Week 1

The diet problem

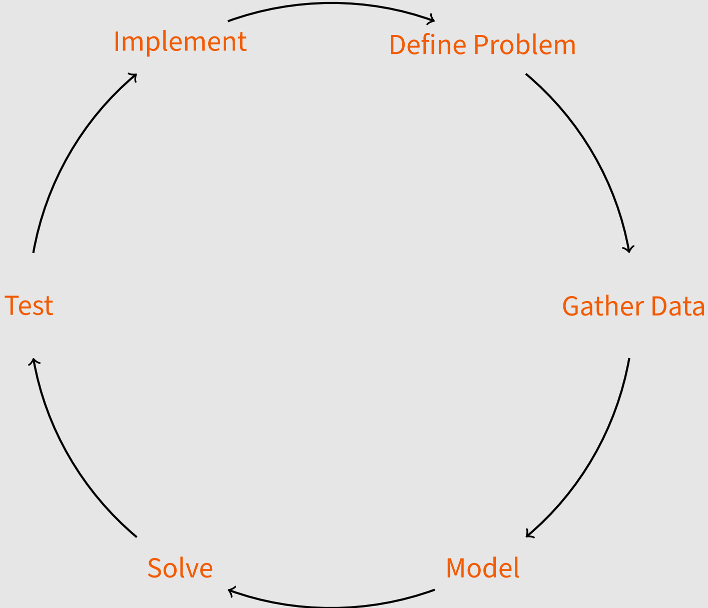
Methodology, Modelling, and Consulting Skills
The University of Edinburgh

The diet problem

Stigler

For a moderately active man (economist) weighing 154 pounds, how much of each of 77 foods should be eaten on a daily basis so that the man's intake of nine nutrients (including calories) will be at least equal to the recommended dietary allowances (RDAs) suggested by the National Research Council in 1943, with the cost of the diet being minimal?

An approach to OR problems



Elements of an optimization problem

In general

Variables	Quantifiable decisions to be taken.
Objective function	What must be optimized?
Constraints	Limitations and requirements.

Linear Programming

- Linear objective function.
- Linear constraints.

The diet problem

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Solution

Collect data

At 1943 prices, \$1 worth of Food f has h_{nf} amount of nutrient n:

	Rice	Liver (beef)	Cabbage	$\dots$
Calories	21200	2200	2600	$\dots$
Protein	460	333	125	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Suppose that we require r_n amount of nutrient n:

	Calories	Protein	$\dots$
r	3000	70	$\dots$

Solution

Define variables and objective

Decision variables

x_1 money spent on rice
x_2 money spent on liver (beef)
x_3 money spent on cabbage

Objective function

$$1x_1 + 1x_2 + 1x_3$$

Solution

Define constraints

Constraints

Calorie requirement

$$21200x_1 + 2200x_2 + 2600x_3 \geq 3000$$

Protein requirement

$$460x_1 + 333x_2 + 125x_3 \geq 70$$

Nonnegativity

$$x_1, x_2, x_3 \geq 0$$

Complete model

$$
\begin{aligned}
\min \quad & 1x_1 + 1x_2 + 1x_3 \\
\text{s.t.} \quad & 21200x_1 + 2200x_2 + 2600x_3 \geq 3000 \\
& 460x_1 + 333x_2 + 125x_3 \geq 70 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Optimal solution

$$x^* = (x_1^*, x_2^*, x_3^*) = (0.153, 0, 0) \qquad z^* = 0.153$$

Result

- Optimal diet consists of spending only \$0.153 on rice
- No other foods are used (Why should we expect this?)
- The optimal cost is \$0.153

Complete model

General form

Index sets

F	set of foods
N	set of nutrients

Variables

x_f	amount of food f $(f \in F)$

Parameters

c_f	cost of one unit of food f $(f \in F)$
h_{nf}	amount of nutrient n in food f $(f \in F, n \in N)$
r_n	required amount of nutrient n $(n \in N)$

$$
\begin{aligned}
\min \quad & \sum_{f \in F} c_f x_f & \\
\text{s.t.} \quad & \sum_{f \in F} h_{nf} x_f \geq r_n & \text{for all } n \in N \\
& x_f \geq 0 & \text{for all } f \in F
\end{aligned}
$$

Complete model

General form – classical formulation

Index sets

F	set of foods
N	set of nutrients

Variables

x_f	dollars spent on food f ($f \in F$)

Parameters

h_{nf}	amount of nutrient n in one dollar worth of food f ($f \in F, n \in N$)
r_n	required amount of nutrient n per day ($n \in N$)

$$
\begin{aligned}
\min \quad & \sum_{f \in F} 1 \cdot x_f \\
\text{s.t.} \quad & \sum_{f \in F} h_{nf} x_f \geq r_n && \text{for all } n \in N \\
& x_f \geq 0 && \text{for all } f \in F
\end{aligned}
$$