



Fundamentals of Optimization

Exercise Set 0

Numerical Problems

1 Simple Optimization Problems

For each of the following three optimization problems, identify the feasible region and the objective function. Determine whether the given optimization problem is *infeasible*, *unbounded*, or *has a finite optimal value*. Find the optimal value using the convention in the lectures and determine the set of all optimal solutions.

$$(1.1) \max\{(x-1)^2 : x^2 \leq -1, \quad x \in \mathbb{R}\}.$$

$$(1.2) \min\{1/(x+1)^2 : x \geq 0, \quad x \in \mathbb{R}\}.$$

$$(1.3) \min\{x^2 - 4x + 6 : |x-2| \geq 1, \quad x \in \mathbb{R}\}.$$

$$(1.4) \max\{x^2 - 3x + 6 : x^2 \geq 1, \quad x \in \mathbb{R}\}.$$

2 Convex Sets and Convex Functions

(2.1) Decide for each of the following sets whether they are *convex* or not.

$$(a) \mathcal{C} = \{x \in \mathbb{R} : x^2 \geq 3\}.$$

$$(b) \mathcal{C} = \{x \in \mathbb{R}^2 : (x_1 + 1)^2 + (x_2 - 1)^2 \leq -3\}.$$

$$(c) \mathcal{C} = \{x \in \mathbb{R}^2 : |x_1| + |x_2| \leq 7\}.$$

(2.2) Decide for each of the following three functions whether they are *convex*, *concave*, *both*, or *neither*.

$$(a) f : \mathbb{R}^2 \rightarrow \mathbb{R}, f(x) = 3x_1 - 2x_2.$$

$$(b) f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = |x|.$$

$$(c) f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 - 4x.$$

Open Ended Problems

3 Epigraphs and Convex Functions

Let $f_1 : \mathbb{R}^n \rightarrow \mathbb{R}$ and $f_2 : \mathbb{R}^n \rightarrow \mathbb{R}$ be two functions. Let $h : \mathbb{R}^n \rightarrow \mathbb{R}$ be a function given by $h(x) = \max\{f_1(x), f_2(x)\}$.

(3.1) Prove the following proposition:

$$\text{epi}(h) = \text{epi}(f_1) \cap \text{epi}(f_2).$$

(Hint: One way of proving that two sets A and B are equal to one another is to show that $A \subseteq B$ and $B \subseteq A$.)

(3.2) Suppose that f_1 and f_2 are convex functions. Show, by using (3.1), that h is a convex function.