Introduction to Real Numbers

Real Numbers

- Real numbers constitute the union of all rational and irrational numbers.
- Any real number can be plotted on the number line.

Euclid’s Division Lemma

- Euclid’s Division Lemma states that given two integers *a* and *b*, there exists a unique pair of integers *q* and *r* such that $a=b\times q+r$ and $0\le r<b$.
- This lemma is essentially equivalent to : *dividend* = *divisor* × *quotient* + *remainder*
- In other words, for a given pair of dividend and divisor, the quotient and remainder obtained are going to be unique.

Euclid’s Division Algorithm

- Euclid’s Division Algorithm is a method used to find the **H.C.F** of two numbers, say *a* and *b* where $a> b$.
- We apply Euclid’s Division Lemma to find two integers *q* and *r* such that $a=b\times q+r$ and $0\le r<b$.
- If r = 0, the H.C.F is b, else, we apply Euclid’s division Lemma to b (the divisor) and r (the remainder) to get another pair of quotient and remainder.
- The above method is repeated until a remainder of zero is obtained. The divisor in that step is the H.C.F of the given set of numbers.

The Fundamental Theorem of Arithmetic

Prime Factorisation

- Prime Factorisation is the method of expressing a natural number as a product of prime numbers.
- Example: $36=2\times2\times3\times3$ is the prime factorisation of 36.

Fundamental Theorem of Arithmetic

- The Fundamental Theorem of Arithmetic states that the prime factorisation for a given number is unique if the arrangement of the prime factors is ignored.
- Example: $36=2\times2\times3\times3$ OR, $36=2\times3\times2\times3$

- Therefore, 36 is represented as a product of prime factors (Two 2s and two 3s) ignoring the arrangement of the factors.

Method of Finding LCM

Example: To find the Least Common Multiple (**L.C.M**) of 36 and 56,

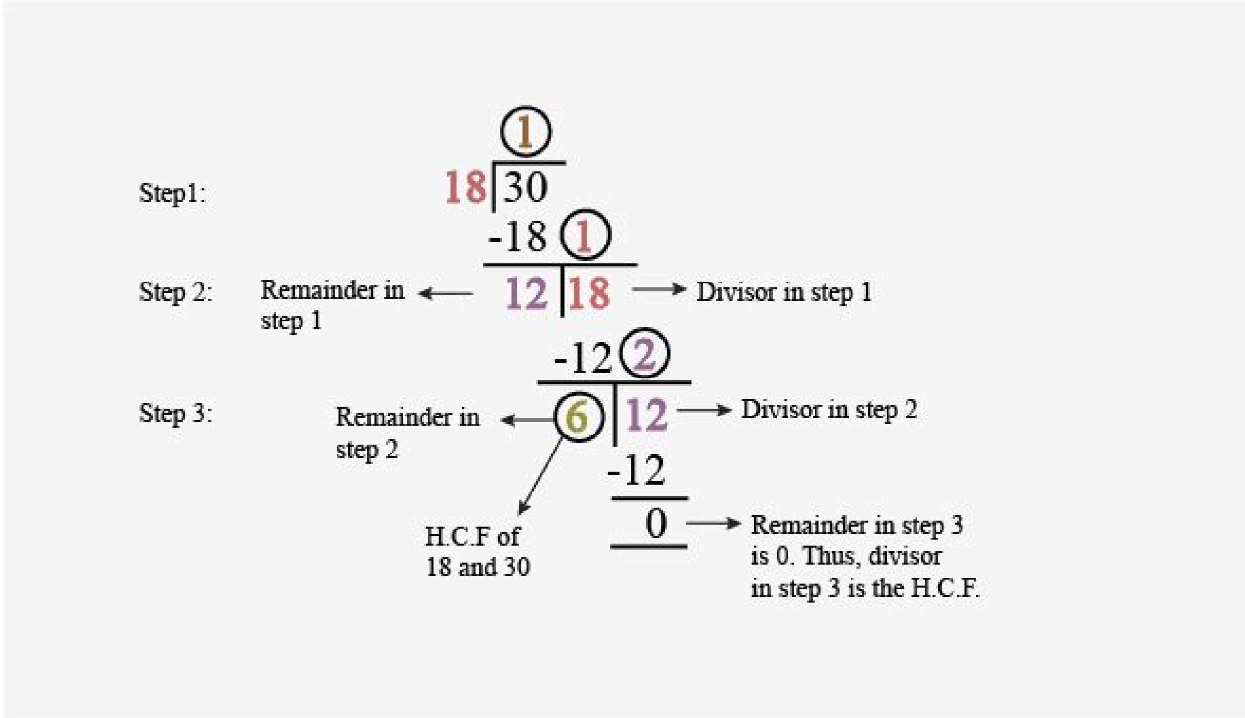
1. $36=2\times2\times3\times3$
   $56=2\times2\times2\times7$
2. The common prime factors are 2×2
3. The uncommon prime factors are 3×3 for 36 and 2×7 for 56.
4. LCM of 36 and 56 = $2\times2\times3\times3\times2\times7$ which is 504

Method of Finding HCF

H.C.F can be found using two methods – Prime factorisation and Euclid's division algorithm.

- Prime Factorisation:
  - Given two numbers, we express both of them as products of their respective prime factors. Then, we select the prime factors that are common to both the numbers
  - Example – To find the H.C.F of 20 and 24
    $20=2\times2\times5$ and $24=2\times2\times2\times3$
  - The factor common to 20 and 24 is 2×2, which is 4, which in turn is the H.C.F of 20 and 24.
- Euclid's Division Algorithm:
  - It is the repeated use of Euclid's division lemma to find the H.C.F of two numbers.

- Example: To find the HCF of 18 and 30



Finding the HCF of 18 and 30

- The required HCF is **6**.

Product of Two Numbers = HCF X LCM of the Two Numbers

- For any **two** positive integers a and b,
  a×b=H.C.F×L.C.M.
- Example – For 36 and 56, the H.C.F is 4 and the L.C.M is 504
  36×56=2016
  4×504=2016
  Thus, 36×56=4×504
- The above relationship, however, doesn’t hold true for 3 or more numbers

Applications of HCF & LCM in Real-World Problems

L.C.M can be used to find the points of common occurrence. This could be the common ringing of bells that ring with different frequencies, the time at which two persons running at different speeds meet, and so on.

Revisiting Irrational Numbers

Irrational Numbers

Any number that cannot be expressed in the form of p/q (where p and q are integers and q≠0.) is an irrational number. Examples √2,π, e and so on.

Number theory: Interesting results

- If a number p (a prime number) divides a^2, then p divides a. Example: 3 divides 6^2 i.e 36, which implies that 3 divides 6.
- The sum or difference of a rational and an irrational number is irrational
- The product and quotient of a non-zero rational and irrational number are irrational.
- √p is irrational when 'p' is a prime. For example, 7 is a prime number and √7 is irrational. The above statement can be proved by the method of "Proof by contradiction".

Proof by Contradiction

In the method of contradiction, to check whether a statement is TRUE

(i) We assume that the given statement is TRUE.

(ii) We arrive at some result which contradicts our assumption, thereby proving the contrary.

Eg: Prove that √7 is irrational.

Assumption: √7 is rational.

Since it is rational √7 can be expressed as

√7 = ab, where a and b are co-prime Integers, b ≠ 0.

On squaring, $a^2/b^2=7$

$\Rightarrow a^2=7b^2$.

Hence, 7 divides a. Then, there exists a number c such that a=7c. Then, $a^2=49c^2$. Hence, $7b^2=49c^2$ or $b^2=7c^2$.

Hence 7 divides b. Since 7 is a common factor for both a and b, it contradicts our assumption that a and b are coprime integers.

Hence, our initial assumption that √7 is rational is wrong. Therefore, √7 is irrational.

Revisiting Rational Numbers and Their Decimal Expansions

Rational Numbers

Rational numbers are numbers that can be written in the form pq, where p and q are integers and $q \neq 0$.

Examples – $12, 45, 1, 0, -3$ and so on.

Terminating and nonterminating decimals

Terminating decimals are decimals that end at a certain point. Example: 0.2, 2.56 and so on.

Non-terminating decimals are decimals where the digits after the decimal point don’t terminate. Example: 0.333333….., 0.13135235343…

Non-terminating decimals can be :

a) Recurring – a part of the decimal repeats indefinitely (0**.**142857***142857****….*)

b) Non-recurring – no part of the decimal repeats indefinitely. Example: $\pi = 3.1415926535\ldots$

Check if a given rational number is terminating or not

If ab is a rational number, then its decimal expansion would terminate if **both** of the following conditions are satisfied :

a) The H.C.F of a and b is 1.

b) b can be expressed as a prime factorisation of 2 and 5 i.e $b = 2_m \times 5_n$ where either m or n, or both can = 0.

If the prime factorisation of b contains any number other than 2 or 5, then the decimal expansion of that number will be recurring

Example:

$140 = 0.025$ is a terminating decimal, as the H.C.F of 1 and 40 is 1, and the denominator (40) can be expressed as $2_3 \times 5_1$.

$37 = 0.428571$ is a recurring decimal as the H.C.F of 3 and 7 is 1 and the denominator (7) is equal to 7_1

Class 10 Real Numbers Important Questions and Answers

Q.1: Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solution: Let x be any positive integer and y =3.

By Euclid's division algorithm;

x =3q +r (for some integer $q \geq 0$ and r = 0, 1, 2 as $r \geq 0$ and $r < 3$)

Therefore,

x = 3q, 3q+1 and 3q+2

As per the given question, if we take the square on both the sides, we get;

$x^2 = (3q)^2 = 9q^2 = 3.3q^2$

Let $3q^2 = m$

Therefore,

$x^2 = 3m$ ………………….(1)

$x^2 = (3q+1)^2 = (3q)^2 + 1^2 + 2 \times 3q \times 1 = 9q^2 + 1 + 6q = 3(3q^2 + 2q) + 1$

Substitute, $3q^2+2q = m$, to get,

$x^2 = 3m + 1$ ……………………………. (2)

$x^2 = (3q+2)^2 = (3q)^2+2^2+2 \times 3q \times 2 = 9q^2 + 4 + 12q = 3 (3q^2 + 4q + 1) + 1$

Again, substitute, $3q^2+4q+1 = m$, to get,

$x^2 = 3m + 1$…………………………… (3)

Hence, from eq. 1, 2 and 3, we conclude that, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Q.2: Express each number as a product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Solutions:

(i) 140

By Taking the LCM of 140, we will get the product of its prime factor.

Therefore, 140 = 2 × 2 × 5 × 7 × 1 = 2^2×5×7

(ii) 156

By Taking the LCM of 156, we will get the product of its prime factor.

Hence, 156 = 2 × 2 × 13 × 3 × 1 = 2^2 × 13 × 3

(iii) 3825

By Taking the LCM of 3825, we will get the product of its prime factor.

Hence, 3825 = 3 × 3 × 5 × 5 × 17 × 1 = 3^2 5^2×17

(iv) 5005

By Taking the LCM of 5005, we will get the product of its prime factor.

Hence, 5005 = 5 × 7 × 11 × 13 × 1 = 5 × 7 × 11 × 13

(v) 7429

By Taking the LCM of 7429, we will get the product of its prime factor.

Hence, 7429 = 17 × 19 × 23 × 1 = 17 × 19 × 23

Q.3: Given that HCF (306, 657) = 9, find LCM (306, 657).

Solutions: As we know that,

HCF×LCM=Product of the two given numbers

Therefore,

9 × LCM = 306 × 657

LCM = (306×657)/9 = 22338

Hence, LCM(306,657) = 22338

Q.4: Prove that $3 + 2\sqrt{5}$ is irrational.

Solutions: Let us say $3 + 2\sqrt{5}$ is rational.

Then the co-prime x and y of the given rational number where ($y \neq 0$) is such that:

$3 + 2\sqrt{5} = x/y$

Rearranging, we get,

$2\sqrt{5}= x/y - 3$

$\sqrt{5}=1/2(x/y-3)$

Since x and y are integers, thus, $1/2(x/y-3)$ is a rational number.

Therefore, $\sqrt{5}$ is also a rational number. But this confronts the fact that $\sqrt{5}$ is irrational.

Hence, we get that $3 + 2\sqrt{5}$ is irrational.

Q.5: Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) 13/3125 (ii) 17/8 (iii) 64/455 (iv) 15/1600

Solution:

Note: If the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has a terminating decimal expansion.

If the denominator has factors other than 2 and 5 then it has a non-terminating decimal expansion.

(i) 13/3125

Factorizing the denominator, we get,

$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Since the denominator has only 5 as its factor, 13/3125 has a terminating decimal expansion.

(ii)17/8

Factorizing the denominator, we get,

$8 = 2\times 2 \times 2=2^3$

Since the denominator has only 2 as its factor, 17/8 has a terminating decimal expansion.

(iii) 64/455

Factorizing the denominator, we get,

$455 = 5\times7\times13$

Since the denominator is not in the form of $2^m \times 5^n$, therefore 64/455 has a non-terminating decimal expansion.

(iv) 15/1600

Factorizing the denominator, we get,

$1600 = 2^6 5^2$

Since the denominator is in the form of $2^m \times 5^n$, thus 15/1600 has a terminating decimal expansion.

Q.6: The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p/q what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000. . .

Solution:

(i) 43.123456789

Since it has a terminating decimal expansion, it is a rational number in the form of p/q and q has factors of 2 and 5 only.

(ii) 0.120120012000120000. . .

Since, it has non-terminating and non- repeating decimal expansion, it is an irrational number.

Q.7: Check whether 6^n can end with the digit 0 for any natural number n.

Solutions: If the number 6n ends with the digit zero (0), then it should be divisible by 5, as we know any number with unit place as 0 or 5 is divisible by 5.

Prime factorization of $6^n = (2\times3)^n$

Therefore, the prime factorization of 6^n doesn't contain prime number 5.

Hence, it is clear that for any natural number n, 6^n is not divisible by 5 and thus it proves that 6^n cannot end with the digit 0 for any natural number n.

Extra questions for Class 10 Maths Chapter 1

Q.1: Find three rational numbers lying between 0 and 0.1. Find twenty rational numbers between 0 and 0.1. Give a method to determine any number of rational numbers between 0 and 0.1.

Q.2: Which of the following rational numbers have the terminating decimal representation?

(i) 3/5

(ii) 7/20

(iii) 2/13

(iv) 27/40

(v) 133/125

(vi) 23/7

Q.3: Write the following rational numbers in decimal form:

(i) 42/100

(ii) 27/8

(iii) 1/5

(iv) 2/13

(v) 327/500

(vi) 5/6

(vii) 1/7

(viii) 11/17

Q.4: If a is a positive rational number and n is a positive integer greater than 1, prove that a^n is a rational number.

Q.5: Show that $^3\sqrt{6}$ and $^3\sqrt{3}$ is not a rational number.

Q.6: Show that $2 + \sqrt{2}$ is not a rational number.

Q.7: Give an example to show that the product of a rational number and an irrational number may be a rational number.

Q.8: Prove that $\sqrt{3} – \sqrt{2}$ and $\sqrt{3} + \sqrt{5}$ are irrational.

Q.9: Express 7/64, 12/125 and 451/13 in decimal form..

Q.10: Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Q.11: Mention whether the following numbers are rational or irrational:

(i) $(\sqrt{2}+2)$

(ii) $(2-\sqrt{2}) \times (2+\sqrt{2})$

(iii) $(\sqrt{2} + \sqrt{3})^2$

(iv) $6/3\sqrt{2}$