Quadratic Equation

Get the complete concepts covered in quadratic equations for class 10 Maths. In this article, let us discuss the complete concept of quadratic equations, which includes the standard, nature of roots, methods for finding the solution for the given quadratic equations with more examples.

Introduction to Quadratic Equations

Quadratic Polynomial

> A polynomial of the form ax^2+bx+c, where a,b and c are real numbers and $a\neq0$ is called a quadratic polynomial.

Quadratic Equation

When we equate a quadratic polynomial to a constant, we get a quadratic equation.

Any equation of the form p(x)=c, where p(x) is a polynomial of degree 2 and c is a constant, is a quadratic equation.

The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2+bx+c=0$, where a,b and c are real numbers and $a\neq0$.

'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

Solving QE by Factorisation

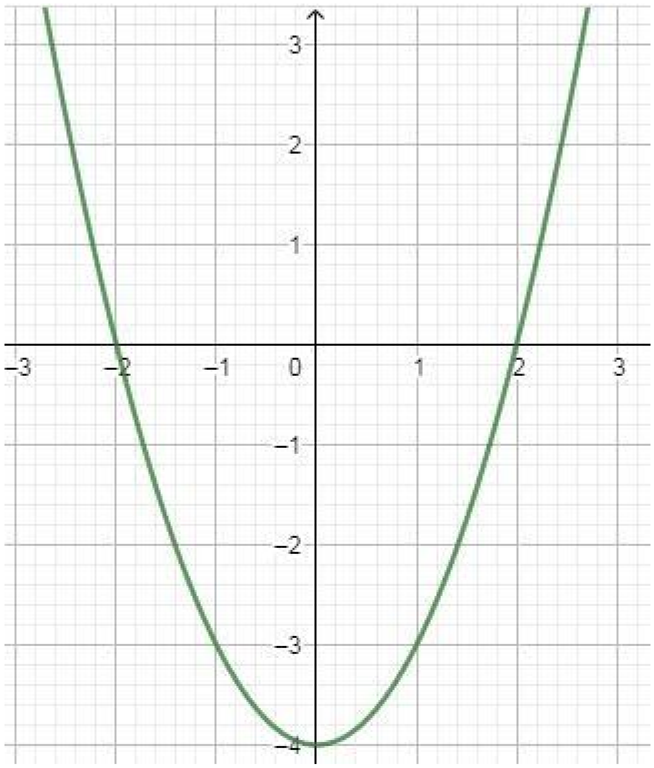
Roots of a Quadratic equation

The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

If α is a root of the quadratic equation $ax^2+bx+c=0$,then, $a\alpha^2+b\alpha+c=0$.

A quadratic equation can have two distinct roots, two equal roots or real roots may not exist.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2-4=0$

Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.
- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

Solving a Quadratic Equation by Factorization method

Consider a quadratic equation $2x^2-5x+3=0$

$\Rightarrow 2x^2-2x-3x+3=0$

This step is splitting the middle term

We split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x^2 and the constant.

$(-2) + (-3) = (-5)$

And $(-2) \times (-3) = 6$

$2x^2-2x-3x+3=0$

$2x(x-1)-3(x-1)=0$

$(x-1)(2x-3)=0$

In this step, we have expressed the quadratic polynomial as a product of its factors.

Thus, x = 1 and x =3/2 are the roots of the given quadratic equation.

This method of solving a quadratic equation is called the factorisation method.

Solving QE by Completing the Square

Solving a Quadratic Equation by Completion of squares method

In the method of completing the squares, the quadratic equation is expressed in the form $(x\pm k)^2=p^2$.

Consider the quadratic equation $2x^2-8x=10$

(i) Express the quadratic equation in standard form.

$2x^2-8x-10=0$

(ii) Divide the equation by the coefficient of x^2 to make the coefficient of x^2 equal to 1.

$x^2-4x-5=0$

(iii) Add square of half of the coefficient of x to both sides of the equation to get an expression of the form $x^2\pm 2kx+k^2$.

$(x^2-4x+4)-5=0+4$

(iv) Isolate the above expression, $(x\pm k)^2$ on the LHS to obtain an equation of the form $(x\pm k)^2=p^2$

$(x-2)^2=9$

(v) Take the positive and negative square roots.

$x-2=\pm3$

$x=-1$ or $x=5$

Solving QE Using Quadratic Formula

Quadratic Formula

Quadratic Formula is used to directly obtain the roots of a quadratic equation from the standard form of the equation.

For the quadratic equation $ax^2+bx+c=0$,

$x= [-b\pm \sqrt{(b^2-4ac)}]/2a$

By substituting the values of a,b and c, we can directly get the roots of the equation.

Discriminant

For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the **discriminant**, (denoted by **D**), of the quadratic equation.

The **discriminant** determines the **nature of roots** of the quadratic equation based on the **coefficients** of the quadratic polynomial.

Solving using Quadratic Formula when D>0

Solve $2x^2-7x+3=0$ using the quadratic formula.

(i) Identify the coefficients of the quadratic polynomial. a = 2,b = -7,c = 3

(ii) Calculate the discriminant, b^2-4ac

$D=(-7)^2-4\times2\times3=25$

$D> 0$, therefore, the roots are distinct.

(iii) Substitute the coefficients in the quadratic formula to find the roots

$x= [-(-7)\pm \sqrt{((-7)^2-4(2)(3))}]/2(2)$

$x=(7 \pm5)/4$

$x=3$ and $x=1/2$ are the roots.

Nature of Roots

Based on the value of the discriminant, $D=b^2-4ac$, the roots of a quadratic equation can be of three types.

Case 1: If **D>0**, the equation has two **distinct real roots**.

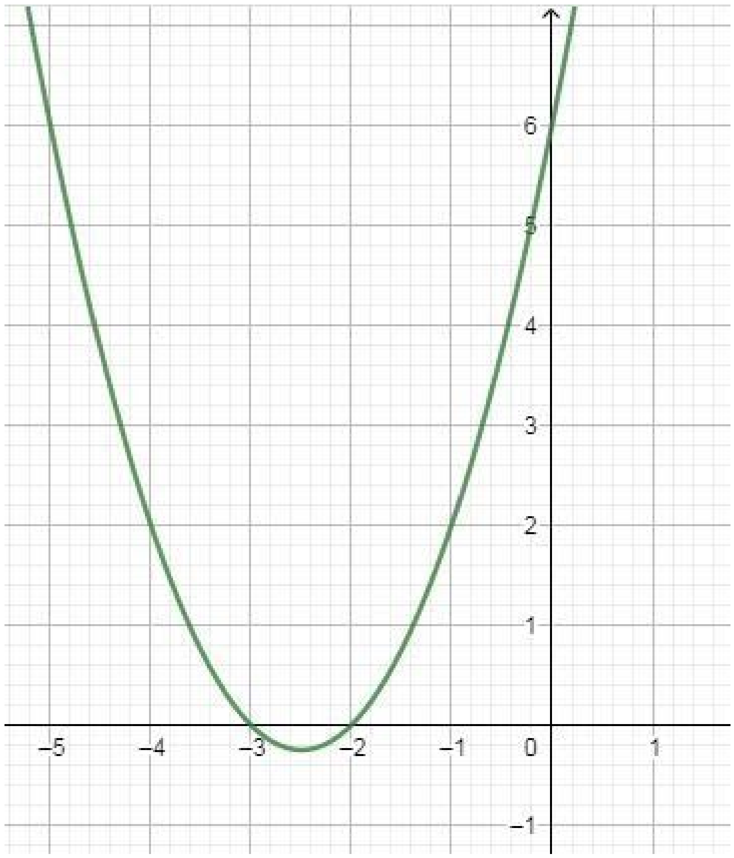
Case 2: If **D=0**, the equation has two **equal real roots**.

Case 3: If **D<0**, the equation has **no real roots**.

Be More Curious

Graphical Representation of a Quadratic Equation

The graph of a quadratic polynomial is a parabola. The roots of a quadratic equation are the points where the parabola cuts the x-axis i.e. the points where the value of the quadratic polynomial is zero.



In the above figure, -2 and -3 are the roots of the quadratic equation

$x^2+5x+6=0$.

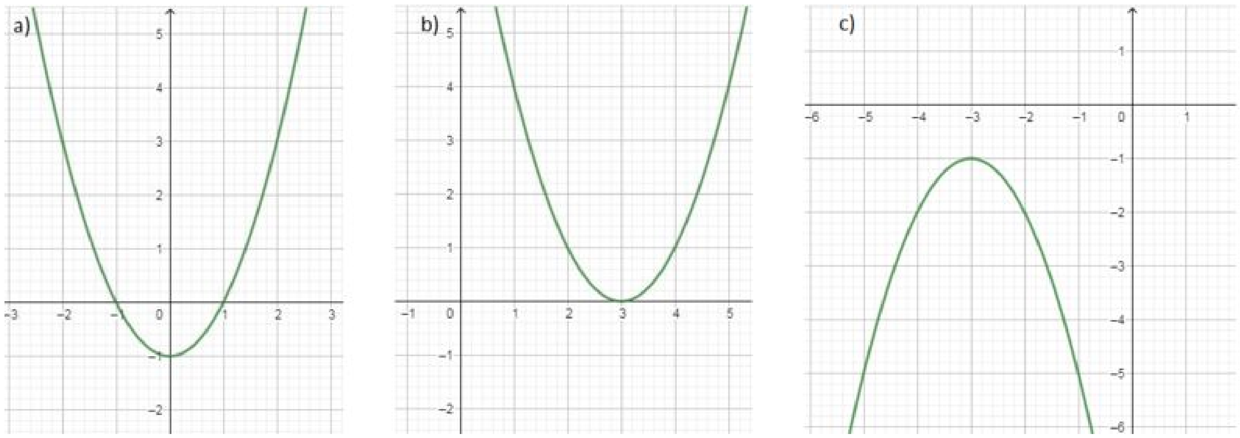
For a quadratic polynomial ax^2+bx+c,

If **a>0**, the parabola opens **upwards**.

If **a<0**, the parabola opens **downwards.**

If **a = 0**, the polynomial will become a first-degree polynomial and its graph is linear.

The discriminant, $D=b^2-4ac$



Nature of graph for different values of D.

If **D>0**, the parabola cuts the x-axis at exactly two distinct points. The roots are distinct. This case is shown in the above figure in a,

where the quadratic polynomial cuts the x-axis at **two distinct points.**

If **D=0**, the parabola just touches the x-axis at one point and the rest of the parabola lies above or below the x-axis. In this case, the roots are equal.

This case is shown in the above figure in b, where the quadratic polynomial touches the x-axis at **only a single point**.

If **D<0**, the parabola lies entirely above or below the x-axis and there is no point of contact with the x-axis. In this case, there are no real roots.

This case is shown in the above figure in c, where the quadratic polynomial **neither cuts nor touches** the x-axis.

Formation of a quadratic equation from its roots

To find out the standard form of a quadratic equation when the roots are given:

Let α and β be the roots of the quadratic equation $ax^2+bx+c=0$. Then,

$(x-\alpha)(x-\beta)=0$

On expanding, we get,

$x^2-(\alpha+\beta)x+\alpha\beta=0$, which is the standard form of the quadratic equation. Here, $a=1, b=-(\alpha+\beta)$ and $c=\alpha\beta$.

Sum and Product of Roots of a Quadratic equation

Let α and β be the roots of the quadratic equation $ax^2+bx+c=0$. Then,

Sum of roots $=\alpha+\beta=-b/a$

Product of roots $=\alpha\beta= c/a$