Some questions

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(Dated: May 30, 2019)

I. A SINGLE TWO-LEVEL SYSTEM (QUBIT): LANDAU-ZENER PROBLEM

Let us consider a quantum system with Hamiltonian (in the unit $\hbar = 1$)

$$\hat{H} = -\frac{1}{2}(\epsilon\hat{\sigma}_z + \Delta\hat{\sigma}_x), \tag{1}$$

where ϵ and Δ are some real numbers.

A. Analytics

1. Suppose $\epsilon = 0$ and $\Delta \neq 0$.
   (a) What are the eigenvectors and corresponding eigenvalues of $\hat{H}$?
   (b) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |0\rangle = (1,0)^T$. Here, ^{T} means transpose. What is the quantum state at time t: $|\psi(t)\rangle$? **Hint:** one have to solve the Schrödinger equation analytically.
   (c) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Here, $|1\rangle = (0,1)^T$. What is the quantum state at time t: $|\psi(t)\rangle$?
2. Suppose $\Delta = 0$ and $\epsilon \neq 0$.
   (a) What are the eigenvectors and corresponding eigenvalues of $\hat{H}$?
   (b) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |0\rangle = (1,0)^T$. What is the quantum state at time t: $|\psi(t)\rangle$?
   (c) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. What is the quantum state at time t: $|\psi(t)\rangle$?
3. Suppose $\epsilon \neq 0$ and $\Delta \neq 0$.
   (a) What are the eigenvectors and corresponding eigenvalues of $\hat{H}$?
   (b) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |0\rangle = (1,0)^T$. What is the quantum state at time t: $|\psi(t)\rangle$?
   (c) In addition to the constraint above, suppose that our quantum system is prepared in the state $|\psi(t=0)\rangle = |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. What is the quantum state at time t: $|\psi(t)\rangle$?

B. Numerics

1. Suppose $\epsilon = 1$. Numerically plot the energy spectrum (eigenvalues) of $\hat{H}$ with respect to the function $\Delta \in [-5, 5]$.
   (a) Numerically find out the ground and first excited states of the quantum system ($\hat{H}$) at $\Delta = -5, 0, 5$.
   (b) Please reminisce the above numerical results with the analytical ones that were found under the section "Analytics".
2. Suppose $\epsilon = 1, \Delta = 0.5$, and the initial quantum state $|\psi(0)\rangle = |1\rangle$. Plot the population of the quantum state in time, i.e., $|\langle\psi(t)|\psi(0)\rangle|^2$, for the duration $4\pi/\epsilon$. In other word, $t : 0 \to 4\pi/\epsilon$.
3. Suppose $\epsilon = 1, \Delta = 0$ and the initial quantum state $|\psi(0)\rangle = |1\rangle$. At time $t > 0$, we change the Hamiltonian slowly (adiabatically) such that $\Delta : 0 \to 1$. Plot $|\langle\psi(t)|\psi(0)\rangle|^2$, for the duration $4\pi/\epsilon$. **Hint:** this is the time-dependent Hamiltonian problem. So far, all the problems above are time-independent, that means there is no parameter in the Hamiltonian which is changing with time. Here, we have Δ which changes in time.

II. ONTOLOGICAL QUERY (OPTIONAL)

We have seen that the entire field of quantum mechanics and quantum information is based on a simple equation called the Schrödinger equation:

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle. \tag{2}$$

Why is this equation fundamental backbone of quantum mechanical dynamics of a quantum system, and not any other form?

III. THERMAL STATE

A quantum three-level system Hamiltonian is given by

$$\hat{H} = \epsilon \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}. \tag{3}$$

The system is in thermal equilibrium with a heat bath at $T = 0.5\epsilon$.

1. What is the state of the system in density matrix? Recall: $\rho = \frac{e^{-\beta\hat{H}}}{Z}$, where $\beta = 1/T$ and $Z = \mathrm{Tr}e^{-\beta\hat{H}}$. In addition, in the eigenbasis of $\hat{H}$, $\rho = \sum_n \omega_n |n\rangle\langle n|$, with $\omega_n = e^{-\beta E_n}/Z$, $Z = \sum_n e^{-\beta E_n}$.
2. What is the probability to find the system in the state: $|n\rangle = (1, 0, 0)^T$?
3. What is the expectation value of the system's energy, i.e., $\langle E\rangle = \mathrm{Tr}[\rho\hat{H}]$?
4. What is the density matrix after time $t = 4 * 2\pi/\epsilon$, given the initial density matrix is $\rho = \frac{e^{-\beta\hat{H}}}{Z}$? In other words, one needs to numerically solve the Schrödinger equation:

$$\frac{d\rho(t)}{dt} = -i[\hat{H}, \rho(t)]. \tag{4}$$

IV. IBM Q EXPERIENCE (FOR FUN)

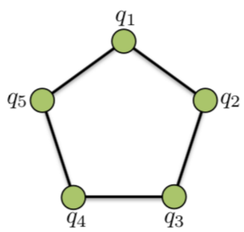

FIG. 1: A five-qubit quantum error correction code. Each dot represents $|+\rangle$ and each bond represents CZ (controlled phase) gate.

Please create the five-qubit quantum error correction code using IBM Q available online for free.

1. What is the resultant $|\psi\rangle$ from IBM machine?
2. Write a python code (if you are bold enough, do it by hand) to create the same state. Check that the two results are the same.