Methodology, Modelling, and Consulting Skills
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# Computer Session 1: Example: A blending problem

## 1 Problem and Data

A chemistry firm can produce two products, A and B, from combining three elements: iron, lead, and tin. What is the blending that maximizes the benefit? Assume that all the production is sold. The data for the problem can be found in Table 1.

## 2 A first model

We have to decide how much of product A and B we want to produce. Hence, we choose our decision variables to be $x_1$ for the amount product A (kg) and $x_2$ for the amount of product B (kg), we want to produce.

Our objective is to maximize our net benefit, we can write the objective as

$$\text{Net benefit: } 10x_1 + 8x_2$$

Now, we have to consider our resource constraints, we have resource constraints for Iron, Lead, and Tin. These can be written as

**Iron**

$$7x_1 + 4x_2 \leq 56,$$

**Lead**

$$3x_1 + 5x_2 \leq 45,$$

**Tin**

$$4x_1 + 3x_2 \leq 48.$$

At the end, we should not forget nonnegativity.

$$x_1, x_2 \geq 0.$$

Table 1: Data for the blending problem.

| Resources | Units per kg of Product | | Available units |
|---|---|---|---|
| | A | B | |
| Iron | 7 | 4 | 56 |
| Lead | 3 | 5 | 45 |
| Tin | 4 | 3 | 48 |

| Product | Net Benefit £/kg |
|---|---|
| A | 10 |
| B | 8 |

In summary, we obtain the following model:

$$
\begin{aligned}
\max \quad & 10x_1 + 8x_2 \\
\text{s.t.} \quad & 7x_1 + 4x_2 \leq 56, \\
& 3x_1 + 5x_2 \leq 45, \\
& 4x_1 + 3x_2 \leq 48, \\
& x_1, x_2 \geq 0.
\end{aligned}
$$

# 3 A general model

To apply our model to generic blending problems, we introduce two sets of indices: $P$ for products and $R$ for resources. We introduce a vector $l \in \mathbb{R}^R$ for the maximum available quantities, a vector $w \in \mathbb{R}^P$ for the per unit benefit for each product, and a matrix $A \in \mathbb{R}^{P \times R}$, such that $A_{pr}$ is the amount of resource $r$ needed to produce one unit of product $p$. We call our variables, how much of a product we produce, $x \in \mathbb{R}^P$.

Then our model can be written as

$$
\begin{aligned}
\max \quad & \sum_{p \in P} w_p x_p \\
\text{s.t.} \quad & \sum_{p \in P} A_{pr} x_p \leq l_r \qquad \text{for all } r \in R \\
& x \geq 0.
\end{aligned}
$$

Or even more compact as

$$
\begin{aligned}
\max \quad & w^\top x \\
\text{s.t.} \quad & Ax \leq l \\
& x \geq 0.
\end{aligned}
$$