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ST2 / 9/1

MA4260 MBOR TUTORIAL 1 GROUP 1

5. a) Based on the information provided in the question and our interpretation, the tableu form of the problem should be as follows.

	Decision variables x_1	Decision variables x_2	Relation	Requirements
Drive-train	1	1	$\leq$	4000
Coachwork	2	1	$\leq$	6000
Final Assembly	1		$\leq$	2500
		1		3000
Objective	590	270	=	z (max)

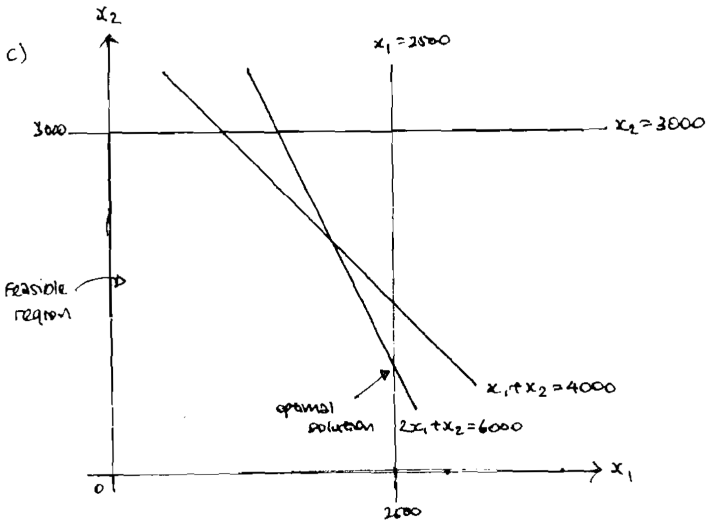
$x_1, x_2 \geq 0$ (integer)

where x_1 represents Aztecs and x_2 represents Broncos

Assumptions: The depreciation and fixed overhead costs are fixed at the allocated price based on the planned production.

b) Objective function for the division: ~~Max~~ $590x_1 + 270x_2$

c)



Optimal solution at (2500, 1000)

∴ Since, there is no prediction for demand, they should produce as many vehicles to maximise profit.

∴ They should produce 2500 Aztecs and 1000 Broncos.

Profit = (2500)(590) + (1000)(270) = \$

6. Consider the sub-contractor costs for each vehicle respectively. i.e. \$300 and \$270. The process of sub-contracting the parts for the drive-trains would imply an increase in the total drive-train capacity of the division. However, it does not change the fact that the final assembly capacity is fixed at 2500 and 3000, respectively. i.e to say, the active constraints as seen for 5(c) is dependent on the coachwork and final assembly of Aztecs, hence as long as these constraints are not affected, the optimal solution will remain as in part (c) of Q^n 5.

Therefore, the division should <u>NOT</u> pay the sub-contractor for the assembly.

(Assumption: 1 vehicle requires exactly 1 part drive-train and 1 part coach work).

7.

Since the question states that there is a 50% overtime labour premium, we assume it refers to the labour costs. i.e to say, the profit for Aztecs and Broncos will be reduced to \$390 and \$45 respectively for every drivetrain produced at overtime.

$\therefore$ Let x_3, x_4 be the numbers of Aztecs and Broncos (respectively) produced in overtime.

Hence, the new objective function is,

$$\text{Max} \quad 590x_1 + 270x_2 + 390x_3 + 45x_4 - 150000x_5$$

(Note: x_5 takes on values 1 or 0, if either x_3 or x_4 is non zero, x_5 takes on value 1, if $x_3 = x_4 = 0$, then $x_5 = 0$, this is the consideration for the increased fixed overhead of 150000)

subjected to $\quad 2x_1 + x_2 + 2x_3 + x_4 \leq 6000$ (coachwork)

$x_1 + x_3 \leq 2500$, $\quad x_1 + x_2 \leq 4000$

$x_2 + x_4 \leq 3000$, $\quad x_3 + x_4 \leq Mx_5$

$x_1, x_2, x_3, x_4 \geq 0$, $\quad x_5 \leq M(x_3 + x_4)$, where M is a very large number

$0 \leq x_5 \leq 1$

$x_i \in \mathbb{Z} \quad \forall\, i = 1,2,3,4,5.$

(Note: Drivetrain is not considered, because we assume that the capacity is unlimited due to the overtime production).

Hence,

in consideration of the new problem, we see that as in Q1 & 6, the active constraints (namely, the coachwork and especially the final assembly capacity) are still not changed in a way such that the capacities are increased to allow more vehicles to be produced. Therefore, together with all the assumptions made, the use of overtime production is not profitable for the division, especially with the 150000 fixed overhead production.

Hence, the division should <u>not</u> go into overtime drivetrain production.

MA 4260 Tutorial 1 ST2 Group5

Qn20 Let x_i^+ and x_i^- be the be the increase and decrease in production from period $i-1$ to period i respectively.

The LP can be summarised in the following table.

Var	x_1^+	x_2^+	x_3^+	x_4^+	x_5^+	x_6^+	x_7^+	x_8^+	x_9^+	x_{10}^+	x_{11}^+	x_{12}^+	x_1^-	x_2^-	x_3^-	x_4^-	x_5^-	x_6^-	x_7^-	x_8^-	x_9^-	x_{10}^-	x_{11}^-	x_{12}^-		Value
d1	1												-1												≥	-2
d2	2	1											-2	-1											≥	0
d3	3	2	1										-3	-2	-1										≥	4
d4	4	3	2	1									-4	-3	-2	-1									≥	12
d5	5	4	3	2	1								-5	-4	-3	-2	-1								≥	24
d6	6	5	4	3	2	1							-6	-5	-4	-3	-2	-1							≥	40
d7	7	6	5	4	3	2	1						-7	-6	-5	-4	-3	-2	-1						≥	56
d8	8	7	6	5	4	3	2	1					-8	-7	-6	-5	-4	-3	-2	-1					≥	64
d9	9	8	7	6	5	4	3	2	1				-9	-8	-7	-6	-5	-4	-3	-2	-1				≥	68
d10	10	9	8	7	6	5	4	3	2	1			-10	-9	-8	-7	-6	-5	-4	-3	-2	-1			≥	70
d11	11	10	9	8	7	6	5	4	3	2	1		-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1		≥	70
d12	12	11	10	9	8	7	6	5	4	3	2	1	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	≥	70
inv1	1												-1												≤	8
inv2	2	1											-2	-1											≤	10
inv3	3	2	1										-3	-2	-1										≤	14
inv4	4	3	2	1									-4	-3	-2	-1									≤	22
inv5	5	4	3	2	1								-5	-4	-3	-2	-1								≤	34
inv6	6	5	4	3	2	1							-6	-5	-4	-3	-2	-1							≤	50
inv7	7	6	5	4	3	2	1						-7	-6	-5	-4	-3	-2	-1						≤	66
inv8	8	7	6	5	4	3	2	1					-8	-7	-6	-5	-4	-3	-2	-1					≤	74
inv9	9	8	7	6	5	4	3	2	1				-9	-8	-7	-6	-5	-4	-3	-2	-1				≤	78
inv10	10	9	8	7	6	5	4	3	2	1			-10	-9	-8	-7	-6	-5	-4	-3	-2	-1			≤	80
inv11	11	10	9	8	7	6	5	4	3	2	1		-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1		≤	80
inv12	12	11	10	9	8	7	6	5	4	3	2	1	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	≤	80
obj	500	500	500	500	500	500	500	500	500	500	500	500	250	250	250	250	250	250	250	250	250	250	250	250	=	*min*

In addition, $\forall\ i$, $x_i^+ \geq 0$, $x_i^- \geq 0$

We use cplex dynamic program, excel, matlab and cplex lp to solve the problem. There are multiple optimal solutions for this question, It might be due to the zero inventory cost assumption.

Qn23 Denote the decision variables as following.$i = 1, 2, 3$

N_i number of new men being hired at the beginning of period i

t_i total number of hours which all trained workers work overtime in period i

l_i total number of radars which are shipped out late in period i

s_i total number of radars which are kept in the storage space in period i

All variables are non-negative.

total amount of revenue generated in these three month$= 500 * 1000$

cost incurred = penalty cost for late shipment + storage cost for excess production + hiring cost + employees' salary

Our ojective is to maximize the net profit which equals to the revenue-cost incurred.In this problem, the revenue is a constant number, to maximize the net profit is equivalent to minimize the cost incurred. There is a production conservation constraint and overtime constraint in each period.For example,in March the production conservation constraint is $300 - l_1 + s_1 = \frac{1}{43}(90 * 172 + 129N_1 + t_1)$,the overtime constraint is $0 \leq \frac{t_1}{90} \leq 35$

There is another constraint about the number of trained men.It requires to have 200 trained men on 1st June. Hence we have $0.95^3(90 + N_1) + 0,95^2 N_2 + 0.95N_3 = 200$.

The LP can be summarised in the following table.

Var	N_1	N_2	N_3	t_1	t_2	t_3	l_1	l_2	l_3	s_1	s_2	s_3		Value
Production1	3			$\frac{1}{43}$			1			-1			=	-60
Production2	3.8	3			$\frac{1}{43}$		-1	1		1	-1		=	58
Production3	3.61	3.8	3			$\frac{1}{43}$		-1	1		1	-1	=	-24.9
Overtime1				1									$\leq$	3150
Overtime1	-33.25				1								$\leq$	2992.5
Overtime1	$-\frac{2527}{80}$	$-\frac{133}{4}$				1							$\leq$	$\frac{22743}{8}$
Trained men	0.95^3	0.95^2	0.95										=	$200 - 0.95^3 * 90$

we use cplexlp and matlab to solve this problem. we have obtained a unique optimal solution.

23. let N_1 denote no. of new men being hired on 1 March

let t_1 denote total no. of hours trained workers work overtime in March

let l_1 denote no. of radars which are shipped out late in March

let s_1 denote no. of radars which are kept in storage space in March

total no. of radars produced in March $= 300 - l_1 + s_1 = \frac{1}{43}(90 \times 172 + 129 N_1 + t_1)$

i.e. $300 - l_1 + s_1 = 360 + 3N_1 + \frac{t_1}{43}$

$3N_1 + \frac{t_1}{43} + l_1 - s_1 = -60$

$0 \le \frac{t_1}{90} \le 35$, $N_1 \geq 0$, $l_1 \geq 0$, $s_1 \geq 0$

Revenue generated in March $= (300 - l_1) \times 500$

cost incurred in March = penalty cost for late shipment + storage cost for excess production + hiring cost + employees' salary

$= 50 l_1 + 10 s_1 + 200 N_1 + 860(90 + N_1) + 7.5 t_1$

Net income in March $= (300 - l_1) \times 500 - 50 l_1 - 10 s_1 - 200 N_1 - 860(90 + N_1) - 7.5 t_1$

$= 72600 - 550 l_1 - 10 s_1 - 1060 N_1 - 7.5 t_1$

Next, we consider April decision.

let N_2 denote no. of new men being hired on 1 April

let t_2 denote total no. of hours trained men work overtime in April.

let l_2 denote no. of radars which are shipped out late in ~~Ma~~ April.

let s_2 denote no. of radars which are kept in storage space in April.

~~total no. of radars produced in April = 400 − l_2 + s_2~~

no. of trained men in April $= (90 + N_1)(1 - 5\%) = 85.5 + 0.95 N_1$

total no. of radars produced in April :

$l_1 - s_1 + 400 - l_2 + s_2 = \frac{1}{43}[172(85.5 + 0.95 N_1) + 129 N_2 + t_2]$

$l_1 - s_1 + 400 - l_2 + s_2 = 342 + 3.8 N_1 + 3N_2 + \frac{t_2}{43}$

$3.8 N_1 + 3N_2 + \frac{t_2}{43} - l_1 + l_2 + s_1 - s_2 = 58$

$0 \le \frac{t_2}{85.5 + 0.95 N_1} \le 35$, $N_2 \geq 0$, $l_2 \geq 0$, $s_2 \geq 0$ i.e. $0 \le t_2 \le 35(85.5 + 0.95 N_1)$

Revenue generated in April $= (400 - l_2) \times 500$

cost incurred in April $= 50 l_2 + 10 s_1 + 200 N_2 + 860[85.5 + 0.95 N_1 + N_2] + 7.5 t_2$

Net income in April $= (400 - l_2) \times 500 - 50 l_2 - 10 s_2 - 200 N_2 - 860[85.5 + 0.95 N_1 + N_2) - 7.5 t_2$

$= 126470 - 550 l_2 - 10 s_2 - 817 N_1 - 1060 N_2 - 7.5 t_2$

Similarly, for May decision

let N_3 denote ~~of~~ no. of new men being hired on 1 May

let t_3 denote total no. of hours trained men work overtime in May

let l_3 denote no. of radars which are shipped out late in May

let S_3 denote no. of radars which are kept in storage space in May

No. of trained man in May $= (90+N_1)(1-5\%)^2 + (1-5\%)N_2 = \frac{3249}{40} + \frac{361}{400}N_1 + \frac{19}{20}N_2$

total no. of radars produced in May :

$$l_2 - S_2 + 300 - l_3 + S_3 = \frac{1}{43}\left[172\left(\frac{3249}{40} + \frac{361}{400}N_1 + \frac{19}{20}N_2\right) + 129N_3 + t_3\right] \cancel{+ l_2 - S_2}$$

$$l_2 - S_2 + 300 - l_3 + S_3 = 324.9 + 3.61N_1 + 3.8N_2 + 3N_3 + \frac{t_3}{43} \cancel{+ l_2 - S_2}$$

$$3.61N_1 + 3.8N_2 + 3N_3 + \frac{t_3}{43} - l_2 + l_3 + S_2 - S_3 = -24.9$$

$$0 \le \frac{t_3}{\frac{3249}{40} + \frac{361}{400}N_1 + \frac{19}{20}N_2} \le 35 \quad \text{i.e.} \quad 0 \le t_3 \le 2842\frac{7}{8} + \frac{2527}{80}N_1 + \frac{133}{4}N_2$$

$$N_3 \ge 0, \quad l_3 \ge 0, \quad S_3 \ge 0$$

Revenue in May $= (300 - l_3) \times 500$

cost incurred in May $= 50l_3 + 10S_3 + 200N_3 + 860\left(\frac{3249}{40} + \frac{361}{400}N_1 + \frac{19}{20}N_2 + N_3\right) + 7.5t_3$

Net income in May $= (300 - l_3) \times 500 - 50l_3 - 10S_3 - 200N_3 - 860\left(\frac{3249}{40} + \frac{361}{400}N_1 + \frac{19}{20}N_2 + N_3\right) - 7.5t_3$

$$= 80146.5 - 550l_3 - 10S_3 - 776.15N_1 - 817N_2 - 1060N_3 - 7.5t_3$$

No. of trained men on 1st June $= (90+N_1)(1-5\%)^3 + N_2(1-5\%)^2 + N_3(1-5\%) = 200$

$$\text{i.e.} \quad 0.95^3 \times 90 + 0.95^3 N_1 + 0.95^2 N_2 + 0.95N_3 = 200$$

$$0.95^3 N_1 + 0.95^2 N_2 + 0.95N_3 = 200 - 0.95^3 \times 90$$

objective function is to maximize the net income in these three months.

$$\max \; 72600 - 550l_1 - 10S_1 - 1060N_1 - 7.5t_1 + 126470 - 550l_2 - 10S_2 - 817N_1 - 1060N_2 - 7.5t_2$$
$$+ 80146.5 - 550l_3 - 10S_3 - 776.15N_1 - 817N_2 - 1060N_3 - 7.5t_3$$

$$\Rightarrow \max \; 279216.5 - 550(l_1+l_2+l_3) - 10(S_1+S_2+S_3) - 7.5(t_1+t_2+t_3) - 2653.15N_1 - 1877N_2 - 1060N_3$$

$$\equiv \min \; 550(l_1+l_2+l_3) + 10(S_1+S_2+S_3) + 7.5(t_1+t_2+t_3) + 2653.15N_1 + 1877N_2 + 1060N_3$$

constraints :

$$3N_1 + \frac{t_1}{43} + l_1 - S_1 = -60$$

$$3.8N_1 + 3N_2 + \frac{t_2}{43} - l_1 + l_2 + S_1 - S_2 = 58$$

$$3.61N_1 + 3.8N_2 + 3N_3 + \frac{t_3}{43} - l_2 + l_3 + S_2 - S_3 = -24.9$$

$$t_1 \le 3150$$

$$t_2 \le 35(85.5 + 0.95N_1) \quad \text{i.e.} \quad -33.25N_1 + t_2 \le 2992.5$$

$$-\frac{2527}{80}N_1 - \frac{133}{4}N_2 + t_3 \le \frac{22743}{8}$$

$$0.95^3 N_1 + 0.95^2 N_2 + 0.95N_3 = 200 - 0.95^3 \times 90$$

```
function [] = T1_23()

%x = (N1,N2,N3,t1,t2,t3,l1,l2,l3,s1,s2,s3,a1,a2,a3) (a1 a2 a3 are slack
%variables)

c = [2653.15;1877;1060;7.5;7.5;7.5;550;550;550;10;10;10;0;0;0];

b = [-60;58;-24.9;3150;2992.5;22743/8;200-0.95^3*90];

A = [3 0 0 1/43 0 0 1 0 0 -1 0 0 0 0 0;3.8 3 0 0 1/43 0 -1 1 0 1 -1 0 0 0 0;3.61 3.8 3 0 0
1/43 0 -1 1 0 1 -1 0 0 0;0 0 0 1 0 0 0 0 0 0 0 0 1 0 0;-33.25 0 0 0 1 0 0 0 0 0 0 0 0 1 0;
-2527/80 -133/4 0 0 0 1 0 0 0 0 0 0 0 0 1;0.95^3 0.95^2 0.95 0 0 0 0 0 0 0 0 0 0 0 0]

[x,y,INFO] = sedumi(A,b,c);
x
optimal = c'*x

maximum_net_income = 279216.5 - optimal
```

```
A =

  Columns 1 through 7

    3.0000         0         0    0.0233         0         0    1.0000
    3.8000    3.0000         0         0    0.0233         0   -1.0000
    3.6100    3.8000    3.0000         0         0    0.0233         0
         0         0         0    1.0000         0         0         0
  -33.2500         0         0         0    1.0000         0         0
  -31.5875  -33.2500         0         0         0    1.0000         0
    0.8574    0.9025    0.9500         0         0         0         0

  Columns 8 through 14

         0         0   -1.0000         0         0         0         0
    1.0000         0    1.0000   -1.0000         0         0         0
   -1.0000    1.0000         0    1.0000   -1.0000         0         0
         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Column 15

         0
         0
         0
         0
         0
    1.0000
         0
```

```
SeDuMi 1.1 by AdvOL, 2005 and Jos F. Sturm, 1998, 2001-2003.
Alg = 2: xz-corrector, Adaptive Step-Differentiation, theta = 0.250, beta = 0.500
eqs m = 7, order n = 16, dim = 16, blocks = 1
nnz(A) = 31 + 0, nnz(ADA) = 39, nnz(L) = 23
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :              7.34E+006 0.000
  1 :  9.28E+005 2.08E+006 0.000 0.2836 0.9000 0.9000   2.35  1  1  2.0E+001
  2 :  3.23E+005 4.75E+005 0.000 0.2282 0.9000 0.9000   1.87  1  1  3.4E+000
  3 :  1.84E+005 1.08E+005 0.000 0.2273 0.9000 0.9000   1.10  1  1  8.1E-001
  4 :  1.57E+005 3.42E+004 0.000 0.3166 0.9000 0.9000   1.13  1  1  2.7E-001
  5 :  1.44E+005 6.28E+003 0.000 0.1836 0.9000 0.9000   1.04  1  1  5.2E-002
  6 :  1.42E+005 5.10E+002 0.000 0.0813 0.9900 0.9900   0.98  1  1  4.4E-003
  7 :  1.42E+005 4.80E+001 0.000 0.0940 0.9120 0.9000   0.86  1  1  4.6E-004
  8 :  1.42E+005 1.04E-002 0.049 0.0002 0.9999 0.9999   0.99  1  1
iter seconds digits       c*x               b*y
  8      0.8    Inf  1.4182743421e+005  1.4182743421e+005
|Ax-b| =   3.4e-013, [Ay-c]_+ =  1.3E-015, |x|= 5.2e+003, |y|= 1.1e+003

Detailed timing (sec)
   Pre          IPM          Post
5.808E-001    7.511E-001    1.702E-001
Max-norms: ||b||=3150, ||c|| = 2.653150e+003,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 1.

x =

  1.0e+003 *

   (3,1)        0.1293
  (10,1)        0.0600
  (11,1)        0.0020
  (12,1)        0.4148
  (13,1)        3.1500
  (14,1)        2.9925
  (15,1)        2.8429


optimal =

  1.4183e+005
```

← maximum of -cost

```
maximum_net_income =

  1.3739e+005
```

← maximum net income

Monday 1-2pm / Group 6 ST1

Guo Wenjin Chenyi U0326181

Ong Bi Hui U0328062

Date : : No.

25. Let x_i = number of rolls of width 100 inches cut as combination i, $i = 1, 2, \ldots, 7$.

Combination	No. of 24-inch	No. of 32-inch	No. of 40-inch	Trim waste (inches)
1	4	0	0	4
2	2	1	0	20
3	2	0	1	12
4	1	2	0	12
5	1	1	1	4
6	0	3	0	4
7	0	0	2	20

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	relation	requirement
24-inch rolls	4	2	2	1	1	0	0	$\geq$	75
32-inch rolls	0	1	0	2	1	3	0	$\geq$	110
40-inch rolls	0	0	1	0	1	0	2	$\geq$	50

$x_i \geq 0, \quad i = 1, 2, \ldots, 7.$

min $4x_1 + 20x_2 + 12x_3 + 12x_4 + 4x_5 + 4x_6 + 20x_7$

$+ 24(4x_1 + 2x_2 + 2x_3 + x_4 + x_5 - 75)$

$+ 32(x_2 + 2x_4 + x_5 + 3x_6 - 110)$

$+ 40(x_3 + x_5 + 2x_7 - 50)$

Optimal solution $x_1 = 4$

$x_3 = 9$

$x_5 = 41$

$x_6 = 23$

$x_2 = x_4 = x_7 = 0$

with optimal objective value 380

26. Let y_j = number of rolls of width 90 inches cut as combination j, $j = 1, 2, \ldots, 6$.

combination	No. of 24-inch	No. of 32-inch	No. of 40-inch	Trim waste (inches)
1	3	0	0	18
2	2	1	0	10
3	2	0	1	2
4	1	2	0	2
5	0	1	1	18
6	0	0	2	10

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	y_2	y_3	y_4	y_5	y_6	relation	requirement
24-inch roll	4	2	2	1	1	0	0	3	2	2	1	0	0	$\geq$	300
32-inch roll	0	1	0	2	1	3	0	0	1	0	2	1	0	$\geq$	180
40-inch roll	0	0	1	0	1	0	2	0	0	1	0	1	2	$\geq$	225
Machine #1	1	1	1	1	1	1	1	0	0	0	0	0	0	$\leq$	400
Machine #2	0	0	0	0	0	0	0	1	1	1	1	1	1	$\leq$	400

$x_i \geq 0$, $i = 1, 2, \ldots, 7$

$y_j \geq 0$, $j = 1, 2, \ldots, 6$

min $4x_1 + 20x_2 + 12x_3 + 12x_4 + 4x_5 + 4x_6 + 20x_7 + 18y_1 + 10y_2 + 2y_3 + 2y_4 + 18y_5 + 10y_6$

$+24(4x_1 + 2x_2 + 2x_3 + x_4 + x_5 + 3y_1 + 2y_2 + 2y_3 + y_4 - 300)$

$+32(x_2 + 2x_4 + x_5 + 3x_6 + y_2 + 2y_4 + y_5 - 180)$

$+40(x_3 + x_5 + 2x_7 + y_3 + y_5 + 2y_6 - 225)$

Optimal solution $x_5 = 150$

$x_6 = 10$

$y_3 = 75$

$x_1 = x_2 = x_3 = x_4 = x_7 = y_1 = y_2 = y_4 = y_5 = y_6 = 0$

optimal objective function 790.

ST2

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MA4260 Model Building in OR Tutorial 1

27a) Each school has a capacity C_{jg} for grade g.
Each of I neighbourhoods has student population S_{ig} for neighbourhood i with grade g.
d_{ij} = distance from neighbourhood i to school j.

Let x_{ijg} represent the number of students from neighbourhood i $(i=1,\dots,I)$ to school j $(j=1,\dots,J)$ with grade g $(g=1,\dots,G)$.

$$\min \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{g=1}^{G} d_{ij}x_{ijg}$$

$$\text{s.t. } \sum_{i=1}^{I} x_{ijg} \le C_{jg} \quad \text{for } \forall j=1,\dots,J,\ \forall g=1,\dots,G \quad \text{(capacity)}$$

$$\sum_{j=1}^{J} x_{ijg} = S_{ig} \quad \text{for } \forall i=1,\dots,I,\ \forall g=1,\dots,G \quad \text{(student population)}$$

$$x_{ijg} \ge 0,\ x_{ijg} \text{ integer}$$

b) S_{ikg} = number of students in neighbourhood i of race k and grade g.
a_k = maximum percent of racial group k assigned to a school.
b_k = minimum percent of racial group k assigned to a school.

Let x_{ijkg} represent the number of students in neighbourhood i $(i=1,\dots,I)$, to school j $(j=1,\dots,J)$ of race k $(k=1,\dots,K)$ with grade g $(g=1,\dots,G)$.
Let T_j represents the total number of students in school j. i.e. $T_j = \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{g=1}^{G} x_{ijkg}$

$$\min \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G} d_{ij}x_{ijkg}$$

$$\text{s.t. } \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijkg} \le C_{jg} \quad \text{for } \forall j=1,\dots,J,\ \forall g=1,\dots,G \quad \text{(capacity)}$$

$$\sum_{j=1}^{J} x_{ijkg} = S_{ikg} \quad \text{for } \forall i=1,\dots,I,\ \forall k=1,\dots,K,\ \forall g=1,\dots,G \quad \text{(student population)}$$

$$\sum_{i=1}^{I}\sum_{g=1}^{G} x_{ijkg} \le \frac{a_k}{100}T_j \quad \text{for } \forall j=1,\dots,J,\ \forall k=1,\dots,K \quad \text{(race upper)}$$

$$\sum_{i=1}^{I}\sum_{g=1}^{G} x_{ijkg} \ge \frac{b_k}{100}T_j \quad \text{for } \forall j=1,\dots,J,\ \forall k=1,\dots,K \quad \text{(race lower)}$$

$$x_{ijkg} \ge 0,\ x_{ijkg} \text{ integer}$$

c) For (a),

Let x_{ighj} represent student h ($h = 1, \dots, S_{ig}$) in neighbourhood i ($i = 1, \dots, I$) to school j ($j = 1, \dots, J$) with grade g ($g = 1, \dots, G$).

Note: h is dependent on i and g.

Let $x_{ighj} = \begin{cases} 1 & \text{if student } h \text{ attends school } j \\ 0 & \text{otherwise} \end{cases}$

Let $z = d_{ij} x_{ighj}$

$$\min \quad z$$

s.t. $z \geq d_{ij} x_{ighj}$

$\sum_{j=1}^{J} x_{ighj} = 1 \quad$ for $\forall i = 1, \dots I$, $\forall g = 1, \dots, G$, $\forall h = 1, \dots, S_{ig}$ (student population)

$\sum_{i=1}^{I} \sum_{h=1}^{S_{ig}} x_{ighj} \leq C_{jg} \quad$ for $\forall j = 1, \dots, J$, $\forall g = 1, \dots, G$ (capacity)

$x_{ighj} \geq 0$, x_{ighj} integer, $z \geq 0$

For (b).

Let x_{ikghj} represent student h ($h = 1, \dots, S_{ikg}$) in neighbourhood i ($i = 1, \dots, I$) to school j ($j = 1, \dots, J$) of race k ($k = 1, \dots, K$) with grade g ($g = 1, \dots, G$).

Note: h is dependent on i, k, g.

Let $x_{ikghj} = \begin{cases} 1 & \text{if student } h \text{ attends school } j \\ 0 & \text{otherwise} \end{cases}$

Let T_j represents the total number of students in school j i.e. $T_j = \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{g=1}^{G} \sum_{h=1}^{S_{ikg}} x_{ikghj}$

Let $z = d_{ij} x_{ikghj}$

$$\min \quad z$$

s.t. $z \geq d_{ij} x_{ikghj}$

$\sum_{j=1}^{J} x_{ikghj} = 1 \quad$ for $\forall i = 1, \dots, I$, $\forall k = 1, \dots, K$, $\forall g = 1, \dots, G$, $\forall h = 1, \dots, S_{ikg}$ (student population)

$\sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{h=1}^{S_{ikg}} x_{ikghj} \leq C_{jg} \quad$ for $\forall j = 1, \dots, J$, $\forall g = 1, \dots, G$ (capacity)

$\sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{h=1}^{S_{ikg}} x_{ikghj} \leq \frac{a_k}{100} T_j \quad$ for $\forall j = 1, \dots, J$, $\forall k = 1, \dots, K$ (race upper)

$\sum_{i=1}^{I} \sum_{g=1}^{G} \sum_{h=1}^{S_{ikg}} x_{ikghj} \geq \frac{b_k}{100} T_j \quad$ for $\forall j = 1, \dots, J$, $\forall k = 1, \dots, K$ (race lower)

$x_{ikghj} \geq 0$, x_{ikghj} integer, $z \geq 0$

MA4260 tutorial 1

Monday group 5
Wang Muran Chew Sze Chong Zhao Ya Goh Siong Thye

Question 28

a) Minimize $\sum_{i=1}^{6}(P_i - b_0 - b_1 L_i - b_2 E_i)^2$

To solve this problem, we can partial differentiate the objective function with respect to b_0, b_1,b_2 respectively and equate them to zero.

Partial differentiate with respect to b_0 and equate it to zero will give us

$$2\sum_{i=1}^{6}(P_i - b_0 - b_1 L_i - b_2 E_i)=0$$

$$\sum_{i=1}^{6} P_i - 6b_0 - b_1\sum_{i=1}^{6} L_i - b_2\sum_{i=1}^{6} E_i = 0 \qquad (1)$$

Similarly for b_1 :

$$\sum_{i=1}^{6}(P_i L_i - b_0 L_i - b_1 L_i^2 - b_2 E_i L_i) = 0$$

$$\sum_{i=1}^{6} P_i L_i - b_0\sum_{i=1}^{6} L_i - b_1\sum_{i=1}^{6} L_i^2 - b_2\sum_{i=1}^{6} E_i L_i = 0 \qquad (2)$$

Similarly for b_2 :

$$\sum_{i=1}^{6}(P_i E_i - b_0 E_i - b_1 L_i E_i - b_2 E_i^2) = 0$$

$$\sum_{i=1}^{6} P_i E_i - b_0\sum_{i=1}^{6} E_i - b_1\sum_{i=1}^{6} L_i E_i - b_2\sum_{i=1}^{6} E_i^2 = 0 \qquad (3)$$

Hence I only need to solve the following linear system of equations :

$$6b_0 + b_1\sum_{i=1}^{6} L_i + b_2\sum_{i=1}^{6} E_i = \sum_{i=1}^{6} P_i$$

$$b_0\sum_{i=1}^{6} L_i + b_1\sum_{i=1}^{6} L_i^2 + b_2\sum_{i=1}^{6} E_i L_i = \sum_{i=1}^{6} P_i L_i$$

$$b_0\sum_{i=1}^{6} E_i + b_1\sum_{i=1}^{6} L_i E_i + b_2\sum_{i=1}^{6} E_i^2 = \sum_{i=1}^{6} P_i E_i$$

b_0=-50234.98695
b_1=16.82767624
b_2=-1.82767624

Excel Output:

P1	155000	L1	12000	E1	350	P1L1	1860000000	P1E1	54250000
P2	120000	L2	10000	E2	300	P2L2	1200000000	P2E2	36000000
P3	100000	L3	9000	E3	100	P3L3	900000000	P3E3	10000000
P4	70000	L4	8000	E4	200	P4L4	560000000	P4E4	14000000
P5	60000	L5	6000	E5	100	P5L5	360000000	P5P5	6000000
P6	100000	L6	9000	E6	200	P6L6	900000000	P6E6	20000000
sum	605000		54000		1250		5780000000		140250000
		L12	144000000	E12	122500	E1L1	4200000		
		L22	100000000	E22	90000	E2L2	3000000		
		L32	81000000	E32	10000	E3L3	900000		
		L42	64000000	E42	40000	E4L4	1600000		
		L52	36000000	E52	10000	E5L5	600000		
		L62	81000000	E62	40000	E6L6	1800000		
		sum	506000000		312500		12100000		
decision variable									
b0	- 50234.98695								
b1	16.82767624								
b2	-1.82767624								
objective function									
min	309530026.1								
constraint									
LHS	RHS								
605000	605000								
5.78E+09	5780000000								
1.4E+08	140250000								

R output

```
P           L      E
155000      12000  350
120000      10000  300
100000      9000   100
70000       8000   200
60000       6000   100
100000      9000   200
> data=read.table("C:/model28.txt",header=T)
> data
    P   L  E
```

```
1 155000 12000 350
2 120000 10000 300
3 100000  9000 100
4  70000  8000 200
5  60000  6000 100
6 100000  9000 200
> attach(data)
> model=lm(P~L+E)
> model

Call:
lm(formula = P ~ L + E)

Coefficients:
(Intercept)          L          E
 -50234.987     16.828     -1.828

> summary(model)
Call:
lm(formula = P ~ L + E)

Residuals:
     1       2       3        4       5       6
3942.6  2506.5 -1031.3 -14020.9  9451.7  -848.6

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -50234.987  25123.348  -2.000  0.1394
L               16.828      4.103   4.101  0.0262 *
E               -1.828     80.408  -0.023  0.9833
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10160 on 3 degrees of freedom
Multiple R-Squared: 0.9477,     Adjusted R-squared: 0.9129
    F-statistic: 27.19 on 2 and 3 DF,  p-value: 0.01195
```

b)

Let $R_i = P_i - \widehat{P}_i = d_i^+ - d_i^i$ where $d_i^+, d_i^- \geq 0$,

Claim 1: For optimal value of $\min\{\sum_{i=1}^{6}(d_i^+ + d_i^-)\}$, we have $d_i^+ d_i^- = 0$ for optimal solution.

Proof:

Suppose on the contrary that d_i^+, d_i^- are both nonzero in the optimal solution. If $R_i = P_i - \widehat{P}_i = d_i^+ - d_i^- \geq 0$, let $\widetilde{d}_i^+ = d_i^+ - d_i^-, \widetilde{d}_i^- = 0$, $\widetilde{d}_i^+ + \widetilde{d}_i^- = d_i^+ - d_i^- < d_i^+ + d_i^-$, contradicting the optimality condition. If $d_i^+ - d_i^i < 0$, let $\widetilde{d}_i^+ = 0, \widetilde{d}_i^- = d_i^- - d_i^+$, $\widetilde{d}_i^+ + \widetilde{d}_i^- = d_i^- - d_i^+ < d_i^+ + d_i^-$, again, contradicting the optimality condition.

Corollory:

$\min\{\sum_{i=1}^{6}(d_i^+ + d_i^-)\} = \min\{\sum_{i=1}^{6}|R_i|\}$ since $d_i^+ d_i^- = 0$ for optimal solution.

Formulation of Linear Programming:

Minimize $\sum_{i=1}^{6}(d_i^+ + d_i^-)$

Subject to:

$d_i^+ - d_i^- + b_0 + b_1 L_i + b_2 E_i = P_i$

$d_i^+, d_i^- \geq 0$ where i takes integer values from 1 to 6.

Optimal solution:

b_0=-55714.28571

b_1=17.14285714

b_2=14.28571429

Excel Output:

P1	155000	L1	12000	E1	350
P2	120000	L2	10000	E2	300
P3	100000	L3	9000	E3	100
P4	70000	L4	8000	E4	200
P5	60000	L5	6000	E5	100
P6	100000	L6	9000	E6	200
decision variable					
b0	-55714.28571				
b1	17.14285714				
b2	14.28571429				
d1+	0				
d1-	0				
d2+	0				
d2-	0				
d3+	0				
d3-	0				
d4+	0				
d4-	14285.71429				
d5+	11428.57143				
d5-	0				
d6+	0				
d6-	1428.571429				
objective function					
min	27142.85714				
constraint					
LHS	RHS				
155000	155000				
120000	120000				
100000	100000				
70000	70000				
60000	60000				
100000	100000				

c)

Notice that $|R_i|=\max\{R_i, -R_i\}$

$$\max_{1\le i\le 6} |R_i| = \max_{1\le i\le 6}\{\max\{R_{i,} - R_i\}\} = \max_{1\le i\le 6}\{R_i, -R_i\}$$

Hence the question posted in part c is a minimax problem.

Minimize z

Subject to

$$z \ge P_i - b_0 - b_1 L_i - b_2 E_i$$

$$z \ge -P_i + b_0 + b_1 L_i + b_2 E_i$$

where i takes integer value from 1 to 6.

It can be rewritten as

Minimize z

Subject to

$$z_i + b_0 + b_1 L_i + b_2 E_i \ge P_i$$

$$z - b_0 - b_1 L_i - b_2 E_i \ge -P_i$$

where i takes integer values from 1 to 6.

b_0=-60000

b_1=20

b_2=-100

Excel Output:

P1	155000	L1	12000	E1	350
P2	120000	L2	10000	E2	300
P3	100000	L3	9000	E3	100
P4	70000	L4	8000	E4	200
P5	60000	L5	6000	E5	100
P6	100000	L6	9000	E6	200
decision variable					
b0	-60000				
b1	20				
b2	-100				
f	10000				
objective function					
min	10000				
constraint					
LHS	RHS				
155000	155000				
-135000	-155000				
120000	120000				
-100000	-120000				
120000	100000				
-100000	-100000				
90000	70000				
-70000	-70000				
60000	60000				
-40000	-60000				
110000	100000				
-90000	-100000				