Fundamentals of Operational Research
Assignment 2
School of Mathematics
The University of Edinburgh
Year 2022/2023

INSTRUCTIONS:

1. **This assignment is individual and it is worth 10% of the mark of the course.**
2. **You must consider this as an exam that you do at home. The same policy than in an exam will be applied for assignments with signs of plagiarism. This is a serious academic offence.**
3. **The submission deadline is Wednesday 16 November at 14:00 on Gradescope.**
4. **Provide clear answers and justify every argument that you use. This does not mean to explain in twenty lines what can be said in two. Neither to explain in two words what requires two or three lines. There is no minimum or maximum number of pages that you can submit, but take into account these guidelines.**

1) (10 marks) A firm has 4 marketing plans and 2 production plans that can adopt. The decisions are taken using an integer programming model. The business analyst in charge of the model only took Fundamentals of Operational Research during his undergraduate studies, which means that sadly the lecturer never taught him to solve a nonlinear optimization model. Answer the first question and formulate the three conditions that follow, but do not use any nonlinear expression:

(a) **(1 mark)** efine the necessary variables that represent the decisions on which plans are adopted.

(b) **(1 mark)** t least one of the marketing plans must be adopted.

(c) **(3 marks)** Only one of the marketing plans ca be implemented by the company. Any additional plan must be outsourced to an external agency at a cost of c monetary units per outsourced plan.

(d) **(5 marks)** Production plan 1 is impossible if marketing plan 2 is adopted and marketing plan 1 is not adopted. *Note: You must use and explain step by step in this question d one or more of the methods seen in class.*

Solution:

(a) We define binary variable M_i that takes value 1 if marketing plan i is adopted, and takes value 0 otherwise, $i = 1, 2, 3, 4$. Likewise we define P_i, $i = 1, 2$, for the production plans. *(1 mark)*

(b) This is a straightforward constraint: $M_1 + M_2 + M_3 + M_4 \geq 1$. *(1 mark)*

(c) We define variable e as a nonnegative integer variable that represents how many marketing plans are outsourced.*(1 mark)*

We add the following constraint to the model: $M_1 + M_2 + M_3 + M_4 \leq 1 + e$.*(1 mark)*

Finally, we add $e \cdot c$ anywhere where the cost of the outsourced plans needs to be included.*(1 mark)*

(d) *Note: If the correct inequality (or inequalities) is (are) obtained, but without using the method seen in class for if-then constraints, only a maximum of 2 marks can be obtained.*

We are modelling "if marketing plan 2 is adopted and marketing plan 1 is not adopted, then production plan 1 is not adopted."*(0.5 marks)*

Equivalently, we are modelling "if $M_2 = 1$ and $M_1 = 0$, then $P_1 = 0$".*(0.5 marks)*

Now, the key to be able to model this easily as an if-then constraint is to define a new variable $\overline{M}_1 = 1 - M_1$, which is the contrary to what M_1 represents.*(0.5 marks)*

Now we have that we are modelling that, if $M_2 = 1$ and $\overline{M}_1 = 1$, then $P_1 = 0$.*(0.5 marks)*

Equivalently, because the variables are binary, if $\overline{M}_1 + M_2 \geq 2$, then $P_1 \leq 0$.*(0.5 marks)*

Or, using again that the variables are binary, if $\overline{M}_1 + M_2 > 1$, then $P_1 \leq 0$.

We can write this as "if $f > 0$, then $g \geq 0$", with $f = \overline{M}_1 + M_2 - 1$ and $g = -P_1$.*(0.5 marks)*

Now, we apply what we know of the if-then constraints. We know that the condition can be modelled through the following constraints:*(1 mark)*

$$
\begin{aligned}
f &\leq k_1(1 - y), \\
-g &\leq k_2 y,
\end{aligned}
$$

where y is an auxiliary binary variable, $k_1 = \max\{f\} = \max\{\overline{M}_1 + M_2 - 1\} = 1$, and $k_2 = \max\{-g\} = \max\{P_1\} = 1$.

Therefore, we write*(0.5 marks)*

$$
\begin{aligned}
\overline{M}_1 + M_2 - 1 &\leq 1 - y, \\
P_1 &\leq y.
\end{aligned}
$$

Finally we undo the change $\overline{M_1} = 1 - M_1$ and we have that the inequalities that we are looking for are*(0.5 marks)*

$$
\begin{aligned}
M_2 + y &\leq 1 + M_1, \\
P_1 &\leq y.
\end{aligned}
$$

Note: It is possible to model this through the single constraint $M_2 + P_1 \leq 1 + M_1$. Giving this solution without using the if-then methodology is worth a maximum of 2 marks.