

DEFENG SUN's Research on Sensitivity Analysis for NLSDP

Consider the perturbed nonlinear semidefinite programming (NLSDP):

$$\min_{x \in \mathbb{R}^n} \{f(x) - \langle a, x \rangle \mid G(x) + b \in K := \{0\}^m \times \mathcal{S}_+^p\}, \quad (1)$$

where f and G are C^2 functions, and (a, b) is the perturbation parameter. For a given (a, b) , let $\mathbb{S}_{\text{KKT}}(a, b)$ denote the set of all solutions (x, y) to the Karush–Kuhn–Tucker (KKT) system:

$$a = \nabla f(x) + \nabla G(x)y = \nabla_x L(x, y), \quad y \in N_K(G(x) + b), \quad (2)$$

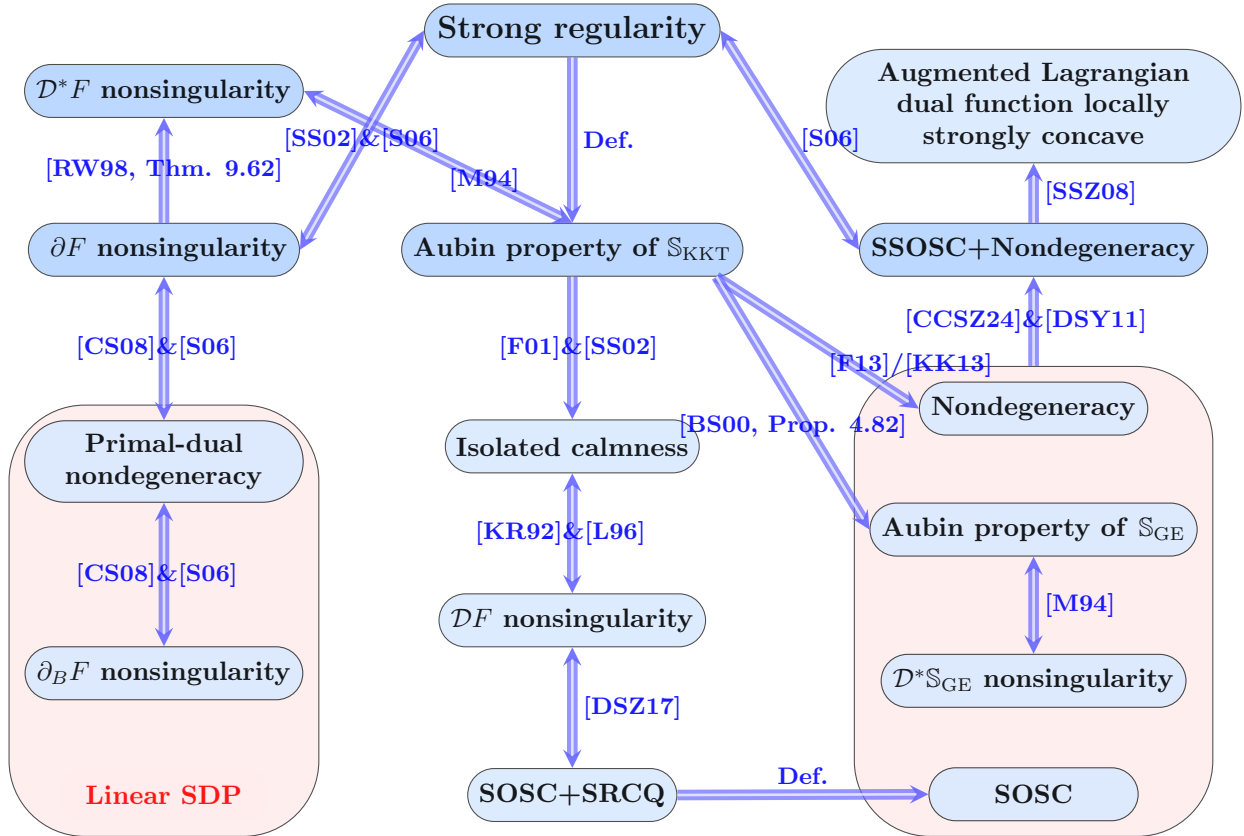
where the Lagrangian function of (1) is defined by $L(x, y) := f(x) + \langle G(x), y \rangle$. For given a , define the set-valued mapping \mathbb{S}_{GE} as

$$\mathbb{S}_{\text{GE}}(a) := \{x \mid a \in \nabla f(x) + \nabla G(x)N_K(G(x))\}. \quad (3)$$

Define the nonsmooth mapping

$$F(x, y) := \begin{pmatrix} \nabla_x L(x, y) \\ G(x) - \Pi_K(G(x) + y) \end{pmatrix}. \quad (4)$$

The following relationships hold at a locally optimal solution of (1) which admits a multiplier.



SOSC: second-order sufficient condition
SSOSC: strong second-order sufficient condition
SRCQ: strict Robinson's constraint qualification

∂_B :	Bouligand subdifferential
∂ :	Clarke's generalized Jacobian
\mathcal{D} :	graphical derivative
\mathcal{D}^* :	Mordukhovich's coderivative

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