NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester 1 (2003/2004) MA4253 Mathematical Programming Tutorial 6

Q1. Consider the following linear programming problem:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{1}$$

and its dual

$$\begin{aligned} \max \quad & b^T y \\ & A^T y \leq c\,, \end{aligned}$$

where A is an $m \times n$ matrix. Show that both the primal and the dual problems have a unique solution under the following conditions:

(i) every basic feasible solution to the primal problem is nondegenerate;

(ii) at every basic feasible solution to the primal problem, the reduced cost of every nonbasic variable is nonzero; and

(iii) there exists an optimal solution to the primal problem.

Q2. Show that problem (1) has an optimal cost $-\infty$ if the constraints in problem (1) contain an extreme ray d such that $c^T d < 0$.

Q3. Consider problem (1). We assume that the rows of A are linearly independent and that (i) every basic feasible solution to problem (1) is nondegenerate and (ii) at every basic feasible solution to problem (1), the reduced cost of every nonbasic variable is nonzero. Let x be some feasible solution to problem (1) and suppose that we have a vector p such that

$$x_i(c_i - A_i^T p) = 0\,, \quad \forall\, i\,.$$

Show that x is a basic feasible solution to problem (1).