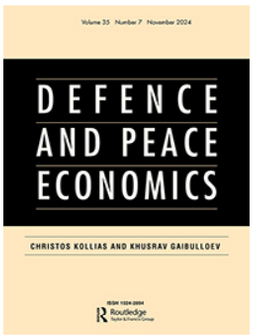

Routledge
Taylor & Francis Group

Defence and Peace Economics

ISSN: 1024-2694 (Print) 1476-8267 (Online) Journal homepage: www.tandfonline.com/journals/gdpe20

Entente versus Alliance: When Should States Be Friends but Not Allies?

Emerson Niou & Sean M. Zeigler

To cite this article: Emerson Niou & Sean M. Zeigler (2024) Entente versus Alliance: When Should States Be Friends but Not Allies?, Defence and Peace Economics, 35:7, 792-808, DOI: 10.1080/10242694.2023.2213468

To link to this article: https://doi.org/10.1080/10242694.2023.2213468

DEFENCE AND PEACE ECONOMICS
2024, VOL. 35, NO. 7, 792–808
https://doi.org/10.1080/10242694.2023.2213468

Routledge
Taylor & Francis Group

Entente versus Alliance: When Should States Be Friends but Not Allies?

Emerson Niou[a] and Sean M. Zeigler[b]

[a]Department of Political Science, Duke University, Durham, North Carolina; [b]RAND Corporation, Defense & Political Sciences Department, Santa Monica, CA, USA

ABSTRACT

When faced with a common threat, states have various alignment choices. Formal alliances offer explicit military obligations of support. Others, such as the Triple Entente that preceded World War I, are more ambiguous understandings. These entente-like alignments make no formal pledges of armed support in the event of hostilities. However, they do not entirely rule out military support either. Why might states embrace this form of strategic ambiguity over firm alliance commitments? Our formal explication addresses this question via the prism of collective action. Our modeling efforts, combined with historical precedents, suggest that an entente might be a more effective alignment choice than a formal alliance for states to balance against powerful threats. An entente permits states to strike the middle ground between entrapment and deterrence. The strategic ambiguity inherent in these two seemingly self-contradictory goals of an entente is a key component of its success as an alignment strategy, especially when external threats are large. However, asymmetry in strength between two states can result in divergent preferences between alliance and entente.

ARTICLE HISTORY
Received 7 October 2022
Accepted 6 May 2023

KEYWORDS
Alliance; entente; collective action; freeriding; contest model

Introduction

Alliances are an integral component of world politics and international conflicts. The first World War is perhaps the most iconic example of the intricate interplay of alliance arrangements and armed conflict between states. It began as a war between Austria-Hungary and Serbia after the assassination of Archduke Francis Ferdinand in Sarajevo on 28 June 1914. Even though the direct interests of Germany, France, and Great Britain were hardly at stake in the Austro-Serbian dispute, all would promptly be drawn into the conflagration, along with Russia and Italy. The reason for this tragic outcome is apparently simple: alliance politics. Germany was the ally of Austria-Hungary; France was allied to Russia; and Great Britain was seemingly bound to France by a 1904 diplomatic agreement known as the *Entente Cordiale* – an arrangement that, while not a formal alliance, carried the 'moral obligation' of one (Schmitt 1924).

These two arrangements preceding WWI are often referred to as the Triple Alliance and the Triple Entente.[1] While an alliance is an explicit pledge of military support among states, an entente is a decidedly – and intentionally – less precise convention. It is not an alliance at all but rather an understanding between states around a set of often contentious issues. Frequently, ententes entail uncertainty or even ambiguity. The 1904 Anglo-French Entente Cordiale illustrates this point. Following French press reports in 1911 contrasting the virility of the Triple Alliance with the moribund state of the Entente, British diplomat Eyre Crowe lamented: 'The fundamental fact of course is that the Entente is not

CONTACT Sean M. Zeigler szeigler@rand.org RAND Corporation, Defense & Political Sciences Department, 1200 South Hayes Street, Arlington, Virginia 22202-5050, USA

This article has been republished with minor changes. These changes do not impact the academic content of the article.

an alliance. For purposes of ultimate emergencies it may be found to have no substance at all. For the Entente is nothing more than a frame of mind, a view of general policy which is shared by the governments of two countries, but which may be, or become, so vague as to lose all content.'[2]

The distinction between crafting firm commitments and informal understandings naturally invites the question: why entente, or why not alliance? Alternatively put, when do states elect to forge formal or rigid alliances rather than vague and flexible arrangements commonly known as ententes? How did Great Britain find it in her interest to craft ambiguous diplomatic overtures to both Russia and France in the face of a rising Germany? And why would France and Russia accept such an offering, rather than a more concrete military pledge? This inquiry offers the motivation for this article.

While the question we address in this essay is straightforward, its answer is complex. The complexity derives from several sources. Alliances, we propose, are strategic calculations. Such calculations often entail decisions addressing longstanding questions in international relations related to deterrence, balancing, and entrapment. Moreover, alliances, in as much as the common defense is a public good, represent a classical instantiation of the collective action problem at work (Olson and Zeckhauser 1966). The collective action problem associated with alliances suggests that alliances are afflicted by the issue of free-riding – a type of market failure where actors are incentivized to underpay or rely on the contributions of others. Finally, the choices made by potential allies depend on the threats they encounter. This issue is often referred to as the endogeneity problem. The combination of these challenges – balancing, collective action, and endogeneity – creates a complex strategic puzzle for nations when considering alliance politics.

To address these issues, we investigate the concept of alliance versus entente from several perspectives. First, we study various alignment arrangements from a historical perspective, which helps us understand the subtleties of the entente strategy. Although not a formal alliance, it still served as a mechanism for France and especially Great Britain to counter or balance against Germany. Second, we develop a formal model to probe the question of entente versus alliance within the collective action framework. This analysis allows us to examine the free-riding consequences of both alignment types. Notably, our model considers external threats endogenously, meaning they are treated as a strategic element for all parties involved. We also place entente within the broader context of alliance classifications, clarifying key differences between ententes and other less formal alliance agreements, such as nonaggression pacts and neutrality agreements. Ententes, as we will see, have significant distinctions from nonaggression and neutrality pacts, which add strategic complexity for both partners and adversaries. Lastly, we draw generalizations from the model and use them to deduce empirical implications. A primary finding is that nations can more effectively balance against larger threats without forming formal alliances. The inherent ambiguity of entente introduces doubt for all parties involved, and this uncertainty helps address the free-riding issue often afflicting formal alliances.

This essay makes several contributions to the literature on alliance politics. First, we formally lay out the conditions under which states find it more favorable to avoid alliances and instead choose the strategic ambiguity provided by ententes. Second, we provide formal clarity to the inherently 'informal' concept of the entente. Entente arrangements are typically not included in alliance datasets, likely due to their conceptual ambiguity. But our work here indicates that they should be. Ententes can serve as an effective alternative institutional design for states facing escalating threats that helps address the collective action issue of free-riding. Third, we show how entente behavior can have balancing effects. The literature exploring the classic question of when states prefer to balance or bandwagon in response to external threats does not consider entente as a balancing strategy, primarily because ententes are not formal alliances. Nevertheless, we show that the most effective way to balance against threats might not involve allying but rather pursuing a mutual understanding.

Alignment Strategies in Historical Context

In international relations, alignments entail various obligations, reflecting the multitude of alternatives available to nation-states. To elucidate the varying strategic selections, it is instructive to examine the

European landscape in the late 19th century and the years leading up to World War I. In this era, the continent's diplomatic tapestry was woven with alliances, neutrality agreements, nonaggression pacts, and ententes.

In 1875, Russia was considering extending its influence into the Balkans, a move that greatly alarmed Austria. Tsar Alexander II inquired of German Chancellor Otto von Bismark if Berlin would remain neutral in an Austro-Russian war. When Bismark made clear to his Russian counterpart that Germany would not acquiesce to such a move, Russia abandoned the idea in favor of a war with Turkey and moved to strike a bargain with Vienna. In January of 1877, at a secret convention in Budapest, Austria consented to adopt a stance of benevolent neutrality in the event of a Russo-Turkish war. In exchange, Russia pledged not to establish a sizable state within the Balkan territory (Schmitt 1924).

The formation of the Austro-German alliance in 1879 serves as a quintessential illustration of two nations uniting against a shared perceived threat in the form of Russia. Faced with overt Russian antagonism, Germany found itself compelled to forge an alliance with Austria. Berlin initially suggested a broad defensive pact to counter aggression from any other power. However, Austria, unwilling to commit against France, insisted on an alliance targeting Russia exclusively. The Germans agreed to this, finalizing the alliance with Austria-Hungary on 7 October 1879. The treaty's terms dictated that if Russia were to attack either party, the signatories were 'bound to come to the assistance of one another with the whole war strength of their Empires.' Notably, the treaty also encompassed a clause stipulating that each state would maintain neutrality should another power (excluding Russia) launch an attack against either party (Schmitt 1924).

Confronted with increasing isolation and the ever-changing landscape of European alliances, Russia sought the support of an ally. The inception of the 'Dual Alliance' between France and Russia can be traced back to August 1891 as a consultative pact in the event of aggression against either party.[3] In August 1892, the agreement was reinforced by a military convention. To maintain secrecy, the alliance was solidified through an exchange of letters between 1893 and 1894. The terms of the agreement specified that if Germany or Italy (with German backing) were to engage in hostilities against France, Russia would provide support with up to 800,000 troops. Conversely, if Germany or Austria-Hungary (with German assistance) were to attack Russia, France would deploy 1.3 million soldiers against Germany. The alliance was renewed in both 1899 and 1912.[4]

The above instances offer valuable insights into a crucial distinction between alliances and ententes. Alliances are conditional on a well-defined threat from a third party to the signatories. It was in the interest of both Germany and Austria to make explicit the threat they saw in Russia. Austria aimed to secure German support should the Austro-Russian rivalry in the Balkans escalate into armed conflict. Berlin pursued alliances as a targeted countermeasure against Moscow and a means of preserving stability within Europe. Russia and France held mutual apprehensions regarding a potential attack from Germany. Their alliance materialized these concerns explicitly. In contrast, ententes frequently lack explicit delineation of the specific threats confronting the partners. To further clarify this distinction, additional historical illustrations are beneficial.

The 1904 Entente Cordiale between Britain and France represented an accord that rectified years of animosity between the two nations, with none of the grievances stemming from Europe. Paradoxically, this Anglo-French understanding enabled both states to collaborate against Germany without an explicit call to arms. The agreement primarily focused on resolving colonial rivalries overseas. France and Britain found themselves in disagreement over their respective interests in Egypt and Morocco. The public declaration they arrived at obliged the two countries to uphold the status quo in these two colonies and settled other longstanding differences in various territories, including Nigeria, French Guinea, Newfoundland, Thailand, Indochina, and the Pacific Islands. The agreement did not constitute an alliance, and there was no mention of Berlin. According to historian Bernadotte Schmitt:

> So far as the written word went, the Anglo-French entente merely effected a liquidation of difficulties beyond the sea. Europe was nowhere mentioned. Yet Europe was inevitably and profoundly affected, and beyond a doubt this was in the back of the minds of the experienced diplomatists who negotiated the agreement. . . .In 1904 their agreement was

> no more than what it pretended to be — a settlement of colonial rivalries. It was not *per se* directed against Germany, but it did impose a barrier to the German policy of the 'free hand'" (Schmitt 1924, 61).

In 1907, the Anglo-Russian Entente presented a comparable counterbalance against Germany without explicitly mentioning Germany or her allies, Italy and Austria-Hungary. The entente significantly eased tensions between Russia and Great Britain following Russia's defeat by Japan, a British ally, in 1904. The 1907 declaration delineated spheres of influence in Persia, specified that neither party would meddle in Tibet's internal affairs, and acknowledged Britain's influence in Afghanistan. While these provisions addressed colonial disputes, both Moscow and London considered Berlin when finalizing the accord. Nevertheless, they assured Germany that the agreement was not aimed against her. '[T]he situation was identical with that created by the Anglo-French entente: Great Britain and Russia, having established cordial relations, could unite if their interests were challenged by Germany.'[5] Although the dual agreements between Britain and her two partners made no references to Germany, predominantly dealt with colonial rivalries, and did not involve commitments of military support, Berlin increasingly perceived the Triple Entente as a threat (Williamson 2011).

The examples highlighted above and many others have significantly influenced and shaped the study of alliances. However, in spite of a wealth of historical episodes, the discipline still lacks an integrated taxonomy of alliance behavior.[6] Importantly, the various and nuanced alignment strategies evident throughout history are not fully reflected in our theoretical or empirical accounting of the subject. These limitations notwithstanding, scholars of alliance politics have identified various types of alignment options and their associated dilemmas. For instance, Snyder (1977) describe a conundrum whereby states '. . .face a choice between firm, explicit alliance commitments that maximize their combined bargaining power vis-à-vis the adversary at the cost of less leverage over each other, and ambiguous understandings to keep their alignment options open and thus maximize their alliance bargaining power. . .' Kann (1976) also noted the distinctions between alliances and their"airtight commitments" in contrast to the more relaxed and flexible relationships between states, often referred to as ententes. Similarly, Snyder (1997) differentiates between alliances and ententes as follows:

> One broad dichotomy is between explicit pledges of mutual assistance and agreements that tacitly raise expectations of mutual support by reducing the amount of conflict between parties, that is, *entente*. Alliances differ from ententes in that their force rests on a promise that is superimposed over existing conflicts rather than on a reduction of the conflicts. . . Sometimes the label 'entente' is given to a disposition to cooperate based simply on shared interest rather than on a negotiated reduction of conflict.[7]

This 'disposition to cooperate' may be driven by the perception of a shared threat, yet the obligations and the exact sources of threats implied by such affinities often remain unspecified. These constitute the key distinguishing characteristics between entente-like arrangements and other alignment options. In alliances, neutrality agreements, non-aggression pacts, and consultation accords, the signatories' obligations are clearly defined. In contrast, the threats and obligations of parties in an entente are left undefined. The strategic intent of this vagueness is to strike a balance between entrapment and deterrence. Put differently, an entente conveys a deceptively simple message to each party in times of crisis: the message to the other signatory is 'don't count on me,' while the message to their mutual threat is 'don't count me out.' The ambiguity of such a mixed signal complicates attempts to interpret or ascribe specific intentions to ententes.

The Anglo-Russian and Anglo-French agreements demonstrate the inherent difficulty in defining the term *entente*. The precise meaning of entente may be less significant than the aspects that differentiate it from an alliance, such as its flexibility, informality, and most importantly, ambiguity. On account of this distinction, Berridge (1989) underscores the need for a comprehensive understanding of entente: 'The corollary of a narrow concept of alliance. . . is a broad concept of entente (p.252).' Many close inter-state relationships have significant military implications, despite the absence of formal provisions for co-belligerency under specific conditions. The mere suggestion that parties *may* be sympathetic to one another when facing hostilities from a third party can instill sufficient doubt in the minds of others.

Numerous instances attest to the assertion that formal alliances are not the sole determinants of state cooperation, particularly in times of conflict. The partnership between the United States and Israel is a prime illustration of collaboration grounded in mutual dedication to shared principles. Although the United States openly pledges its commitment to Israel's security, the absence of explicit alliance obligations leaves the exact nature and scope of potential support in a conflict situation ambiguous.

Similarly, the United States upholds an indistinct relationship with Saudi Arabia and other Gulf states, devoid of a formal treaty alliance. The absence of an Article 5-like defense agreement with Saudi Arabia, in particular, is noteworthy. Instead, a more subtle understanding emerges through a bilateral 'Agreement Concerning Mutual Defense Assistance,' whose specifics remain undisclosed. The U.S. intervention following Iraq's invasion of Kuwait in August of 1990 highlights this point, as no formal obligation to provide military assistance existed.

France's engagement with its former colonies further exemplifies the prevalence of such ambiguity. Likewise, Russia's current relationship with Syria, where it maintains a military base but lacks a formal alliance with Damascus, attests to the same phenomenon. Russia's military intervention on behalf of the Syrian government during the 2015 civil war occurred without an explicit alliance. Of note, however, Moscow and Damascus maintain a Soviet-Syrian "Treaty of Friendship and Cooperation", signed in 1980. That treaty, which remains in force as of 2023, deals with consultation mechanisms rather than military support.

Finally, the contemporary understanding between Moscow and Beijing underscores the limitations of formal alliances in defining state cooperation. In February 2022, China and Russia announced a 'friendship between the two states with no limits,' encompassing numerous cooperative undertakings but conspicuously omitting explicit military commitments. The precise implications of this friendship in any given situation remain discerningly absent from their joint declaration. Indeed, Russia's invasion of Ukraine, in February of 2022, has greatly complicated the Sino-Russian relationship. The war has demonstrated that the Russian-Chinese partnership is not, in fact, an alliance.[8]

The elusive nature of entente is further reflected in the different ways it is treated in datasets classifying alliance arrangements. The Correlates of War (COW) data project (Singer and Small 1966) categorizes three distinct types of alliance obligations: defense pacts; neutrality and non-aggression pacts; entente.[9] An entente obligates parties to 'consult and/or cooperate in a crisis, including armed attack (p. 5).' This definition incorrectly conflates ententes with formal consultation agreements, similar to security clauses in other treaties.[10] It is puzzling, then, to find the Entente Cordiale (1904) and the Anglo-Russian Entente (1907) among the observations in the Singer and Small (1966) data, though these ententes in no way satisfy the definition provided above.

The more recent and comprehensive Alliance Treaty Obligations and Provisions (ATOP) data project provides an alternative coding scheme to the COW obligation typology (Leeds et al. 2002).[11] The ATOP dataset categorizes alliances of the following types: offense pacts, defense pacts, neutrality pacts, nonaggression pacts, and consultation pacts. Of note, the ATOP data explicitly exclude ententes, described as a state of friendly relations between two states that gives rise to a tacit expectation of diplomatic and/or military cooperation. Therefore, the Entente Cordiale (1904) and the Anglo-Russian Entente (1907) are not included in the ATOP data. However, the Franco-Russian 'Dual Alliance' (1893) – a component of the Triple Entente – is included.

Given their historical role and continued salience, we believe treating informal or entente-like relationships as a distinct alignment option is essential. This entente frame-of-mind may be difficult to delineate precisely, yet it merits formal investigation as a strategic option among other alignment choices.

Theoretical Models

Our modeling effort is motivated and influenced by the pioneering line of alliance scholarship by Glenn Snyder. Advocating for a more comprehensive theory of alliances, Snyder (1991) urged scholars to create a framework that encompasses the"entire domain of security policy choice" and bridges the two prominent fields of interaction – alliance relations and adversary relations.[12] This specific context is of

paramount importance, as it examines the intricate relationships of power and interdependence – not only among allies but also in their interactions with adversaries. Snyder (1984) further delves into the competitive dynamics between allies, presenting a "security dilemma" intrinsic to an alliance game.

When entering into alliances, states must concurrently tackle at least two optimization challenges. The first is to decide which type of alignment arrangement to embrace, as varying arrangements entail distinct degrees of commitment and accommodation. But this optimization problem between the two parties is in turn affected by the degree of threat posed by the non-aligned adversary. It is evident that threats differ in nature, and this variation should be endogenously considered. Snyder advocates for elucidating the trade-offs states confront both within and outside their coalitions. Consequently, analyses should encompass decision-making at both intra- and inter-alliance levels. In our study, we endeavor to incorporate these aspects.

The first subsection presents an alliance model that endogenizes a state's relationship with its enemy and ally within a bipolar system. The second subsection introduces a model of entente wherein two nations agree to diminish internal conflict levels without forming an alliance. After deriving the equilibrium payoffs for states in both alliance and entente systems, we outline the conditions under which states confronting a shared adversary would favor an entente over an alliance. This comparative analysis constitutes the third subsection. We incorporate numerical examples throughout the discussion to facilitate the comprehension of diverse outcomes.

Before presenting our modeling effort, we highlight two prior studies that have formally scrutinized ententes and alliance behavior. Conybeare, and Sandler (1990) provides both a theoretical and empirical assessment of the Triple Alliance and Triple Entente, developing an n-country joint product model of alliance behavior to examine an 'allied' nation's demand for military activity. Their empirical investigation explores differences in the alliance behaviors among the six countries constituting the Triple Entente and Triple Alliance. Overall, they find little evidence of free-riding, with the exception of France and Russia. A key distinction between their work and ours is that their joint project model is applied to nations in both alliance and entente arrangements. In contrast, we formulate separate models for each scenario.

A related study by Conybeare et al. (1994) employs Hirshleifer's (1983) classic differentiation between 'weakest link' and 'best shot' to determine the provision of public goods in a noncooperative setting.[13] Their research, also informed by a joint product model, suggests that the Triple Alliance's behavior aligned with a best-shot representation of defense provisions (exemplified by Germany), while the Triple Entente's behavior corresponded to weakest-link representations (epitomized by Great Britain). This investigation offers interesting empirical analysis probing the key distinctions between the Triple Alliance and Triple Entente. In contrast, our models aim to identify a more general differentiation between the concepts of alliance and entente.

Alliance Model

Alliances serve as prominent manifestations of collective action in world politics (Olson 1965; Olson and Zeckhauser 1966).[14] Given that alliance contributions are typically regarded as a public good, issues of collective action plague coalitions, despite the presence of common threats and shared adversaries. Central to this research area are concerns of cooperation, exploitation, and free-riding, as scholars investigate the conditions and factors associated with sheltering and shirking (Kim and Sandler 2020; Hartley and Sandler 1999). The majority of this research, situated within the Olson (1965) approach, treats external threats as an exogenous factor. Nonetheless, the magnitude and scope of external threats are far from irrelevant in alliance calculus. Endogenizing a state's relationship with both its enemy and ally is crucial for improving theories of alliance politics (Skaperdas 1998; Esteban and Ray 2001; Niou and Tan 2005). In the following, we present an alliance model wherein the payoffs to allies are endogenously determined by the external threat they confront.

In our three-player alliance model, players 1 and 2 form an alliance, A, against the nonaligned player 3.[15] Let the strength of each player i be represented by the resources or wealth, w_i, at its

disposal. We can normalize the total resources such that $w_1 + w_2 + w_3 = 1$. For example, when all three players possess equal relative capabilities $w_1 = w_2 = w_3 = \frac{1}{3}$. Each player must decide how much of their respective resources to allocate to military capability, q_i, for defense. We assume military hardware is only useful to prevent others from procuring a player's own resources or to take resources from others, and cannot be converted back or transferred into other consumable goods. These assumptions imply that players prefer to retain as many resources as possible for productive purposes while still requiring a certain level of defensive capabilities. Consequently, each player faces the classic guns versus butter dilemma: how much of their respective resources w_i should they allocate to fighting capacity q_i?

In addressing this problem, the allied parties (1 and 2) take into account both the capacity of the nonaligned player (3) and the relative strength of their respective ally. For player 1, this implies that both w_2 and w_3 should have some bearing on its decision. Likewise, player 3 will want to consider both the combined and relative strength of the two players within alliance A. If we preclude players from borrowing resources from external sources, it must be the case that $q_i \leq w_i$ for all players. That is, parties cannot procure more defensive resources than their endowments allow. The military capacity of the alliance will be the joint defensive capabilities of its members, formally, $q_1 + q_2 = Q_A$.

In the event of war, the prize for victory is appropriating all the defeated side's unconverted resources (i.e. non-military). In the case where the alliance defeats player 3, we assume that players 1 and 2 divide the spoils of victory in direct proportion to their respective defensive contributions to alliance A. For instance, if both players contributed equal amounts to the alliances, then upon victory, player 3's resources $(w_3 - q_3)$ are perfectly halved between them. Alternative division rules are conceivable, but this one has the advantage of creating positive incentives to contribute to the alliance – the more a player contributes to an alliance, the more that player earns in the event of alliance success.[16] As is common in conflict and rent-seeking literature (Hirshleifer 1989; Skaperdas 1992; Neary 1997; Powell 1999; Garfinkel 2004), we assume the 'contest success function' to be probabilistic and directly proportional to the fighting capacities of each side. This implies that the probability that alliance A prevails in a conflict against player 3 is $(q_1 + q_2)/(q_1 + q_2 + q_3)$. The probability that player 3 wins is analogously defined as $q_3/(q_1 + q_2 + q_3)$.

At this point, we have a sufficient foundation to define distinct payoff functions for allied players 1 and 2 and for the nonaligned player 3. We employ an expected utility function to represent the payoffs in a contest. The utility for each member of the alliance is as follows:

$$U_1 = \frac{q_1 + q_2}{q_1 + q_2 + q_3}[(w_1 - q_1) + \frac{q_1}{q_1 + q_2}(w_3 - q_3)] \tag{1}$$

$$U_2 = \frac{q_1 + q_2}{q_1 + q_2 + q_3}[(w_2 - q_2) + \frac{q_2}{q_1 + q_2}(w_3 - q_3)] \tag{2}$$

The payoff function for the nonaligned player is analogously specified as:

$$U_3 = \frac{q_3}{q_1 + q_2 + q_3}(1 - q_1 - q_2 - q_3) \tag{3}$$

Both functions are the product of two factors: the probability of victory and the spoils. In the alliance payoff functions, the probability of victory is represented by the term outside the square brackets, while the spoils are enclosed within the brackets. A closer examination of equation 1 reveals some inherent trade-offs and demonstrates how defense contributions for player 1 (q_1) impact payoffs in three different ways. First, the fewer productive resources player 1 allocates to the alliance's defense, the greater player 1's direct utility component $(w_1 - q_1)$. The incentive imparted by this term is to invest less. However, the more player 1 contributes, the higher the probability of alliance success $(\frac{q_1+q_2}{q_1+q_2+q_3})$. The incentive from this term is to invest more in defense. Finally, more defensive contributions for player 1 will translate to that player earning a larger share $(\frac{q_1}{q_1+q_2})$ of the

potential spoils $(w_3 - q_3)$.[17] These trade-offs demonstrate the conflicting incentives involved in contributing to alliance defense, and players must strike the proper balance between them.

The payoff functions of the basic alliance model coalesce the internal and external connections inherent to alliance decision-making. Within the alliance, parties 1 and 2 choose q_1 and q_2 to optimize their respective payoff functions (subject to their resource constraints), while taking into account the investment strategies of the external player. Formally, alliance utility depends not only on q_1 and q_2, but also on q_3. Similarly, the nonaligned player, 3, chooses q_3 to maximize its payoff (subject to its resource constraint), given the investment strategy of the alliance. The simultaneous optimization of these payoff functions produces solutions $\hat{q}_1$, $\hat{q}_2$ and $\hat{q}_3$ as specified below, addressing the question of how much each player should optimally invest in fighting capacity.[18]

$$\hat{q}_1 = \frac{1 + w_3}{9w_3}[(2 - w_3)w_1 + 2w_3 - 1] \tag{4}$$

$$\hat{q}_2 = \frac{1 + w_3}{9w_3}[(2 - w_3)w_2 + 2w_3 - 1] \tag{5}$$

$$\hat{q}_3 = \frac{1}{9}(1 + w_3)(2 - w_3) \tag{6}$$

These values may in turn be used to determine the utility to each player in equilibrium $\hat{U}_1$, $\hat{U}_2$ and $\hat{U}_3$ as specified below.[19]

$$\hat{U}_1 = \frac{(1 - 2w_3)^2}{9w_3} + \frac{-2 + 8w_3 + w_3^2}{9w_3} \times w_1 \tag{7}$$

$$\hat{U}_2 = \frac{(1 - 2w_3)^2}{9w_3} + \frac{-2 + 8w_3 + w_3^2}{9w_3} \times w_2 \tag{8}$$

$$\hat{U}_3 = \frac{(2 - w_3)^2}{9} \tag{9}$$

Numerical examples can shed light on how the strength of the non-aligned player influences alliance decision-making. Consider the case where $w = (.20, .20, .60)$. Here, player 3 has three times the resources of each allied player. In equilibrium, each player's military spending is $\hat{q}_i = (.142, .142, .249)$, and the expected utility for each player is $\hat{U}_i = (.124, .124, .218)$. In this situation, player 3's resource advantage, w_3, leads to substantial potential rewards for players 1 and 2 if the alliance succeeds. Consequently, players 1 and 2 have a strong incentive to invest in combat capabilities, with each converting precisely 71 percent (.142/.20) of their respective resources for the alliance. However, when player 3 is less powerful, this dynamic changes. We consider the case where $w = (.35, .25, .40)$. In equilibrium, $\hat{q}_i = (.140, .078, .249)$, and the expected utility for each player is $\hat{U}_i = (.143, .106, .284)$. In this case, the incentive to freeride within the alliance reduces military spending. Player 1 now invests approximately 40 percent of its resources, and player 2 invests just over 31 percent – a notable decrease compared to the previous example.

The basic alliance model endogenizes intra- and inter-alliance factors. It folds both ally and adversary into the same analytical plane. This allows player 1 to specifically determine a proper division of resources, based on the strategic decisions of ally 2 and 'enemy' 3 alike. Player 2 does the same. Finally, player 3 is similarly able to allocate resources, taking into account the combined defense allocation of players 1 and 2.

Entente Model: Friends, Not Allies

In this section, we present a model in which two players, 1 and 2, agree to reduce internal conflicts and adopt a friendlier attitude toward each other without forming a formal alliance against the third player in the event of hostilities. The inherent ambiguity of this entente arrangement means that players 1 and 2 are unsure if they can rely on the other's military support, in the event of conflict with player 3. At the same time, the lack of a formal commitment to intervene if the other entente member enters a war with player 3 sends a mixed message to player 3. Essentially, while the unclear nature of military support conveys a 'don't count on me' message between players 1 and 2, it simultaneously communicates the noted 'don't count me out' message to player 3. In the event of war, players 1 and 2 may or may not join forces against player 3. As a result of this uncertainty, player 3 faces a genuine risk of engaging in war with one or both of the other players. The following three payoffs for each player reflect this scenario:

$$U_1 = \frac{q_1}{q_1 + q_3}(w_1 - q_1 + w_3 - q_3) \tag{10}$$

$$U_2 = \frac{q_2}{q_2 + q_3}(w_2 - q_2 + w_3 - q_3) \tag{11}$$

$$U_3 = \frac{q_3}{q_1 + q_2 + q_3}(1 - q_1 - q_2 - q_3) \tag{12}$$

A few details about these equations, as they relate to the alliance model, are worth discussing. Similar to the alliance model, the functions above combine the spoils of victory (enclosed within the parentheses) with a contest success component. As shown in Equation 10, player 1 assumes the likelihood of defeating player 3 depends on both players' defense investments. Equation 11 presents a corresponding calculation for player 2, the other member of the entente. Equation 12 reveals that player 3 assumes the most pessimistic scenario – a conflict involving both members of the entente. The spoils of victory are the resources not allocated by each player for defensive purposes. This feature makes the prize of conflict dependent on each player's relative endowments.[20]

In the entente setting, each player $i = 1, 2, 3$ selects q_i in order to maximize its payoff, subject to resource constraints. We employ q_i^* to represent the optimal military expenditure for player i under the entente arrangement. Taking the derivative of U_i with respect to q_i for $i = 1, 2, 3$, we derive the first-order conditions for an interior solution:

$$q_1 = \sqrt{(w_1 + w_3)q_3} - q_3 \tag{13}$$

$$q_2 = \sqrt{(w_2 + w_3)q_3} - q_3 \tag{14}$$

$$q_3 = \sqrt{q_1 + q_2} - (q_1 + q_2) \tag{15}$$

It may be verified that the second-order conditions are satisfied. Substituting the equivalence relations for q_1 and q_2 from equations (13) and (14) into equation (15) generates the following:

$$q_3 = (\sqrt{w_1 + w_3} + \sqrt{w_2 + w_3})\sqrt{q_3} - \sqrt{(\sqrt{w_1 + w_3} + \sqrt{w_2 + w_3})\sqrt{q_3} - 2q_3} \tag{16}$$

From equation 16 we can derive q_3^* as a function of $w = (w_1, w_2, w_3)$.

$$q_3^* = \frac{1}{6}\left(4(-2 + a^2) + \frac{2\left(4 - 16a^2 + a^4\right)}{\left(8 + \frac{147a^2}{2} - 30a^4 - a^6 + \frac{3}{2}\sqrt{3}\sqrt{a^2(3 + 2a^2)^2(32 - 13a^2 + 4a^4)}\right)^{1/3}} +\right.$$

$$2^{2/3}\left(16+147a^2-60a^4-2a^6+3\sqrt{3}\sqrt{a^2(3+2a^2)^2(32-13a^2+4a^4)}\right)^{1/3}\Bigg), \text{where } a = \sqrt{w_1+w_3}+\sqrt{w_2+w_3}. \tag{17}$$

In the Appendix, we show that for every $w=(w_1,w_2,w_3)$, there exists a unique $0<q_3^*<1$. Once q_3^* is determined, q_1^*, q_2^*, U_1^*, U_2^*, and U_3^* can be computed accordingly.

Similar to the alliance model, we simplify the presentation of results by excluding from the subsequent discussion those distributions $w=(w_1,w_2,w_3)$ that do not satisfy the resource-constraint condition $w_i \geq q_i^*$.

Alliance versus Entente: Comparative Statics and Implications

In this subsection we utilize the equilibrium results obtained from subsections 3.1 and 3.2 to explore the conditions under which a country prefers entente over a formal alliance pledge. Formally, this entails a direct comparison between $\hat{U}_i$ and U_i^* for players 1 and 2 for any distribution of resources $w=(w_1,w_2,w_3)$. Should the payoffs for countries 1 and 2 be greater in the entente than in the alliance system, they would choose entente over the alliance. On the contrary, if their payoffs within the entente are less than those in the alliance system, they would opt for an alliance. A third possibility may arise, where countries 1 and 2 hold divergent preferences – one favoring entente while the other prefers an alliance. In this case, countries 1 and 2 would not form an alliance, and their entente-like relationship would exhibit differing inclinations: one player desires an alliance, while the other does not.

For clarity of presentation, we display the preferences (alliance versus entente) of each player graphically. This allows for a visual representation of the expected alignment outcomes. To achieve this, we employ a barycentric coordinate system. Barycentric coordinates can be used to represent any point's position within a triangle using three scalars. In our context, this is depicted by an equilateral triangle with its three sides corresponding to the initial endowments, w_i, of the three players. Therefore, the sides or axes of the triangle range from 0 to 1, corresponding to the range of endowment values. In Figure 1, to enhance visual clarity, we use the range from 0 to 100 to represent the model's normalized range (w_i) from 0 to 1. A point within the triangle represents the distribution of resources $w=(w_1,w_2,w_3)$.

Figure 1 displays the relative sizes of the three players (w_i) and the resulting equilibria when each player optimizes its respective utility functions in the alliance and entente models. The process involves determining the optimal levels of defense expenditures ($\hat{q}_i$, q_i^*) under both scenarios and their associated payoffs ($\hat{U}_i$, U_i^*). We distinguish players 1 and 2's equilibrium preferences for entente and alliance according to the colored areas in Figure 1. The purple area signifies that both players 1 and 2 prefer entente. The blue area represents distributions where players 1 and 2 favor a formal alliance. The green and red areas indicate divergent preferences between players 1 and 2. In the green section, player 1 leans towards the entente, while player 2 opts for the alliance. Conversely, the red area showcases the opposite situation: player 2 prefers entente, and player 1 favors alliance. The white sections of the graphic denote resource distributions that fail to meet the resource constraints in equilibrium for at least one player. The diamond-shaped, color-coded region in the triangle's center signifies distributions where players have adequate resources to respond optimally to threats.

Before diving into specific numerical examples, several key insights and general patterns can be observed from the graphic. First, when $w_3 \approx .42$, both players prefer entente (the purple area). In situations where player 3 possesses relatively abundant resources, entente is generally the optimal choice for players 1 and 2. On the other hand, when facing less powerful or smaller adversaries, the alliance strategy (the blue area) is more favorable than the entente strategy. Although the relationship is curvilinear, this trend generally holds. The red and green sections of the graphic reveal that preferences between players 1 and 2 differ most when resource disparities between them are most

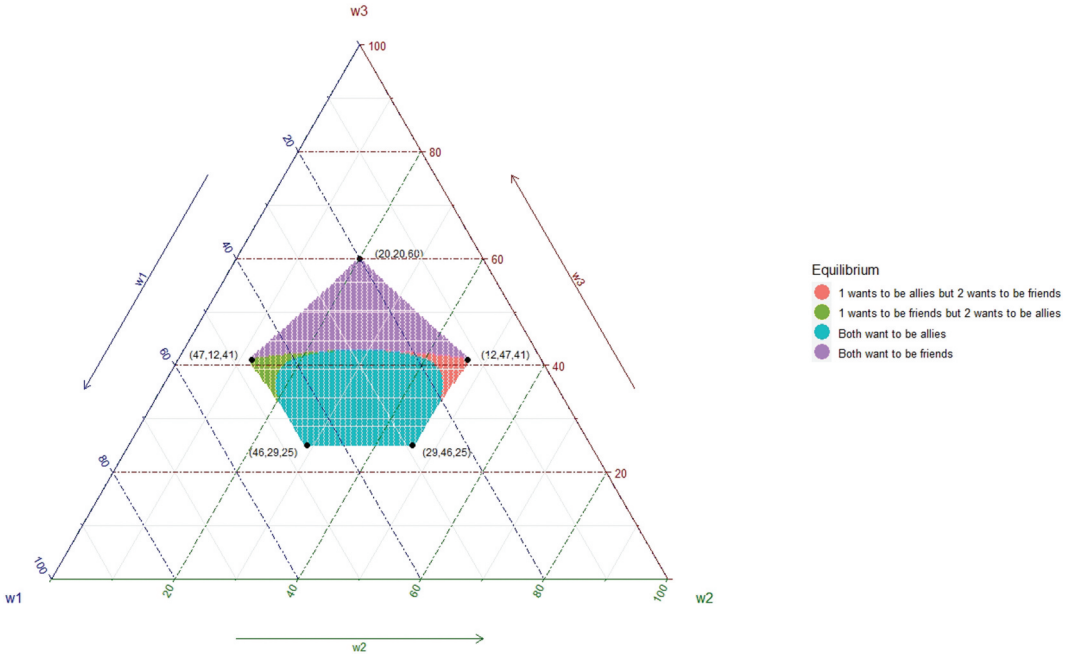

Figure 1. Three Player Simplex.

pronounced. When w_1 significantly exceeds w_2, player 1 opts for entente, while player 2 leans toward alliance. Conversely, preferences switch when w_1 is substantially less than w_2. In general, the smaller player prefers a formal alliance, while the larger player prefers entente.

Below, we take advantage of numerical examples to demonstrate how external threats and internal resource disparities influence alignment choices. First, we examine the impact the external threat (w_3) has on the decision to form an alliance or pursue an entente. It is helpful to start with a scenario where all players have equal initial resources, $w = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. When players 1 and 2 are allies, the expected payoff in equilibrium for both players is $\hat{U}_i$ =.123. But if they choose entente, the payoff is $U_i^* = .103$. In this situation, an alliance is a more desirable alignment option than an entente. However, when the external player is more powerful, for example, $w = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$, the dynamics shift. In this case, the expected utility for each player is $\hat{U}_i = .125$ in an alliance and $U_i^* = .143$ in an entente. This example suggests that when the external threat from player 3 intensifies, players 1 and 2 find it more advantageous to be 'friends' rather than formal allies.

We now interrogate how resource disparities between players 1 and 2 influence their alignment preferences. When their resources are evenly distributed, for example, $w = (.30, .30, .40)$, if players 1 and 2 are allies, $\hat{q} = (.109, .109, .249)$ and $\hat{U} = (.124, .124, .284)$. By contrast, in an entente, the equilibrium outcomes are $q^* = (.169, .169, .243)$ and $U^* = (.118, .118, .175)$. Clearly, both players 1 and 2 prefer to ally. However, the outcome changes as a disparity arises between their respective resources (while holding player 3's resources constant). For instance, when $w = (.45, .15, .40)$, if players 1 and 2 are allies, $\hat{q} = (.202, .016, .249)$ and $\hat{U} = (.181, .068, .284)$. By contrast, in an entente, the equilibrium outcomes are $q^* = (.211, .122, .244)$ and $U^* = (.183, .061, .178)$. It is evident from these payoffs that player 1 (the larger of the two) prefers an entente, while player 2 (the smaller of the two) opts for an alliance.

This outcome warrants further exploration. First, it reveals that under this asymmetrical resource distribution, split preferences prevail. Why is it the case that the smaller player wants the security of a formal alliance, while the larger player prefers the opposite? The answer is that the alliance affords

Table 1. Equilibrium defense allocations, equilibrium utility, and system preferences.

	Allies				Friends				
w_1, w_2, w_3	$\hat{q}_1$	$\hat{q}_2$	$\hat{U}_1$	$\hat{U}_2$	q_1^*	q_2^*	U_1^*	U_2^*	Preferences
1/3, 1/3, 1/3	0.099	0.099	0.123	0.123	0.159	0.159	0.103	0.103	Allies, Allies
.30, .30, .40	0.109	0.109	0.124	0.124	0.169	0.169	0.118	0.118	Allies, Allies
.45, .15, .40	0.202	0.016	0.181	0.068	0.211	0.122	0.183	0.061	Friends, Allies
.25, .25, .50	0.125	0.125	0.125	0.125	0.184	0.184	0.143	0.143	Friends, Friends

the smaller player a chance to benefit from freeriding on the larger player. The degree of freeriding by player 2 is evident in its defense contributions in each scenario. When $w = (.45, .15, .40)$, under the alliance, $\hat{q}_2 = .016$; and in the entente, $q_2^* = .122$. Clearly, player 2 wants to contribute very little defense to the alliance (just over 10 percent of its total resources). But in an entente, this player is forced to greatly increase defensive investments (more than 81 percent of its endowment). The free-riding tendency, rational within the alliance, proves too risky for the smaller player in the entente.

For the larger player, the decision-making process differs. Player 1's expected payoff is greater in an entente than in an alliance. This outcome arises from several factors. First, this player spends roughly the same amount in defense under both scenarios ($\hat{q}_1 = .202$, $q_1^* = .211$). However, the combined military spending of players 1 and 2 in equilibrium is higher under entente than in the alliance (.333 versus .218), on account of defense increases from player 2. As a result, player 3 has less to gain in terms of spoils ($W_A - \hat{Q}_A$) in the event of victory and therefore invests less than in the alliance context (.249 versus .244). This, in turn, leaves more resources ($W_3 - \hat{Q}_3$) available for player 1 to claim under the entente arrangement.

Table 1 presents results from four of the numerical examples noted above. It lists the equilibrium values for players 1 and 2, under both the alliance and entente settings. The final column of the table lists each player's alignment preferences under a given resource distribution. We selected these distributions because they neatly illustrate the changing preference dynamics inherent to the model.

Concluding Discussion

This essay is motivated by a simple question: 'When should states be friends but not allies?' Our theoretical approach, along with significant historical events, offer several insights and implications. First, the prevailing assumption that balancing necessitates or compels states to form alliances may warrant reconsideration. The classic balancing versus bandwagoning distinction (Walt 1985) may misrepresent or not accurately reflect the strategic choices states have in making alignment decisions. That is, it is likely not an either-or proposition parties face. Our analysis suggests that two states confronting a more powerful adversary can effectively balance by establishing an entente instead of a rigid alliance. The fluid alliance politics leading up to WWI lend support to this notion. As Germany rose to prominence before the war, Britain and France chose to sign the Entente Cordiale of 1904 rather than a defensive alliance; three years later, Britain and Russia signed the Anglo-Russian Entente of 1907. None of these states could be certain if the others would come to their aid in case of hostilities with Germany. Furthermore, Germany was left to speculate how many adversaries it might face should a conflict with one of its opponents erupt.

Another expectation arising from our study is that entente-like arrangements, with their inherent ambiguity, should deter freeriding. The free-rider problem has long been recognized and extensively documented in alliance politics. In fact, this general result aligns with the empirical finding by Conybeare, Sandler, and Sandler (1990) of freeriding between France and Russia (the two formal allies) during World War I within the Triple Entente. The other member countries reacted positively to defense investments made by others, which is consistent with the behavior expected in an entente. In other words, the free-riding behavior exhibited by France and Russia perfectly aligns with their mutual promises of military support.

Another crucial insight from our model pertains to the very concept of entente itself. Indeed, an entente is an elusive idea, reflecting only a 'state of mind' or a 'disposition.' However, this may be where its strategic advantage over an alliance lies. An entente could serve as a clever institutional design. The purpose of choosing an entente over an alliance is to strike a balance between entrapment and deterrence. An entente sends one message to potential partners: don't count on me. Simultaneously, it conveys another message to potential aggressors: don't count me out. The ambiguity inherent in these two self-contradictory objectives of an entente is a vital aspect of its effectiveness. In this sense, an entente embodies a policy of *strategic ambiguity*. Our effort here is, of course, but one formal interpretation of an entente. Future work should look at alternative specifications.

Our largely theoretical offering also presents a challenge to those who conduct empirical studies of politics. Our comparative modeling effort effectively demonstrates that split preferences between states might not be rare in alliance politics: smaller states may prefer alliances with larger states who themselves wish to remain friends rather than announce any alliance. The empirical upshot of this observation is that instances of split preferences between countries may be difficult to observe. However, this does not imply that such instances have not existed or do not exist currently. As mentioned in Section 2, the Sino-Russian treaty of friendship, initially announced in February 2022, may now represent an example of emerging split preferences after more than a year of Russia's war against Ukraine, which has strained the relationship. Identifying and quantifying these mixed relationships is an area for future research in the discipline.

Taken together, these observations suggest another takeaway from our endeavor. In reality, entente-like relationships are not uncommon in international politics, and thus they merit attention in the study of alliance and alignment strategies. This pursuit can generate testable hypotheses, such as the possibility that ententes might discourage free-riding, leading to superior military capabilities compared to alliances. Expanding our understanding of alignment politics to fully incorporate ententes can unveil this and other empirical implications, paving the way for future research in the discipline.

Notes

1. The Triple Alliance was based on formal agreements between Germany, Austria-Hungary and Italy, initially signed in 1882 and renewed periodically until WWI. The Triple Entente was the product of three bilateral accords. The first was a traditional military convention between France and Russia signed in December of 1893. Each party pledged military support should either be attacked by a Triple Alliance power, provided Germany was involved directly or in a supportive role. The second was the Anglo-French Entente Cordiale, signed in 1904, designed to defuse colonial tensions between the two powers. The third was the Anglo-Russian Entente of 1907, aimed at reducing mutual tensions in Asia. Thus, the Triple Entente was in fact the byproduct of a military accord between France and Russia and the two entente arrangements Great Britain maintained with both France and Russia. See Conybeare, and Sandler (1990) for a general discussion and formal treatment of the Triple alliance and Triple Entente.
2. Quoted in Hamilton (1977), (p.324).
3. On the evolution of the Dual Alliance, particularly from the Russian perspective, see Packard (1920).
4. It would also constitute the only overt military pledge within what would come to be called the Triple Entente (France, Great Britain and Russia).
5. Cited from Schmitt (1924), p. 65.
6. Wilkins (2012) makes a similar point in his insightful critique of alliance scholarship. He challenges the field to broaden its conception and understanding of the 'phenomenon of alignment as a whole' and not confine its attention to formal military alliance pacts (p. 54).
7. Emphasis in original. Snyder (1997), 53-54.
8. More than a year into the Russian war in Ukraine, the degree of 'friendship' between Beijing and Moscow is under strain, as China has adopted a more cautious tone when describing its relationship with Russia. This may be an evolving instance of diverging preferences between parties. Moscow clearly would benefit from military support from Beijing, which, thus far, has not materialized.
9. Defense pacts obligate signatories to intervene militarily on the side of any treaty partner attacked militarily. Neutrality and non-aggression pacts require parties to remain militarily neutral if any co-signatory is attacked.
10. Berridge (1989) makes this point as well, characterizing the COW definition as 'wrong' (p. 252).
11. The ATOP definition of alliance is as follows: Alliances are written agreements, signed by official representatives of at least two independent states, that include promises to aid a partner in the event of military conflict, to

remain neutral in the event of conflict, to refrain from military conflict with one another, or to consult/cooperate in the event of international crises that create a potential for military conflict (p. 238).

12. Snyder (1991), p. 141.
13. The 'weakest link' reflects a situation where the weakest element of a chain determines the maximum force it can sustain. The 'best shot' is when the highest or most effective contribution determines the aggregate level of the public good.
14. We are motivated here and draw upon the rich literature related to the economics of alliances. This field has played a fundamental role in understanding and applying collective action problems to real-world applications such as alliances. Sandler and Hartley (2001) offer a comprehensive review of scholarship on alliance theory and public goods. Sandler (1993) also presents an earlier survey on the economic theory of alliances. See also Garfinkel (2007) for a broader examination of the economics of conflict as well as Hirshleifer's (2001) collection of his pioneering work on the topic.
15. We present only the basic model setup and equilibrium results. For more details and equilibrium calculations, see Niou and Zeigler (2019), which offers a generalized treatment of various alliance systems.
16. This feature of the model implies that military expenditure (q_i) serves as both a private and public good within the alliance. Sandler (1977) and Sandler and FORBES (1980) also extend traditional models of alliances to allow for national security expenditures to provide both public and private benefits to allied members. In particular, Sandler (1977) allows for jointly produced ally-specific benefits that can help mitigate free-riding by larger states over smaller allies.
17. The literature has explored alternative sharing rules. For example, Ueda (2002) examines a sharing rule that is a weighted average between equal sharing and sharing proportional to effort contributions. Nitzan and Ueda (2011) investigate the characteristics of endogenously-determined sharing rules in collective contests.
18. While our aim in this essay is to contrast this alliance model with a model of entente, it is worth noting that the equilibrium results we derive align with key aspects of related economic models of alliances. Free-riding and suboptimality characterize the equilibrium results. Sandler (1977) provides an insightful discussion on the extent and origins of suboptimality linked to defense provision within the framework of Olson and Zeckhauser (1966).
19. For simplification, we restrict our equilibrium analysis to the universe of cases meeting the resource-constraint condition $w_i \geq \hat{q}_i \geq 0$. In those cases where $w_i < \hat{q}_i$, we set $\hat{q}_i = w_i$ and recompute the other players' best responses. And in those cases where $\hat{q}_i < 0$, we set $\hat{q}_i = 0$ and recompute the equilibrium military spending for the other players. The equilibrium results are consistent with the main theoretical findings that we present in Section 3.3. The proof of the adjusted equilibrium results is available upon request.
20. This design feature stands in contrast to contests with fixed prizes. For details on optimal contest designs, see Letina, Liu Nick Netzer, and Netzer (2023).
21. We use Mathematica software (Wolfram Research, Inc., Champaign, IL) to determine the value of q_3^* from equation 24.

Acknowledgement

We would like to extend our deepest gratitude towards Jidong Chen for his invaluable help in developing the mathematical results in the Appendix. We thank Austin Horng-En Wang for his contribution to the creation of Figure 1. We are indebted to Samuel Charap and Jeffrey Martini for their added insights.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Berridge, G. R. 1989. "Ententes and Alliances." *Review of International Studies* 15 (3): 251–260. doi:10.1017/S0260210500112860.

Conybeare, J. A., J. C. Murdoch, and T. Sandler. 1994. "Alternative Collective-Goods Models of Military Alliances: Theory and Empirics." *Economic Inquiry* 32 (4): 525–542. doi:10.1111/j.1465-7295.1994.tb01349.x.

Conybeare, J. A., and T. Sandler. 1990. "The Triple Entente and the Triple Alliance 1880–1914: A Collective Goods Approach." *The American Political Science Review* 84 (4): 1197–1206. doi:10.2307/1963259.

Esteban, J. D. R., and D. Ray. 2001. "Collective Action and the Group Size Paradox." *The American Political Science Review* 95 (3): 663–672. doi:10.1017/S0003055401003124.

Garfinkel, M. R. 2004. "On the Stability of Group Formation: Managing the Conflict Within." *Conflict Management and Peace Science* 21 (1): 43–68. doi:10.1080/07388940490433918.

Garfinkel, M. R. S. S. 2007. "Economics of Conflict: An Overview." *Handbook of Defense Economics* 2: 649–709.
Hamilton, K. A. 1977. "Great Britain and France, 1911—1914." In *British Foreign Policy Under Sir Edward Grey*, edited by Francis Harry Hinsley, 336–337, Cambridge University Press.
Hartley, K. T. S., and T. Sandler. 1999. "NATO Burden-Sharing: Past and Future." *Journal of Peace Research* 36 (6): 665–680. doi:10.1177/0022343399036006004.
Hirshleifer, J. 1983. "From Weakest-Link to Best-Shot: The Voluntary Provision of Public Goods." *Public Choice* 41 (3): 371–386. doi:10.1007/BF00141070.
Hirshleifer, J. 1989. "Conflict and Rent-Seeking Success Functions: Ratio Vs. Difference Models of Relative Success." *Public Choice* 63 (2): 101–112. doi:10.1007/BF00153394.
Hirshleifer, J. 2001. *The Dark Side of the Force: Economic Foundations of Conflict Theory*. Cambridge, UK: Cambridge University Press.
Kann, R. A. 1976. "Alliances Versus Ententes." *World Politics* 28 (4): 611–621. doi:10.2307/2010069.
Kim, W. T. S., and T. Sandler. 2020. "NATO at 70: Pledges, Free Riding, and Benefit-Burden Concordance." *Defence and Peace Economics* 31 (4): 400–413. doi:10.1080/10242694.2019.1640937.
Leeds, B., J. Ritter, S. Mitchell Andrew Long, and A. Long. 2002. "Alliance Treaty Obligations and Provisions, 1815-1944." *International Interactions* 28 (3): 237–260. doi:10.1080/03050620213653.
Letina, I., S. Liu Nick Netzer, and N. Netzer. 2023. "Optimal Contest Design: Tuning the Heat." *Journal of Economic Theory* 105616. doi:10.1016/j.jet.2023.105616.
Neary, H. M. 1997. "A Comparison of Rent-Seeking Models and Economic Models of Conflict." *Public Choice* 93 (3/4): 373–388. doi:10.1023/A:1017901524758.
Niou, E. M. S., and G. Tan. 2005. "External Threat and Collective Action." *Economic Inquiry* 43 (3): 519–530. doi:10.1093/ei/cbi035.
Niou, E., and S. M. Zeigler. 2019. "External Threat, Internal Rivalry, and Alliance Formation." *The Journal of Politics* 81 (2): 571–584.
Nitzan, S. K. U., and K. Ueda. 2011. "Prize Sharing in Collective Contests." *European Economic Review* 55 (5): 678–687. doi:10.1016/j.euroecorev.2010.09.005.
Olson, M. 1965. *Logic of Collective Action: Public Goods and the Theory of Groups*. Cambridge, MA: Harvard University Press.
Olson, M. R. Z., and R. Zeckhauser. 1966. "An Economic Theory of Alliances." *The Review of Economics and Statistics* 48 (3): 266–279. doi:10.2307/1927082.
Packard, L. B. 1920. "Russia and the Dual Alliance." *The American Historical Review* 25 (3): 391–410. doi:10.2307/1836878.
Powell, R. 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton, NJ: Princeton University Press.
Sandler, T. 1977. "Impurity of Defense: An Application to the Economics of Alliances." *Kyklos* 30 (3): 443–460. doi:10.1111/j.1467-6435.1977.tb02203.x.
Sandler, T. 1993. "The Economic Theory of Alliances: A Survey." *The Journal of Conflict Resolution* 37 (3): 446–483. doi:10.1177/0022002793037003003.
Sandler, T. J. F. F., and J. F. Forbes. 1980. "Burden Sharing, Strategy, and the Design of Nato." *Economic Inquiry* 18 (3): 425–444. doi:10.1111/j.1465-7295.1980.tb00588.x.
Sandler, T. K. H., and K. Hartley. 2001. "Economics of Alliances: The Lessons for Collective Action." *Journal of Economic Literature* 39 (3): 869–896. doi:10.1257/jel.39.3.869.
Schmitt, B. E. 1924. *Triple Alliance and Triple Entente*. New York, NY: Henry Holt And Company.
Singer, J. D. M. S., and M. Small. 1966. "Formal Alliances, 1815—1939: A Quantitative Description." *Journal of Peace Research* 3 (1): 1–31. doi:10.1177/002234336600300101.
Skaperdas, S. 1992. "Cooperation, Conflict, and Power in the Absence of Property Rights." *The American Economic Review* pp. 720–739.
Skaperdas, S. 1998. "On the Formation of Alliances in Conflict and Contests." *Public Choice* 96 (1/2): 25–42. doi:10.1023/A:1004912124496.
Snyder, G. H. 1991. "Alliances, Balance, and Stability." *International Organization* 45 (1): 121–142. doi:10.1017/S0020818300001417.
Snyder, G. H. 1997. *Alliance Politics*. Ithaca, NY: Cornell University Press.
Snyder, G. H. P. D. 1977. *Conflict Among Nations: Bargaining, Decision Making, and System Structure in International Crises*. Princeton, NJ: Princeton University Press.
Ueda, K. 2002. "Oligopolization in Collective Rent-Seeking." *Social Choice and Welfare* 19 (3): 613–626. doi:10.1007/s003550100140.
Walt, S. M. 1985. "Alliance Formation and the Balance of World Power." *International Security* 9 (4): 3–43. doi:10.2307/2538540.
Wilkins, T. S. 2012. "'Alignment', Not 'Alliance'–The Shifting Paradigm of International Security Cooperation: Toward a Conceptual Taxonomy of Alignment." *Review of International Studies* 38 (1): 53–76. doi:10.1017/S0260210511000209.
Williamson, S. R., Jr. 2011. "German Perceptions of the Triple Entente After 1911: Their Mounting Apprehensions Reconsidered." *Foreign Policy Analysis* 7 (2): 205–214. doi:10.1111/j.1743-8594.2011.00133.x.

Appendix

The Optimal q_i^* for $i = 1, 2, 3$ in the Entente System:

In the entente system the three countries' payoff functions are represented as follows.

$$U_1 = \frac{q_1}{(q_1 + q_3)}(w_1 - q_1 + w_3 - q_3) \tag{18}$$

$$U_2 = \frac{q_2}{(q_2 + q_3)}(w_2 - q_2 + w_3 - q_3) \tag{19}$$

$$U_3 = \frac{q_3}{(q_1 + q_2 + q_3)}(1 - q_1 - q_2 - q_3) \tag{20}$$

Taking the derivative of U_i with respect to q_i for $i = 1, 2, 3$, we obtain the following first-order conditions for an interior solution:

$$q_1 = \sqrt{(w_1 + w_3)q_3} - q_3 \tag{21}$$

$$q_2 = \sqrt{(w_2 + w_3)q_3} - q_3 \tag{22}$$

$$q_3 = \sqrt{q_1 + q_2} - (q_1 + q_2) \tag{23}$$

Substituting the equivalence relations for q_1 and q_2 from equations (21) and (22) into equation (23), we obtain

$$q_3 = (\sqrt{w_1 + w_3} + \sqrt{w_2 + w_3})\sqrt{q_3} - \sqrt{(\sqrt{w_1 + w_3} + \sqrt{w_2 + w_3})\sqrt{q_3} - 2q_3} \tag{24}$$

Let $a = \sqrt{w_1 + w_3} + \sqrt{w_2 + w_3}$ and $x = \sqrt{q_3}$. Equation (24) can be written as follows:

$$aX - X^2 = \sqrt{aX - 2X^2} \tag{25}$$

Upon multiplying, rearranging, and simplifying terms within Equation 25, we arrive at the following equation:

$$X^3 - 2aX^2 + (2 + a^2)X - a = 0 \tag{26}$$

We are now able to define the LHS of the equation above as $f(X)$ such that:

$$f(X) = X^3 - 2aX^2 + (2 + a^2)X - a \tag{27}$$

In the following Lemma 1, we show that $f(x)$ is strictly increasing.

Lemma 1:$f(X) = X^3 - 2aX^2 + (2 + a^2)X - a$ is strictly increasing.

Proof: The derivative of $f(X)$ is as follows:

$$f'(X) = 3X^2 - 4aX + (2 + a^2) \tag{28}$$

Note that the derivative $f'(x)$ has the standard quadratic form $g(x) = Ax^2 + Bx + C$. Remember that for equations with this form, if $A > 0$ and $B^2 - 4AC < 0$, then $g(x) > 0$ for all x. In our case, $A = 3 > 0$ and $B^2 - 4AC = 16a^2 - 4(3)(2 + a^2) = 4(a^2 - 6)$. Given that $w_1 > 0, w_2 > 0, w_3 > 0$, and $w_1 + w_2 + w_3 = 1$, it is straightforward to show that a lies in the interval (1, 2), which implies that $4(a^2 - 6) < 0$ and, subsequently, that $f'(x) > 0$. We can thus conclude that $f(x)$ is strictly increasing.

By Lemma 1, we know that $f(x)$ is strictly increasing. Furthermore, note that $f(0) = -a = -(\sqrt{w_1 + w_3} + \sqrt{w_2 + w_3}) < 0$ and that $f(1) = 1 - 2a + (2 + a^2) - a = (a - 3/2)^2 + 3/4 > 0$. Therefore, we can conclude that $f(x) = 0$ has a unique solution x^*, within the interval (0,1). And given that $x = \sqrt{q_3}$, it follows that

equation 24 has a unique solution q_3^*, within the interval (0,1). q_3^* can be represented as follows[21]:

$$q_3^* = \frac{1}{6}\left(4(-2+a^2) + \frac{2\left(4-16a^2+a^4\right)}{\left(8+\frac{147a^2}{2}-30a^4-a^6+\frac{3}{2}\sqrt{3}\sqrt{a^2(3+2a^2)^2(32-13a^2+4a^4)}\right)^{1/3}} + \right.$$

$$\left. 2^{2/3}\left(16+147a^2-60a^4-2a^6+3\sqrt{3}\sqrt{a^2(3+2a^2)^2(32-13a^2+4a^4)}\right)^{1/3}\right),$$

$$\text{where } a = \sqrt{w_1+w_3}+\sqrt{w_2+w_3}. \tag{29}$$