NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester I (2009/2010) MA4254 Discrete Optimization Tutorial 2

Q1. Let

$$P = \{x \in \Re^3 \,|\, Ax = b\,, x \geq 0\}\,,$$

where

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$

Let y be an extreme point of P. Show that y is an integer.

Q2. Are all optimal solutions to

$$\begin{array}{ll} \min & x_1 + 2x_2 + 3x_3 + 4x_4 \\ \text{s.t.} & x_1 + x_2 + x_3 + x_4 = 7 \\ & x_1 \leq 2 \\ & x_2 \leq 4 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

integer? Justify your answer.

Q3. Suppose that $A \in \Re^{m\times n}$ is of full row rank ($m \leq n$). Show that any extreme point x of the polyhedron

$$S = \{x \in \Re^n \,|\, Ax = b, x \geq 0\}$$

can be decomposed into two parts $x_{\mathcal{B}} \geq 0$ and $x_{\mathcal{N}} = 0$ such that

$$Bx_{\mathcal{B}} = b\,,$$

where B is an $m \times m$ nonsingular submatrix of A.

Q4. Prove, for a given graph $G = (V, E)$, that the following are equivalent:

(i) G is a tree;

(ii) G contains no cycles and $|E| = |V| - 1$;

(iii) G is connected and $|E| = |V| - 1$;

(iv) any two vertices of G are connected by exactly one path;

(v) after removing of any verge, G becomes unconnected.

Q5. Let $S(b) = \{x \in \Re^n \,|\, Ax \leq b, x \geq 0\}$, where $A \in \Re^{m\times n}$ and $b \in \Re^m$. Show that the extreme points of $S(b)$ correspond to the basic feasible solutions of the system

$$Ax + Is = b\,,\ x \geq 0,\ s \geq 0\,.$$