**Fundamentals of Operational Research**
**Tutorial 2**
**School of Mathematics**
**The University of Edinburgh**
**Year 2022/2023**

1. The maintenance of a pump requires the jobs listed in the table below, which also gives the jobs that must be completed before each job can start and the duration in hours of each job.

| Job No. | Job title | Duration | Predecessor jobs |
|---|---|---|---|
| 1 | Remove cover | 2 | – |
| 2 | Disconnect pipes | 2 | – |
| 3 | Assemble test rig | 3 | 2 |
| 4 | Remove seals | 1 | 1 |
| 5 | Remove damaged blades | 2 | 1 |
| 6 | Replace damaged blades | 3 | 5 |
| 7 | Analyse damaged blades | 5 | 5 |
| 8 | Lubricate | 2 | 4 |
| 9 | Test electrics | 4 | 4 |
| 10 | Test running | 3 | 3, 6, 8 |
| 11 | Replace seals and cover | 2 | 9, 10 |
| 12 | Reconnect pipes | 2 | 9, 10 |

(a) Draw a project network showing the above jobs as nodes.

(b) Find the earliest start time, the latest finish time and the slack for each job.

2. An industrial pump manufacturer has one pump in stock at the start of month 1 and has orders for $d_t$ pumps in months $t = 1, 2, 3, 4$. The company wishes to have one pump in stock at the start of month 5. Orders in a given month may be met from stock or from that month production.

The cost of producing $x$ pumps in any month is $r(x)$ and at most 3 pumps can be produced per month. The cost of having $y$ pumps in stock at the start of any month is $s(y)$. There is no space for more than 2 pumps in stock at the start of any month.

Here is the different data:

- $d_1 = 2,\ d_2 = d_3 = d_4 = 1.$
- $r(0) = 4,\ r(1) = 14,\ r(2) = 20,\ r(3) = 24.$
- $s(0) = 0,\ s(1) = 4,\ s(2) = 7.$

The order of actions at each month to determine costs is:

1. Pay inventory cost.
2. Produce pumps.

3. Deliver orders.

You are asked the following:

(a) Identify the states of the problem and write down a dynamic programming recurrence.

(b) Draw a network and find a shortest path that corresponds to the cheapest production schedule.