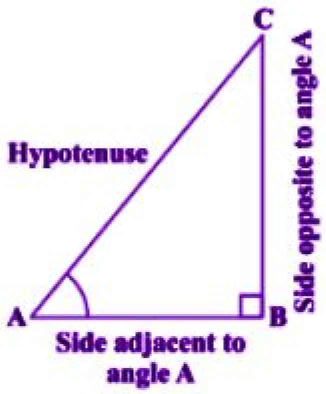
Trigonometric Ratios

Opposite & Adjacent Sides in a Right Angled Triangle

In the ΔABC right-angled at B, BC is the side opposite to ∠A, AC is the hypotenuse and AB is the side adjacent to ∠A.



Trigonometric Ratios

For the right ΔABC, right-angled at ∠B, the trigonometric ratios of the ∠A are as follows:

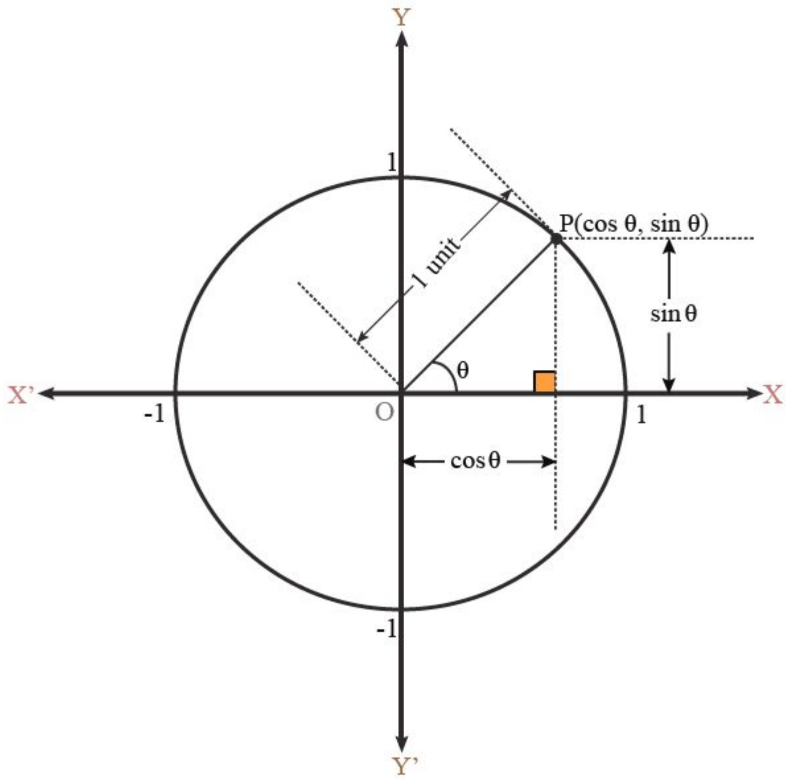
- sin A=opposite side/hypotenuse=BC/AC
- cos A=adjacent side/hypotenuse=AB/AC
- tan A=opposite side/adjacent side=BC/AB
- cosec A=hypotenuse/opposite side=AC/BC
- sec A=hypotenuse/adjacent side=AC/AB
- cot A=adjacent side/opposite side=AB/BC

Visualization of Trigonometric Ratios Using a Unit Circle

Draw a circle of unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre which makes an angle θ with the x-axis. Draw a perpendicular from P to the x-axis to cut it at Q.

- sinθ=PQ/OP=PQ/1=PQ
- cosθ=OQ/OP=OQ/1=OQ
- tanθ=PQ/OQ=sinθ/cosθ
- cosecθ=OP/PQ=1/PQ
- secθ=OP/OQ=1/OQ

- cotθ=OQ/PQ=cosθ/sinθ



Visualisation of Trigonometric Ratios Using a Unit Circle

Relation between Trigonometric Ratios

- cosec θ =1/sin θ
- sec θ = 1/cos θ
- tan θ = sin θ/cos θ
- cot θ = cos θ/sin θ=1/tan θ

Trigonometric Ratios of Specific Angles

Range of Trigonometric Ratios from 0 to 90 degrees

For $0^\circ \leq \theta \leq 90^\circ$,

- $0 \leq \sin\theta \leq 1$
- $0 \leq \cos\theta \leq 1$

- $0 \le \tan\theta < \infty$
- $1 \le \sec\theta < \infty$
- $0 \le \cot\theta < \infty$
- $1 \le \mathrm{cosec}\theta < \infty$

$\tan\theta$ and $\sec\theta$ are not defined at 90°.

$\cot\theta$ and $\mathrm{cosec}\theta$ are not defined at 0°.

Variation of trigonometric ratios from 0 to 90 degrees

As θ increases from 0° to 90°

- sinθ increases from 0 to 1
- cosθ decreases from 1 to 0
- tanθ increases from 0 to ∞
- cosecθ decreases from ∞ to 1
- secθ increases from 1 to ∞
- cotθ decreases from ∞ to 0

Standard values of Trigonometric ratios

∠A	0º	30º	45º	60º	90º
sin A	0	1/2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	not defined
cot A	not defined	√3	1	1/√3	0

Trigonometric Ratios of Complementary Angles

Complementary Trigonometric ratios

If θ is an acute angle, its complementary angle is $90^{\circ} - \theta$. The following relations hold true for trigonometric ratios of complementary angles.

- $\sin (90^{\circ} - \theta) = \cos \theta$
- $\cos (90^{\circ} - \theta) = \sin \theta$
- $\tan (90^{\circ} - \theta) = \cot \theta$
- $\cot (90^{\circ} - \theta) = \tan \theta$
- $\text{cosec} (90^{\circ} - \theta) = \sec \theta$
- $\sec (90^{\circ} - \theta) = \text{cosec}\, \theta$

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \cot^2\theta = \text{coesc}^2\theta$
- $1 + \tan^2\theta = \sec^2\theta$