Summary of H. P. Williams' Book for MA4260 Model Building in Operations Research

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Chapter 2 Solving Mathematical Programming Models:

Algorithms

- Revised simplex algorithm for LP models; (interior point methods).
- The separable extension of the revised simplex algorithm for separable programming models.
- The branch and bound algorithm for integer programming models.

Packages

1) Reduction: detecting and removing redundancies.

2) Starting solutions. User specified starting point.

3) Simple bound constraints

$$x \leq u$$

as a bound of variable rather than the conventional constraints.
[Bounded variable revised simplex method]

4) Ranged constraints

$$\begin{cases} \sum_j a_j x_j \le b_1 \,, \\ \sum_j a_j x_j \ge b_2 \,. \end{cases}$$

One can specify only the first constraint together with a range of $b_1 - b_2$ on the constraint. This is to limit slack variables

$$\begin{cases} \sum_j a_j x_j + y = b_1 \,, \\ 0 \le y \le b_1 - b_2 \,. \end{cases}$$

5) Generalized upper bounding constraints such as

$$x_1 + x_2 + \cdots + x_n \leq M$$

are common in linear programming models.

6) Sensitivity analysis

Practical Considerations

Let us consider the following blending problem:

A company manufactures a blending oil by refining raw oils and blending them together.

The raw oils come in two categories: vegetable oils (VEG 1 and VEG 2) and non-vegetable oils (OIL 1, OIL 2, and OIL 3).

Vegetable oils and non-vegetable oils require different production lines for refining. In any month, it is not possible to refine more than 200 tons of vegetable oil and more than 250 tons of non-vegetable oil.

The hardness of the final product in units, in which the hardness is measured, must lie between 3 and 6.

It is assumed that hardness blends linearly. The costs (per ton) and the hardness of the raw oils are

VEG 1	VEG 2	OIL 1	OIL 2	OIL 3
£110	£120	£130	£110	£115
8.8	6.1	2.0	4.2	5.0

The final product sells for £150 per ton.

Mathematically,

$$
\begin{aligned}
\max \quad & -110x_1 - 120x_2 - 130x_3 - 110x_4 - 115x_5 + 150y & \\
\text{s.t.} \quad & x_1 + x_2 & \le 200 \\
& x_3 + x_4 + x_5 & \le 250 \\
& 8.8x_1 + 6.1x_2 + 2x_3 + 4.2x_4 + 5x_5 - 6y & \le 0 \\
& 8.8x_1 + 6.1x_2 + 2x_3 + 4.2x_4 + 5x_5 - 3y & \ge 0 \\
& x_1 + x_2 + x_3 + x_4 + x_5 - y & = 0 \\
& x_1, x_2, x_3, x_4, x_5, y & \ge 0\,.
\end{aligned}
$$

Variables:

$$x_1 = \text{VEG1}$$
$$x_2 = \text{VEG2}$$
$$x_3 = \text{OIL1}$$
$$x_4 = \text{OIL2}$$
$$x_5 = \text{OIL3}$$
$$y = \text{PROD}\,.$$

Objective = PROF (profit)

Constraints

- VVEG (vegetable refining)
- NVEG (non-vegetable refining)
- UHAR (upper hardness)
- LHAR (lower hardness)
- CONT (continuity)

Right-hand side coefficients are regarded as a column and named CAP (capacity).

In the table format,

	VEG 1	VEG 2	OIL 1	OIL 2	OIL 3	PROD		CAP
PROF	-110	-120	- 130	- 110	- 115	150		
VVEG	1	1					$\leq$	200
NVEG			1	1	1		$\leq$	250
UHAR	8.8	6.1	2.0	4.2	5.0	-6.0	$\leq$	
LHAR	8.8	6.1	2.0	4.2	5.0	-3.0	$\geq$	
CONT	1.0	1.0	1.0	1.0	1.0	-1.0	$=$	

Blank cells indicate a zero coefficient.

More conveniently, we use MPS (mathematical programming system) format to represent the data in the table.

Data are divided into three main sections:

- ROWS section
- COLUMNS section
- RHS section

After naming the problem BLEND,

The ROWS section consists of a list of the rows in the model together with a designator N, L, G, or E:

- N: non-constraint row
- COLUMNS section
- L: less-than-or-equal ($\leq$) row
- G: greater-than-or-equal ($\geq$) row
- E: stands for an equality ($=$) constraint

The COLUMNS section contains the body of the matrix coefficients. These are scanned column by column with up to two nonzero coefficients in a statement (zero coefficients are ignored). Each statement contains the column name, row names, and the corresponding matrix coefficients.

The RHS section is regarded as a column using the same format as the columns section

The ENDDATA entry indicates the end of the data.

OTHER DATA possible such as bounds. Found in the manual.

```
NAME              BLEND
ROWS
  N PROF
  L VVEG
  L NVEG
  L UHRD
  G LHRD
  E CONT
COLUMNS
      VEG       01      PROF    -110.000000    VVEG      1.000000
                                    .
                                    .
                                    .
      PROD              LHRD      -3.000000    CONT     -1.000000
RHS
      RHS       01      VVEG     200.000000    NVEG     250.00000
ENDDATA
```