NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester 1 (2003/2004) MA4253 Mathematical Programming Tutorial 2

Q1. Suppose that y^* is an optimal basic feasible solution to

$$\begin{array}{ll} \max & b^T y \\ \text{s.t.} & A^T y \le c\,, \end{array}$$

where $A \in \Re^{m\times n}$, $b \in \Re^m$ and $c \in \Re^n$. Assume that there are exactly m inequalities which are active at y^*, i.e., $|I| = m$, where

$$I := \{i \,|\, a_i^T y^* = c_i\,\}$$

and a_i is the ith column of A, $i = 1, \ldots, n$. Give an optimal solution to

$$\begin{array}{lrcl} \min & c^T x & & \\ \text{s.t.} & Ax & = & b \\ & x & \ge & 0\,. \end{array}$$

Q2. Let $P = \{x \in \Re^n \mid Ax = b, x \ge 0\}$. Then, there exist $x^1, \ldots, x^q, d^1, \ldots, \ldots d^r$ in $\Re^n$ such that

$$P = \mathrm{conv}\{x^1, \ldots, x^q\} + \mathrm{cone}\{d^1, \ldots, d^r\}\,.$$

Consider the following linear programming problem

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & x \in P\,. \end{array} \tag{1}$$

(i) Show that if P is a polytope and nonempty, then to solve (1) is equivalent to solve

$$\min\{c^T x^i \,|\, i = 1, \ldots, q\}\,. \tag{2}$$

In particular, the optimal solution of (1) can be found at an extreme point of P.

(ii) The optimal value in (1) is bounded if and only if

$$c^T d^i \ge 0,\ i = 1, \ldots, r\,.$$

Q3. Solve the following problem by the simplex method with bounded variables technique:

$$\begin{array}{ll} \max & 4x_1 + 2x_2 + 6x_3 \\ \text{s.t.} & 4x_1 + x_2 \leq 9 \\ & x_1 - x_2 + 2x_3 \leq 8 \\ & 1 \leq x_1 \leq 3 \\ & 0 \leq x_2 \leq 5 \\ & 0 \leq x_3 \leq 2\,. \end{array}$$

Q4. (Optional) Write a computer code to solve **Q3**.