Model Building in Operations Research

MA4260

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Objectives and scope. This course is intended to give a global view of Operations Research from the perspective of applications. The emphasis is to illustrate how application problems can be modeled mathematically as optimization problems. The first part of the course introduces basic models essential to model building. The second part is on the formulation of a variety of realistic problems.

The focus of this module is on modelling instead of methods for solving the formulated problems. Students are encouraged to use computers to solve the modelled problems.

Contents

1. Overview
2. Linear, integer linear and nonlinear programming (deterministic and stochastic)
3. Semidefinite and second order cone programming
4. Complementarity and variational inequality problems (static and dynamic)
5. Case Studies.

Assessment (subject to changes at Instructor's discretion).

* 20% on assignments, including tutorial-class performance.
* 20% on one mid-term test (1 hour).
* 60% on the final examination (2.5 hours).

Recommended texts

1. H.P. Williams, *Model Building in Mathematical Programming*, John Wiley, 1999.
2. S.P. Bradley, A.C. Hax an T.L. Magnantee, *Applied Mathematical Programming*, Adison-Wesley, 1977.
3. Dimitris Bertsimas and John Tsitsiklis, *Introduction to Linear Optimization*, Athema Scientific, Belmont, Mass., 1997.
4. R.W. Cottle, J-S. Pang and R.E. Stone, *The Linear Complementarity Problem*, Academic Press, 1992.

5. S. Boyd and and L. Vandenberghe, *Convex Programming,* Cambridge University Press, 2004.

6. M.S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, “Applications of second-order cone programming,” Linear Algebra and Its Applications, 284:193-228, 1998.

- Books 1-4 are reserved for you in the “RBR” section of Science Library. The first part of this course will be based on Book 1 and Chapter 1 of Book 2. Item 5 and 6 can be found at Professor Lieven Vandenberghe’s webpage

http://www.ee.ucla.edu/~vandenbe/publications.html

1 Overview

1.1 An overview of OR

Operations research = Management science

* OR is characterized by the use of mathematical models in providing guidelines to decision makers for making effective decisions within the current state of informatioin.
* A key feature is in the use of mathematical methods and the capabilities of computers to solve complex decision problems confronting modern managers.
* Mathematical programming is one of the best developed and most widely used branches of OR. It concerns the optimum allocation

of scarce resources among competing activities, under a set of constraints (financial, technological, social, marketing, etc) imposed by the nature of the problem being analyzed. Broadly speaking, MP is a mathematical representation aimed at planning the best possible allocation of scarce resources.

* An abstract MP model has the form:

$$\begin{aligned} \min / \max \;& f(x) \\ \text{subject to } \;& g_i(x) \leq b_i, \quad i = 1, 2, \ldots, m \\ & x \in X \subseteq R^n, \end{aligned}$$

where f, g_i are real-valued functions defined on X.

The emphasis in this course is on learning principles and techniques that lead to **good formaulations** of actual decision problems.

1.2 Basic models

Models are **building blocks** for formulating complex OR problems.

1. Linear programming
2. network models
3. integer and mixed-integer programming
4. nonlinear programming
5. complementarity and variational inequality problems
6. semidefinite and second order cone programming
7. dynamix programming
8. stochastic programming
9. dynamic complementarity problems
10. many others

1.3 Modeling principles

* A practical problem can often be modelled in more than one way.

  Compare and contrast results from different types of formulation.

* Realism vs. solvability

  A model should be realistic enough to capture the main features of the problems being studied, yet it should be "simple" enough to allow computation of its solutions within a reasonable amount of computer resources (CPU time and memory).

* Model-design effort tries to provide guidelines to managers (at the same time, increase their understanding of the consequences) in decision making. This is to support, but not to replace, management

actions. Thus modellers and managers should work closely.

Managers should formulate the basic questions to be addressed by the model, and then interpret the model's solutions in light of their own experience and intuition, taking into account of model's limitations.

* Sensitivity analysis

– Is the model robust under perturbation of data?

– How does the results change when data is modified?

1.4 Main steps in formulation

(1) Identify the decision variables.

– Elements within our control.

– Continuous or discrete

– Range of values for the solution

(2) Determine the objective function to be optimized

– Are there alternative criteria?

(3) Identify the constraints

– Restrictions on the values of the decision variables

– Write down even the obvious constraints such as non-negativity

– Some of the constraints may be less critical then others, or even redundant. These may be revealed in the analysis stage.

1.5 Algorithms and packages

* Optimization technology center – software, research, and online computing (NEOS).
  http://www.ece.northwestern.edu/OTC
* Benchmarks for optimization software
  http://plato.la.asu.edu/bench.html

Other relevant sites:

* An eprint site for optimization community – research reports.
  http://www.optimization-online.org
* Linear and nonlinear programming frequently

asked questions

http://www-unix.mcs.anl.gov/otc/Guide/faq