BT_Workshop_2

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Problem 1:

Bayes Theorem: travel times and routes (data and parameters)

```
route.prob <- c(0.25,0.25,0.25,0.25)
names(route.prob) <- LETTERS[1:4]
cond.prob.time <- c(0.2,0.5,0.8,0.9)
marg.prob.time <- sum(cond.prob.time*route.prob)
cat("marginal probability time <= 1.5 hrs=",marg.prob.time,"\n")
```

```
## marginal probability time <= 1.5 hrs= 0.6
```

```
#posterior probabilities per route
post.prob <- route.prob*cond.prob.time/marg.prob.time
cat("posterior probabilities for each route \n")
```

```
## posterior probabilities for each route
```

```
print(post.prob)
```

```
##          A          B          C          D
## 0.08333333 0.20833333 0.33333333 0.37500000
```

Problem 2:

Bayes Theorem: Covid test results and infectious status (data and parameters)

```
bt <- function(Infect,Sensitivity,Specificity) {
  p1 <- Sensitivity*Infect
  p2 <- (1-Specificity)*(1-Infect)
  out <- p1/(p1+p2)
  return(out)
}
cat("Part a: posterior prob Infected=\n")
```

```
## Part a: posterior prob Infected=
```

```r
bt(Infect=0.0222,Sensitivity = 0.75, Specificity=0.99)
```

```
## [1] 0.6300136
```

```r
cat("Part b: post prob if Moderate risk \n")
```

```
## Part b: post prob if Moderate risk
```

```r
bt(Infect=0.1,Sensitivity = 0.75, Specificity=0.99)
```

```
## [1] 0.8928571
```

```r
cat("Part c: post prob if Medium Risk \n")
```

```
## Part c: post prob if Medium Risk
```

```r
bt(Infect=0.3,Sensitivity = 0.75, Specificity=0.99)
```

```
## [1] 0.9698276
```

```r
cat("Part c: post prob if High Risk \n")
```

```
## Part c: post prob if High Risk
```

```r
bt(Infect=0.7,Sensitivity = 0.75, Specificity=0.99)
```

```
## [1] 0.9943182
```

Problem 3:

Bayesian inference for Poisson parameter

```r
y <- c(5,6,5,6,7,5,4,5,3,6)
n <- length(y)
cat("Total ladybirds=",sum(y))
```

```
## Total ladybirds= 52
```

```r
#Gamma posterior shape and rate
post.shape <- sum(y)+ 0.5
post.rate  <- n
cat("Posterior shape=",post.shape,"rate=",post.rate,"\n")
```

```
## Posterior shape= 52.5 rate= 10
```

```r
# Posterior Expected Value & Std Deviation
post.E <- post.shape/post.rate
post.SD <- sqrt(post.shape/post.rate^2)
cat("Posterior Expectation=",post.E,"SD=",post.SD,"\n")
```

```
## Posterior Expectation= 5.25 SD= 0.7245688
```

Problem 4

Prior predictive for discrete rv with discrete parameter

```r
theta  <- c(0.2,0.4,0.6)
pi.theta <- c(0.50,0.25,0.25)

cond.prob.y.theta <- function(y,theta) {
  if(y==0) out <- theta
  if(y==1) out <- theta^2
  if(y==2) out <- 1-theta-theta^2
  return(out)
}

#prior predictive for each possibility for y
for(y in 0:2) {
 cat("For y=",y,"Prior predictive probability=",
     sum(cond.prob.y.theta(y=y,theta=theta)*pi.theta),
    "\n")

}
```

```
## For y= 0 Prior predictive probability= 0.35
## For y= 1 Prior predictive probability= 0.15
## For y= 2 Prior predictive probability= 0.5
```