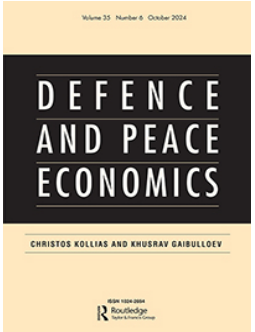

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Concentration and Innovation in the Defence Industry: A Stochastic Game

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ABSTRACT
The relationship between industry structure, aggregate R&D activity and the quality level of the resulting innovation is investigated in a simple stochastic one-shot game in which all firms share the same probability of attaining the innovation concerning a new weapon system and therefore obtaining the contract from the government. The main findings of the game can be summarised as follows. Although only performing numerical calculations, the model predicts the existence of a concave and single-peaked relationship between aggregate R&D and industry structure, making it desirable to adjust concentration in order to reach that peak. However, the degree of concentration that maximises the final quality level of the innovation is higher than that maximises industry-wide R&D efforts, except in the special case in which technical knowledge freely spills over across firms, thanks to the creation of a research joint venture involving all firms.

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defence industry; innovation; concentration; technological spillovers

Introduction

The traditional debate concerning the opposite views of Schumpeter (1942) and Arrow (1962) about the relationship between industry structure and the pattern of aggregate innovation efforts has been revived by the relatively recent contribution of Aghion et al. (2005; 2015; 2019), where empirical analysis, accompanied by a model generating the same outcome, has highlighted the possible emergence of an inverted-U-shaped R&D curve, establishing the existence of a specific industry structure maximising aggregate R&D efforts. All of this is in contrast with the monotone patterns dating back to Schumpeter (1942) and Arrow (1962), as well as a large number of follow-ups.[1]

This amounts to saying that it may not be true that market power (as the Schumpeterian position holds) or the absence thereof (as the Arrovian objection claims) favour the pace of innovation, with relevant implications concerning any decisions concerning a horizontal merger in a sector of strategic relevance and already characterised by some non-negligible and possibly very high degree of concentration (this being certainly the case in the US): in a nutshell, if the industry structure at which the peak of collective R&D efforts is reached is unknown, proceeding with any given merger may compromise the whole industry's technological development in the long run, i.e. for several decades.[2]

Here, I adapt this approach to the relationship between industry structure and innovation incentives, typical of the theory of industrial organization, to the case in which the industry does not supply a traditional market expressing a downward sloping demand, but a single client, namely,

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a government. To this aim, I propose a simple model, taking the form of a one-shot stochastic and noncooperative innovation race with a generic number of firms, each of which is aware of having the same probability of innovating and delivering a new weapon system to a government. Two essential features of the game are worth mentioning explicitly. On the one hand, the fact that we are dealing with innovation in the defence sector implies that the price is not determined along a market demand curve, as it is the outcome of a negotiation (which is intentionally left unmodelled) between the government and firms, and therefore the only strategic variable manoeuvred by each firm is the intensity of R&D. Incidentally, in this respect, it may also be added that the model could serve as well the purpose of investigating an innovation race under a regulation scheme that fixes market price at a given level.[3] On the other hand, the only concern of the government is not welfare, as in the traditional approach of the theory of industrial organization, but the impact of industry fragmentation on the quality level of the innovation and the possible existence of a peak of aggregate R&D in correspondence of a certain industry structure.[4]

Among other things, the model accounts for the presence of industrial offsets from the winner to the $n-1$ losers in such a way that once the race is over and the identity of the winner is known, any other firm in the industry has a share of production (taking the form of a certain number of either products or parts) and thus a positive payoff.

The results emerging from the analysis of the game are the following. There exists a number of firms that maximise aggregate R&D efforts; one clearcut message emerging from the model is that the number of firms maximising the intrinsic quality of the innovation will not maximise the aggregate R&D effort. In particular, there always exists a region of parameters in which the industry structure that maximises quality is more concentrated than that maximises aggregate R&D. The second property worth stressing is that, if an RJV involving all firms in the industry should take place, then the number of firms that maximises total R&D expenditure also maximises the quality level of the innovation. Consequently, we may also draw the conclusion that too little concentration simultaneously means lower innovation quality and inefficient effort duplication.

The remainder of the paper is structured as follows. A brief account of the long-standing debate on innovation models and the alternative approaches characterise it is presented in section 2. The layout of the model is illustrated in section 3. Section 4 contains the analysis of the equilibrium behaviour of firms, while section 5 illustrates the basic properties of the industry-wide research joint venture (RJV). A few concluding remarks are provided in section 6.

A Brief Recollection of the Debate on Innovation Races

The traditional debate in the industrial organization literature on (possibly, but not necessarily, stochastic) innovation games triggered by the Arrow vs Schumpeter debate, took three different forms, which can be summarised in the following terms.

The first, which can be traced back to Scherer (1967), has almost immediately taken the form of an auction (Gilbert and Newbery 1982 ; Reinganum 1983), leaving technological uncertainty aside. In this strand of research, at least two firms bid for an innovation of a given size and, at equilibrium (i) only the winner bids, and (ii) all firms' profits are nil at equilibrium, precisely because the winner's bid entirely dissipates the following rent generated by the innovation.

The second stream of research is systematically characterised by uncertainty, as it treats the innovation date as a random variable. Consider n fully symmetric firms investing in R&D to achieve a new product or technology that is patentable forever. Firms behave noncooperatively, each choosing the R&D investment x_i so as to maximise its discounted expected profit flow. The uncertainty affecting R&D technology takes an exponential form, whereby the expected discovery date is described by a Poisson 'memoryless' exponential distribution function.

In the seminal paper by Loury (1979; see also Dasgupta and Stiglitz 1980), firm i's objective function is

$$\Omega_i^L = \int_0^\infty e^{-(\rho+H)t}[h(x_i)V_i + H_{-i}v_i + \pi_i]dt - x_i, \tag{1}$$

where x_i is a one-off investment borne at $t = 0$, whereby this approach is traditionally associated with the choice of a *contractual* R&D effort. Moreover, $\rho > 0$ is the common discount rate, the hazard rate $h(x_i)$ measures firm i's the instantaneous probability of innovating conditional upon the fact that nobody has innovated before[5] and $H \equiv \sum_{i=1}^n h(x_i)$, $H_{-i} \equiv H - h(x_i)$. π_i are firm i's current gross profits, and V_i (respectively, V_i) is the discounted continuation value of the game if firm i wins (loses) the innovation race.

In Lee and Wilde (1980), as well as in a large part of the ensuing literature (see Delbono and Denicolò 1991, and many others up to Delbono and Lambertini 2022), Loury's (1979) approach is modified as far as the nature of the R&D cost is concerned. Here, the innovation effort is non-contractual, taking the form of a fixed effort x_i (i.e. optimally chosen but time-invariant) that drops to zero as soon as a firm does innovate and therefore gets the patent. Accordingly, firm $i = 1, 2, \ldots n$ must choose its efforts to maximise the following payoff:

$$\Omega_i^{LW} = \int_0^\infty e^{-(\rho+H)t}[h(x_i)V_i + H_{-i}v_i + \pi_i - x_i]dt \tag{2}$$

The implications of the maximands appearing in (1-2) deserve some brief remarks. If we look at the standpoint chosen by Loury's (1979) and Dasgupta and Stiglitz (1980), we may see an underlying mechanism whereby any increase in industry fragmentation reduces the expected profits from R&D investment without modifying the related R&D costs, which are paid at the initial date. This drives firms to reduce investments as n increases. Conversely, in the formulation proposed by Lee and Wilde (1980), the flow of non-contractual R&D efforts enters the expected value of the net individual payoff, and both gross instantaneous profits and R&D costs shrink as n increases. The balance of these effects is that firms respond to increasing fragmentation or competition by investing more (for additional elements on this, see Reinganum 1984, 1989).

Now it is useful to stress two aspects that may explain why the possible emergence of non-monotone patterns has been overlooked for a long time. The first is a direct consequence of the nature of the Schumpeter vs Arrow debate, as the related flow of research has focused for a long time on two polar views, both taking the form of a monotone behaviour but with opposite slopes. The second has a technical nature and can be appreciated by looking at the first-order conditions generated, respectively, by (1) and (2):

$$\frac{\partial \Omega_i^L}{\partial x_i} = \frac{h'(x_i)[\rho V_i + H(V_i - v_i) - \pi_i]}{(\rho + H)^2} - 1 = 0 \tag{3}$$

$$\frac{\partial \Omega_i^{LW}}{\partial x_i} = \frac{h'(x_i)[\rho V_i + x_i + H(V_i - v_i) - \pi_i]}{(\rho + H)^2} - \frac{1}{\rho + H} = 0 \tag{4}$$

In general, explicit solutions w.r.t. x_i cannot be obtained, and therefore these two models and the large ensuing literature have pointed at identifying necessary and sufficient conditions for monotonicity, which were then explicitly replicated many times in more specialised models adopting a set of restrictive assumptions about the functional forms of the hazard rate and payoffs, as well as the type of market competition. This search for monotone reactions of aggregate R&D to industry structure was, of course, aligned with the desire to deliver either an Arrovian or a Schumpeterian solution. A relevant feature of this stream of literature, which also emerges from an equally large amount of research using static multistage games under perfect certainty, from d'Aspremont and Jacquemin (1988) onwards, is the excess investment characterising the industry at equilibrium, whose social inefficiency can be mitigated – if not altogether eliminated – resorting to R&D cartels, research joint ventures (RJVs) or consortia (see Amir 2000). As we shall see, this perspective is also relevant in terms of the problem treated in the present paper, although for different reasons.

Last but not least, if the time horizon is infinite, the memoryless nature of the Poisson process has the desirable consequence of driving to zero the exponential expressions that would instead appear if one were to assume the presence of a finite delivery date, as is usually the case for procurement contracts in general and, in particular, for those concerning weapon systems. This very fact prevents the use of these settings to analyse the problem at stake in the present paper.

Additionally, it remains to be pointed out that V_i and V_i can be specified in terms of the profits generated by either price or quantity competition in markets in which firms face downward-sloping demand functions generated by consumers. It is pretty intuitive that this kind of scenario is not appropriate to tackle the issue at stake here, for at least two quite solid reasons, namely, (i) once again, firms operating in the defence sector face a predetermined and finite time horizon, with possibly very strict delivery dates, and (ii) the mechanism determining prices cannot be likened to (or mimicked by) market demand functions, although of course a demand for weapon systems is expressed by governments.

The third avenue of research encompasses the so-called tournament games in which the identity of the innovating firm is uncertain, the innovation date being instead certain. The departure point of this literature can be traced back to Futia (1980), Hartwick (1982) and Rogerson (1982), stretching at least to Taylor (1995).[6] This approach, which has also been empirically tested using data related to the aerospace sector (Rogerson 1989), promotes the adoption of research tournaments as a solution to the problems associated with unobservable research efforts and outcomes that are unverifiable in court. Concerning public procurement contracts, this is particularly true for the quality of the final product being supplied to the monopsonist sponsoring the tournament. Accordingly, the essential elements distinguishing R&D tournaments from races can be summarised as follows: While in R&D races, the level or quality of the innovation is exogenous and the date at which it is achieved is endogenous, this is not the case in tournaments; consequently, innovation races focus on the intensity of R&D efforts, leaving aside the predetermined size of the innovation at stake, which is instead emphasised in tournament models. Therefore, the systematic adoption of games defined as tournaments in the literature dealing with public procurement in general and the defence sector, in particular, appears as a natural outcome of these features.

What follows is indeed connected with this strand of literature, while coupling it with the spirit of the recent debate about the possibility of a non-monotone relationship between innovation incentives and industry concentration.

A Stochastic Game with Industrial Offsets

Consider $n \geq 2$ firms operating in the defence sector, racing for an innovation in a static stochastic game in which innovation efforts, for the sake of simplicity, are one-off choices. At the beginning of the single period in which the game takes place, all firms simultaneously and non-cooperatively decide the intensity of their respective R&D efforts; at the end of the period, the innovator (or, the winner) gets the patent, or a pool thereof, as well as the contract from the government. The individual investment of firm i is $k_i > 0,$ while the quality of the resulting final product is $q_i > 0$. The model, as will be clarified below, is built up so as to account for the plausible presence of industrial offsets. The stochastic nature of innovation is formulated in a way that closely reflects that appearing in Shy (1995, pp. 224–27), adapting it to the aims of the present model.

Let the profit function of firm i be

$$\pi_i = px_i - C(k_i) \tag{5}$$

where $p > 0$ is the contractual unit price, fixed by the government in such a way to ensure a positive profit to the innovator; as stressed by Rogerson (1994, pp. 68–69), the task of contracting officers does not consist in obtaining the lowest possible price, as the latter has to be fairly negotiated, accounting for estimated costs. The magnitude $x_i = x + q_i - \sigma \sum_{j \neq i} q_j$ is the amount of units firm i expects to sell, which is determined by a benchmark volume $x \geq 1$ (for example, the minimum number of tanks needed to equip the planned or existing armoured brigades or divisions) and the balance of its quality q_i and the quality levels attained by the $n - 1$ rivals, weighted by parameter $\sigma \in [0, 1/(n-1))$; finally, $C(k_i) = bk_i^2$ is the cost function related to the firm's R&D project.

The quality level q_i characterising the innovation attained by firm i must satisfy a lower bound q_0, and it ultimately depends on the effort of firm i plus the technological spillovers (if any) that it receives from its $n - 1$ rivals:

$$q_i = q_0 + k_i + \sigma \sum_{j \neq i} k_j \tag{6}$$

Intuitively, in (6), there appears the same parameter $\sigma \in [0, 1/(n-1))$, which here measures the intensity of positive technological externality across firms, if any: indeed, if $\sigma = 0$, there is no information leakage from firm i towards its rivals, and conversely. The upper bound of σ at $1/(n-1)$ reflects the reasonable assumption that a firm cannot acquire more technological information from others than it can develop in-house because of its own R&D division. As a result, the individual profit function can be rewritten more explicitly as

$$\pi_i = p\left(x + q_i - \sigma \sum_{j \neq i} q_j\right) - bk_i^2 \tag{7}$$

wherein every single quality level can be specified as in (6).

Now a few words about the stochastic side of the game. As in Shy (1995), the probability of innovating, $\mathfrak{p} \in [0, 1]$, is the same for all firms, each of which acting as if it were inserting a coin of its own choice (the profit maximising level of k_i for every i) into a slot machine hoping for a favourable outcome.[7] The fact that probability $\mathfrak{p}$ is independent of innovation efforts (unlike what happens in the literature on stochastic innovation races summarised above) can be interpreted as a consequence of the secrecy typically characterising R&D activity in the defence industry. An additional reason is that including this feature would involve the appearance of a hazard function, which would preclude the analytical solution of the game, as explained in section 2.

The backbone of the game is the following assumption, defining the common knowledge firms share about what is going to happen once the outcome of the race is in the public domain. In this respect, it is stipulated that each of the n firms knows that, if anyone else innovates, the innovator will pay off to each of the losers a compensation accounting for the industrial offset (or, production sharing),[8] which one may represent in the form of a slice of the winner's profits (for instance because part the production of the x units of the final product will be allocated to each loser). As a consequence, the maximand of firm i is the following expected profit function:

$$E(\pi_i) = \mathfrak{p}(1 - \mathfrak{p})^{n-1}\pi_W + \gamma(1 - \mathfrak{p})[1 - (1 - \mathfrak{p})^n]\pi_L \tag{8}$$

where π_W is the profit of the winner, while π_L is the profit of a loser receiving an offset. In (8), the definition of the expected profit of firm i relies on (i) the probability of winning the race, $\mathfrak{p}(1 - \mathfrak{p})^{n-1}$, i.e. the probability of innovating, $\mathfrak{p}$, multiplied by the probability that none of the rivals do, $(1 - \mathfrak{p})^{n-1}$; and (ii) the probability of being one of the $n - 1$ losers, $(1 - \mathfrak{p})[1 - (1 - \mathfrak{p})^n]$, itself multiplied by the individual share of industrial offsets γ. In order for the winner's profit not to evaporate completely because of industrial offsets, one has to pose $\gamma \in (0, 1/(n-1))$.

Before proceeding with the construction of the generic firm's expected profit function, it is appropriate to discuss the attitude of the government expecting the delivery of the new weapon system. One may define the government's utility function as $U_G = S(q)x_i$, with $S(q)$ measuring the degree of safety and $E = px_i$ being the expenditure respectively associated with the acquisition of the weapon system itself, and I will assume that the available amount of public funds is a budget $B \geq E$. Moreover, borrowing the non-satiation axiom from microeconomics, one may also assume that $\partial S(q)/\partial q > 0$, while $\partial x_i/\partial q > 0$ by construction. In real-world situations, budget constraints will ultimately be binding, but a strict preference for the highest feasible quality level is a *leitmotiv* of military procurement, on which I will come back in section 5, dealing with the properties of an RJV in this setting.

Now it is time to define the problem of the individual firm. The initial step consists in imposing explicitly $\pi_W = \pi_i$ and $\pi_L = \pi_j$, with $j \neq i$ representing the identity of a generic loser being assigned an offset, in such a way that (8) becomes

$$E(\pi_i) = \mathfrak{p}(1-\mathfrak{p})^{n-1}\left[p\left(x + q_i - \sigma\sum_{j\neq i} q_j\right) - bk_i^2\right] \\ + \gamma(1-\mathfrak{p})[1-(1-\mathfrak{p})^n]\left[p\left(x + q_j + \sigma\sum_{\ell\neq j} q_\ell\right) - bk_j^2\right] \tag{9}$$

in which q_i appears in $\sum_{\ell\neq j} q_\ell$.

Before proceeding with the equilibrium analysis, it is appropriate to dwell upon a few preliminary observations concerning the key properties that must hold in order for the industry-wide R&D effort to exhibit a concave and single-peaked pattern w.r.t. industry structures. To this aim, we define the aggregate R&D effort as $K^*(n) = nk^*(n)$, $k^*(n)$ being the symmetric individual effort at equilibrium. The essential properties of the relationship between total R&D and industry structure can be grasped by looking at the following partial derivatives:

$$\frac{\partial K^*(n)}{\partial n} = k^*(n) + n \cdot \frac{\partial k^*(n)}{\partial n} \tag{10}$$

$$\frac{\partial^2 K^*(n)}{\partial n^2} = 2 \cdot \frac{\partial k^*(n)}{\partial n} + n \cdot \frac{\partial^2 k^*(n)}{\partial n^2} \tag{11}$$

Now, from (10) we immediately learn that if $\partial k^*(n)/\partial n > 0$, then necessarily also $\partial K^*(n)/\partial n > 0$. In view of (6), this implies the following Arrovian conclusion:

Remark 1 *If $\partial k^*(n)/\partial n > 0$, then the equilibrium levels of aggregate R&D and innovation quality monotonically increase in the number of firms.*

If, conversely, $\partial k^*(n)/\partial n < 0$, then (leaving aside the integer problem) there may exist a number $n_K \geq 2$ in correspondence of which $\partial K^*(n)/\partial n = 0$. Indeed, we may implicitly solve (10) to obtain

$$n_K = -\frac{k*(n)}{k'(n)} \tag{12}$$

with $k'(n) = \partial K^*(n)/\partial n$, and plug it into (11), which simplifies as follows:

$$\frac{\partial^2 K^*(n)}{\partial n^2} = \frac{2[k^*(n)]^2 - k*(n) \times k''(n)}{k'(n)} \tag{13}$$

with $k''(n) = \partial^2 k^*(n)/\partial n^2$. Since in this case $k'(n) < 0$, n_K determines a maximum of $K^*(n)$ provided that $\partial^2 K^*(n)/\partial n^2 \leq 0$ when evaluated at $n = n_K$. In turn, this is true if and only if $k''(n) \leq 2[k'(n)]^2/k*(n)$. This delivers the following

Lemma 2 *The necessary and sufficient conditions for aggregate R&D to be concave and single-peaked in the number of firms are as follows: (i) $\partial k^*(n)/\partial n < 0$; (ii) $-k*(n)/k'(n) \geq 2$; and $\partial^2 k^*(n)/\partial n^2 \leq 2[\partial k^*(n)/\partial n]^2/k^*(n)$.*

Accordingly, a crucial step in the analysis of the game consists in assessing the sign (and possibly the size) of the first and second partial derivatives of the individual equilibrium investment with respect to the numerosity of firms.

Equilibrium Analysis

To begin with, one may focus upon the scenario in which offsets are absent. If so, $\gamma = 0$ and firm i's first order condition (FOC) w.r.t. k_i is

$$\left.\frac{\partial E(\pi_i)}{\partial k_i}\right|_{\gamma=0} = \mathfrak{p}(1-\mathfrak{p})^{n-1}\left[p\left(1-\sigma^2(n-1)\right)-2bk_i\right] = 0 \tag{14}$$

whereby the equilibrium individual investment is $k^*(n)|_{\gamma=0} = p[1-\sigma^2(n-1)]/(2b)$ and the collective R&D effort is $K^*(n)|_{\gamma=0} = nk^*(n)|_{\gamma=0} = np[1-\sigma^2(n-1)]/(2b)$, which is strictly concave in n and reaches its maximum at $n_K|_{\gamma=0} = (1+\sigma^2)/(2\sigma^2)$. Since $\sigma \in [0, 1/(n-1))$, and excluding monopoly because it is incompatible with the existence of spillovers, this means that $K^*(n)|_{\gamma=0}$ may only reach its peak at $n_K|_{\gamma=0} = 2$ (with $\sigma = \sqrt{3}/3 \simeq 0.577$) and $n_K|_{\gamma=0} = 3$ (with $\sigma = \sqrt{5}/5 \simeq 0.447$), as the equation $n - n_K|_{\gamma=0} = 0$ yields $\sigma = 1/\sqrt{2n-1} > 1/(n-1)$ for all $n \geq 4$.

An analogous approach can be taken to deal with the resulting equilibrium quality,

$$q^*|_{\gamma=0} = q_0 + [1+\sigma(n-1)]k^*(n)|_{\gamma=0} = q_0 + \frac{p[1+\sigma(n-1)][1-\sigma^2(n-1)]}{2b} \tag{15}$$

which is maximised at $n_q|_{\gamma=0} = 1 + (1-\sigma)/(2\sigma^2) > n_K|_{\gamma=0}$ for all admissible spillover levels. Taking into account the integer problem, this means that the admissible industry structure delivering the highest quality level will be reached in correspondence with less intense spillovers, and indeed the peaks are attained at $n_q|_{\gamma=0} = 2$ (with $\sigma = 1/2$) and $n_K|_{\gamma=0} = 3$ (with $\sigma = \left(\sqrt{17}-1\right)/8 \simeq 0.390$) and the admissible additional case $n_K|_{\gamma=0} = 4$ (with $\sigma = 1/3$).

This discussion deserves to be compacted into the following

Remark 3 *In the absence of offsets, quality can be maximised at* $n_K|_{\gamma=0} = 2, 3, 4,$ *in correspondence to spillover levels at which the R&D effort at the industry level is still increasing in the number of firms.*

As we are about to see, this result stretches beyond the special case where $\gamma = 0$. If offsets are present, the characterisation of firms' equilibrium behaviour requires solving the following set of FOCs:

$$\begin{aligned}\frac{\partial E(\pi_i)}{\partial k_i} = (1-\mathfrak{p})\{p\left[\left(1-\sigma^2(n-1)\right)(1-\gamma(n-1))\mathfrak{p}(1-\mathfrak{p})^{n-2} - (n-2)\gamma\sigma(1-(1-\mathfrak{p})^n)\right] \\ -2bk_i[1-\gamma(n-1)]\mathfrak{p}(1-\mathfrak{p})^{n-2}\} = 0\end{aligned} \tag{16}$$

Then, disregarding the second-order condition, which is clearly satisfied given that the r.h.s. of (16) is linear and monotonically decreasing in k_i, and imposing symmetry across the population of firms, we obtain

$$k* = \frac{p\left[(1-\sigma^2(n-1))\Gamma(1-)^{n-2} - (n-2)\gamma\sigma^2(1-(1-)^n)\right]}{2b\Gamma(1-)^{n-2}} \tag{17}$$

where $\Gamma \equiv 1-\gamma(n-1)$. Accordingly, the equilibrium quality level characterising the innovation patented by the winner is

$$q^*(n) = q_0 + [1+\sigma(n-1)]k^*(n) \tag{18}$$

Intuitively, the definition of the individual profit function (7) implies the fact that the FOC and the equilibrium expressions of individual R&D and innovation quality are independent of the contractual quantity x.

Having characterised the equilibrium level of the individual investment effort, we may pay attention to the properties highlighted in Lemma 1. To begin with, a monopolist does invest a positive amount, since

$$\lim_{n\to 1}\left(\lim_{\sigma\to 0} k^*(n)\right) = p \tag{19}$$

and indeed, the sign of $\partial^2 k^*(n)/\partial n^2$ can be evaluated in general as

$$\frac{\partial^2 \mathrm{k}*(\mathrm{n})}{\partial \mathrm{n}^2} = -\frac{p\gamma\sigma^2[(1-)^n(+\gamma) - \Gamma\ln(1-)(2(1-\gamma)-(n-2)\Gamma\ln(1-))]}{2b(1-)^{n-2}\Gamma^3} \tag{20}$$

is clearly negative for all admissible $\{b, n, p, \gamma, \sigma, \mathfrak{p}\}$. The sign of

$$\frac{\partial \mathrm{k}*(\mathrm{n})}{\partial \mathrm{n}} = -\frac{p\gamma\sigma^2\left[2\gamma(1-\gamma\Phi)^n + \gamma(1-)^2(1-\gamma+(n-2)\Gamma\ln(1-))\right]}{2b(1-)^n\Gamma^2} \tag{21}$$

where $\Phi \equiv \gamma[1+\mathfrak{p}(\mathfrak{n}(\mathfrak{n}-2)+\mathfrak{p}-1)] - 1 - \mathfrak{p}[2(\mathfrak{n}-2)+\mathfrak{p}]$, can be established by noting that

$$2\gamma(1-\gamma\Phi) + \gamma(1-\mathfrak{p})^2(1-\gamma+(n-2)\Gamma\ln(1-\mathfrak{p})) > 0 \tag{22}$$

for all $\gamma \in (0, \min\{1/(n-1), \widehat{\gamma}\})$, where

$$\widehat{\gamma} = \frac{(1-\mathfrak{p})^{n-1}[1+\mathfrak{p}(2(\mathfrak{n}-2)+\mathfrak{p})] - (1-\mathfrak{p})[1-(n-2)\ln(1-\mathfrak{p})] - \sqrt{F}}{2\left[(1-\mathfrak{p})^{n-1}(1+\mathfrak{p}(\mathfrak{n}(\mathfrak{n}-2)+\mathfrak{p}-1)) - (1-\mathfrak{p})(1-(n-1)(n-2)\ln(1-\mathfrak{p}))\right]} \tag{23}$$

and

$$\begin{aligned} F \equiv\ & (1-\mathfrak{p})^2 - 2(1-\mathfrak{p})^n[1+\mathfrak{p}(2(\mathfrak{n}-3)+\mathfrak{p})] + (1-\mathfrak{p})^{2n}[1+\mathfrak{p}(2(\mathfrak{n}-5)+\mathfrak{p})] \\ & -(n-2)(1-\mathfrak{p})^2\ln(1-\mathfrak{p})[2(1-(1-\mathfrak{p})^n)-(n-2)\ln(1-\mathfrak{p})] \end{aligned} \tag{24}$$

with $1/(n-1) > \widehat{\gamma}$ in $n=1$ and $\widehat{\gamma} \geq 1/(n-1)$ for all $n \geq 2$ for all $\mathfrak{p} \in (0, 1]$. The two curves are portrayed in Figure 1, where the region below the lower envelope of $1/(n-1)$ and $\widehat{\gamma}$ identifies infinitely many points in the space (n, γ) in correspondence of which $\partial k^*(n)/\partial n < 0$.

Now suppose indeed $K^*(n)$ takes its maximum at $n=2$. This holds iff $-k*(n)/k'(n) = 2$ and $\partial^2 K^*(n)/\partial n \leq 0$. The first requirement entails

$$\frac{1-\gamma-\sigma^2[3+\gamma(1-2\mathfrak{p})]}{\sigma^2[1+\gamma(1-\mathfrak{p})]} = 0 \tag{25}$$

with $3+\gamma(1-2\mathfrak{p}) > 0$ for all admissible values of γ and $\mathfrak{p}$. Hence, the above equation is solved by $\sigma = \sqrt{1-\gamma}/\sqrt{3+\gamma(1-2\mathfrak{p})} < 1/2$. The second, instead, amounts to evaluating the sign of

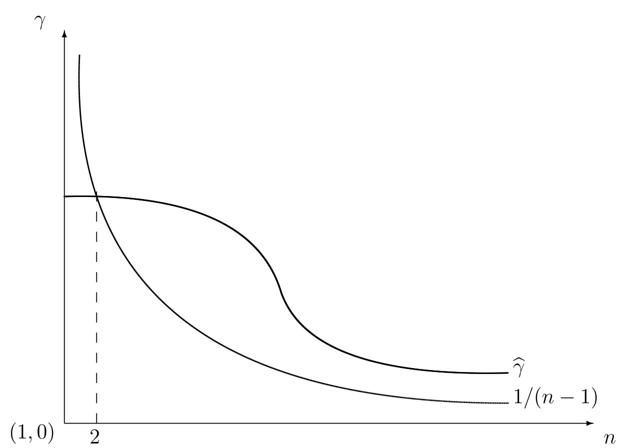

Figure 1. The sign of $\partial k^*/\partial n$ in the space (n, γ).

$$\left.\frac{\partial^2 K^*(n)}{\partial n^2}\right|_{n=2} = \frac{\sigma^2 \mathfrak{p}[\mathfrak{p}(\gamma(\mathfrak{p}-\gamma(3-\mathfrak{p}))-1)+2\gamma(1-\gamma)\ln(1-\mathfrak{p})]}{b\mathfrak{p}(1-\gamma)^2} \tag{26}$$

which is indeed negative everywhere. The corresponding individual R&D effort is $k^* = \mathfrak{p}(1-\sigma^2)/(2\mathfrak{b})$.

The foregoing discussion leads to the following:

Remark 4 *If the industry is at least triopolistic, then any $\gamma \in (0, 1/(n-1))$ combined with a spillover level $\sigma = \sqrt{1-\gamma}/\sqrt{3+\gamma(1-2\mathfrak{p})}$ ensures that the aggregate R&D effort is concave and single-peaked at $n = 2$.*

This exercise can be repeated by taking any finite $n \geq 3$ as a candidate industry structure maximising the aggregate R&D effort. It remains true, however, that (21) is not an algebraic equation and cannot be solved analytically with respect to n (even leaving aside the integer constraint). Yet, some additional hints are obtained by looking at the admittedly special case in which the spillover level, the individual probability of success and the individual share of offsets have the same size. To do so, we may pose $\gamma, \mathfrak{p} = \sigma,$ whereby the expression of the equilibrium effort can be rewritten as follows:

$$k^*(n)|_{\gamma,\mathfrak{p}=\sigma} = \frac{p\omega^n[(n-2)\omega(1-\omega^n)\sigma^2 - (1-\sigma^2(n-1))\omega^{n-1}(1-\sigma(n-1))]}{2b[1-\sigma(n-1)]} \tag{27}$$

where $\omega \equiv 1-\sigma,$ and numerical methods show that $k^*(n)|_{\gamma,\mathfrak{p}=\sigma}$ is decreasing and concave in n for all $n \geq 1$ and all $\sigma \in [0, 1/(n-1))$ for which $k^*(n)|_{\gamma,\phi=\sigma} > 0$. Of course, this is a very special case, but imposing $\gamma, \mathfrak{p} = \sigma$ amounts to requiring that the three parameters must belong to $[0, 1/(n-1)),$ which is an admissible restriction identifying a plausible parameter region wherein $\partial k^*(n)/\partial n$ and $\partial^2 k^*(n)/\partial n^2$ are both strictly negative, meeting thus the requirements highlighted in Lemma 1.

Sticking to $\gamma, \mathfrak{p} = \sigma,$ numerical simulations illustrate the presence of an inverted-U shaped aggregate R&D curve. For instance, consider an industry made up by four firms, so that $\gamma, \mathfrak{p}, \sigma \in [0, 1/3),$ and impose $\sigma = 1/4$. At this spillover level (which determines also γ and $\mathfrak{p}$), the maximum of $K^*(n)$ is attained at $n = 3.0096$. Taking into account the integer number, this means that the industry structure maximising the total investment in R&D is a triopoly. Or, equivalently, a regulator interested in maximising the industry-wide R&D effort should authorise a merger involving two firms out of four if this chance materialises.

Countless analogous cases arise if one drops the simplifying assumption used thus far. Before delving into other interesting numerical exercises, one may examine the effect of an increase in the level of offsets on the optimal individual investment, observing that

$$\frac{\partial k^*(n)}{\partial \gamma} = -\frac{p\sigma^2(n-2)[1-(1-\mathfrak{p})^n]}{2b[1-\gamma(n-1)]^2 \mathfrak{p}(1-\mathfrak{p})^{n-2}} < 0 \tag{28}$$

and therefore also $\partial K^*(n)/\partial \gamma = n \cdot \partial k^*(n)/\partial \gamma < 0$ for all $n \geq 2$. For obvious reasons, the same qualitative property applies also to $q^*(n)$:

$$\frac{\partial q^*(n)}{\partial \gamma} = -\frac{p\sigma^2(n-2)[1-\sigma(n-1)][1-(1-\mathfrak{p})^n]}{2b[1-\gamma(n-1)]^2 \mathfrak{p}(1-\mathfrak{p})^{n-2}} < 0$$

This proves the following claim, which grasps the intuitive negative effect on investment incentives induced by the presence of offsets and any increase thereof:

Remark 5 *Any increase in the offset level decreases k^*, K^* and $q^*,$ with $\partial K^*(n)/\partial \gamma < \partial q^*(n)/\partial \gamma < \partial k^*(n)/\partial \gamma < 0$*

Consider the alternative triple $\{\gamma = 8/50, \sigma, \mathfrak{p} = 1/5\}$. In this case, the maximum number of firms compatible with the binding assumption $\sigma \in [0, 1/(n-1))$ is $n = 6$, and the peak of $K^*(n)$ is reached at $n = 4.0733$, which implies $n_K = 4$. An interesting property can be singled out, concerning the impact of variations of probability $\mathfrak{p}$ on the industry structure maximising $K^*(n)$, for a given pair $\{\gamma,\sigma\}$. For $\gamma=1/10$ and $\beta=1/8$, so that at most $n=9$, numerical calculations show the following:

- if $\mathfrak{p}=1/4$, $n_K=6$ and $K^*\simeq 4.755$;
- if $\mathfrak{p}=1/2$, $n_K=5$ and $K^*\simeq 4.082$;
- if $\mathfrak{p}=3/4$, $n_K=4$ and $K^*\simeq 3.433$;
- if $\mathfrak{p}=24/25$, $n_K=3$ and $K^*\simeq 2.759$.

This pattern, which can be replicated for other values of $\{\gamma,\sigma\}$, illustrates two features of the equilibrium, namely, that $\partial n_K/\partial\mathfrak{p}<0$ and $\partial K^*/\partial\mathfrak{p}<0$ as well. How come? Observing

$$\frac{\partial k^*(n)}{\partial\mathfrak{p}}=-\frac{(n-2)p\gamma\sigma^2[\mathfrak{p}((1-\mathfrak{p})^{\mathfrak{n}}+\mathfrak{n}-1)+(1-\mathfrak{p})^{\mathfrak{n}}-1]}{2b\mathfrak{p}^2(1-\mathfrak{p})^{n-1}\Gamma}\leq 0 \tag{29}$$

for all $n\geq 1$, being zero at $n=2$. This implies that every firm shrinks its own investment as uncertainty fades away because increasing the probability of obtaining the patent amounts to diminishing the need to outperform rivals. This, in turn, shifts the curve of aggregate investment down and moves and its peak necessarily to the left, decreasing n_K to the left as $\mathfrak{p}$ goes up. Although this result cannot be claimed formally, it deserves to be stressed:

Remark 6 *Any increase in probability $\mathfrak{p}$ reduces $k^*(n)$, with the exception of the duopoly. Consequently, any decrease in uncertainty is a driver of industry concentration because $\partial n_K/\partial\mathfrak{p}<0$ and $\partial K^*(n)/\partial\mathfrak{p}<0$.*

This fact, however, also has the unpleasant consequence of diminishing the resulting quality of the innovation $q^*(n)=q_0+[1+\sigma(n-1)]k^*(n)$ for any given pair $\{n,\sigma\}$:

Corollary 7 *By reducing $k^*(n)$, any increase in probability $\mathfrak{p}$ reduces $k^*(n)$ for any level of technological spillovers and any industry structure.*

Using the same numerical values as above, and defining as n_q as the number of firms maximising the quality level of the innovation at equilibrium, we find the following (in this case, I have intentionally listed the exact numerical values of n_q):

- if $\mathfrak{p}=1/4$, $n_q=5.162$ and $q^*\simeq q_0+1.323$;
- if $\mathfrak{p}=1/2$, $n_q=4.277$ and $q^*\simeq q_0+1.269$;
- if $\mathfrak{p}=3/4$, $n_q=3.5$ and $q^*\simeq 1.218$;
- if $\mathfrak{p}=24/25$, $n_q=2.764$ and $q^*\simeq q_0+1.165$.

This numerical exercise shows that as the probability of success increases, the peak of the quality level becomes progressively lower and takes place in correspondence with a more concentrated industry. It is worth observing that the difference between n_K and n_q shrinks as $\mathfrak{p}$ approaches one – and the same must also apply if σ tends to one, as this implies that $q^*(n)=nk^*(n)$. This poses a further question, namely, whether a government may stimulate any form of cost- or information-sharing in the organization of industry-wide R&D activities, so as to diminish or eliminate the misalignment between the degrees of concentration maximising, respectively, aggregate R&D and product quality. This aspect, which in Amir's (2000) jargon implies the presence of an industry-wide RJV wherein any technological advancements freely circulate as a public good, is the subject matter of the next section.

The Effects of an Industry-Wide RJV

The possibility of organizing R&D activities in a partially or totally cooperative way has been a long-standing issue in the related literature belonging to the theory of industrial organization, at least since d'Aspremont and Jacquemin (1988). The general idea is that since R&D competition may almost systematically yield inefficient outcomes characterised by excess investment as compared to the social optimum, resorting to R&D cartels or RJVs may mitigate or even eliminate this inefficiency by inducing firms to properly internalise technological externalities. The essence of this debate has been accurately reconstructed by Amir (2000).

Here, in order to conclude the analysis of the game, we may take a closer look at what happens if firms, keeping the individual expected profit function defined as in (9), share their full technological advancements through an RJV in which $\sigma = 1$, this being the case labelled as a proper RJV in Amir's (2000), with n labs operating noncooperatively but fully sharing technological advancements. For this purpose, I would like to focus on a relevant and intuitive property of the equilibrium magnitudes, which emerges when K^* is monotonically increasing in n. The functional form of q^* as defined in (18) suggests the following: whenever $\sigma > 0$, in terms of its relationship with industry structure, the equilibrium quality level q^* has *almost* the same shape as $K^* = nk^*$.[9] Essentially, to see this, we may write

$$\frac{\partial q^*(n)}{\partial n} = [1 + \sigma(n-1)] \cdot \frac{\partial k^*(n)}{\partial n} + \sigma k^*(n) = 0 \tag{30}$$

and

$$\frac{\partial K^*(n)}{\partial n} = \frac{\partial (nk^*(n))}{\partial n} = k^*(n) + n \cdot \frac{\partial k^*(n)}{\partial n} = 0 \tag{31}$$

to find out that the first is satisfied at the level of n such that

$$k^*(n) = -\frac{[1 + \sigma(n-1)]}{\sigma} \cdot \frac{\partial k^*(n)}{\partial n} > 0, \tag{32}$$

while the second requires n to satisfy

$$k^*(n) = -n \cdot \frac{\partial k^*(n)}{\partial n} > 0 \tag{33}$$

since $\partial k^*(n)/\partial n < 0$ from Lemma 1. All else equal, (32) and (33) jointly entail that the curves $q^*(n)$ and $K^*(n)$ will have exactly the same shape (with $q^*(n)$ shifted above $K^*(n)$ by the exogenous amount q_0) and will consequently take their respective maxima in correspondence to the same n if and only if

$$\frac{[1 + \sigma(n-1)]}{\sigma} = n \Leftrightarrow 1 + \sigma(n-1) = \sigma n \Leftrightarrow \sigma = 1 \tag{34}$$

In practice, the peaks coincide only when spillovers are full, i.e. under an RJV involving the whole industry. To summarise:

Proposition 8 *The degree of concentration maximising the quality of the innovation exceeds the degree of concentration driving the industry to maximise R&D investments, except in the special but relevant case in which technical knowledge is freely exchanged within an industry-wide RJV.*

A more explicit formulation of this result consists in saying that a generic industry structure may create a consortium with the explicit twofold aim of fully internalising technological spillover and eliminating the discrepancy between the behaviour of innovation quality and that of aggregate investment. This may also respond to the demand for higher quality expressed by the government in view of the non-satiation axiom mentioned in section 3, while at the same time eliminating the excess investment necessarily arising under independent ventures.

Concluding Remarks

The take-home message emerging from the foregoing discussion (and largely independent of the modelling strategies) is twofold: (i) the innovation level (i.e. its quality) and the aggregate R&D investment are both concave and single-peaked in the number of firms operating in the industry; and (ii) in general, the peak of innovation quality takes place at a higher degree of concentration than that of aggregate R&D, irrespective of whether industrial offsets are planned or not. This means that reducing the number of firms via horizontal mergers (or acquisitions) from an initially fragmented state may trigger an increase in aggregate research activities without maximising innovation quality, which, being the real target, will require a more pronounced concentration process. However, if the initial industry structure is already quite concentrated, any further mergers might compromise both innovation quality and aggregate R&D.

Notes

1. For the details of the lively and intense discussion on this issue, see Kamien and Schwartz (1982), Tirole (1988); Reinganum (1989); Martin (1993, 2001) and Scotchmer (2004), *inter alia*.
2. This crucial aspect of the matter is well known to antitrust authorities and has been debated in detail in the extant literature (see Motta 2004; Gilbert 2006 and Shapiro 2012), in particular after the Dow-DuPont case (see Federico, Langus, and Valletti 2017, 2018; Denicolò and Polo 2018; Jullien and Lefouili 2018; Haucap, Rasch, and Stiebale 2019 and Delbono and Lambertini 2022).
3. For instance, a regulator may set market price having in mind the containment of emissions and climate change, and use this instrument to spur firms to invest in green technologies. See, among others, the deterministic differential game by Feichtinger et al. (2016), in which aggregate R&D for emission abatement exhibits a concave and single-peaked pattern.
4. A related matter, systematically addressed in industrial economics when assessing the associated welfare level, is the need of avoiding or reducing wasteful duplication efforts, by either R&D cartelss or RJVs. This possibility has appeared first in the NCRA (National Cooperative Research Act) in the US back in 1984, and then also in other legal systems, and has of course received due attention in the literature (see d'Aspremont and Jacquemin 1988; Kamien, Muller, and Zang 1992; Suzumura 1992 and Kamien and Zang 2000, among others).
5. The hazard function is increasing and strictly concave in x_i, with $h(0) = 0 = \lim_{x_i \to \infty} h'(x_i)$ and $\lim_{x_i \to 0} h'(x_i) = \infty$ These conditions (usually labelled as the *Inada conditions*) ensure the concavity of maximands (1) and (2) as well as the existence of an inner solution.
6. An exhaustive discussion of research tournaments can be found in Rogerson (1994). See also Baye and Hoppe (2003) for a very interesting synthesis of the parallel literatures on innovation races and tournaments.
7. This is a crucial aspect differentiating the present formulation of the one-shot innovation race from Shy's (1995), in which the R&D cost is exogenously fixed.
8. In terms of the analytical properties of the model, what I consistently refer to as industrial offsets is equivalent to profit or production sharing. In the European Union, this might reflect a compensation mechanism accompanying a multilateral adoption of a weapon system by several governments across EU.
9. More explicitly, here I mean the general qualitative properties determined by n, that is, by industry structure (while being unaffected by p, x and q_0).

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Ethical approval

This article does not include any studies with human participants or animals performed by any of the authors.

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