Exercises sheet 3
School of Mathematics
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Exercise 1 *Model*

As a warm-up exercise, let us look at some abstract constraints that we can represent with binary variables and linear constraints.

Let $x_1, x_2, \dots, x_n \in \{0, 1\}$ and $z \in \{0, 1\}$. For each of the following examples, find one or more linear constraints that express the following conditions:

1. At most one of $x_1, x_2, \dots, x_n$ is 1.
2. Exactly one of $x_1, x_2, \dots, x_n$ is 1.
3. At least one of $x_1, x_2, \dots, x_n$ is 1.
4. $z = 0$ implies $x_1 = 0$.
5. $z = 0$ implies that all of $x_1, \dots, x_n$ are 0.

Exercise 2 *Model + Solve*

Continuing with simple integer programming models, we look at a classical game.

Sudoku is a classical puzzle game, where the goal is to fill in a 9×9 grid with digits from 1 to 9. The rules are that in every row, column, and nine 3×3 subgrids ("boxes") each digit has to appear exactly once.

The standard puzzle consists of a grid with some numbers filled with the promise that this partial solution can be uniquely completed to a full solution.

Model a standard Sudoku game as an integer program. Pick an example from eg a newspaper and solve it with Xpress.
Note Use binary variables! General integer variables are less helpful than you might think.

Exercise 3 *Model + Solve*

We look at a simplified shift planning example for a bus company. The bus company wants to make a shift plan that takes into account the varying demand over the day. Each bus driver has a shift of 8 h. Of this shift, 1 h is reserved time for a break which should be roughly in the middle of the shift. The company operates between 06:00 and 24:00 every day. The different shifts have different costs associated with them due to extra payments in the early morning/late evening.

To simplify the problem we have made the following assumptions: We have decided to divide the day into 1 h segments. We allow shifts only to start on the full hour and allow only two shift patterns; Pattern 1: 3 h work, 1 h break and 4 h or Pattern 2: 4 h work, 1 h break and 3 h. We have numbered the shift starting times by assigning the time slot at hour h the number $h - 5$.

We have been given data for the demand and the shift costs.

Time	Demand
06:00	3
07:00	9
08:00–10:00	10
10:00–12:00	6
12:00–14:00	7
14:00–16:00	6
16:00–17:00	8
17:00–19:00	10
19:00–20:00	8
20:00–23:00	3
23:00–24:00	3

Time slot	Costs
1	100
2	90
3	80
4–8	50
9	60
10	70
11	100

Write down a model for this shift planning problem on paper. Explain the meaning of your variables and constraints. Implement your model in Xpress and compute an optimal solution. What happens if you decide that all drivers that start their shift at a specific time should also have their break at the same time. How does this change your model? How much more does it cost? What is the minimum amount of drivers needed to fulfill the demand? Additional exercise: Investigate which variables should be integer and which you can keep continuous without changing the solution.