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Trade Policy for Dual-Use Technology

Leo J. Blanken[a] and Jason J. Lepore[b]

[a]Department of Defense Analysis, Naval Postgraduate School, Monterey, CA, USA; [b]Department of Economics, Orfalea College of Business, California Polytechnic State University, San Luis Obispo, CA, USA

ABSTRACT
We consider trade policy for a private market good that is also valuable for the production of military force. In a two-country model with both contested and uncontested resources, we show necessary and sufficient conditions for the importing country to restrict trade with quota and subsidy combination in equilibrium. Equilibrium can involve subsidization by the exporting country with equilibrium total of the importing country increasing in this subsidy.

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Introduction

In recent years, the US Department of Defense (DoD) has shown growing concern about US dependence on the foreign production of private goods used as key components in military weapons systems.[1] This concern is driven by the lack of a robust US production base for these products, which could become problematic if there is a conflict with the country exporting the good to the US. We use the term 'dual-use' good to identify a good that is produced for private market consumption and has the added feature that its national production base is valuable in the generation of armed force in time of conflict. In a prolonged conflict in which trade between two nations is disrupted, the ability to produce the dual-use good at home becomes seminal to military success. This is because the production of new military capabilities, which require the dual-use good as an input, are necessary during such a conflict. An important contemporary example is the production of microelectronics, which are key input components for military capabilities. Although the DoD has made an effort to understand this problem,[2] existing economic theory does not provide clear guidance for policy.

To contextualize the specification of our model and help provide intuition for our results, we use the historical example of the United States and England in the decades after the American Revolution. At the time, these were two nations that had asymmetric production technology to manufacture a strategic, dual-use good while existing under the threat of war. On the one hand, having just won its independence from England in a bloody and protracted war, the fledgling United States worried considerably about its safety in the face of a British military that remained the global leader. Those fears proved justified in the early years of the 19th century as war broke out again between the two nations in 1812, resulting in British troops capturing Washington and burning the White House and Capitol building. On the other hand, Britain and the United States were important trading partners for this period as well. The United States sent between 25 and 50% of its exports to Great Britain over this period, though 95% of these exports were unfinished goods (Lipsey 1994: 2-3).

The founding fathers – Alexander Hamilton, Thomas Jefferson, and James Madison – debated this problem exhaustively (Irwin 2004; Nelson 1979; Peskin 2002). They were concerned with the optimal

CONTACT Jason J. Lepore jlepore@calpoly.edu Department of Economics, Orfalea College of Business, California Polytechnic State University, 1 Grand Avenue, 93407, San Luis Obispo, CA, USA

policy tools to promote American manufacturing, increase national wealth, and support the nation's defense. The three tools that were debated included: subsidies to domestic manufacturers, tariffs on imports, as well as restrictions (quotas) on imports. Over the course of their political careers, each of these thinkers worked actively to shape and enact policy to use these tools to the best of their understanding in the interest of the nation.

Let us consider shipbuilding as the relevant dual-use technology. A robust shipbuilding industry not only provides merchant vessels for trade and fishing in peace time but also supplies warships in wartime. Hamilton in particular was very interested in the health of American shipbuilding as a dual-use input for national security and recognized the circular causation between naval power and economic power: 'The necessity of naval protection to external or maritime commerce does not require elucidation, no more than the conduciveness of that species of commerce to the prosperity of the navy' (quoted in Earle 1986: 237). It was specifically the shipbuilding industry production base that impacted future wartime military capabilities, because storage of ships in drydock or otherwise, was prohibitively costly. Further, technological progress was moving fast enough that old stock could become obsolete quickly. Thus, the primary policy debate was about how to use national-level policy tools to address the problem of promoting a dual-use production base.

In this paper, we construct a trade model to answer the two related questions: Under what circumstances should a country enact trade restrictions for a dual-use good and what does an optimal policy look like? Answering these questions requires a model that goes beyond the assumptions of perfect security, which is standard in most of the extant trade literature.[3] Particularly, our questions require a setting in which the value of military expenditure to create security is formalized.[4] We utilize a two-country Ricardian model of trade and conflict with both secure and contested resources following Garfinkel, Skaperdas, and Syropoulos (2012). Producers across the two countries have access to the same technology for the production of the pure consumption good (we call 'butter'), while the dual-use good is produced with different technology. The country with less efficient dual-use production has a comparative disadvantage in the dual-use good and under free trade is an importer of the dual-use good. Conflict is captured in the model as a standard contest over a fixed quantity of contested resources used to produce the two traded goods. The contested resources are divided based on a contest function that depends on the two countries' military capabilities. The countries invest in military capabilities (' guns') by considering the value of more guns via gains in contested resources against the cost of guns, which is the direct loss of using these resources for production. More guns allow a country to get more of the contested resources while depleting uncontested resources.

Our primary finding is to show the conditions for trade restriction by the importing country and that optimal intervention involves a production quota paired with a production subsidy. Such a policy increases the home production of the dual-use good generating a benefit by increasing efficiency of the production of guns which increases the quantity of contested resources for this country. The marginal cost of increasing home production is the difference between the marginal cost of producing at home and the marginal cost of importing the dual-use good. The optimal quota level is found at the point in which the marginal benefit of home production (in terms of contested resources) equals the marginal cost of home production (the difference in marginal costs of the two nations including subsidies). Thus, the importing country restricts trade with a quota if and only if their marginal gain in contested resources at zero production is greater than their technology disadvantage in marginal cost of production.

In the case that restrictions are used, there is always an equilibrium with the importing country using a quota combined with a subsidy that makes the marginal cost of production for firms in the importing country equal to marginal cost of the exporting country's firms. There is also a continuum of equilibria with the exporting country using a positive subsidy and the importing country using a quota with the subsidy that equates the importing firms' marginal cost minus their subsidy to the exporting firms' marginal cost minus their subsidy. The intuition for the continuum of equilibrium is based on 'in equilibrium' incentives of each nation. The importing country will find it optimal to use a subsidy that makes the price of their home dual-use producers the same as price of the exporting country. The exporting country will find using a subsidy to match the dual-use price of the importing country optimal. This matching feature of the optimal subsidy choices leads to the multiplicity of equilibrium. We show that the set of equilibria is ordered in terms of the welfare of the importing country. The higher the equilibrium subsidy of the exporting country, the greater the importing country's total surplus.

Before proceeding, it is important to discuss why optimal policy involves only quotas and subsidies, not tariffs. Since the choice of trade restriction is done as part of a strategic game, it is important to see explicitly why a tariff system is a less effective policy tool for controlling the home production level of the dual-use good. If the importing country restricts dual-use imports to some fixed level via a tariff the exporting country can always respond by subsidizing home production and increase their dual-use exports beyond the desired level of the other country. In contrast, by definition, an import quota provides no possibility for the exporting country to increase their dual-use exports. Further, in our model, combining a tariff with a binding quota creates a welfare reducing trade distortion for the importer.

Now we can revisit the nineteenth century debate of Hamilton, Jefferson, and Madison using the example of shipbuilding. Our model tells us that if market restrictions are to be used, then it should be an import quota on British ships and subsidization of home ship producers so that their marginal cost equals the marginal cost of the ships being exported from England. Interestingly, this policy prescription does not exactly coincide with either the home industry subsidization and tariff pairing advocated by Hamilton, or the import restriction (quota) and tariff combination advocated by Jefferson and Madison. Instead, the policy we show to be optimal is a combination of the two policy suggestions without the use of tariffs.

Framework

Consider a global economy with two countries indexed by $i = 1, 2$. We denote country 1 as the 'importer' country and country 2 the 'exporter.' Each country can convert its resources into two consumption goods, the first good is the numeraire that we call 'butter', while the second good is a 'dual-use' consumption good. We assume that each country can convert their raw resources into butter at a one-to-one rate. That is, the resources are in units of butter. The production of the dual-use good also follows constant returns-to-scale with the marginal cost of transformation for country i denoted by γ_i.

Country 2 has superior production technology for the dual-use good, formally $\gamma_1 > \gamma_2$. We assume that each country i has a group of identical consumers with quasi-linear preferences represented by the utility function $u_i(b_i, x_i) = b_i + v_i(x_i)$, where b_i is the total quantity of butter consumed by country i and x_i is total quantity of dual-use consumed by country i. The function $v_i(x_i)$ is increasing and strictly concave in $x_i \geq 0$.

All markets are assumed to be perfectly competitive. In this setting, the technology superiority in the production of the dual-use good for country 2 means that without trade restrictions country 1 exclusively imports and country 2 exclusively exports the dual-use good.

Each country i has r_i units of secure resources. There exist additional resources r_0 that are the subject of dispute. The additional resources are divisible and the precise division between the two countries is based on a bargaining process. Policymakers use arming or 'guns' to gain control of these disputed resources with the goal of maximizing their nation's total surplus. We specify the technology of conflict in what follows.

Technology of Conflict

The share of the contested resources r_0 that goes to each country i is dependent on the military capabilities of each nation i denoted by m_i, and determined by the contest success function:

$$\varphi_i(m_i, m_j) = \begin{cases} \frac{m_i}{m_1+m_2} & \text{if}(m_1, m_2) \gg (0,0), \\ \frac{1}{2} & \text{if}(m_1, m_2) = (0,0). \end{cases} \quad (1)$$

This share can be viewed as the reduced form of a bargaining process in which military capabilities determine the division of the resources. Since better military capabilities give a country a larger division of contested resources, each country can benefit from investment in such capabilities. But this military investment has a cost that comes in the form of fewer secure resources being converted into consumption.

Now we specify the way in which military investment translates into military capabilities. Denote by g_i, the military investment, or 'guns' of country i. This is an anarchic setting in which writing enforceable contracts on the proliferation of arms and the division of r_0 is not possible. The investment in guns acts as the enforcement mechanism that determines the shares of the contested resource. The military investment is in units of ' butter'. Each country's production base of dual-use good in part determines its translation of military investment to military capabilities. Denote by y_i the total production of the dual-use good in country i. An investment of g_i translates into military capabilities m_i based on the production function $m_i = a_i(y_i)g_i$. We label $a_i(y_i)$ the rate of translation of military investment (g_i) to military capabilities (m_i). The idea behind this abstract specification is that the capability of production is important in a conflict with a trading partner because access to trade will be cut off and the production base will be necessary for producing continued military force as the conflict persists.

An astute reader might have noticed the omission of military consumption of the dual-use good in the model formalization. We have chosen the specification with only the home production of dual-use impacting the production of military capabilities for a few reasons. First, the primary concern of national security policymakers is with the dual-use production base and prolonged conflict, because in this scenario it is the ability to convert the dual-use good production base into specific military production during the conflict that is important. That is, although some of the traded dual-use good could be used in current military capabilities, this is second order compared to the need to be able to produce the dual-use good and more specific military versions in time of conflict. Second, we leave military consumption out of the formulation because it makes the model more cumbersome without adding any real conceptual value for the policy analysis. We discuss the basic effects of the inclusion of military consumption in the model in the conclusion when we discuss relevant extensions of the model.

Formally, we assume that $a_i'(y_i) \geq 0$ and $a_i''(y_i) \leq 0$ for all $y_i \geq 0$. The ability to access country j's production via trade is limited in the event of conflict between the two nations. The idea that the process of country i arming itself and executing a conflict with country j will be aided by its own production base of this technology is formalized by the assumption that $a_i(y_i)$ is non-decreasing in y_i. The concavity of $a_i(y_i)$ formalizes the idea that the dual-use industrial base has decreasing marginal product of military capabilities.

Timing of the Model

The timing of our model is as follows:

1. The two countries simultaneously and independently choose trade policies. The trade policy for each country is determined by the following choices. For country 1, the policymakers must choose the two objects: a per-unit subsidy to their dual-use production $s_1 \geq 0$, and an import quota $q \geq 0$. The policymakers of country 2 only choose a per-unit subsidy $s_2 \geq 0$ on exports.[5] Its worth noting that the choices $s_1 \leq \gamma_1 - \gamma_2$, $s_2 = 0$, and q sufficiently large equates to unrestricted trade.
2. The two countries simultaneously and independently choose their investment in guns.
3. Trade occurs between the two countries, with consumption and production determined by private market decisions. The division of the contested resource r_0 is determined.

As is evident, we do not formally include tariffs in the analysis, because for any meaningful trade restriction the use of a binding quota is optimal and a tariff in conjunction with a binding quota is never useful in our model. This is because with the use of a binding import quota a tariff only serves as a distortionary transfer of wealth from country 1 consumers to country 1 producers.

Now we move to the calculation of the market outcome for both countries given arbitrary fixed trade policies and investments in guns.

Preliminary Analysis

The price that consumers pay for the dual-use good from the producers of country 2 is denoted by $p_2 > 0$. Since there is a possibly positive subsidy $s_2 \geq 0$, the price the producers in country 2 receive in selling to the consumers of country 2 is $p_2 + s_2$. In the case of a binding import quota, the consumers of country 1 can pay two different prices for the dual-use good. The price consumers pay buying from country 1 producers is denote by $p_1 > 0$. The home producers of country 1 receive the price $p_1^1 + s_1$. We assume that the consumers of country 1 buy up to the quota from country 2 producers before buying from country 1 producers in that case that $p_1 \geq p$.[6]

We assume that each country i has sufficient secure resources such that the representative consumer's most preferred outcome is found by picking the quantity that maximizes the representative consumer's utility minus the cost. This assumption amounts to each r_i being large enough that additional contested resources are always consumed as butter.

Specifically, for country 2 the representative consumer's demand for the dual-use good is

$$\delta_2(p_2) = \arg\max\{v_2(x_2) - p_2 x_2\}.$$

For country 1, the representative consumer's dual-use demand from country 1 and country 2 producers are found by solving the following maximization problem

$$\left(d_1^1(p_1,p_2,q), d_1^2(p_1,p_1,q)\right) = \arg\max_{x_1^1,x_1^2}\{v_1(x_1^1 + x_1^2) - p_1 x_1^1 - p_2 x_1^2 | x_1^2 \leq q\}. \tag{2}$$

The total demand of the representative consumer of country 1 is $d_1^1(p_1,p_2,q) + d_1^2(p_1,p_1,q)$. The notation $\delta_1(p_1) = \arg\max\{v_1(x_1) - p_1 x_1\}$ is useful for the description of the solution.

The solution to the problem in (2) is different depending on which price is larger. Now we will specify the solution of this problem for the two cases that $p_1 < p_2$ (no imports) and $p_1 \geq p_2$ (positive imports).

In the case that $p_1 < p_2$, the consumers of country 1 only buy from their home producers. Thus, the demand of the representative consumer of country 1 is simply the demand from country 1's producers $d_1^1(p_1,p_2,q) = \delta_1(p_1)$ with $d_1^2(p_1,p_2,q) = 0$.

In the case that $p_1 \geq p_2$, the consumers of country 1 purchase everything they can from country 2 producers. Formally, $d_1^2(p_1,p_2,q) = \min\{\delta_1(p_2), q\}$. Then, the residual demand of country 1's consumers from country 1's producers is $d_1^1(p_1,p_2,q) = \max\{\delta_1(p_1) - d_1^2(p_1,p_2,q), 0\}$.

To summarize, the total demand of the consumers of country 1 is

$$D_1(p_1,p_2,q) = \begin{cases} \delta_1(p_2) & \text{if} p_1 \geq p_2 \text{ and } \delta_1(p_2) < q \\ \delta_1(p_1) & \text{otherwise} \end{cases}.$$

We now compute the competitive equilibrium of stage 3 for arbitrary fixed (q,s_1,g_1,s_2,g_2). Denote the realized resources of country i by

$$R_i(g_i,g_j) = r_i + \varphi_i(g_1,g_2)r_0 - g_i.$$

Since production has constant returns-to-scale, the competitive equilibrium prices are $\widehat{p}_1 = \gamma_1 - s_1$, and $\widehat{p}_2 = \gamma_2 - s_2$. The competitive equilibrium production and consumption of each nation are

$$\hat{x}_1 = D_1(\gamma_1 - s_1, \gamma_2 - s_2, q)$$

$$\hat{x}_2 = \delta_2(\gamma_2 - s_2),$$

$$\hat{y}_1 = d_1^1(\gamma_1 - s_1, \gamma_2 - s_2, q),$$

$$\hat{y}_2 = \delta_2(\gamma_2 - s_2) + d_1^2(\gamma_1 - s_1, \gamma_2 - s_2, q).$$

The total surplus of each country is composed of three separable parts. The first term is the representative consumer's utility in terms of butter, which includes the nations share of the resources, the consumer's payment for the dual-use good, and the subsidy to home production. The second term is utility from the consumption of the dual-use good. The final term is the profits of the country's firms, which in competitive equilibrium is always zero. Below we simplify the competitive equilibrium total surplus for both countries, where we use the notation $\hat{z} = d_1^2(\gamma_1 - s_1, \gamma_2 - s_2, q)$ for equilibrium imports of the dual-use good to country 1.

$$TS_1 = \underbrace{R_1(g_1, g_2) - \hat{p}_1\hat{y}_1 - \hat{p}_2\hat{z} - s_1\hat{y}_1}_{\text{utility from butter}} + \underbrace{v_1(\hat{x}_1)}_{\text{utility from dual-use}} + \underbrace{(\hat{p}_1 + s_1 - \gamma_1)\hat{y}_1}_{\text{profits}}$$

$$= R_1(g_1, g_2) - \gamma_1\hat{y}_1 - (\gamma_2 - s_2)\hat{z} + v_1(\hat{x}_1),$$

$$TS_2 = \underbrace{R_2(g_2, g_1) - \hat{p}_2\hat{x}_2 - s_2\hat{y}_2}_{\text{utility from butter}} + \underbrace{v_2(\hat{x}_2)}_{\text{utility from dual-use}} + \underbrace{(\hat{p}_2 + s_2 - \gamma_2)\hat{y}_2}_{\text{profits}}$$

$$= R_2(g_2, g_1) - \gamma_2\hat{x}_2 - s_2\hat{z} + v_2(\hat{x}_2).$$

Next, we consider the equilibrium of the armament subgame.

Equilibrium of the Armament Subgame

In the stage 2 subgame, policymakers of each country choose their investment in guns. Notice that the total surplus of each country i is an additively separable function with country i's surplus from contested resources $R_i(g_i, g_j)$ separated from the surplus from the uncontested resources and, as the notation suggests, that the choice of guns for each country only impacts the total surplus of the countries via the contested resources $R_i(g_i, g_j)$. Thus, the first-order condition for an optimal choice of guns for country i is

$$\frac{\partial TS_i}{\partial g_i} = r_0 \frac{\partial \varphi_i(m_i, m_j)}{\partial g_i} a_i(y_i) - 1 = 0. \tag{3}$$

Solving for the unique Nash equilibrium resources of armament subgame we have

$$\hat{R}_i = r_i + r_0 \left(\frac{a_i(y_i)}{a_1(y_1) + a_2(y_2)} \right)^2. \tag{4}$$

The derivation of this is shown in the first subsection of the appendix. Using this expression, we can now analyze the countries choices of trade restrictions.

Equilibrium Trade Restrictions

We will analyze the subgame perfect equilibrium in which each country implements the Nash equilibrium level of guns in the stage 2 subgame. Before proceeding, we add three important assumptions since we now have the notation to state them clearly.

First, we assume that $a_2(y_2) = a_2$ for all $y_2 \geq d_2(\gamma_2)$. This imposes that the exporting country has a sufficient industrial base from supplying its own consumers that the production from exporting the dual-use good does not increase its military capabilities. Based on this assumption, we will write both countries' equilibrium portion of contested resources as functions of country 1′s production capacity: $\widehat{R}_i(y_1)$. This assumption accords with our focus on the policy actions of the importing country.

Second, we assume that country 1′s marginal rate of translation of military investment to military capabilities is greater than half country 2′s; formally, $a_1(y_1) > a_2/2$ for all $y_1 \geq 0$. This assumption bounds the importance of the dual-use good for production of military capabilities and makes the equilibrium of the model more tractable. Based on our previous assumption that production of military capabilities is increasing at a decreasing rate in dual-use home production, this assumption only implies that country 2′s maximal productive efficiency of military capabilities is less than twice as large as that of country 1. The specific case in which the maximal efficiency of production of military capabilities is symmetric in the two countries clearly satisfies this assumption. To be more specific about the technical usefulness of this assumption, this condition is sufficient for country 1 to have a unique best response in terms of quota (home production level) and subsidy.

Third, we assume that the importer's marginal gain in contested resources at production $\delta_1(\gamma_1)$ is greater than their disadvantage in marginal cost of production:

$$\widehat{R}_1'(\delta_1(\gamma_1)) \leq \gamma_1 - \gamma_2.$$

This assumption is made to eliminate the case of full autarky.

It will be useful to define the implicit function $\widetilde{y}(s_2)$ such that $\widehat{R}_i'(\widetilde{y}(s_2)) = \gamma_1 - \gamma_2 + s_2$ if $\widehat{R}_i'(0) > \gamma_1 - \gamma_2 + s_2$ and $\widetilde{y}(s_2) = 0$ otherwise.

In the following proposition, we establish the exact conditions under which trade restriction by country 1 are part of all equilibria.

Proposition 1 *No binding trade restrictions are used in all subgame perfect equilibrium if and only if*

$$\widehat{R}_1'(0) \leq \gamma_1 - \gamma_2. \tag{†}$$

If the condition (†) does not hold, then all subgame perfect equilibrium involve binding trade restrictions and the set of equilibrium restrictions is (s_1^*, q^*, s_2^*) *defined by*:

$$s_2^* < \widehat{R}_1'(0) - \gamma_1 + \gamma_2, \tag{5}$$

$$s_2^* \leq \frac{\widehat{R}_2(\widetilde{y}(s_2^*)) - \widehat{R}_2(\delta_1(\gamma_2 - s_2^*)) + v_2(\delta_2(\gamma_2 - s_2^*)) - v_2(\delta_2(\gamma_2)) - \gamma_2(\delta_2(\gamma_2 - s_2^*) - \delta_2(\gamma_2))}{\delta_1(\gamma_2 - s_2^*) - \widetilde{y}(s_2^*)}, \tag{6}$$

$$q^* = \delta_1(\gamma_2 - s_2^*) - \widetilde{y}(s_2^*), \tag{7}$$

$$s_1^* = \gamma_1 - \gamma_2 + s_2^*. \tag{8}$$

All proofs of propositions are provided in the Appendix. One useful interpretation of Proposition 1 is that holding all else constant, trade restrictions are part of equilibrium if and only if the size of the contested resources is sufficiently large.

Now we turn to understanding the optimal intervention of country 1 by answering a few key questions. The first question to answer is: why would country 1 ever use a binding import quota? The simple answer is that it is when the marginal benefit of gaining contested resources out ways marginal cost of paying more for some units of the dual-use good. This is only true at home production level of zero exactly when condition (†) does not hold. The right-hand side of (†) is the marginal benefit in terms of contested resources at home production zero and the left-hand side is marginal cost increase of producing the good at home instead of buying them overseas.

The second question is: given the use of a quota by country 1, why use a subsidy? To best understand the equilibrium use of subsidization by country 1 we need to think of this country using a quota and subsidy scheme to implement a precise home production level. At any fixed home production level y_1 there is a set of quota-subsidy pairs that result in that home production level. More precisely, this is the set of quotas and subsidies such that country 1′s demand for the dual-use good minus the quota ($\delta_1(\gamma_1 - s_1) - q$) equals the desired home production level (y_1). Based on the properties of country 1′s dual-use good demand function, at any fixed $s_1 \geq 0$ there is, at most, one quota such that this equality holds. Given a fixed home production quantity, the optimal quota and subsidy is the pair that maximizes the total surplus of country 1 and satisfies the above equality. On the one hand, since the home production is fixed, the cost of industry subsidization is exactly compensated by the benefit to consumers via lower prices. On the other hand, a positive subsidy increases the total consumption of country 1′s consumers, which requires a larger quota (than with no subsidy). That is, subsidization creates no distortion (deadweight loss) when allowing the quota to appropriately move with the subsidy. Taking this into account, country 1′s total surplus is maximized at the total dual-use quantity such that the marginal utility of the dual-use good equals the marginal cost of importing the good from the low cost country. To implement this total surplus maximizing dual-use quantity ($\delta_1(\gamma_2 - s_2)$), at a fixed home production level (y_1), country 1 can only use the subsidy $s_1 = \gamma_1 - \gamma_2 + s_2$ paired with the quota $q = \delta_1(\gamma_2 - s_2) - y_1$. Figure 1 below provides a graphical illustration of this idea. In the figure, the area shaded with the slanted lines country 1′s total surplus with no subsidy (and the paired quota), while the country 1′s total surplus with the optimal subsidy (and the paired quota) also includes the triangle shaded with boxes.

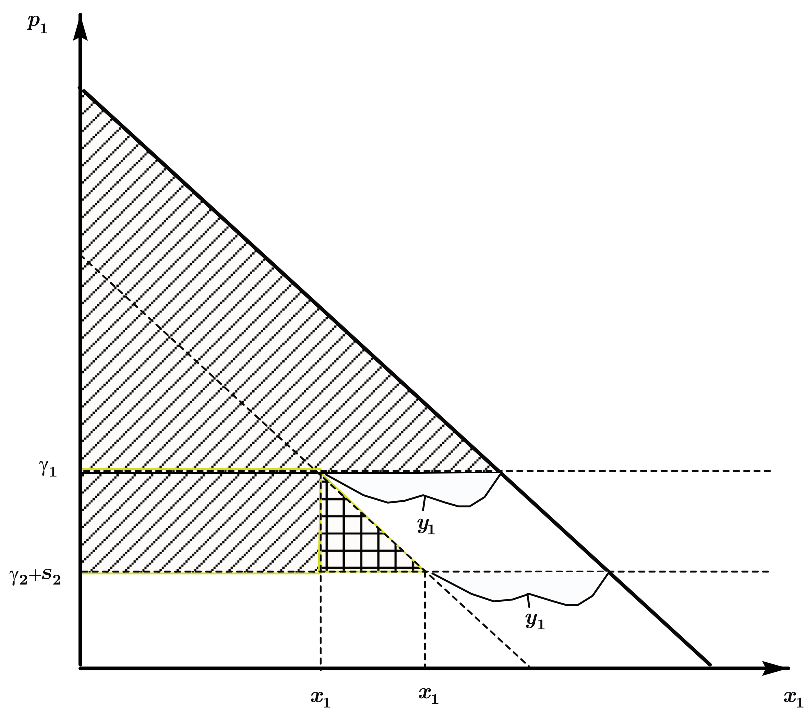

Figure 1. Country 1′s total surplus with the fixed production quantity y_1 for the cases of no subsidy and an optimal subsidy.

The final step in solving for the optimal quota-subsidy pair for country 1 is finding the optimal home production level. It turns out that the optimal home production level is a unique quantity y_1 that equates the marginal benefit of additional contested resources ($\widehat{R}_1'(y_1)$) to the marginal cost of producing from home instead of importing ($\gamma_1 - \gamma_2 + s_2$). This is exactly the definition of $\widetilde{y}(s_2)$. Following from the reasoning in the previous paragraph, the optimal quota-subsidy pair is $s_1 = \gamma_1 - \gamma_2 + s_2$ and $q = \delta_1(\gamma_2 - s_2) - \widetilde{y}(s_2)$.

The multiplicity of equilibrium is a consequence of the nature of the subsidy best response of country 2. Given country 1 uses some quota-subsidy pair $s_1 > \gamma_1 - \gamma_2$, $q < \delta_1(\gamma_1 - s_1)$, country 2 has some limited control over the home production of country 1 by way of its own subsidy. If the subsidy of country 2 is less than $\gamma_2 - \gamma_1 + s_1$, then the consumers of country 1 will not import and the home production of country 1 will be $\delta_1(\gamma_1 - s_1)$. In contrast, if the subsidy of country 2 is greater than or equal to $\gamma_2 - \gamma_1 + s_1$, then the consumers of country 1 will import up to the quota q and the home production of country 1 will be $\delta_1(\gamma_2 - s_2) - q$. Therefore, the optimal choice of country 2 is either pick $s_2 = 0$ and receive the benefit of no subsidization while losing contested resources, or pick $s_2 = \gamma_2 - \gamma_1 + s_1$ which is a costly subsidy. The multiple equilibrium described in the statement of the proposition 1 capture the range subsidies for country 2 that give a greater benefit in terms of contested resources than the cost of subsidization when compared to no subsidy.

Our second primary result shows that the equilibrium total surplus of country 1 is increasing in the equilibrium subsidy of country 2.

Proposition 2 *If (†) does not holds, then the set of equilibrium is strictly ordered in terms of importer's welfare as follows: Consider two equilibrium subsidies for the exporter $s_2^{*'}$ and s_2^*. If $s_2^{*'} > s_2^*$, then $TS_1^{*'} > TS_1^*$.*

The result shows that subsidization by country 2 creates more gains from trade to country 1 by lowering the price on the dual-use good than losses by lowering country 1's share of the contested resources. The intuition for this is based on the fact that with a high subsidy for country 2, country 1 can use the same subsidy and home production level as is optimal with the lower subsidy by the country 2. The total surplus of the second case is higher than the first because the price paid for the imports by country 1 consumers is lower, while everything else is the same for country 1. Note this option is no better than the best response of quota and subsidy for the high subsidy of country 2. Therefore, the high subsidy equilibrium must result in higher total surplus for country 1.

Conclusion

This section is used to contextualize the policy implications of our main results. First, we discuss the impact of two relevant extensions of the model. Second, we discuss how our results can be used to advise policy decisions.

The two extensions are: (i) including the dual-use good as part of the production of force purchased by the government, (ii) adding a third middle efficiency country to the model. We argue that neither of these additions changes the qualitative structure of the results, instead they only make the equilibrium use of trade restriction more or less common in terms of model parametrizations. In this discussion, we try to explicate the reasoning as clearly as possible without all the formalities of models, although clearly these arguments are not as complete as would be the case with the additional formality.

As discussed briefly in the text, adding military consumption of the dual-use good (imported or produced at home) into the model lowers the importance of the home dual-use production capacity. This is because the importing nation's production of force, hence grab of contested resources, is greater with more military dual-use consumption. Low-cost imports of the dual-use good can be used to increases the militaries dual-use consumption. Therefore, adding a quota on imports could lead to less military consumption of the dual-use good. This creates an additional marginal cost of increasing home production via import quota. Thus, we would have an additional term on the left-hand side of the inequality (†) that is negative. Based on the negative effect of this term on country 1's optimal home production choice, we can assert with reasonable confidence that the overall impact of this addition is that trade restrictions become less common in equilibrium and, in the case that they are used, the quota is relatively larger than in the base model.

Now let us consider the case that a third country is added to model with a marginal cost of production strictly between the other two countries and this country does not have contested resources with the other two nations. With no trade restrictions, all country's import the dual good from country 1. If we assume that trade between country 1 and country 3 is not disrupted in time of conflict between the first two countries, then instead of implementing an import quota and producing for itself, country 1 can put an import quota only on country 2 and import some of the dual-use good from country 3. In this extension, the production of country 3 serves the same function as the home production in the base model. In comparison to home production, the production of country 3 is done at a lower cost, changing the right-hand side of expression (†) to the smaller difference in marginal cost. Therefore, we expect this extension to make trade restrictions on imports of country 2 more common and with smaller quotas than in the base model.

Now we turn to contextualizing our results for policy. Thinking in terms of pragmatic policy there must be some way to approximately measure the marginal effects of expression (†). Empirical analysis should be relatively straight forward to approximately measure the marginal cost difference on the right-hand side of (†), as this is a somewhat standard economic problem. By contrast, measuring the left-hand side of (†), the marginal gain in contested resources, is conceptually more difficult. This exercise first requires that the contested resources of the two nations are clearly understood. The second conceptual step in measurement is assessing how exactly increasing the home production base of the dual-use good would impact a conflict over these resources. If the conflict is likely to be short and not rely pivotally on weapons in which the dual-use home production matters, then the marginal benefit from increasing the base is low. In contrast, if such a conflict is more likely to be drawn out and determined by the production capacity of dual-use good, then the marginal benefit could be extremely high. As outlined here, the relevant contribution for policy-makers is to provide some clear structure to guide reasoning about such intrusive policies.

Notes

1. See for example: Adams (2018), Barfield (2021), Congressional Research Service report IF11897 (2021), Goodman (2021), Hunter (2018), Rasser and Lamberth (2021), Roberts, Moraes, and Ferguson (2018), and Werner (2018).
2. See for example Defense Science Board Task Force (2005).
3. For example the entire treatment of the most prominent graduate textbook in international trade Feenstra (2016) does not consider the problem of insecure international property.
4. Some important contributions to this literature include Charles, Anderton, and Carter (1999), Skaperdas (1992), Skaperdas and Syropoulos (2002)), Anderson and Marcouiller (2005), Acemoglu et al. (2012), Garfinkel and Skaperdas (2007) and Garfinkel, Skaperdas, and Syropoulos (2015).
5. An export quota can be useful as a way to take advantage of the nation's market power to improve terms of trade. In our model this would only occur when the value of contested resources between the two nations is sufficiently small compared to the gains from trade of such a policy. Since we are interested in the baseline free trade case in which the importer has no production base and the value of the contested resources is relatively large, we gain parsimony by this omitting the export quota from the formal model.
6. The reason that country 1's consumers of buy from country 2 producers first in the case that $p_1 = p_2$ is purely technical, as it closes the model so that the equilibrium is in pure strategies. A different, more cumbersome, modeling choice that results in an arbitrarily close equilibrium to ours, is to discretize the set of prices to an arbitrary fine finite space.

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6 Appendix

6.1 Derivation of Section 3.2

In the derivation we omit the argument y_i from the function a_i to simplify the expressions. We begin with computing the derivative of $\partial\varphi_i/\partial g_i$:

$$\frac{\partial\varphi_i}{\partial g_i}=\frac{a_i}{a_1g_1+a_2g_2}-\frac{a_i^2g_i}{(a_1g_1+a_2g_2)^2}$$

$$=\frac{a_1a_2g_j}{(a_1g_1+a_2g_2)^2}.$$

Plugging this into (3) we have,

$$r_0\frac{a_1a_2g_j^*}{\left(a_1g_1^*+a_2g_2^*\right)^2}-1=0. \tag{9}$$

Using the condition (3) for both i, it must be that $g_1^*=g_2^*$. Inputting this equality into (9) we solve for g_j^*:

$$g_j^*=r_0\frac{a_1a_2}{(a_1+a_2)^2}.$$

Plugging back into the total surplus R_i of country i we have

$$\widehat{R}_i=r_i+r_0\frac{a_i}{a_1+a_2}-r_0\frac{a_1a_2}{(a_1+a_2)^2} \tag{10}$$

$$=r_i+r_0\left(\frac{a_i}{a_1+a_2}\right)^2. \tag{11}$$

6.2 Proofs of Propositions

Before moving to the proof of Proposition 1 we state two lemmas used in the proof. We begin with the following lemma that specifies the optimal trade restrictions of country 1, given a fixed size of production y_1.

Lemma 1 *Given some fixed $s_2\geq 0$, any production $y_1\leq\delta_1(\gamma_2-s_2)$ is implemented at the maximal total surplus for country 1 with $s_1^{'}=\gamma_1-\gamma_2+s_2$, and $q^{'}=\delta_1(\gamma_2-s_2)-y_1$.*

Proof of Lemma 1. Consider any subsidy $s_1^{''}\geq 0$, it must pair with $q^{''}=\delta_1(\gamma_1-s_1^{''})-y_1$ to implement y_1. We calculate the total surplus of this policy

$$TS_1^{''}=\widehat{R}_1(y_1)-\gamma_1y_1-(\gamma_2-s_2)q^{''}+v_1(\delta_1(\gamma_1-s_1^{''}))$$

$$=\widehat{R}_1(y_1)-(\gamma_2-s_2)\delta_1(\gamma_1-s_1^{''})-(\gamma_1-\gamma_2+s_2)y_1+v_1(\delta_1(\gamma_1-s_1^{''}))$$

By the definition of $\delta_1(p_1)$ as the unique maximizer of $u(x_1)-p_1x_1$, the unique subsidy and quota pair that maximizes total surplus at y_1 must be is $s_1^{'}=\gamma_1-\gamma_2+s_2$, and $q^{'}=\delta_1(\gamma_2-s_2)-y_1$.■

The second lemma characterizes the best response of country 1 in the trade restriction subgame.

Lemma 2 *For any $s_2\geq 0$,*
1) if $\widehat{R}_1^{'}(0)\leq\gamma_1-\gamma_2+s_2$, then the best response of country 1 is $s_1<\gamma_1-\gamma_2+s_2$, and $q\geq\delta_1(\gamma_2-s_2)$;
2) if $\widehat{R}_1^{'}(0)>\gamma_1-\gamma_2+s_2$, then the best response of country 1 is $s_1=\gamma_1-\gamma_2+s_2$, and $q=\delta_1(\gamma_2-s_2)-\widetilde{y}(s_2)$.

Proof of Lemma 2. First we show that $\widehat{R}_1^{''}(y_1)\leq 0$ and $\widehat{R}_1^{''}(0)<0$.

$$\widehat{R}_1''(y_1) = \frac{2r_0(a_1'(y_1)(a_2 - 2a_1(y_1)) + a_1''(y_1)(a_1(y_1) + a_2)a_1(y_1))}{(a_1(y_1) + a_2)^4}$$

$$\leq \frac{2r_0a_1'(y_1)(a_2 - 2a_1(y_1))}{(a_1(y_1) + a_2)^4}$$

Based on the fact that $a_1(y_1)$ is non-decreasing and concave, and by assumption $a_2 < 2a_1(y_1)$, it must be that $\widehat{R}_1''(y_1) \leq 0$. Since $a_1'(0) > 0$, $\widehat{R}_1''(0) = 0$.

Now, we prove Part 1 of the lemma. Since $\widehat{R}_1(y_1)$ is concave for any $y_1 \geq 0$ and strictly concave at $y_1 = 0$, total surplus of country 1 is uniquely maximized at $y_1 = 0$. All subsidies less than $\gamma_1 - \gamma_2 + s_2$ do not impact trade and have no effect on the market outcome. Any subsidy strictly larger than $\gamma_1 - \gamma_2 + s_2$ switches to full production which has lower total surplus for country 1 based on the fact that $\widehat{R}_1'(0) \leq \gamma_1 - \gamma_2 + s_2$ and the zero production is optimal.

Part 2. The condition in 2 makes a positive level of production optimal. Based on the assumption that $\widehat{R}_i'(\delta_1(\gamma_1)) \leq \gamma_1 - \gamma_2$, this y_1 is less than $\delta_1(\gamma_1)$. Its uniquely defined by $\widetilde{y}(s_2)$ and the subsidy the lemma statement are based on Lemma 1.■

Now we can prove Proposition 1.

Proof of Proposition 1. Part 1. First, based on Lemma 2, $s_1 < \gamma_1 - \gamma_2$ and $y_1 = 0$ is the only best response for country 1 to $s_2 = 0$. Now we show that $s_2 = 0$ is a best response for country 2. For any $s_2 > 0$

$$\widehat{TS}_2(s_2) = \widehat{R}_2(0) - \gamma_2\delta_2(\gamma_2 - s_2) - s_2\delta_1(\gamma_2 - s_2) + v_2(\delta_2(\gamma_2 - s_2))$$

$$< \widehat{R}_2(0) - \gamma_2\delta_2(\gamma_2 - s_2) - s_2\delta_1(\gamma_2) + v_2(\delta_2(\gamma_2 - s_2))$$

$$< \widehat{R}_2(0) - \gamma_2\delta_2(\gamma_2) - s_2\delta_1(\gamma_2) + v_2(\delta_2(\gamma_2))$$

$$= \widehat{TS}_2(0).$$

The second strict inequality follows from the definition of $\delta_1(\gamma_2)$ as the unique maximizer of $v_2(x_2) - -\gamma_2 x_2$.

Second, we show that in this case there are no other equilibria. Consider any $s_2 > 0$ and based on Lemma 2 all best responses of country 1 are of the form $s_1 < \gamma_1 - \gamma_2 + s_2$ and $y_1 = 0$. So again the total surplus of country 2 can be written

$$\widehat{TS}_2(s_2) = \widehat{R}_2(0) - \gamma_2\delta_2(\gamma_2 - s_2) - s_2\delta_1(\gamma_2 - s_2) + v_2(\delta_2(\gamma_2 - s_2))$$

Any slight decrease in s_2 to $s_2 - \in$ for $\in > 0$ such that $s_1 < \gamma_1 - \gamma_2 + s_2 - \varepsilon$ gives country 2 total surplus

$$\widehat{TS}_2(s_2 - \in) = \hat{R}_2(0) - \gamma_2\delta_2(\gamma_2 - s_2 + \in) - (s_2 - \in)\delta_1(\gamma_2 - s_2 + \in) + v_2(\delta_2(\gamma_2 - s_2 + \in))$$

$$\leq \hat{R}_2(0) - \gamma_2\delta_2(\gamma_2 - s_2 + \in) - s_2\delta_1(\gamma_2 - s_2 + \in) + \in\delta_1(\gamma_2 - s_2 + \in) + v_2(\hat{x}_2)$$

Since $\delta_1(\gamma_2 - s_2 + \in) \leq \delta_1(\gamma_2 - s_2)$ and $v_2(\delta_2(\gamma_2 - s_2 + \in)) - \gamma_2\delta_2(\gamma_2 - s_2 + \in) \geq v_2(\delta_2(\gamma_2 - s_2)) - \gamma_2\delta_2(\gamma_2 - s_2)$, this is strictly more than the total surplus with subsidy s_2. Thus, any such trade restrictions cannot be an equilibrium.

Part 2. We begin this part by showing that $s_1 = \gamma_1 - \gamma_2$, $q = \delta_1(\gamma_2) - \overline{y}_1(0)$ and $s_2 = 0$ is an subgame perfect equilibrium. Given these subsidies and that (†) does not hold, based on Lemma 2 the total surplus of country 1 is maximized at this policy. It remains to show that $s_2 = 0$ is a best response for country 2. The total surplus of country 2 at $s_2 = 0$ is

$$\widehat{TS}_2(0) = \widehat{R}_2(\overline{y}_1(0)) - \gamma_2\widehat{x}_2 + v_2(\widehat{x}_2)$$

For any $s_2 > 0$, the total surplus of country 2 is

$$\widehat{TS}_2(s_2) = \widehat{R}_2(\overline{y}_1(0)) - \gamma_2\delta_2(\gamma_2 - s_2) - s_2\delta_1(\gamma_2 - s_2) + v_2(\delta_2(\gamma_2 - s_2)),$$

which is strictly less.

Next, we show that the set of trade restrictions described in the proposition are equilibria. Given condition (†) does not hold and (5) in the statement of the proposition, Lemma 2 tells us that $q^* = \delta_1(\gamma_2 - s_2^*) - \widetilde{y}(s_2^*)$, $s_1^* = \gamma_1 - \gamma_2 + s_2^*$ is the best response of country 1. For country 2, the only possible profitable defection from s_2^* is to subsidize less than s_2^* and have no exports to country 1. At s_2^* total surplus of country 2 is

$$\widehat{R}_2(\widetilde{y}(s_2^*)) - \gamma_2\delta_2(\gamma_2 - s_2^*) - s_2^*\big(\delta_1(\gamma_2 - s_2) - \widetilde{y}(s_2^*)\big) + v_2(\delta_2(\gamma_2 - s_2^*)),$$

while for any $s_2^{'} < s_2^*$ total surplus is

$$\widehat{R}_2(\delta_1(\gamma_2 - s_2^*)) - \gamma_2\delta_2(\gamma_2 - s_2^{'}) + v_2(\delta_2(\gamma_2 - s_2^{'})).$$

This is largest at $s_2^{'} = 0$. The total surplus at s_2^* is weakly greater than at any $s_2^{'} \in [0, s_2^*]$ if

$$\widehat{R}_2(\widetilde{y}(s_2^*)) - \gamma_2\delta_2(\gamma_2 - s_2^*) - s_2^*\big(\delta_1(\gamma_2 - s_2) - \widetilde{y}(s_2^*)\big) + v_2(\delta_2(\gamma_2 - s_2^*))$$

$$\geq \widehat{R}_2(\delta_1(\gamma_2 - s_2^*)) - \gamma_2\delta_2(\gamma_2 - s_2^{'}) + v_2(\delta_2(\gamma_2 - s_2^{'})),$$

which is equivalent to the inequality in condition (6).

Finally, we argue that no other equilibria exist. Note that condition (†) not holding rules out the possibility of any equilibrium without binding restriction holding since, based on Lemma 2, country 1 will respond with binding trade restrictions to all $s_2 \in (0, \widehat{R}_1^{'}(0) - \gamma_1 + \gamma_2)$. If $s_2 \geq \widehat{R}_1^{'}(0) - \gamma_1 + \gamma_2$, then country 1 will not use restrictions, but country 2 will never find it optimal to subsidize. This is because at any $s_2 > 0$, the total surplus of country 2 is strictly decreasing in s_2 (using the same argument as in Part 1 of the proof).■

Proof of Proposition 2. Consider two equilibrium with subsidies for country 2: $s_2^{'} > s_2^*$. Based on the fact that $\widehat{R}_1(y_1)$ is non-decreasing and concave in y_1, $\widetilde{y}(s_2^{'}) \leq \widetilde{y}(s^*)$. The total surplus of country 1 can be written

$$\begin{aligned}
TS_1^{'} &= \widehat{R}_1\left(\widetilde{y}(s_2^{'})\right) - \gamma_1\widetilde{y}(s_2^{'}) - \left(\gamma_2 - s_2^{'}\right)\left(\delta_1(\gamma_2 - s_2^{'}) - \widetilde{y}(s_2^{'})\right) + v_1(\delta_1(\gamma_2 - s_2^{'})) \\
&\geq \widehat{R}_1\left(\widetilde{y}(s_2^*)\right) - \gamma_1\widetilde{y}(s_2^*) - \left(\gamma_2 - s_2^{'}\right)\left(\delta_1(\gamma_2 - s_2^{'}) - \widetilde{y}(s_2^*)\right) + v_1(\delta_1(\gamma_2 - s_2^{'})) \\
&\geq \widehat{R}_1\left(\widetilde{y}(s_2^*)\right) - \gamma_1\widetilde{y}(s_2^*) - \left(\gamma_2 - s_2^{'}\right)\left(\delta_1(\gamma_2 - s_2^*) - \widetilde{y}(s_2^*)\right) + v_1(\delta_1(\gamma_2 - s_2^*)) \\
&> \widehat{R}_1\left(\widetilde{y}(s_2^*)\right) + \gamma_1\widetilde{y}(s_2^*) - \left(\gamma_2 - s_2^*\right)\left(\delta_1(\gamma_2 - s_2^*) - \widetilde{y}(s_2^*)\right) + v_1(\delta_1(\gamma_2 - s_2^*)) \\
&= TS_1^*.
\end{aligned}$$

The first inequality follows from the suboptimality fro country 1 of $\widetilde{y}(s_2^*)$ for the equilibrium with subsidy $s_2^{'}$. The second inequality follows $\delta_1(\gamma_2 - s_2^{'})$ as the unique maximizer of $v_1(x_1) - (\gamma_2 - s_2^{'})x_1$. The third inequality is simply based on the fact that $s_2^{'} > s_2^*$.