Exercises sheet 4
School of Mathematics
The University of Edinburgh
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Exercise 1 *Model*

We look at the *simple plant location problem* (SPLP). We have a company producing one product and a set of customers that need this product. The goal of the company is to decide where to locate their warehouses and which warehouse should serve which customer, so that the overall costs are minimal.

More formally, we are given a set of customers J and a set of possible facilities (warehouses) I. We are given the demand d_j of every customer $j \in J$ and the combined cost of production and transport c_{ij} if we serve one unit of demand of customer j from facility i. We are also given costs f_i for each facility $i \in I$ that we incur if we decide to open facility i.

We need to decide which of the facilities $i \in I$ we want to open and how much demand of customer j will be served by facility i. We can only serve customers from a facility if the facility is opened.

Model the SPLP as a mixed-integer optimization problem.

Exercise 2 *Model + Solve*

Mato Brothers produce tomato sauce in five plants (P1, ..., P5). They have four customers (C1, ...C4).

Their annual production capacities of the plants are as follows:

	P1	P2	P3	P4	P5
Capacity (t)	300	200	300	200	400
Fixed Cost (£)	35,000	45,000	40,000	42,000	40,000

The demand of the customers is

	C1	C2	C3	C4
Demand (t)	200	300	200	250

Production and shipping costs (in £) are

		To			
		C1	C2	C3	C4
	P1	1,180	1,160	1,190	1,200
	P2	810	800	850	760
From	P3	850	830	890	840
	P4	770	750	810	780
	P5	800	770	820	830

Compute an optimal production plan for Mato Brothers.

Exercise 3 *Model + Solve*

This problem originates from a real-life consultancy project.

The Scottish government is considering a plan to roll out a hydrogen refuelling infrastructure in Scotland. To do this they have identified a list of 25 candidate refuelling demand sites each with an associated demand. The government is happy to consider a case where, if any candidate sites are within 20km radius of one another candidate site, then the demand of each candidate site can be satisfied at only a single site. This means that if $\mathcal{I} = \{1, \dots, \mathcal{I}^{tot}\}$ denotes the set of demand sites, D_c denotes the maximum coverage distance, $d^{\text{sites}}_{j,i}$ denotes the distance from demand site j to site i, then we define $a_{i,j} = 1$ for $i, j \in \mathcal{I}$ if $d^{\text{sites}}_{j,i} \leq D_c$, which implies that the demand of site j can be satisfied at candidate site i. To the contrary, we define $a_{i,j} = 0$ for $i, j \in \mathcal{I}$ if $d^{\text{sites}}_{j,i} > D_c$ which implies that the demand of site j cannot be satisfied at candidate site i. The adjacency matrix $\boldsymbol{A} = (a_{i,j})_{i,j \in \mathcal{I}}$ can be found in `Dataset1_N25_students.txt`. Assume that there is a fixed cost of £50 000 for opening a refuelling station. The demand for each candidate site can be found in `Dataset1_N25_students.txt`.

The government is considering two possible hydrogen delivery methods. The first method is to produce the hydrogen at a centralised production plant and transport the hydrogen to the refuelling stations in trucks. The government is considering opening 3 different centralised production facilities at a cost of £25 000. It costs £40 per km for a tube trailer delivery if the trailer is full and is 75% less if the tube trailer is empty. The hydrogen from a centralised production facility costs £2 per kg. The capacity of a single tube trailer is 1 000 kg. If tube trailers are used to deliver the hydrogen then a maximum of 7 tube trailer deliveries can be made at each site. The distance from each candidate facility to each production facility can be found in `Dataset1_N25_students.txt`.

The second option is to produce the hydrogen locally at the refuelling station. If they decide to install a local production facility then the government must decide between a large and a small localised production facility. A large localised production facility costs £6 000 and can produce a maximum of 9 500 kg of hydrogen a year. A small localised production facility costs £3 500 and can produce a maximum of 5 000 kg of hydrogen a year. If a localised production facility is established then it must produce at least 100 kg of hydrogen a year. Only one type of localised production facility can be established.

Write and solve a MILP that recommends the refuelling infrastructure that minimises the cost of satisfying all the demand.