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NATO's Approach to Multi-Domain Operations: From the Perspective of the Economics of Alliances

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ABSTRACT
This study presents a model that incorporates the characteristics of multi-domain operations (MDO) and analyzes the impact of the North Atlantic Treaty Organization's (NATO) approach to MDO on the behavior of its member countries from the perspective of the economics of alliances. NATO is wary of authoritarian states and prepared for the great power competition with them, as shown in its current strategic concept. To prepare for a possible future war against peer competitors, NATO is strengthening its capabilities to conduct MDO, in which cross-domain synergy is a key factor. The model presented in this study relates the cross-domain synergy in MDO and the complementarity between the defense activities in multiple domains using organizational and public economics methods. The implication of this model is that if the defense activities in different domains are complementary, one member country's increasing defense activities in one domain can increase another's defense activity in another domain that is complementary to that domain. It is particularly crucial for NATO's future military buildup: NATO's approach to MDO will make defense burden-sharing fairer because one member's military buildup will motivate other members to build up theirs.

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multi-domain operations; cross-domain synergy; NATO burden sharing; complementarity; weaker-link; supermodular games

Introduction

In June 2022, at the Madrid Summit, the North Atlantic Treaty Organization (NATO) adopted its current strategic concept. It stipulates NATO's three essential core tasks – collective defense, crisis prevention and management, and cooperative security – and states that NATO's key purpose and greatest responsibility is to ensure collective defense, against all threats, from all directions (NATO 2022c). Moreover, it is noteworthy that the latest strategic concept mentions, for the first time, the need to prepare for high-intensity, multi-domain warfighting against nuclear-armed peer-competitors (NATO 2022c). Multi-domain operations (MDO) are operations conducted in multiple domains.[1] In MDO, cyberspace and space are also operational domains, in addition to the traditional land, maritime, and air domains. The urgency for NATO to conduct MDO is based on the rapidly changing security environment, which requires the alliance to re-think warfighting and warfare development in the short, medium, and long term to maintain NATO's military credibility, especially in the context of Russia's war of aggression against Ukraine (NATO 2022b). Indeed, the capabilities to conduct MDO are essential in modern warfare. In the war in Ukraine, MDO have constituted part of the war efforts and have had a significant impact

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on its trends (e.g. kinetic military operations combined with non-kinetic cyber attacks; missile and artillery strikes based on information collected by space satellites). In addition to stating that the Russian Federation is the most significant and direct threat to the allies' security, peace, and stability in the Euro-Atlantic area, the 2022 Strategic Concept shows caution toward the People's Republic of China's coercive tactics and efforts to divide the alliance (NATO 2022c). Amid this growing concern about China's coercive policies, the leaders of the four Asia-Pacific partners of NATO (AP4: Australia, Japan, New Zealand, and the Republic of Korea) participated in the Madrid Summit (Japan Ministry of Foreign Affairs 2022). These developments imply that the most serious challenge for NATO has been the great power competition with China and Russia since the 2010s, and that in a possible future war with such peer competitors, MDO will have a significant impact on the outcome of the war. Thus, the capabilities to conduct MDO are critically important in the current security environment.

This study presents a model that incorporates the characteristics of MDO and analyzes the impact of NATO's approach to MDO on the behavior of its member countries from the perspective of the economics of alliances. The novelty of the model is that it incorporates not only the relations between each ally's defense activities within a domain (i.e. the within-domain relations) but also the relations between the defense activities in multiple domains (i.e. the between-domain relations). Synergy occurs when two or more actions combine to produce an effect greater than the sum of their individual effects (U.S. Joint Chief of Staff 2016). In MDO, it is crucial to create synergy between forces in multiple domains (i.e. cross-domain synergy).[2] This is because cross-domain synergy makes it possible to surpass an adversary in terms of integrated forces in multiple domains even when one's forces in a certain domain are inferior to those of the adversary. From the perspective of organizational economics, complementarity is a near synonym for synergy. This is because, if two inputs are complementary, the increase in outputs from raising both inputs exceeds the sum from increasing either separately (Brynjolfsson and Milgrom 2013). Hence, the forces in different domains that create cross-domain synergy are interpreted as complementary to each other. To represent the complementary relations between forces of multiple domains within the alliance, the model uses several types of public goods aggregation technology, especially the weaker-link type. This is because the weaker-link type is a supermodular function and hence represents the complementary relations between inputs.

The remainder of this paper is organized as follows. Section 2 explains NATO's approach to MDO. Section 3 provides a brief review of previous studies on the economics of alliances, with a particular focus on the burden sharing in NATO. Section 4 describes the relation between MDO, cross-domain synergy, and weaker-link aggregation technology. Section 5 introduces a supermodular game in which aggregation technology is a key factor. Section 6 presents the theoretical model of this study. Section 7 provides examples based on the theoretical model of how alliance members' enhanced capacities in MDO interact and how they affect defense burden-sharing within the alliance. Section 8 concludes the paper.

NATO's Approach to MDO

The concept of MDO is an evolved form of that of joint operations, in which land, maritime, and air military services are closely coordinated while cooperating under a centralized chain of command. This is because joint operations emphasize the integration of forces in different domains, although this is limited to the traditional domains. One of the origins of the concept of joint operations is the AirLand Battle Doctrine, which was established by the U.S. Army in the 1980s to counter Warsaw Treaty Organization forces (Johnson 2018; Jones and de Leon 2020; Leon 2021). Moreover, NATO's 1991 Strategic Concept pointed out the importance of close cooperation by ground, maritime, and air forces (NATO 1991). The 1999 Strategic Concept stated that the allies' forces must be interoperable and have appropriate doctrines and technologies to conduct a wide range of complex joint and combined operations (NATO 1999). In 2003, Allied Command Transformation, which has three subordinate entities for joint operations (the Joint Warfare Center, Joint Force Training Center, and Joint Analysis and Lessons Learned Center), was created to be responsible for the continuing transformation of military capabilities and for the promotion of the interoperability of the allies' forces (NATO 2002).

Since the 2010s, NATO has expanded its operational domains and strengthened its capabilities to conduct MDO. The 2010 Strategic Concept indicated the importance of cyber defense in addition to defense in the traditional domains (NATO 2010). NATO recognized cyberspace as an operational domain in 2016 (NATO 2022a). It subsequently recognized space as an operational domain in 2019 (NATO 2021b). The 2022 Strategic Concept states that operations in cyberspace or space against NATO allies, and hostile hybrid operations, can reach the level of an armed attack and prompt the allies to take a response based on Article 5 of the North Atlantic Treaty. It also shows caution toward hybrid tactics by authoritarian actors through diverse means, including malicious activities in cyberspace and space, as a threat to the allies (NATO 2022c). With the expansion of operational domains, NATO has focused on building forces to conduct MDO through planning, training, and exercises (NATO 2006, 2021a). The NATO Warfighting Capstone Concept also notes the importance of integrated multi-domain defense because the threats that the alliance faces are no longer in any one domain and a joint and flexible approach to a fluid environment is required to protect the alliance's integrity against all threats, regardless of their origin or nature (Tammen 2021).

As detailed in the subsequent sections, to incorporate these MDO characteristics into a model of the economics of alliances, this study focuses specifically on the weaker-link type among the types of public goods aggregation technology. This is because the weaker-link type is best suited to the characteristics of MDO, in which the combination of forces from multiple domains allows a country to be superior in overall strength even if it is inferior to the enemy in one domain.

Burden Sharing in NATO and the Economics of Alliances

Olson and Zeckhauser (1966) developed an economic theory of alliances and presented the pure public good model. The pure public good model makes a theoretical assumption that the defense goods produced by each member country brings non-rival (i.e. consumption opportunity related to a good's unit does not reduce with the number of allies) and non-excludable (i.e. available to all allies once provided) benefits to all members. Furthermore, it assumes substitutional relations between member countries' defense activities, in that a member responds negatively to spillovers from another member (Gonzalez and Mehay 1991). These assumptions were based on the fact that NATO's deterrence missions depended mainly on the U.S. nuclear forces during the period when NATO was under the massive retaliation doctrine (Hartley and Sandler 1999; Sandler and Hartley 2001). Applying the pure public good model to NATO, Olson and Zeckhauser (1966) revealed that the allies' burden sharing and their country size were positively correlated in the early 1960s. Thus, the so-called exploitation hypothesis, in which larger countries bear more burden than smaller countries, held for NATO during its massive retaliation doctrine period.

However, this correlation became insignificant in the latter half of the 1960s, when NATO adopted the flexible response doctrine. Furthermore, free-riding was no longer a major problem in NATO in the 1990s (Hartley and Sandler 1999). Numerous subsequent studies found no evidence of a disproportional defense burden in NATO (Khanna and Sandler 1996; Oneal and Elrod 1989; Russett 1970; Sandler and Forbes 1980; Sandler and Murdoch 2000). In this context, the joint product model, which was the second major step in the economics of alliances and an improvement of the traditional pure public good model, was presented (Khanna and Sandler 1996; Sandler and Cauley 1975; Sandler, Cauley, and Forbes 1980). In the joint product model, a member country's defense activity produces both private and public outputs; these characteristics applied to the flexible response doctrine, which emphasized the appropriate combination of conventional and nuclear forces (Hartley and Sandler 1999; Murdoch and Sandler 1982; Sandler 1993). This is because while conventional forces bring private and country-specific benefits, nuclear forces provide public and alliance-wide benefits. These studies show that if the complementarity between private and public outputs is sufficiently strong, an ally increases its defense activities in response to the increase in another ally's defense activities (Cornes and Sandler 1984, 1996; Murdoch and Sandler 1982). Based on this implication, some empirical studies argued that NATO allies were increasing defense activities in response to the increase in spillover from other allies (Hansen, Murdoch, and Sandler 1990; Murdoch, Sandler, and Hansen 1991; Sandler and Murdoch 1990). The same trend was reported in UN and non-UN peacekeeping missions conducted by various countries, including NATO allies (Gaibulloev, Sandler, and Shimizu 2009). Previous studies also reported that many NATO and European Union (EU) members increased their military expenditure in response to the increase in other members' military expenditure (Nikolaidou 2008; Odehnal et al. 2021), and that the change in the U.S. military expenditure as a share of gross domestic product (GDP) had a positive impact on the U.S. allies (Smith 1989). Several studies argued that the exploitation hypothesis was true in the later stages of the Cold War era because the U.S. kept a position of hegemony and bore a disproportional burden, while the smaller allies were free-riding (Oneal and Elrod 1989; Plümper and Neumayer 2015). Regarding defense burden-sharing within NATO during the late Cold War, the influence of the Strategic Defense Initiative (SDI) promoted by the Reagan administration, the so-called Star Wars project, is noteworthy. In SDI, missile defense by the largest contributor (i.e. the U.S.) was critically important to deter enemy attacks against the entire alliance. Therefore, theoretically, the best-shot type would be applied for SDI (de la Rochère, Ghislain, and Rocaboy 2014; Sandler 2006; Sandler and Hartley 2001). Such SDI characteristics might be factors for supporting the exploitation hypothesis.

Since the 1990s, NATO has conducted various operations and missions worldwide. It implies that NATO allies' defense activities have become more public since then because those various out-of-area operations and missions have brought non-rival and non-excludable benefits to all allies (Hartley and Sandler 1999; Sandler and Hartley 1999; Shimizu and Sandler 2010). Regarding the increasing public nature of NATO missions and operations, some studies showed that NATO allies had been free-riding since 2014 (George and Sandler 2018; Kim and Sandler 2020; Zannella 2020). The same trend was reported for EU member countries (George and Sandler 2021). Additionally, it was reported that burden sharing for peacekeeping missions and operations conducted by various countries, including NATO allies, was shouldered by the large countries for the small countries (Khanna, Sandler, and Shimizu 1998; Shimizu and Sandler 2002, 2010). Several studies analyzed burden sharing, focusing not only on military but also on governments. Balcaen, Du Bois, and Buts (2023) presented the joint product model, which encompasses non-military government activities to counter hybrid threats. Haesebrouck (2022) argued that NATO allies' defense budget and burden sharing were influenced by each government's political ideology, budgetary constraints, and the Russian threat. Alozious (2022) argued that public debt constrained development in NATO allies' military expenditure. Furthermore, contributions to NATO missions were proposed as an indicator of defense burden-sharing within the alliance other than military expenditure (Kivimäki 2019; Oma 2012; Ringsmose 2010; Robison 2020).

MDO, Cross-Domain Synergy, and Weaker-Link

Aggregation technologies of public goods other than simple summation are also practically important (Hirshleifer 1983). They also have profound implications in defense economics (Sandler 2006). The majority of the studies on the economics of alliances have used the summation type to aggregate member countries' defense activities, while some have focused on merging individual member allies' contributions and applied aggregation technologies other than summation, such as the weakest-link, weaker-link, better-shot, and best-shot types (Bogers, Beeres, and Bollen 2022). Conybeare, Murdoch, and Sandler (1994) applied the weakest-link and best-shot types to the Triple Alliance, the Triple Entente, Warsaw Pact, and NATO. Siqueira and Sandler (2001) analyzed NATO multi-lateral rapid reaction forces using the weakest-link and best-shot types. Sandler (2006) stated that the weakest- and weaker-link types were suitable for analyzing counter-terrorism actions.

Nakagawa, Sejima, and Fujimoto (2010) also applied the better-shot type to NATO. de la Rovchère, Ghislain, and Rocaboy (2011) argued that the best-shot type was better suited to NATO in the early Cold War period than summation. de la Rochère, Ghislain, and Rocaboy (2014) investigated the impact of SDI on NATO allies using the best-shot type and concluded that the expected shift in type from summation to best-shot following SDI did not take place. Balcaen, Du Bois, and Buts (2023) argued that the outputs for increasing the resilience of each NATO ally against hybrid threats should be aggregated with the weakest-link type.

As shown above, existing studies have used various types of aggregation technology other than summation to represent the characteristics of warfare and operations. For example, Sandler (2006) noted that such types of aggregation technology are useful for the study of terrorism, peacekeeping, and new security challenges. Similarly, this paper uses several types of aggregation technology to present the models that are suitable for MDO. In MDO, due to the diversity of operational domains, the type of aggregation technology suitable for representing domain-specific characteristics can vary. In the traditional land, sea, and air domains, the summation type can be suitable because of the importance of force size. In cyberspace, electronic (e.g. radio base stations) and physical networks (e.g. undersea cables) are important, and a disruption of one such network point can be fatal to the entire network. Therefore, the weakest-link type can be suitable. Regarding the space domain, while activities (e.g. launching and operating space satellites) require large investments, it is relatively easy to share the results of such activities (e.g. information collected by satellites) with other countries. Thus, the best-shot type can be suitable.

In addition to focusing on domain-specific characteristics, MDO also require attention to the relations between operational domains. MDO are operations conducted across multiple domains to overcome an adversary's strengths by presenting them with several operational/tactical dilemmas (U.S. Army 2018). In MDO, cross-domain synergy, which is the synergy between forces in multiple domains, is a key factor to determine the overall strengths. This is because it results in the complementary, vice merely additive, employment of capabilities in different domains so that each enhances the effectiveness and compensates for the vulnerabilities of the others (U.S. Army 2018; U.S. Department of Defense 2012). Furthermore, the complementary provision of domain capabilities between allies strengthens cross-domain synergies (UK Ministry of Defence 2020).

These characteristics of MDO differ from those of traditional single-domain operations. In single-domain operations, the within-domain relations are important. However, in MDO, in addition to the within-domain relations, the between-domain relations are also important. This difference between single-domain operations and MDO can be compared to that between individual and group sports. In individual sports, the competitiveness of the player is crucial. However, in group sports, in addition to the competitiveness of individual players, teamwork among those players is essential. Similarly, in MDO, it is not only the forces in individual domains, but also the combination of them that determines the overall strength.

An example of an actual multi-domain operation creating cross-domain synergy can be seen in the Russian war campaign against Georgia in August 2008. In the campaign, Russia coordinated cyberspace attacks synchronized with major combat actions in the other warfighting domains, primarily land and air. They were MDO actually conducted for the first time in history. The Russian cyberspace attacks degraded and disrupted the Georgian command and communication network and limited Georgian military responses (U.S. Joint Chief of Staff 2016).

From the point of view of the economics of alliances, such nature of cross-domain synergy in MDO is interpreted as the complementarity between defense activities in different domains. Assuming public goods aggregation technology with a generalized mean, the types of aggregation technology correspond to the nature of defense goods. Among such types of aggregation technology, the one that expresses the complementarity between the defense activities is the weaker-link type. This is because weaker-link aggregation function is supermodular. Given these relations, this paper presents a model that relates cross-domain synergy in MDO, the complementarity between the defense activities in different domains, and the weaker-link aggregation technology.

Table 1. The form of the aggregation function.

Parameter values	Aggregation technology	Form of aggregation function
$a = 1,\ v \to -\infty$	Weakest-link	$Q = \min\{q_1, q_2, \cdots, q_n\}$
$a = 1,\ -\infty < v < 1$	Weaker-link	
$a = 1,\ v = 1$	Average	$Q = (1/n)(q_1 + q_2 + \cdots + q_n)$
$a = 1,\ 1 < v < \infty$	Better-shot	
$a = 1,\ v \to \infty$	Best-shot	$Q = \max\{q_1, q_2, \cdots, q_n\}$
$a = n,\ v = 1$	Summation	$Q = q_1 + q_2 + \cdots + q_n$

Supermodularity and Weaker-Link

To analyze the complementary relations between the alliance member countries' defense activities in different domains, this study uses organizational and public economics methods. Some researchers have applied organizational economics methods – such as organizational design (Sandler and Forbes 1980) and transaction cost economics (Sandler 1999; Sandler and Hartley 1999; Weber 2000) – to the economics of alliances. This paper uses complementarity in organizations, especially the theory of supermodular games (Milgrom and Roberts 1990a, 1990b; Topkis 1998). Supermodularity represents the economic notion of complementary inputs. In a supermodular game, more activity by some members of the economy raises the returns to the increased levels of activity by others (Milgrom and Roberts 1990b); that is, the actions of each player are mutually complementary.

Assuming public goods aggregation technology with a generalized mean, the types of aggregation technology correspond to the nature of public goods. This is because the aggregation technology function can be either supermodular or submodular, depending on the value of the exogenous parameter of the generalized mean. Aggregation technology with a generalized mean is represented by the following function:

$$Q = a\left[\left(\tfrac{1}{n}\right)\sum_{k=1}^{n}(q_k)^v\right]^{\frac{1}{v}} \tag{1}$$

where Q denotes the level of the aggregated public goods produced by each player, q_k denotes the kth player's public goods production, n denotes the number of players, and a and v are the exogenous parameters (Cornes and Sandler 1996). Table 1 summarizes the types of aggregation technology and their parameter values. Mathematically, the aggregation function is concave if $v < 1$ and convex if $v \geq 1$ (see appendix for details).

This aggregation function is twice continuously differentiable, implying that it is supermodular if and only if its second partial derivative with respect to q_i and q_j is greater than 0 (Milgrom and Roberts 1990b):

$$\frac{\partial^2 Q}{\partial q_i \partial q_j} = a\left(\tfrac{1}{n}\right)^2(1-v)\left[\left(\tfrac{1}{n}\right)\sum_{k=1}^{n}(q_k)^v\right]^{\frac{1}{v}-2} q_i^{v-1}q_j^{v-1} > 0, i, j = 1, \cdots, n; i \neq j. \tag{2}$$

Then, the aggregation function is supermodular if $v < 1$ and submodular if $v \geq 1$. This shows that the weakest- and weaker-link aggregation functions are supermodular, whereas the better- and best-shot aggregation functions are submodular. In this study, I specifically focus on the weaker-link technology because it is a supermodular function and therefore serves the economic representation of complementarity between the public goods produced by each player.

Another interesting feature of the weaker-link aggregation function is that it converges to the geometric mean when the exogenous parameter v converges to 0 (Cornes and Sandler 1996):

$$\lim_{v \to 0} a\left[\left(\tfrac{1}{n}\right)\sum_{k=1}^{n}(q_k)^v\right]^{\frac{1}{v}} = a\prod_{k=1}^{n} q_k^{\frac{1}{n}} \tag{3}$$

Therefore, the geometric mean is a specific form of the weaker-link aggregation function.

Theoretical Model

In the theoretical model of this study, there are m operational domains, and n allies produce defense outputs through the defense activities in those domains. The alliance-wide defense level is represented as the aggregate of the defense activities in each domain:

$$Q = m\left[\left(\frac{1}{m}\right)\sum_{k=1}^{m}(q_k)^{\mu}\right]^{\frac{1}{\mu}} \tag{4}$$

where Q denotes the alliance-wide defense level, q_k denotes the defense activities in kth domain, and μ denotes the exogenous parameter. Let $I = \left[\underline{q}, \bar{q}\right]$ be an interval in $\mathbb{R}^m_+$; then, Q is twice continuously differentiable on some open set containing I. The second partial derivative of Q with respect to q_d and q_e is

$$\frac{\partial^2 Q}{\partial q_d \partial q_e} = m\left(\frac{1}{m}\right)^2(1-\mu)\left[\left(\frac{1}{m}\right)\sum_{k=1}^{m}(q_k)^{\mu}\right]^{\frac{1}{\mu}-2} q_d^{\mu-1}q_e^{\mu-1}, d, e = 1, \cdots, m; d \neq e. \tag{5}$$

Thus, Q is supermodular on I if and only if $\mu < 1$. It implies that the defense activities in each domain are complementary to each other if they are aggregated with the weaker-link or weakest-link type. Furthermore, q_d, the defense level in dth domain, is represented as the aggregate of each member country's defense activity in the domain:

$$q_d = n\left[\left(\frac{1}{n}\right)\sum_{l=1}^{n}(q_{dl})^{v_d}\right]^{\frac{1}{v_d}} \tag{6}$$

where q_{dl} denotes lth ally's defense activity in dth domain, and v_d denotes the exogenous parameter of dth domain.

Examples

To explain the implications of the theoretical model on the behavior of NATO allies and defense burden-sharing within the alliance, this section presents three examples, as shown in Table 2. In these three examples, it is commonly assumed that there are three operational domains. The value of μ (the parameter used to aggregate the defense activity in each domain) and that of v_d (the parameter used to aggregate each ally's defense activity in dth domain) differ from example to example. These assumptions are based on the fact that the concept of MDO inherently emphasizes the relations between different domains rather than the relations within a domain.

In Example 1, the defense activities in each domain are aggregated with the summation type technology, and each ally's defense activity in one domain is also aggregated with the summation. In this instance, Q is not supermodular on I because $\mu = 1$, which shows that

Table 2. Examples.

		Aggregator	
Examples	Parameter values	Between-domain	Within-domain
Example 1	$\mu = 1$ $v_d = 1, d = 1, 2, 3$	$Q = \sum_{k=1}^{3} q_k$	$q_d = \sum_{l=1}^{n} q_{dl}, d = 1, 2, 3$
Example 2	$\mu \to 0$ $v_d = 1, d = 1, 2, 3$	$Q = 3\prod_{k=1}^{3}(q_k)^{\frac{1}{3}}$	$q_d = \sum_{l=1}^{n} q_{dl}, d = 1, 2, 3$
Example 3	$\mu \to 0$ $v_1 = 1$ $v_2 \to -\infty$ $v_3 \to \infty$	$Q = 3\prod_{k=1}^{3}(q_k)^{\frac{1}{3}}$	$q_1 = \sum_{l=1}^{n} q_{1l}$ $q_2 = \min\{q_{21}, \cdots, q_{2n}\}$ $q_3 = \max\{q_{31}, \cdots, q_{3n}\}$

defense activities in each domain are not complementary to each other, and therefore, there is no cross-domain synergy. Consequently, one member country's increased defense activity in one domain does not motivate another member to increase its defense activity in another domain.

In Example 2, the defense activities in each domain are aggregated with the weaker-link type, while each member country's defense activity within a domain is aggregated with the summation. In this instance, Q is supermodular on I because $\mu < 1$. Due to this supermodularity, the defense activities in each domain are complementary to each other and create cross-domain synergy. Thus, in Example 2, unlike Example 1, one member country's increased defense activity in one domain motivates another member to increase its activity in another domain.

In Example 3, as in Example 2, the defense activities in each domain are aggregated with the weaker-link type. However, each member country's defense activity within a domain is aggregated by different methods for each domain. In the first domain, the member countries' defense activities are aggregated with the summation (e.g. the traditional domain). In the second domain, they are aggregated with the weakest-link type (e.g. cyberspace). In the third domain, they are aggregated with the best-shot type (e.g. the space domain). Again, in Example 3, the defense activities in each domain are complementary to each other. However, the effect of an increase in one member's activity in one domain motivating an increase in another's activity in another domain is less than in Example 2. This is because the defense level in the second domain increases only with the increase in the least contributing member's defense activity, and that in the third domain increases only with the increase in the best contributing member's activity.

These examples suggest that the behavior of the member countries varies depending on how the forces in the different domains relate (i.e. the between-domain relations), and how each member's forces in the same domain relate (i.e. the within-domain relations). If the forces in different domains are sufficiently complementary, even if the forces within the same domain are substitutional, one member's strengthening defense forces motivates another's strengthening defense forces. In MDO, the forces in different domains can be complementary to each other as each ally seeks to create cross-domain synergy. In other words, when one member increases its forces in one domain, another member can increase its forces in another domain that is complementary to that domain. For example, enhanced cyberspace defense by one member could improve the alliance-wide command and communications network, leading to more efficient operation of conventional forces. A member's enhanced intelligence-gathering capabilities through space satellites could enable other members to conduct more precise attacks in traditional land, maritime, and air domains.

Furthermore, these examples imply that defense burden-sharing in MDO can be fairer than in single-domain operations. This is because larger allies' strengthening defense forces in one domain can encourage smaller allies' strengthening defense forces in another domain owing to the complementarity between the defense activities in different domains. Consequently, the smaller allies can act as cooperators. This result contrasts with that of single-domain operations, where the larger allies' defense activity in the domain substitutes for that of the smaller allies in the same domain; hence, the smaller allies act as free-riders.

Concluding Remarks

As shown in the 2022 Strategic Concept, NATO is wary of authoritarian states and prepared for the great power competition with them. To prepare for a possible future war against peer competitors, NATO is strengthening its capabilities to conduct MDO, in which cross-domain synergy is a key factor. In the context of NATO's approach to MDO, this study presented a model that incorporates the within-and between-domain relations. Using organizational and public economics methods, the model relates cross-domain synergy in MDO, the complementarity between the defense activities in different domains, and weaker-link aggregation technology.

The model has a crucial implication for the economics of alliances. If the defense activities in different domains are complementary, when one member country increases its defense activity in one domain, another member increases its activity in another domain that is complementary to that domain. It enables smaller allies to act as cooperators rather than free-riders. Consequently, defense burden-sharing within the alliance in MDO can be fairer than in single-domain operations.

At the 2014 NATO Summit, the allies endorsed the Defense Investment Pledge, which recognized that fair burden-sharing was the foundation of the alliance. It also called for all allies to meet the NATO-agreed guideline of spending 2% of GDP on defense by 2024 (NATO 2022d). Following Russia's invasion of Ukraine, NATO allies are taking relevant measures, such as increasing military expenditure. Germany, for example, has made a historic decision to raise its military expenditure as a share of GDP to more than 2% by 2024 (Scholz 2022). Given this security environment, the implication for the economics of alliance is particularly crucial for NATO's future military buildup: NATO's approach to MDO will make defense burden-sharing fairer because one member's military buildup will motivate other members to build up theirs. This, in turn, will facilitate the achievement of the 2% target.

Notes

1. The U.S. Army (2018) defines MDO as 'operations conducted across multiple domains and contested spaces to overcome an adversary's (or enemy's) strengths by presenting them with several operational and/or tactical dilemmas through the combined application of calibrated force posture; employment of multi-domain formations; and convergence of capabilities across domains, environments, and functions in time and spaces to achieve operational and tactical objectives.'
2. The U.S. Army (2018) defines cross-domain synergy as 'the employment of mutually supporting lethal and nonlethal capabilities of multiple domains to create conditions designed to generate overmatch, present multiple dilemmas to the enemy, and enable Joint Force freedom of movement and action.'

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References

Alozious, J. 2022. "Nato's Two Percent Guideline: A Demand for Military Expenditure Perspective." *Defence and Peace Economics* 33 (4): 475–488. https://doi.org/10.1080/10242694.2021.1940649.

Balcaen, P., C. Du Bois, and C. Buts. 2023. "Sharing the Burden of Hybrid Threats: Lessons from the Economics of Alliances." *Defence and Peace Economics* 34 (2): 142–159. https://doi.org/10.1080/10242694.2021.1991128.

Bogers, M., R. Beeres, and M. Bollen. 2022. "NATO Burden Sharing Research Along Three Paradigms." *Defence and Peace Economics* 33 (5): 534–547. https://doi.org/10.1080/10242694.2020.1819135.
Brynjolfsson, E., and P. Milgrom. 2013. "Complementarity in Organizations." In *The Handbook of Organizational Economics*, edited by R. Gibbons and J. Roberts, 11–55. Princeton, NJ: Princeton University Press. https://doi.org/10.1515/9781400845354-003.
Conybeare, J. A. C., J. C. Murdoch, and T. Sandler. 1994. "Alternative Collective-Goods Models of Military Alliances: Theory and Empirics." *Economic Inquiry* 32 (4): 525–542. https://doi.org/10.1111/j.1465-7295.1994.tb01349.x.
Cornes, R., and T. Sandler. 1984. "Easy Riders, Joint Production, and Public Goods." *The Economic Journal* 94 (375): 580–598. https://doi.org/10.2307/2232704.
Cornes, R., and T. Sandler. 1996. *The Theory of Externalities, Public Goods, and Club Goods*. 2nd ed. Cambridge, UK: Cambridge University Press. https://doi.org/10.1017/CBO9781139174312.
De la Rochère, D., J.-M. J. Ghislain, and Y. Rocaboy. 2014. "SDI, NATO, and the Social Composition Function." *Defence and Peace Economics* 25 (2): 85–95. https://doi.org/10.1080/10242694.2013.774772 .
De la Rovchère, D., J.-M. J. Ghislain, and Y. Rocaboy. 2011. "The Role of Aggregation Technologies in the Provision of Supranational Public Goods: A Reconsideration of Nato's Strategies." *The Review of International Organizations* 6 (1): 85–103. https://doi.org/10.1007/s11558-010-9098-x.
Gaibulloev, K., T. Sandler, and H. Shimizu. 2009. "Demands for UN and Non-UN Peacekeeping: Nonvoluntary versus Voluntary Contributions to a Public Good." *Journal of Conflict Resolution* 53 (6): 827–852. https://doi.org/10.1177/0022002709338509.
George, J., and T. Sandler. 2018. "Demand for Military Spending in NATO, 1968–2015: A Spatial Panel Approach." *European Journal of Political Economy* 53 (July): 222–236. https://doi.org/10.1016/j.ejpoleco.2017.09.002.
George, J., and T. Sandler. 2021. "EU Demand for Defense, 1990–2019: A Strategic Spatial Approach." *Games* 12 (1): 13. Article 13. https://doi.org/10.3390/g12010013 .
Gonzalez, R. A., and S. L. Mehay. 1991. "Burden Sharing in the NATO Alliance: An Empirical Test of Alternative Views." *Public Choice* 68 (1): 107–116. https://doi.org/10.1007/BF00173822 .
Haesebrouck, T. 2022. "NATO Burden Sharing After the Wales Summit: A Generalized Set Qualitative Analysis." *Defence and Peace Economics* 33 (6): 637–654. https://doi.org/10.1080/10242694.2021.1928435.
Hansen, L., J. C. Murdoch, and T. Sandler. 1990. "On Distinguishing the Behavior of Nuclear and Non-Nuclear Allies in NATO." *Defence Economics* 1 (1): 37–55. https://doi.org/10.1080/10430719008404649.
Hartley, K., and T. Sandler. 1999. "NATO Burden-Sharing: Past and Future." *Journal of Peace Research* 36 (6): 665–680. https://doi.org/10.1177/0022343399036006004.
Hirshleifer, J. 1983. "From Weakest-Link to Best-Shot: The Voluntary Provision of Public Goods." *Public Choice* 41 (3): 371–386. https://doi.org/10.1007/BF00141070 .
Japan Ministry of Foreign Affairs. 2022. "NATO Asia-Pacific Partners (AP4) Leaders' Meeting." June 29. https://www.mofa.go.jp/a_o/ocn/ki/page1e_000413.html.
Johnson, D. 2018. *Shared Problems: The Lessons of AirLand Battle and the 31 Initiatives for Multi-Domain Battle*. RAND Corporation. https://doi.org/10.7249/PE301.
Jones, M. A., and J. D. de Leon. 2020. "Multi-Domain Operations." *The Three Sword Magazine* 36 (November): 38–41.
Khanna, J., and T. Sandler. 1996. "Nato Burden Sharing: 1960–1992." *Defence and Peace Economics* 7 (2): 115–133. https://doi.org/10.1080/10430719608404846.
Khanna, J., T. Sandler, and H. Shimizu. 1998. "Sharing the Financial Burden for U.N. and NATO Peacekeeping, 1976-1996." *Journal of Conflict Resolution* 42 (2): 176–195. https://doi.org/10.1177/0022002798042002003.
Kim, W., and T. Sandler. 2020. "NATO at 70: Pledges, Free Riding, and Benefit-Burden Concordance." *Defence and Peace Economics* 31 (4): 400–413. https://doi.org/10.1080/10242694.2019.1640937.
Kivimäki, T. 2019. "Power, Contribution and Dependence in NATO Burden Sharing." *European Security* 28 (1): 66–84. https://doi.org/10.1080/09662839.2019.1578750.
Leon, J. D. D. 2021. "Understanding Multi-Domain Operations in NATO." *The Three Sword Magazine* 37 (October): 91–94.
Milgrom, P., and J. Roberts. 1990a. "The Economics of Modern Manufacturing: Technology, Strategy, and Organization." *The American Economic Review* 8 (3): 511–528.
Milgrom, P., and J. Roberts. 1990b. "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities." *Econometrica* 58 (6): 1255–1277. https://doi.org/10.2307/2938316.
Murdoch, J. C., and T. Sandler. 1982. "A Theoretical and Empirical Analysis of NATO." *Journal of Conflict Resolution* 26 (2): 237–263. https://doi.org/10.1177/0022002782026002003.
Murdoch, J. C., T. Sandler, and L. Hansen. 1991. "An Econometric Technique for Comparing Median Voter and Oligarchy Choice Models of Collective Action: The Case of the NATO Alliance." *The Review of Economics and Statistics* 73 (4): 624–631. https://doi.org/10.2307/2109401.
Nakagawa, S., M. Sejima, and S. Fujimoto. 2010. "Alliance Formation and Better-Shot Global Public Goods: Theory and Simulation." *Evolutionary and Institutional Economics Review* 6 (2): 201–225. https://doi.org/10.14441/eier.6.201.
NATO. 1991. "The Alliance's New Strategic Concept (1991)." November 7. https://www.nato.int/cps/en/natohq/official_texts_23847.htm.

NATO. 1999. “The Alliance’s Strategic Concept (1999).” April 24. https://www.nato.int/cps/en/natohq/official_texts_27433.htm.
NATO. 2002. “The Prague Summit and Nato’s Transformation.” June 25. https://www.nato.int/docu/rdr-gde-prg/rdr-gde-prg-eng.pdf.
NATO. 2006. *Interoperability for Joint Operations*. Brussels: NATO Public Diplomacy Division.
NATO. 2010. *Strategic Concept: Active Engagement, Modern Defence*. Brussels: NATO Public Diplomacy Division.
NATO. 2021a. “Multi-Domain Exercise on Norwegian Coast.” August 19. https://ac.nato.int/archive/2021/NATO_NOR_US_MDE.
NATO. 2021b. “Nato’s Approach to Space.” December 2. https://www.nato.int/cps/en/natohq/topics_175419.htm .
NATO. 2022a. “Cyber Defence.” March 23. https://www.nato.int/cps/en/natohq/topics_78170.htm.
NATO. 2022b. “Multi-Domain Operations: Enabling NATO to Out-Pace and Out-Think Its Adversaries.” July 29. https://www.act.nato.int/articles/multi-domain-operations-out-pacing-and-out-thinking-nato-adversaries.
NATO. 2022c. NATO 2022 Strategic Concept. https://www.nato.int/strategic-concept/.
NATO. 2022d. *The Secretary General’s Annual Report 2021*. https://www.nato.int/nato_static_fl2014/assets/pdf/2022/3/pdf/sgar21-en.pdf.
Nikolaidou, E. 2008. “The Demand for Military Expenditure: Evidence from the EU15 (1961–2005).” *Defence and Peace Economics* 19 (4): 273–292. https://doi.org/10.1080/10242690802166533.
Odehnal, J., J. Neubauer, A. Olejníček, J. Boulaouad, and L. Brizgalová. 2021. “Empirical Analysis of Military Expenditures in NATO Nations.” *Economies* 9 (3): 107. https://doi.org/10.3390/economies9030107.
Olson, M., Jr., and R. Zeckhauser. 1966. “An Economic Theory of Alliances.” *The Review of Economics and Statistics* 48 (3): 266–279. https://doi.org/10.2307/1927082.
Oma, I. M. 2012. “Explaining States’ Burden-Sharing Behaviour within NATO.” *Cooperation and Conflict* 47 (4): 562–573. https://doi.org/10.1177/0010836712462856.
Oneal, J. R., and M. A. Elrod. 1989. “NATO Burden Sharing and the Forces of Change.” *International Studies Quarterly* 33 (4): 435–456. https://doi.org/10.2307/2600521.
Plümper, T., and E. Neumayer. 2015. “Free-Riding in Alliances: Testing an Old Theory with a New Method.” *Conflict Management and Peace Science* 32 (3): 247–268. https://doi.org/10.1177/0738894214522916.
Ringsmose, J. 2010. “NATO Burden-Sharing Redux: Continuity and Change After the Cold War.” *Contemporary Security Policy* 31 (2): 319–338. https://doi.org/10.1080/13523260.2010.491391.
Robison, R. R. 2020. “NATO Burden-Sharing: A Comprehensive Framework for Member Evaluation.” *Comparative Strategy* 39 (3): 299–315. https://doi.org/10.1080/01495933.2020.1740574.
Russett, B. M. 1970. *What Price Vigilance?: The Burdens of National Defense*. New Haven, CT: Yale University Press.
Sandler, T. 1993. “The Economic Theory of Alliances: A Survey.” *Journal of Conflict Resolution* 37 (3): 446–483. https://doi.org/10.1177/0022002793037003003.
Sandler, T. 1999. “Alliance Formation, Alliance Expansion, and the Core.” *Journal of Conflict Resolution* 43 (6): 727–747. https://doi.org/10.1177/0022002799043006003.
Sandler, T. 2006. “Hirshleifer’s Social Composition of Function in Defense Economics.” *Defence and Peace Economics* 17 (6): 645–655. https://doi.org/10.1080/10242690601025583.
Sandler, T., and J. Cauley. 1975. “On the Economic Theory of Alliances.” *Journal of Conflict Resolution* 19 (2): 330–348. https://doi.org/10.1177/002200277501900207.
Sandler, T., J. Cauley, and J. F. Forbes. 1980. “In Defense of a Collective Goods Theory of Alliances.” *Journal of Conflict Resolution* 24 (3): 537–547. https://doi.org/10.1177/002200278002400308 .
Sandler, T., and J. F. Forbes. 1980. “Burden Sharing, Strategy, and the Design of NATO.” *Economic Inquiry* 18 (3): 425–444. https://doi.org/10.1111/j.1465-7295.1980.tb00588.x.
Sandler, T., and K. Hartley. 1999. *The Political Economy of NATO: Past, Present, and into the 21st Century*. Cambridge, UK: Cambridge University Press.
Sandler, T., and K. Hartley. 2001. “Economics of Alliances: The Lessons for Collective Action.” *Journal of Economic Literature* 39 (3): 869–896. https://doi.org/10.1257/jel.39.3.869.
Sandler, T., and J. C. Murdoch. 1990. “Nash-Cournot or Lindahl Behavior?: An Empirical Test for the NATO Allies.” *The Quarterly Journal of Economics* 105 (4): 875–894. https://doi.org/10.2307/2937877.
Sandler, T., and J. C. Murdoch. 2000. “On Sharing NATO Defence Burdens in the 1990s and Beyond.” *Fiscal Studies* 21 (3): 297–327. https://doi.org/10.1111/j.1475-5890.2000.tb00026.x.
Scholz, O. 2022. “Policy Statement by Olaf Scholz, Chancellor of the Federal Republic of Germany and Member of the German Bundestag, 27 February 2022 in Berlin.” https://www.bundesregierung.de/breg-en/news/policy-statement-by-olaf-scholz-chancellor-of-the-federal-republic-of-germany-and-member-of-the-german-bundestag-27-february-2022-in-berlin-2008378.
Shimizu, H., and T. Sandler. 2002. “Peacekeeping and Burden-Sharing, 1994-2000.” *Journal of Peace Research* 39 (6): 651–668. https://doi.org/10.1177/0022343302039006001.
Shimizu, H., and T. Sandler. 2010. “Recent Peacekeeping Burden Sharing.” *Applied Economics Letters* 17 (15): 1479–1484. https://doi.org/10.1080/13504850903049593.

Siqueira, K., and T. Sandler. 2001. "Models of Alliances: Internalizing Externalities and Financing." *Defence and Peace Economics* 12 (3): 249–270. https://doi.org/10.1080/10430710108404987.

Smith, R. P. 1989. "Models of Military Expenditure." *Journal of Applied Econometrics* 4 (4): 345–359. https://doi.org/10.1002/jae.3950040404.

Tammen, J. W. 2021. "Nato's Warfighting Capstone Concept: Anticipating the Changing Character of War." *NATO Review*, July 9. https://www.nato.int/docu/review/articles/2021/07/09/natos-warfighting-capstone-concept-anticipating-the-changing-character-of-war/index.html.

Topkis, D. M. 1998. *Supermodularity and Complementarity*. Princeton, NJ: Princeton University Press.

UK Ministry of Defence. 2020. *Joint Concept Note 1/20, Multi-Domain Integration*. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/950789/20201112-JCN_1_20_MDI.PDF.

U.S. Army. 2018. *The U.S. Army in Multi-Domain Operations 2028*. https://www.army.mil/article/243754/the_u_s_army_in_multi_domain_operations_2028.

U.S. Department of Defense. 2012. *Capstone Concept for Joint Operations: Joint Force 2020*. https://www.ndu.edu/Portals/59/Documents/Incoming/ccjo_2012.pdf.

U.S. Joint Chief of Staff. 2016. *Cross-Domain Synergy in Joint Operations*. https://www.jcs.mil/Portals/36/Documents/Doctrine/concepts/cross_domain_planning_guide.pdf?ver=2017-12-28-161956-230.

Weber, K. 2000. *Hierarchy Amidst Anarchy: Transaction Costs and Institutional Choice*. Albany, NY: State University of New York Press.

Zannella, A. 2020. "An Analysis of Burden Sharing in NATO and the Problem of Free Riding." *Political Analysis* 21: 1–19.

Appendix

This appendix introduces the proof of the following theorem related to the concavity/convexity of the aggregation technology function.

Theorem: Consider the aggregation technology function $Q(q_1, \cdots, q_n) : \mathbb{R}_+^n \to \mathbb{R}_+$ defined by

$$Q(\cdot) = a\left[\left(\frac{1}{n}\right)\sum_{k=1}^{n} q_k^v\right]^{\frac{1}{v}}$$

where $a > 0$. Then, $Q(\cdot)$ is a concave function if $v < 1$ and a convex function if $v \geq 1$.

Proof: The second partial derivative of $Q(\cdot)$ with respect to q_i is

$$\frac{\partial^2 Q}{\partial q_i^2} = a\left(\frac{1}{n}\right)^2 (1-v)\left[\left(\frac{1}{n}\right)\sum_{k=1}^{n} q_k^v\right]^{\frac{1}{v}-2} q_i^{v-2}\left(-\sum_{k\neq i} q_k^v\right).$$

The second partial derivative with respect to q_i and q_j is

$$\frac{\partial^2 Q}{\partial q_i \partial q_j} = a\left(\frac{1}{n}\right)^2 (1-v)\left[\left(\frac{1}{n}\right)\sum_{k=1}^{n} q_k^v\right]^{\frac{1}{v}-2} q_i^{v-1} q_j^{v-1}.$$

Then, H, the Hessian matrix of $Q(\cdot)$ is

$$H = \begin{pmatrix} \frac{\partial^2 Q}{\partial q_1^2} & \cdots & \frac{\partial^2 Q}{\partial q_1 \partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 Q}{\partial q_n \partial q_1} & \cdots & \frac{\partial^2 Q}{\partial q_n^2} \end{pmatrix}$$

$$= a\left(\frac{1}{n}\right)^2 (1-v)\left[\left(\frac{1}{n}\right)\sum_{k=1}^{n} q_k^v\right]^{\frac{1}{v}-2} \begin{pmatrix} q_1^{v-2}\left(-\sum_{k\neq 1} q_k^v\right) & \cdots & q_1^{v-1} q_n^{v-1} \\ \vdots & \ddots & \vdots \\ q_n^{v-1} q_1^{v-1} & \cdots & q_n^{v-2}\left(-\sum_{k\neq n} q_k^v\right) \end{pmatrix}.$$

Let $x \in \mathbb{R}^n$ be a non-zero real column vector, and x^t be transpose of x. That is, $x^t = (x_1, \cdots, x_n)$. The quadratic form is

$$x^t H x = a\left(\frac{1}{n}\right)^2 (1-v)\left[\left(\frac{1}{n}\right)\sum_{k=1}^{n} q_k^v\right]^{\frac{1}{v}-2} x^t \begin{pmatrix} q_1^{v-2}\left(-\sum_{k\neq 1} q_k^v\right) & \cdots & q_1^{v-1} q_n^{v-1} \\ \vdots & \ddots & \vdots \\ q_n^{v-1} q_1^{v-1} & \cdots & q_n^{v-2}\left(-\sum_{k\neq n} q_k^v\right) \end{pmatrix} x.$$

And,

$$x^t \begin{pmatrix} q_1^{v-2}\left(-\sum_{k\neq 1} q_k^v\right) & \cdots & q_1^{v-1} q_n^{v-1} \\ \vdots & \ddots & \vdots \\ q_n^{v-1} q_1^{v-1} & \cdots & q_n^{v-2}\left(-\sum_{k\neq n} q_k^v\right) \end{pmatrix} x$$

$$
\begin{aligned}
&= q_1^{v-2}\left(-\sum_{k\neq 1} q_k^v\right)x_1^2 + q_1^{v-1}q_2^{v-1}x_1x_2 + \cdots + q_n^{v-2}\left(-\sum_{k\neq n} q_k^v\right)x_n^2 \\
&= \sum_{k=1}^{n}\left[q_k^{v-2}\left(-\sum_{l\neq k} q_l^v\right)x_k^2\right] + \sum_{k=1}^{n}\left[\sum_{l\neq k}\left(q_k^{v-1}q_l^{v-1}x_kx_l\right)\right] \\
&= \sum_{k=1}^{n-1}\sum_{l>k}\left(-q_k^{v-2}q_l^v x_k^2 - q_l^{v-2}q_k^v x_l^2\right) + 2\sum_{k=1}^{n-1}\sum_{l>k}\left(q_k^{v-1}q_l^{v-1}x_kx_l\right) \\
&= \sum_{k=1}^{n-1}\sum_{l>k}\left[\left(-\frac{q_l}{q_k}q_k^{v-1}q_l^{v-1}x_k^2 - \frac{q_k}{q_l}q_l^{v-1}q_k^{v-1}x_l^2\right) + 2\left(q_k^{v-1}q_l^{v-1}x_kx_l\right)\right] \\
&= \sum_{k=1}^{n-1}\sum_{l>k}\left[q_k^{v-1}q_l^{v-1}\left(-\frac{q_l}{q_k}x_k^2 - \frac{q_k}{q_l}x_l^2 + 2x_kx_l\right)\right] \\
&= -\sum_{k=1}^{n-1}\sum_{l>k}\left[q_k^{v-1}q_l^{v-1}\left(\frac{q_l^2x_k^2 + q_k^2x_l^2 - 2q_kq_lx_kx_l}{q_kq_l}\right)\right] \\
&= -\sum_{k=1}^{n-1}\sum_{l>k}\left[q_k^{v-2}q_l^{v-2}\left(q_lx_k - q_kx_l\right)^2\right] \\
&\leq 0
\end{aligned}
$$

Thus, H is negative semidefinite if $v < 1$, and positive semidefinite if $v \geq 1$. Consequently, the aggregation function $Q(\cdot)$ is concave if $v < 1$ and convex if $v \geq 1$.