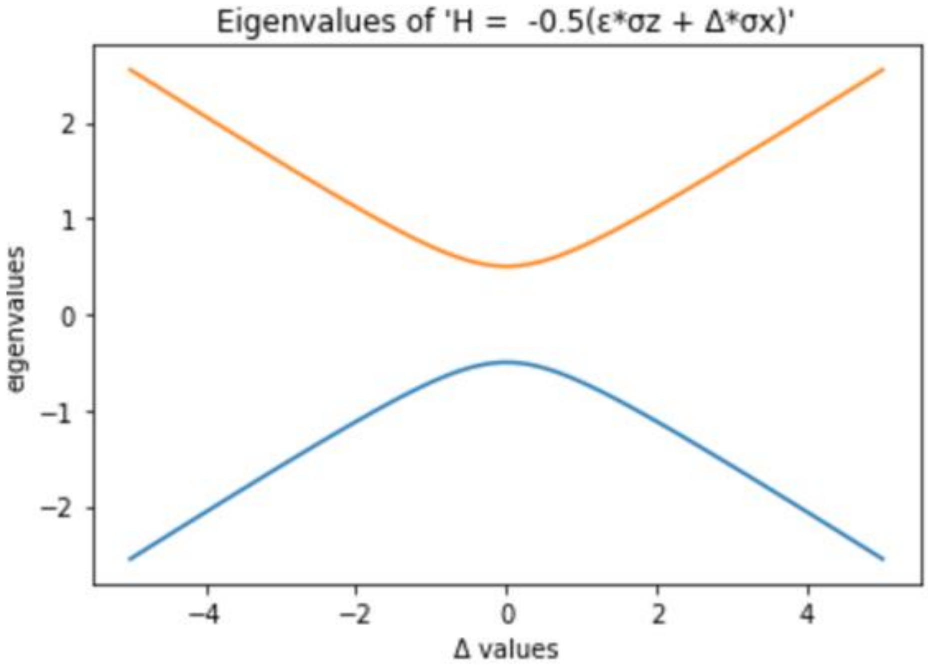
- For time-dependent Hamiltonians, Schrödinger’s Equation is valid for continuous time evolutions.
- According to the quantum adiabatic theorem, a quantum system that begins in the non-degenerate ground state of a time-dependent Hamiltonian will remain in the instantaneous ground state provided the Hamiltonian changes sufficiently slowly.

Childs, Andrew, Prof. "LECTURE 18: The Quantum Adiabatic Theorem." Www.math.uwaterloo.ca. January/February 2008. Accessed July 23, 2019. https://www.math.uwaterloo.ca/~amchilds/teaching/w08/l18.pdf.

Background Information

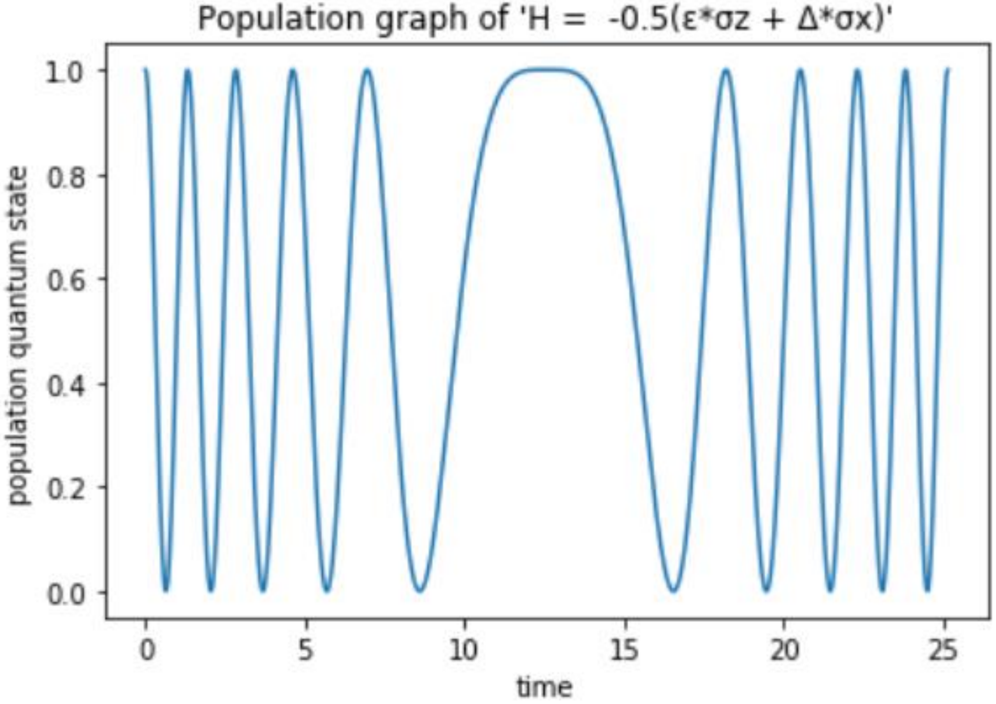
Adiabatic Evolution

- **Adiabaticity depends on the energy gap between ground and first excited states**



Background Information

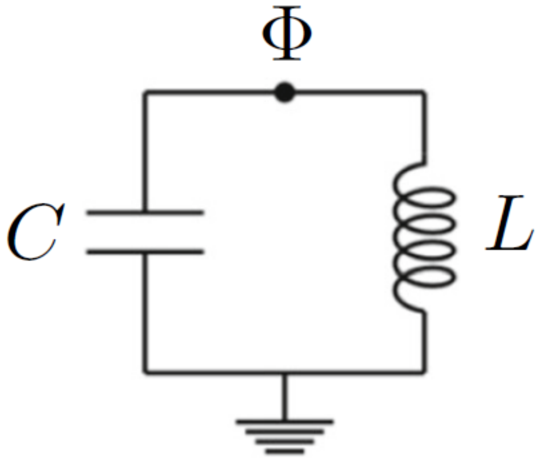
Unitary Time Evolution with Time-Dependent Hamiltonian Simulations



Δ(t) = 10t/(8π) - 5

Circuit Models

Classical LC Circuits



$$\boxed{\Phi(t) = \int_{-\infty}^{t} V(t')dt'}$$

$$L = 10 \text{ nH}, C = 1 \text{ pF}$$

$$\omega_0/2\pi = \frac{1}{2\pi\sqrt{LC}} \simeq 1.6 \text{ GHz}$$

Circuit Models

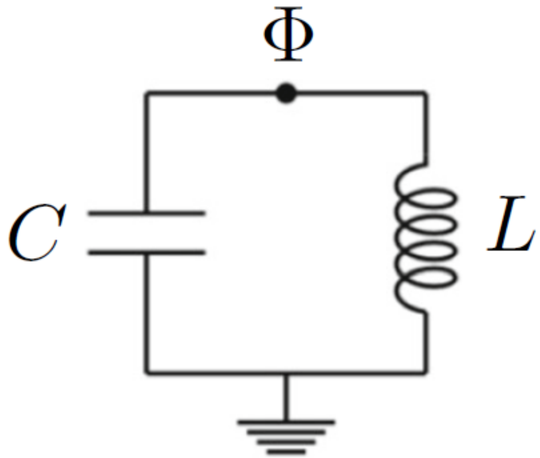
Quantum Circuit Requirements

- The temperature of the dilution refrigerator should be 20 mK.
- Thermal fluctuation energy should be smaller than quantum energy associated with the natural frequency of the circuit $\rightarrow k_B T \ll h\omega_0$

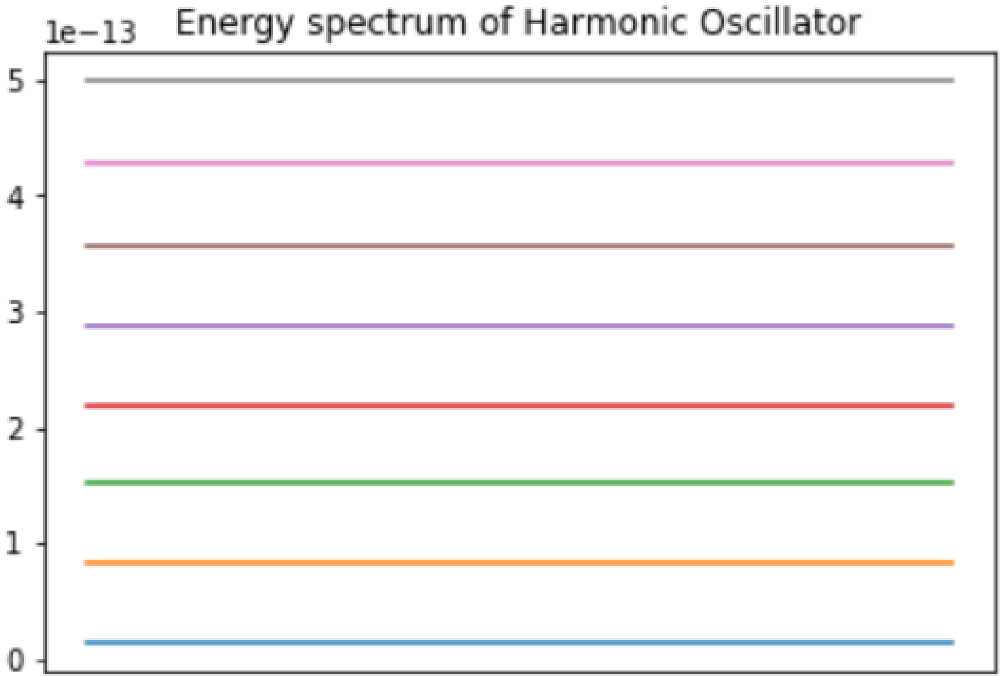
Vool, Uri and Michel Devoret. “Introduction to quantum electromagnetic circuits.” I. J. Circuit Theory and Applications 45 (2016): 897-934.

Circuit Models

LC Oscillator Simulations



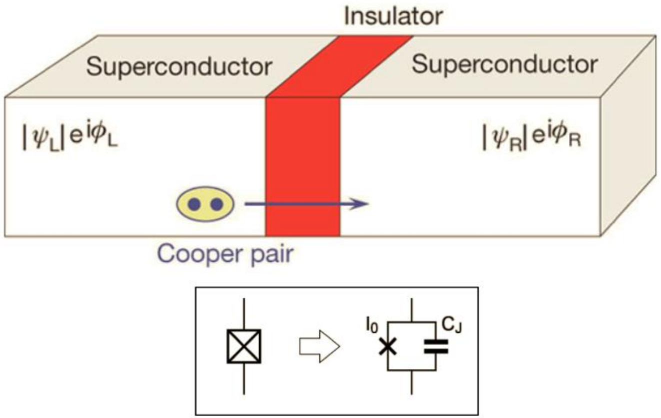
$$\hat{H} = \hbar\omega_0 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$



Kyaw, Thi Ha. (2019). Towards a Scalable Quantum Computing Platform in the Ultrastrong Coupling Regime. 10.1007/978-3-030-19658-5.

Circuit Models

Josephson Junction



Schematic Diagram of Josephson Junction[2]

- When there is voltage V, the supercurrent oscillates at frequency[1]:

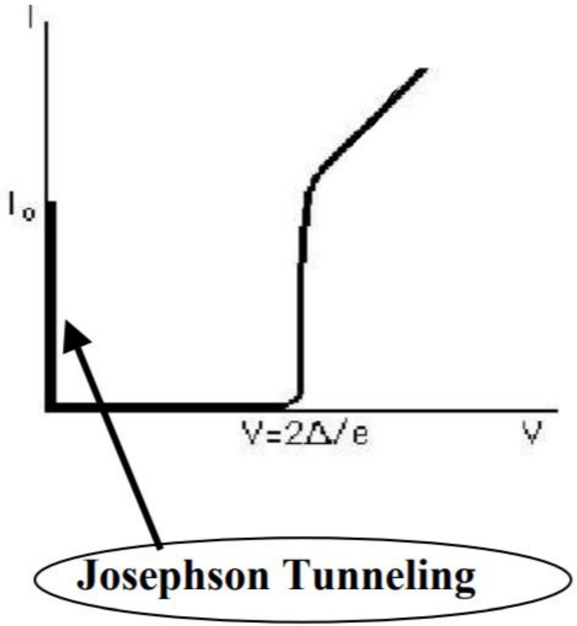
$$\nu = 2eV/h$$

- Quantum Tunneling: Cooper pairs can pass through a potential barrier even if they do not have enough energy. This is because of the phase difference between two superconductor layers.[2]

[1]: M. Chauvin's PhD thesis, Université Paris 6 (2005)
[2]:Clarke, John. (1970). The Josephson Effect and e/h. American Journal of Physics - AMER J PHYS. 38. 1071-1095. 10.1119/1.1976556.

Circuit Models

Josephson Junctions



Voltage-Current Relationship [1]

Josephson Inductance[2]:

$$L_J(\phi) = \frac{\Phi_0}{2\pi I_c \cos\phi} = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{E_J \cos\phi} \qquad E_J = \hbar I_c/(2e)$$

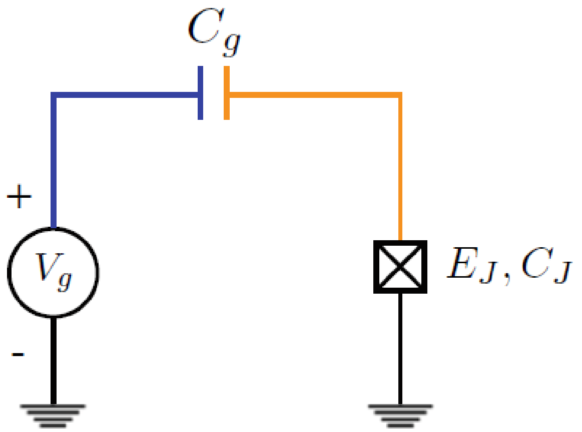
[1]: "Generalized Josephson Junctions." 6.763 2003 Lecture 13. 6.763 2003 Lecture 13, 19 July 2019, Boston, Massachusetts Institute of Technology.
[2]: Kyaw, Thi Ha. (2019). Towards a Scalable Quantum Computing Platform in the Ultrastrong Coupling Regime. 10.1007/978-3-030-19658-5.

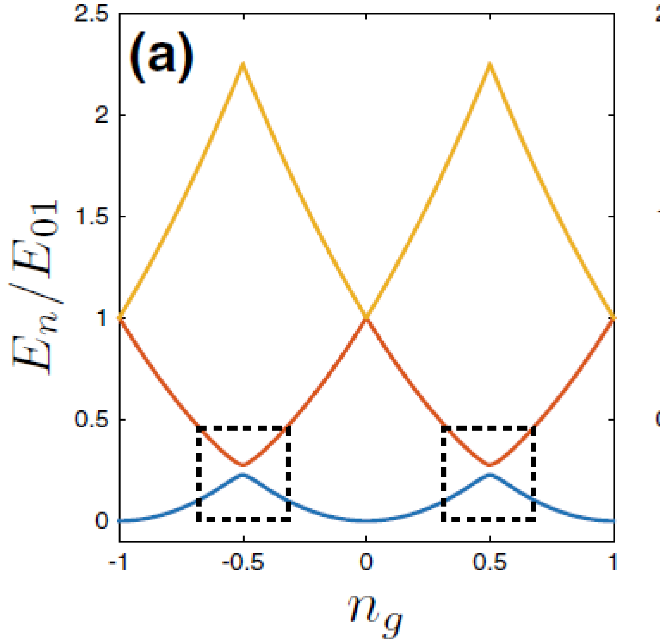
Circuit Models

Single Cooper-Pair Box

Hamiltonian:

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\left(2\pi \frac{\Phi_J}{\Phi_0}\right)$$





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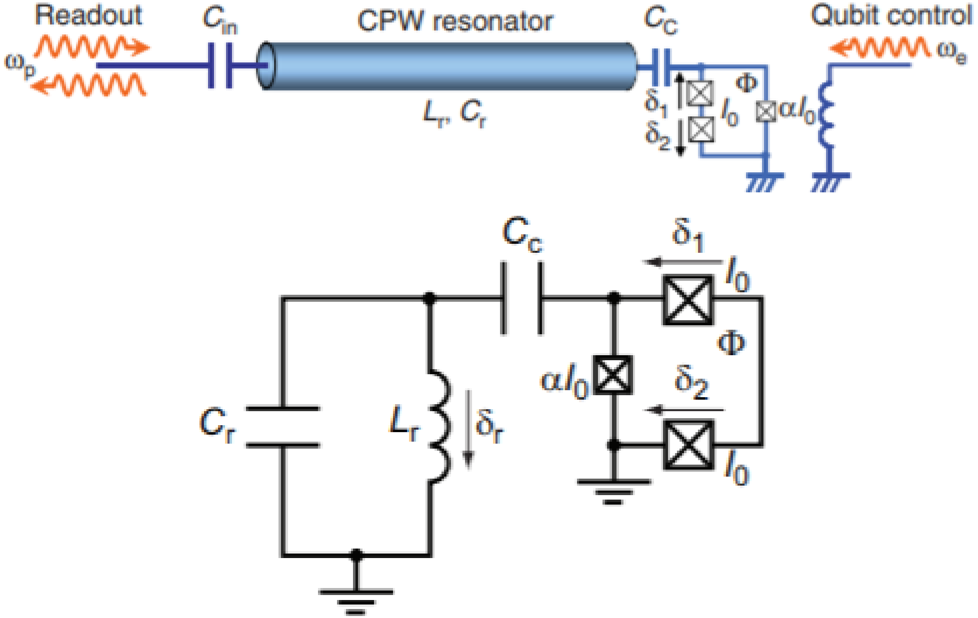
Circuit Models

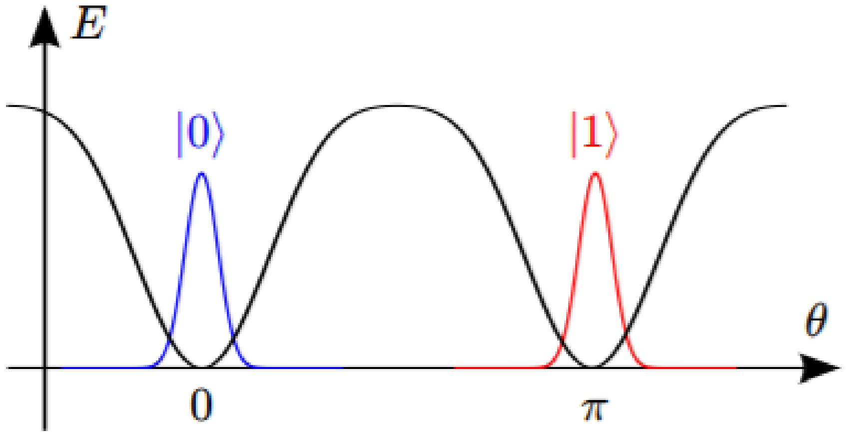
A Typical Quantum Computing Architecture



Inomata, Kunihiro & Yamamoto, T & M. Billangeon, P & Nakamura, Y & Tsai, J. (2012). Large Dispersive Shift of Cavity Resonance Induced by a Superconducting Flux Qubit in the Straddling Regime. Physical Review B. 86. 10.1103/PhysRevB.86.140508.

Towards robust quantum hardware:

0-π Qubit



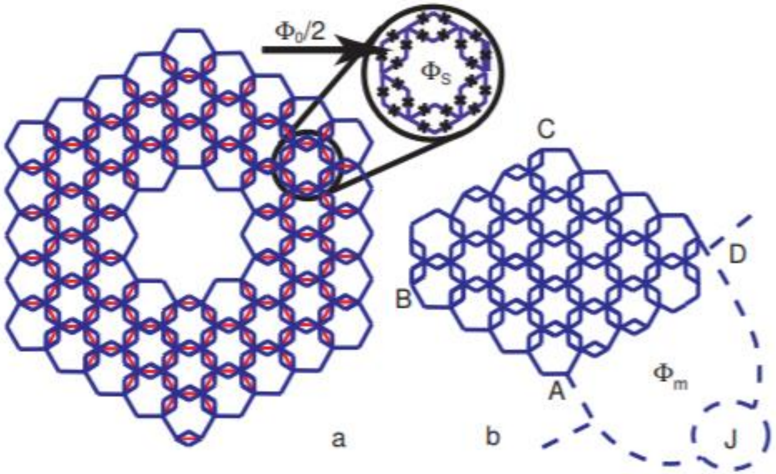
0-π Qubit

History of protected qubits

PHYSICAL REVIEW B **66**, 224503 (2002)

Possible realization of an ideal quantum computer in Josephson junction array

L. B. Ioffe[1,*] and M. V. Feigel'man[2]
[1]*Center for Materials Theory, Department of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Road, Piscataway, New Jersey 08854*
[2]*Landau Institute for Theoretical Physics, Kosygina 2, Moscow, 117940 Russia*
(Received 3 July 2002; published 11 December 2002)



Protected qubit based on a superconducting current mirror

Alexei Kitaev
Microsoft Project Q, UCSB, Santa Barbara, California 93106, USA
California Institute of Technology, Pasadena, California 91125, USA

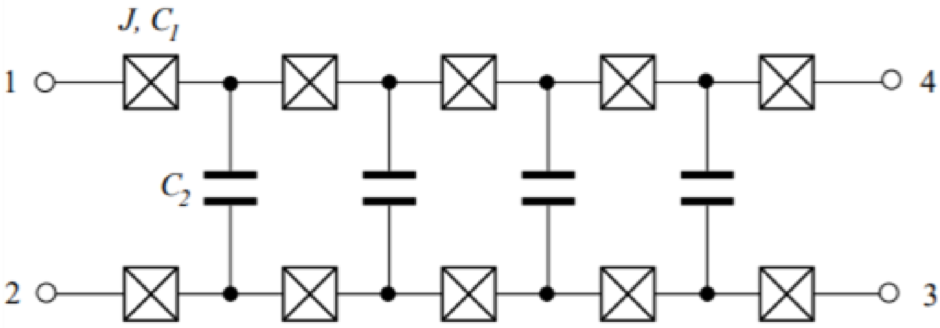


Figure 1: The superconducting current mirror.

0-π Qubit

Hamiltonian

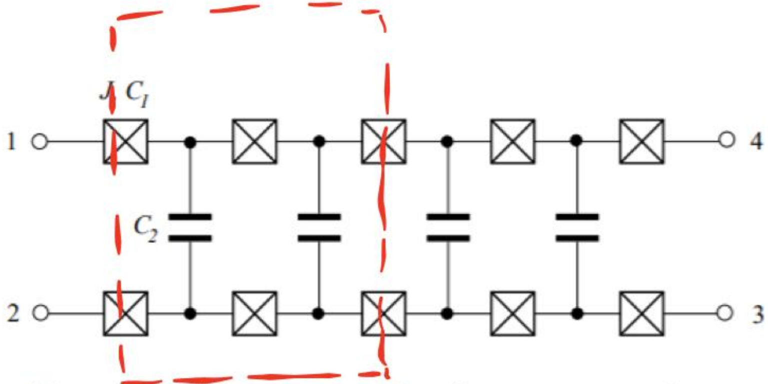
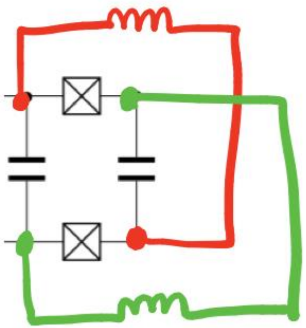
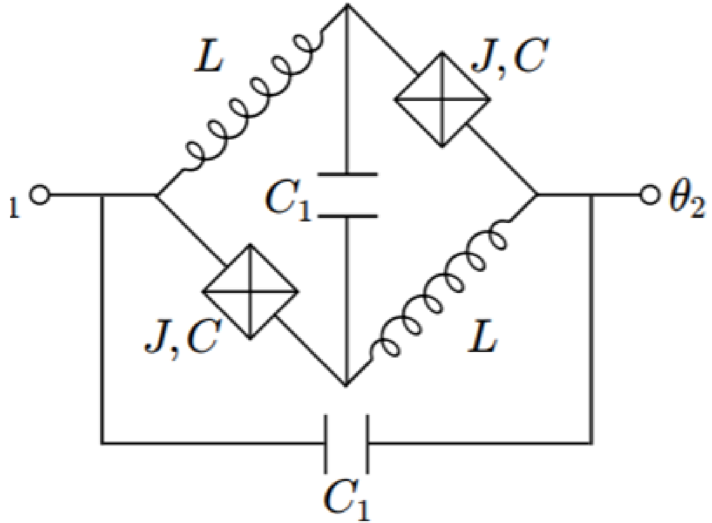


Figure 1: The superconducting current mirror.





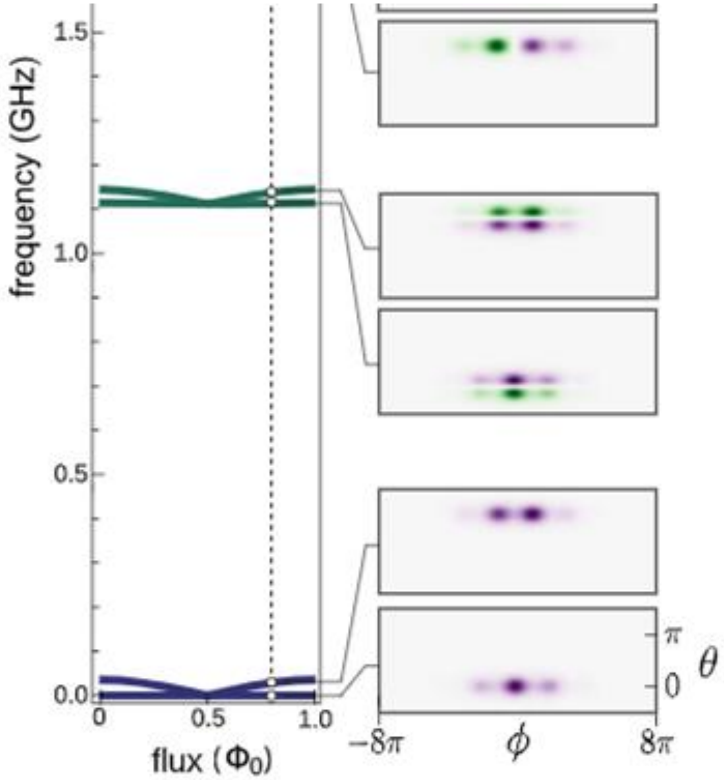
$$H = H_{0-\pi} + H_{\zeta} + H_{\text{int}}$$

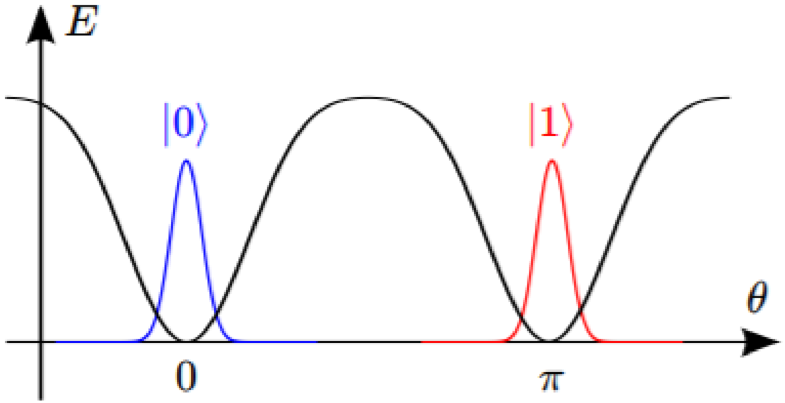
[1]: Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.

[2]: Brooks, Peter & Kitaev, Alexei & Preskill, John. (2013). Protected gates for superconducting qubits. Physical Review A. 87. 10.1103/PhysRevA.87.052306.

0-π Qubit

Eigenspectrum
And Energy



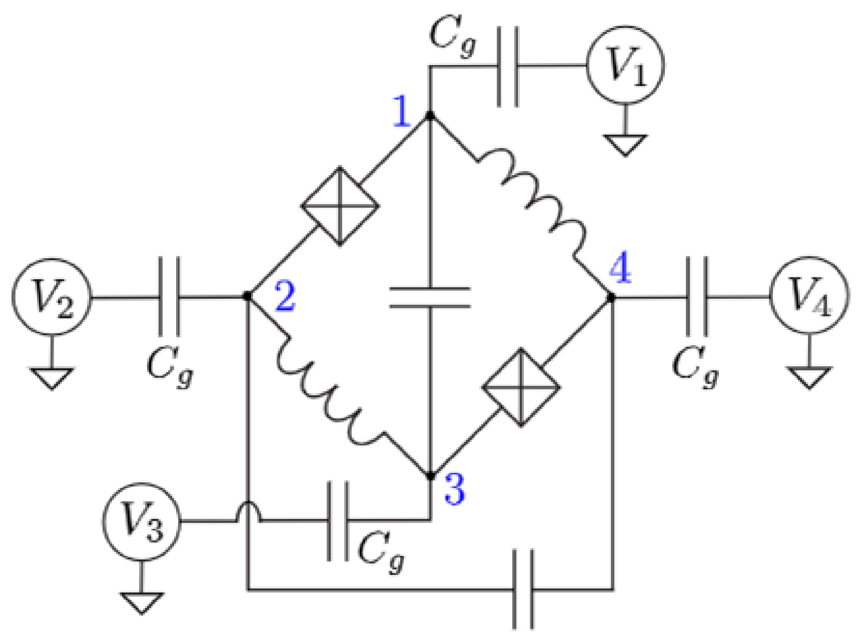


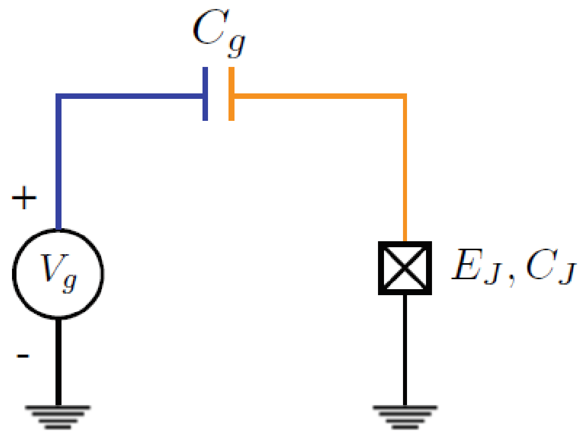
[1]: Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.

[2]: Brooks, Peter & Kitaev, Alexei & Preskill, John. (2013). Protected gates for superconducting qubits. Physical Review A. 87. 10.1103/PhysRevA.87.052306.

0-π Qubit

Charge Noise Analysis

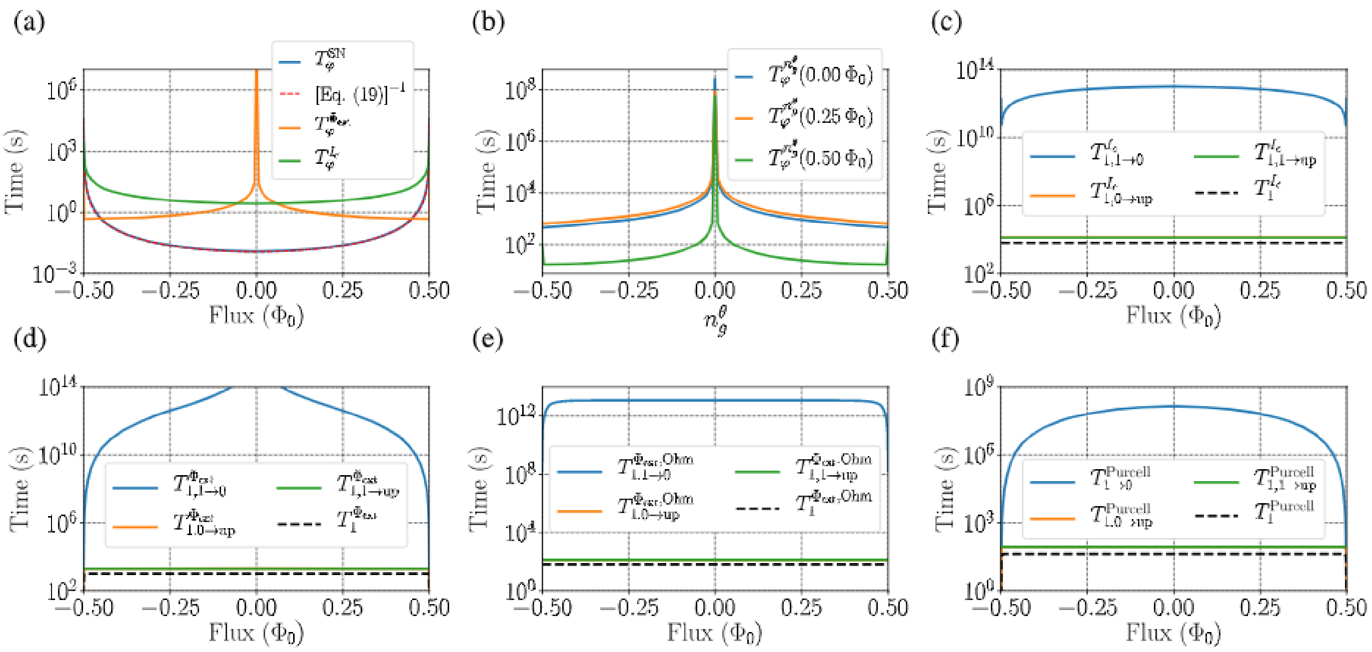




[1]: Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.
[2]: Kyaw, Thi Ha. (2019). Towards a Scalable Quantum Computing Platform in the Ultrastrong Coupling Regime. 10.1007/978-3-030-19658-5.

Future Steps

1 Reproducing open quantum system simulation results from [1]



Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.

Future Steps

2. Analyzing Holonomic Processes [2]: tuning parameters adiabatically to realize protected gates

3. Applying Fast-Holonomic Scheme on 0-π qubit and simulating results

Set of Holonomic and Protected Gates on Topological Qubits for Realistic Quantum Computer

Andrey R. Klots[1], Lev B. Ioffe[1,2]

[1]Department of Physics, University of Wisconsin – Madison, Madison, WI 53706 USA

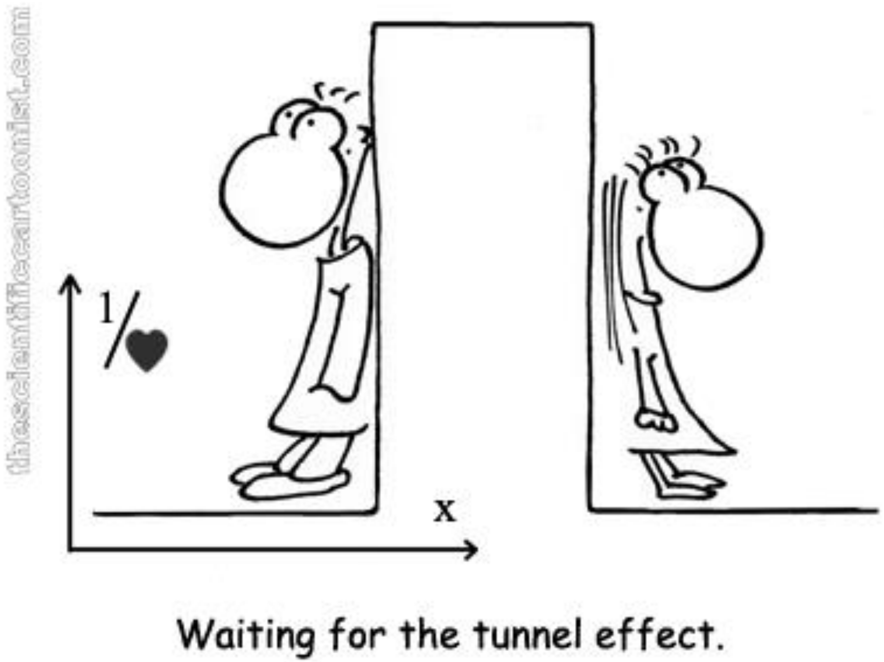
[2]Google Inc., Venice, CA 90291 USA

R. Klots, Andrey & B. Ioffe, Lev. (2019). Set of Holonomic and Protected Gates on Topological Qubits for Realistic Quantum Computer.

Special Thanks

- Centre for Quantum Information and Quantum Control for Summer Prize Scholarship
- Prof. Alán Aspuru-Guzik
- Dr. Thi Ha Kyaw
- University of Toronto Computer Science and Chemistry Departments

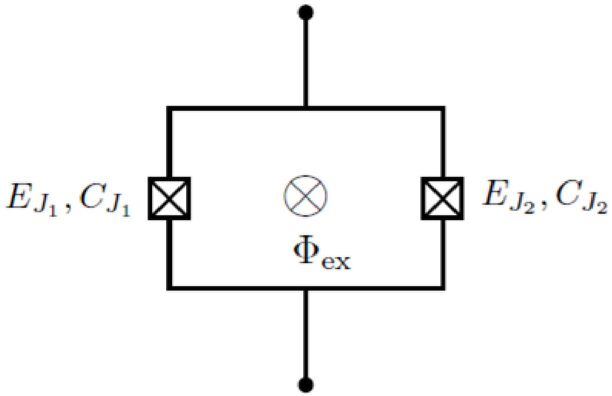
Questions?



[1]: “The Scientific Cartoonist.” The Scientific Cartoonist » The Quantum Mechanics of Love - Scientific Humor - Scientific Cartoons - Humor y Ciencia, www.thescientificcartoonist.com/?p=124.
[2]: Lian, Evan. “WANTED Dead or Alive - Schrodinger's Cat.” Cartoon Collections, www.cartooncollections.com/cartoon?searchID=CX304978.

Appendix

DC-SQUID Calculations



Flux:

$$\sum_i \Phi_i + \Phi_{ind} + \Phi_{ex} = 0$$

Lagrangian:

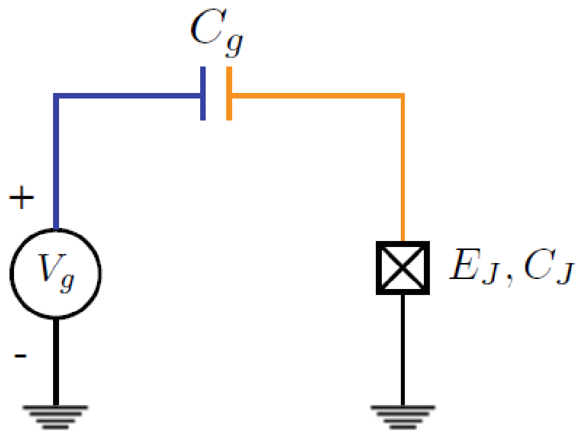
$$\hat{\mathcal{L}} = \frac{1}{2} C_{J_1} \dot{\hat{\Phi}}_1^2 + \frac{1}{2} C_{J_2} \dot{\hat{\Phi}}_2^2 + E_{J_1} \cos\left(2\pi \frac{\hat{\Phi}_1}{\Phi_0}\right) + E_{J_2} \cos\left(2\pi \frac{\hat{\Phi}_2}{\Phi_0}\right)$$

where $L_J = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{E_J \cos\phi}$

Kyaw, Thi Ha. (2019). Towards a Scalable Quantum Computing Platform in the Ultrastrong Coupling Regime. 10.1007/978-3-030-19658-5.

Appendix

Single Cooper-Pair Box Calculations



Lagrangian:

$$\hat{\mathcal{L}} = \frac{1}{2} C_J \dot{\hat{\Phi}}_J^2 + \frac{1}{2} C_g (\dot{\hat{\Phi}}_J + V_g)^2 + E_J \cos\left(2\pi \frac{\hat{\Phi}_J}{\Phi_0}\right)$$

Conjugate Momentum:

$$\hat{Q}_J = \frac{d\hat{\mathcal{L}}}{d\dot{\hat{\Phi}}_J} = C_J \dot{\hat{\Phi}}_J + C_g(\dot{\hat{\Phi}}_J + V_g)$$

Hamiltonian:

$$\hat{H} = 4E_c(\hat{n} - n_g)^2 - E_J \cos\left(2\pi \frac{\hat{\Phi}_J}{\Phi_0}\right),$$

where $E_c = \frac{e^2}{2(C_g + C_J)}$ [charging energy], $n_g = \frac{C_g V_g}{2e}$ [dimensionless gate charge]

$$\hat{H} = \sum_n \left[4E_c(\hat{n} - n_g)^2 |n\rangle\langle n| - \frac{1}{2} E_J (|n+1\rangle\langle n| + |n-1\rangle\langle n|)\right]$$

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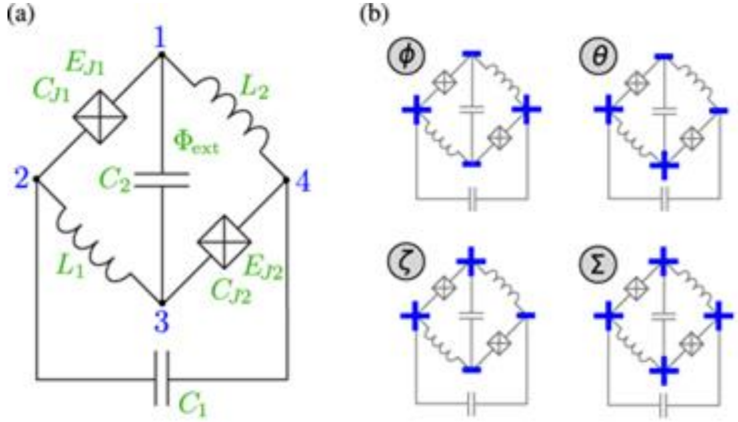
Appendix

0-π Qubit Calculations



$$2\phi = (\varphi_2 - \varphi_3) + (\varphi_4 - \varphi_1), \quad 2\zeta = (\varphi_2 - \varphi_3) - (\varphi_4 - \varphi_1),$$

$$2\theta = (\varphi_2 - \varphi_1) - (\varphi_4 - \varphi_3), \quad 2\Sigma = \sum_j \varphi_j.$$

$$H = H_{0-\pi} + H_\zeta + H_{\text{int}}$$

$$H_{0-\pi} = H_{\text{sym}} + 2E_{\text{Cs}}\mathrm{d}C_{\text{J}}\partial_\phi\,\partial_\theta + E_{\text{J}}\,\mathrm{d}E_{\text{J}}\sin\theta\,\sin\left(\phi - \frac{\varphi_{\text{ext}}}{2}\right) + \mathcal{O}(\mathrm{d}C^2, \mathrm{d}C_{\text{J}}^2),$$

$$H_{\text{sym}} = -2E_{\text{CJ}}\,\partial_\phi^2 - 2E_{\text{Cs}}\,\partial_\theta^2 - 2E_{\text{J}}\cos\theta\,\cos\left(\phi - \frac{\varphi_{\text{ext}}}{2}\right) + E_{\text{L}}\phi^2 + H_\zeta,$$

$$H_\zeta = -2E_{\text{C}}\,\partial_\zeta^2 + E_{\text{L}}\,\zeta^2,$$

$$H_{\text{int}} = 2E_{\text{Cs}}\mathrm{d}C\partial_\theta\,\partial_\zeta + E_{\text{L}}\mathrm{d}E_{\text{L}}\phi\,\zeta + \mathcal{O}(\mathrm{d}C^2, \mathrm{d}C_{\text{J}}^2),$$

$$E_{\text{L}} = (\Phi_0/2\pi)^2/2\text{L}$$

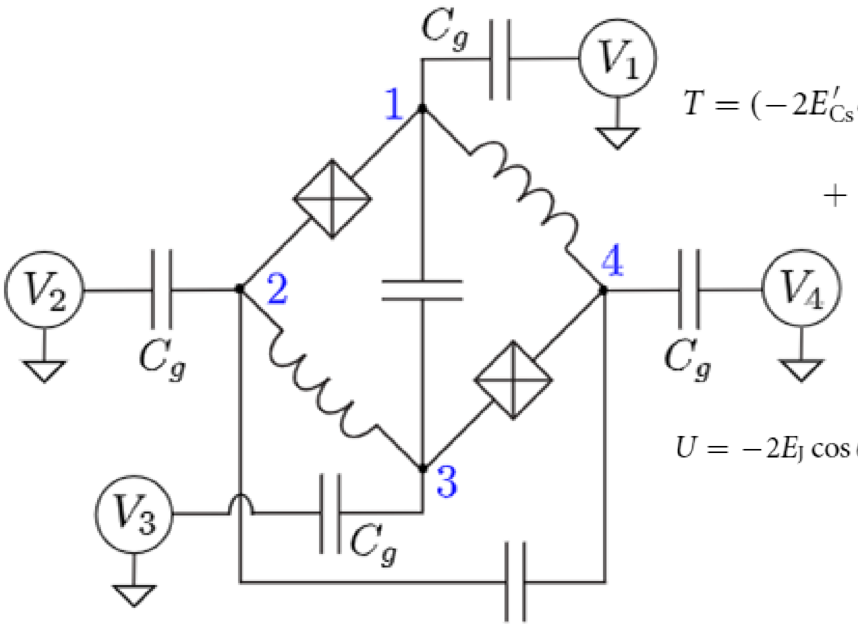
$$E_{\text{C}} = e^2/2C,\, E_{\text{CJ}} = e^2/2C_{\text{J}},\, E_{\text{Cs}} = (1/E_{\text{C}} + 1/E_{\text{CJ}})^{-1} = e^2/2C_{\text{s}},$$

$$C_{\text{s}} = C + C_{\text{J}}.$$

Groszkowski, Peter & Di Paolo, Agustín & L Grimsmo, Arne & Blais, Alexander & Schuster, David & Houck, Andrew & Koch, Jens. (2018). Coherence properties of the 0-π qubit. New Journal of Physics. 20. 10.1088/1367-2630/aab7cd.

Appendix

0-π Qubit with Charge Noise Calculations



$$H' = T + U$$

$$T = (-2E'_{\mathrm{Cs}}\partial_\theta^2 - 2E'_{\mathrm{CJ}}\partial_\phi^2 - 2E'_{\mathrm{C}}\partial_\zeta^2) + \left(2E'_{\mathrm{Cs}}\frac{E'_{\mathrm{CJ}}}{E_{\mathrm{CJ}}}\,\mathrm{d}C_{\mathrm{J}}\,\partial_\theta\,\partial_\phi + 2E'_{\mathrm{Cs}}\frac{E'_{\mathrm{C}}}{E_{\mathrm{C}}}\,\mathrm{d}C\partial_\theta\,\partial_\zeta\right)$$
$$+ (-4\mathrm{i}E'_{\mathrm{Cs}}n_g^\theta\,\partial_\theta - 4\mathrm{i}E'_{\mathrm{CJ}}n_g^\phi\,\partial_\phi - 4\mathrm{i}E'_{\mathrm{C}}n_g^\zeta\,\partial_\zeta).$$

$$U = -2E_{\mathrm{J}}\cos\theta\,\cos\left(\phi - \frac{\varphi_{\mathrm{ext}}}{2}\right) + E_{\mathrm{L}}\,\phi^2 + E_{\mathrm{L}}\zeta^2 + E_{\mathrm{J}}\,\mathrm{d}E_{\mathrm{J}}\sin\theta\,\sin\left(\phi - \frac{\varphi_{\mathrm{ext}}}{2}\right) + E_{\mathrm{L}}\mathrm{d}E_{\mathrm{L}}\phi\,\zeta$$

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