Fundamentals of Optimization (MATH11111)

Optimization is concerned with finding the best solution of a decision problem among a set of candidate solutions. Optimization problems arise in many different settings, ranging from energy and communication networks to machine learning and treatment planning.

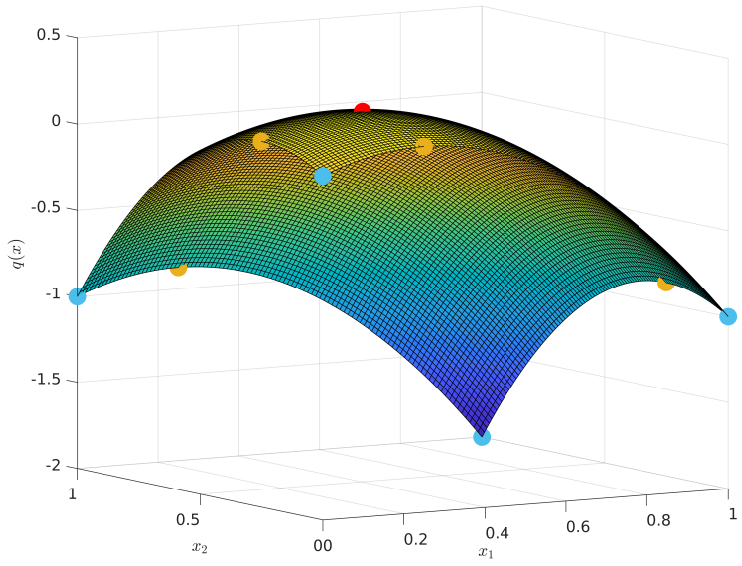

Figure 1: An optimization problem in two variables (the blue dots denote local minimizers, the red dot denotes the local maximizer, and the yellow dots denote the saddle points)

Course Information

Time:	Semester 1, 2022/2023
Course Organiser:	Dr E. Alper Yıldırım (JCMB 5415)
E-mail:	`E.A.Yildirim@ed.ac.uk`
Course Secretary:	Ms Gemma Aitchison, e-mail: `Gemma.Aitchison@ed.ac.uk`
SCQF Credits:	10
Credit Level:	SCQF Level 11 (Postgraduate)
Prerequisites:	None (some background in proof methods, multivariable calculus and linear algebra will be assumed)

Course Teaching Team

Name	Role	E-mail
Dr E. Alper Yıldırım	Course Organiser	E.A.Yildirim@ed.ac.uk
Bárbara Rodrigues	Tutor	b.c.rodrigues@sms.ed.ac.uk
Yuzhou Qiu	Tutor	y.qiu-16@sms.ed.ac.uk
Filippo Zanetti	Tutor	f.zanetti@sms.ed.ac.uk

Course Information on the Web

http://www.drps.ed.ac.uk/22-23/dpt/cxmath11111.htm

Course Description

MATH11111 is designed to expose students to different types of optimization problems and to introduce appropriate solution approaches for each type. The role of convexity in optimization is emphasised. The course provides an in-depth treatment of linear programming and solving linear programming problems using the simplex method. The students will be exposed to the theoretical foundations of linear programming problems. The role of duality and sensitivity analysis for linear programming problems are examined. Alternative solution approaches for large-scale linear programming are discussed. The course gives a brief introduction to nonlinear optimization and introduces a few basic algorithms for unconstrained optimization.

Tentative Outline

A tentative list of course topics is as follows:

- Introduction to optimization and convexity
- Introduction to convex optimization, vertices of convex sets, polyhedra, and basic feasible solutions
- Graphical solution method, vertex enumeration, polyhedra in standard form, and linear programming in standard form
- Basic solutions of polyhedra in standard form, fundamental theorem of linear programming, optimality conditions, and degeneracy
- Optimality conditions under nondegeneracy, development of the simplex method, and the simplex method in dictionary form
- Simplex method and degeneracy, two-phase method, and efficiency of the simplex method
- Introduction to duality theory and weak duality theorem, primal-dual symmetry and strong duality theorem, complementary slackness property

- Economic interpretation of the dual variables, a dual perspective on the simplex method, and the dual simplex method
- Sensitivity analysis and reoptimization
- Continuity and differentiability
- Optimality conditions in unconstrained optimization, convexity and unconstrained optimization

Learning Outcomes:

On the completion of this course, students will be able to

- Identify different types of optimization problems, and be able to connect these with the available methods for their solution.
- Apply appropriate optimization techniques to solve small optimization problems by hand.
- Discuss and interpret the sensitivity of a solution of an optimization problem to changes in the parameter values of the problem.

Assessment:

Coursework 40% (four assessments of 10% each), Exam 60% (in December).

Recommended Textbooks

1. *Introduction to Linear Optimization*, Dimitris Bertsimas and John N. Tsitsiklis, Athena Scientific, Dynamic Ideas, LLC, Belmont, Massachusetts, 1997, ISBN: 1886529191 (Hard copies are available for a one-week loan at Hub Reserve at the Ground Floor of the Main Library.)
2. *Linear Programming: Foundations and Extensions*, Robert J. Vanderbei; Fred Hillier (Editor); Robert J. Vanderbei (Editor), Springer US, Boston, Massachusetts, 2008, Third Edition, International Series in Operations Research & Management Science, ISBN: 0387743871 (E-book is available through the University Library website.)
3. *Linear Programming and Network Flows*, Mokhtar S. Bazaraa, John J. Jarvis, Hanif D. Sherali, Hoboken, N.J, John Wiley & Sons, 2010, Fourth edition, ISBN: 0471485993 (E-book is available through the University Library website.)
4. *Linear and Nonlinear Programming*, David G. Luenberger, Yinyu Ye, Springer US, New York, NY, 2008, Third Edition, International Series in Operations Research & Management Science, ISBN: 0387745025 (E-book is available through the University Library website.)

We will mostly follow the treatment in the first textbook, i.e., *Introduction to Linear Optimization* by Dimitris Bertsimas and John N. Tsitsiklis. This textbook is only available as a hard copy in the library. However, the course will be self-contained, in the sense that all course materials that will be necessary and relevant for the course will be provided on the Learn page. You may download and use the e-books of the second, third, and fourth references above if you need to consult any supplementary material.

Here is the University Library website:

`https://www.ed.ac.uk/information-services/library-museum-gallery`

The Learn Page

Learn is the primary Virtual Learning Environment at the University of Edinburgh. You can access your courses directly from the Learn channel in MyEd, or login at `https://www.learn.ed.ac.uk`. You can also download the Blackboard app to access your courses from a phone or tablet device. The course will have a Learn page that will contain all teaching materials, exercise sheets, homework assessments, solution sets, announcements, and links to tools for submissions of assessments. It will be our primary channel for general course-related announcements.

Teaching Materials

All teaching materials will be available on the Learn page and can be accessed by navgiating to "Course Materials → Teaching Materials."

1. **Lecture Slides and Lecture Notes:** Each set of slides used in the lectures as well as the associated lecture notes will be made available on the Learn page on a week-by-week basis. The lecture notes will also contain some review problems at the end of each chapter. The slides will be almost identical to those used in the lectures with one difference: There will be no hand-written annotations. Instead, the proofs of propositions, lemmas, and corollaries will be typed. Lecture slides and lecture notes can be accessed by navigating to "Course Materials → Teaching Materials → Lecture Slides and Lecture Notes" on the Learn page. The links to each set of slides and review problems will be available in the relevant week folder.

2. **Handouts:** Several handouts will be made available on the Learn page in an attempt to refresh your background on certain topics, such as proof techniques and basic material from linear algebra. **Note that this material will not be reviewed in lectures but will be assumed as background information.** Handouts can be accessed by navigating to "Course Materials → Teaching Materials → Useful Handouts on Background Material" on the Learn page.

Lectures

On-campus lectures will take place on Thursdays at 11:10 - 13:00 at the Joseph Black Building, Theatre 250. The Joseph Black Building is located at the King's Buildings Campus in the KB Square opposite from the King's Building's House. All lectures will be recorded and the recorded material will be subsequently be made available on the course Learn page.

Workshops and Exercise Sheets

Workshops will be conducted on campus in odd-numbered weeks (Weeks 1, 3, 5, 7, 9, and 11). There will be three sessions and you will need to attend only one of them:

1. Friday, 10:00 - 10:50, JCMB Lecture Theatre C
2. Friday, 11:10 - 12:00, JCMB Lecture Theatre C
3. Friday, 12:10 - 13:00, JCMB Lecture Theatre C

Approximately one week before each workshop, an exercise sheet will be released on the Learn page, which can be accessed at "Course Materials → Exercise Sheets and Solution Sets." You will be expected to work on the provided exercise sets before each workshop and have a chance to ask questions and have discussions during the workshops.

Other Useful Tools

1. **Piazza:** We have set up a Piazza forum for the course. To enrol to the forum, please visit:

   `https://piazza.com/ed.ac.uk/fall2022/math1111120223ss1sem1/`

   You can also access the link on the Learn page at "Course Information → How do I Access?"

   The interface should be fairly intuitive. We will be conducting all class-related discussion here this term. The quicker you begin asking questions on Piazza (rather than via emails), the quicker you will benefit from the collective knowledge of your classmates and instructors. We encourage you to ask questions when you are struggling to understand a concept.

   When creating a new post, you can decide whether you want to post anonymously or not. In the current setting, "anonymous" is only anonymous to your fellow students. Members who are registered as "instructors" will be able to see your name.

   Moreover, when creating a new post, please select a folder where it fits best. When you first visit the page, posts across all folders are shown on the left-hand side. To see only the posts in a specific folder, you have to click on the name of a folder on the top of the screen (just below the blue banner on the very top). Using folders makes it much easier to find things later on.

2. **Top Hat:** Top Hat is a course management tool that offers several capabilities. We will mainly use it for attendance purposes in lectures and/or workshops. In addition, we will rely on Top Hat for the multiple choice review questions at the end of each lecture. There is a mobile app available. You have been automatically added or sent an invitation to join the course Top Hat page. For further information, please see

   `https://www.ed.ac.uk/information-services/learning-technology/electronic-voting-sys`
   `students`

If you have a Top Hat account, you can also join the course manually by using the course code "918495".

3. **STACK:** STACK (the System for Teaching and Assessment using a Computer algebra Kernel) is an online assessment package for mathematics:

   `https://www.ed.ac.uk/maths/stack`

   We will be using STACK as part of homework assessments and some of the exercise sheets. STACK-based problems can be accessed by navigating to "Course Information → How do I Access?" on the Learn page.

4. **Gradescope:** Gradescope is the main tool you will use to submit digital copies of your solutions to open-ended questions in homework assessments:

   `https://www.gradescope.com/`

   Submissions of assignments will be carried out by navigating to "Assessments → Assignment Submission".

Assessments and Feedback

Each of the four assessments will consist of a combination of STACK-based problems and open-ended problems. In terms of the organisation and content, each assessment will be very similar to the preceding exercise set. Therefore, working on exercise sets will be very useful both for benefitting from the discussions during the workshops and for your preparation for the actual assessment. STACK-based problems will be assessed by the software and feedback will be provided immediately after the deadline. Open-ended problems will be marked by the course team and feedback will be provided on your Gradescope page.

Homework assignments can be accessed by navigating to "Assessment → Homework Assignment XX" on the Learn page, where XX denotes the assignment number. STACK-based problems can be accessed by navigating to "Assessment → Assignment Submission → STACK - Exercises and Assignments" on the Learn page.

Homework Submission

Each homework assignment consists of a combination of STACK-based problems and open-ended problems. Solutions of STACK-based problems should be submitted on the relevant STACK page by following "Assessment → STACK - Exercises and Assignments" on the Learn page. **You do not need to submit any additional work for STACK-based problems.**

The open-ended problems can be either typeset using LaTeX and submitted as a PDF file, or you can solve your problems on paper and scan them using your mobile devices. **The instructions for creating a pdf file is available on the Learn page at "Course Information → Tips for Creating a PDF File for Online Submission". Note that you should submit a single PDF file by navigating to "Assessment → Assignment Submission → Gradescope" on the Learn page.**

What to Expect in This Course

- Mathematical derivations and rigour (proof-based)
- Development of the theory and solution methods
- Linear algebra is crucial for optimization.

What Not to Expect in This Course

- Optimization modelling (see MATH11007 Methodology, Modelling and Consulting Skills)
- Applications (see MATH11007 Methodology, Modelling and Consulting Skills, MATH10065 Fundamentals of Operational Research in Semester 1; or MATH11183 Topics in Applied Operational Research, MATH11193 Operational Research in the Energy Industry, MATH11190 Risk and Logistics, MATH11158 Optimization Methods in Finance in Semester 2)
- Solvers (see MATH11007 Methodology, Modelling and Consulting Skills in Semester 1; MATH11147 Large Scale Optimization for Data Science, MATH11158 Optimization Methods in Finance in Semester 2)
- Large-scale optimization (MATH11147 Large Scale Optimization for Data Science in Semester 2)
- Integer optimization (MATH11192 Integer and Combinatorial Optimization in Semester 2)

Important Dates

- *Week 0 (12 September – 16 September)*
  - Thursday, 15 September, by 17:00: Exercise Set 0 to be released
- *Week 1 (19 September – 23 September)*
  - Thursday, 22 September, 11:10 – 13:00: Lecture
  - Friday, 23 September: Workshop 1

- *Week 2 (26 September – 30 September)*
  - Thursday, 29 September, 11:10 – 13:00: Lecture
  - Thursday, 29 September, by 17:00: Exercise Set 1 to be released
- *Week 3 (3 October – 7 October)*
  - Thursday, 6 October, 11:10 – 13:00: Lecture
  - Friday, 7 October: Workshop 2
  - Friday, 7 October, by 13:00: Homework Assignment 1 to be released
- *Week 4 (10 October – 14 October)*
  - Thursday, 13 October, 11:10 – 13:00: Lecture
  - Thursday, 13 October, by 17:00: Exercise Set 2 to be released
  - Friday, 14 October, by 12 pm (noon): Homework Assignment 1 is due
- *Week 5 (17 October – 21 October)*
  - Thursday, 20 October, 11:10 – 13:00: Lecture
  - Friday, 21 October: Workshop 3
  - Friday, 21 October, by 13:00: Homework Assignment 2 to be released
- *Week 6 (24 October – 28 October)*
  - Thursday, 27 October, 11:10 – 13:00: Lecture
  - Thursday, 27 October, by 17:00: Exercise Set 3 to be released
  - Friday, 28 October, by 12 pm (noon): Homework Assignment 2 is due
- *Week 7 (31 October – 4 November)*
  - Thursday, 3 November, 11:10 – 13:00: Lecture
  - Friday, 4 November: Workshop 4
  - Friday, 4 November, by 13:00: Homework Assignment 3 to be released
- *Week 8 (7 November – 11 November)*
  - Thursday, 10 November, 11:10 – 13:00: Lecture
  - Thursday, 10 November, by 17:00: Exercise Set 4 to be released
  - Friday, 11 November, by 12 pm (noon): Homework Assignment 3 is due
- *Week 9 (14 November – 18 November)*
  - Thursday, 17 November, 11:10 – 13:00: Lecture
  - Friday, 18 November: Workshop 5

  - Friday, 18 November, by 13:00: Homework Assignment 4 to be released

- *Week 10 (21 November – 25 November)*
  - Thursday, 24 November, 11:10 – 13:00: Lecture
  - Thursday, 24 November, by 17:00: Exercise Set 5 to be released
  - Friday, 25 November, by 12 pm (noon): Homework Assignment 4 is due
- *Week 11 (28 November – 2 December)*
  - Thursday, 1 December, 11:10 - 13:00: Lecture
  - Friday, 2 December: Workshop 6
- *Week 12 (5 December – 8 December) Revision Period*
- *Weeks 12/13/14 (9 December – 21 December) Examination Period*

Academic Misconduct

Academic misconduct includes plagiarism, collusion, falsification, deceit, cheating and personation. The University takes all reported incidences of academic misconduct seriously and seeks to ensure that they are dealt with efficiently and appropriately. Please see the following webpage for further information about the University policies on academic misconduct:

`https://www.ed.ac.uk/academic-services/staff/discipline/academic-misconduct`