NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester 1 (2003/2004) MA4253 Mathematical Programming Tutorial 1

Q1. Let A be an $m \times n$ matrix and b be a vector in $\Re^m$. Show that

$$S := \{x \in \Re^n \mid \|Ax - b\| \leq 1\}$$

is a convex set, where $\|\cdot\|$ is the Euclidean norm in $\Re^m$.

Q2. Which of the following sets S are polyhedra? If possible, express S in the form of $S = \{x \mid Ax \leq b, Fx = g\}$.

(a) $S = \{y_1 a_1 + y_2 a_2 \mid -1 \leq y_1 \leq 1, -1 \leq y_2 \leq 1\}$, where $a_1, a_2 \in \Re^n$.

(b) $S = \{x \in \Re^n \mid x \geq 0, e^T x = 1, \sum_{i=1}^n x_i a_i = b_1, \sum_{i=1}^n x_i a_i^2 = b_2\}$, where $a_1, \ldots, a_n, b_1, b_2 \in \Re$ and $e = (1, \ldots, 1)^T$.

(c) $S = \{x \in \Re^n \mid x \geq 0, x^T y \leq 1 \ \forall\ y \text{ with } \|y\| = 1\}$.

(d) $S = \{x \in \Re^n \mid x \geq 0, x^T y \geq 1 \ \forall\ y \text{ with } \|y\| = 1\}$.

(e) $S = \{x \in \Re^n \mid x \geq 0, x^T y \leq 1 \ \forall\ y \text{ with } \sum_i |y_i| = 1\}$.

(f) $S = \{x \in \Re^n \mid x \geq 0, x^T y \geq 1 \ \forall\ y \text{ with } \sum_i |y_i| = 1\}$.

Q3. Find all extreme points of the following problem:

$$\begin{array}{ll} \min & 2x_1 + 4x_2 + 7x_3 \\ \text{s.t.} & 2x_1 + x_2 + 6x_3 \geq 5 \\ & 4x_1 - 6x_2 + 5x_3 \geq 8 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

Q4. Let $P = \{x \in \Re^3 \mid 2x_3 \geq 1,\ 4x_1 \leq 3,\ x_1 + x_2 + x_3 = 1,\ x \geq 0\}$. Give two basic feasible solutions of P.

Q5. Let $P = \{x \in \Re^n \mid Ax = b,\ x \geq 0\}$, where $A \in \Re^{m \times n}$ is of full row rank. Show that for any basic feasible solution x of P there exists a submatrix $B \in \Re^{m \times m}$ of A such that B is nonsingular, $B^{-1}b \geq 0$ and x is of the form $((B^{-1}b)^T\ 0)^T$.