DEFENG SUN's Research on Sensitivity Analysis for NLSDP

Consider the perturbed nonlinear semidefinite programming (NLSDP):

$$\min_{x\in\mathbb{R}^n} \{f(x) - \langle a, x\rangle \mid G(x) + b \in K := \{0\}^m \times \mathcal{S}^p_+\}, \tag{1}$$

where f and G are C^2 functions, and (a, b) is the perturbation parameter. For a given (a, b), let $\mathbb{S}_{\rm KKT}(a, b)$ denote the set of all solutions (x, y) to the Karush–Kuhn–Tucker (KKT) system:

$$a = \nabla f(x) + \nabla G(x)y = \nabla_x L(x, y), \quad y \in N_K(G(x) + b), \tag{2}$$

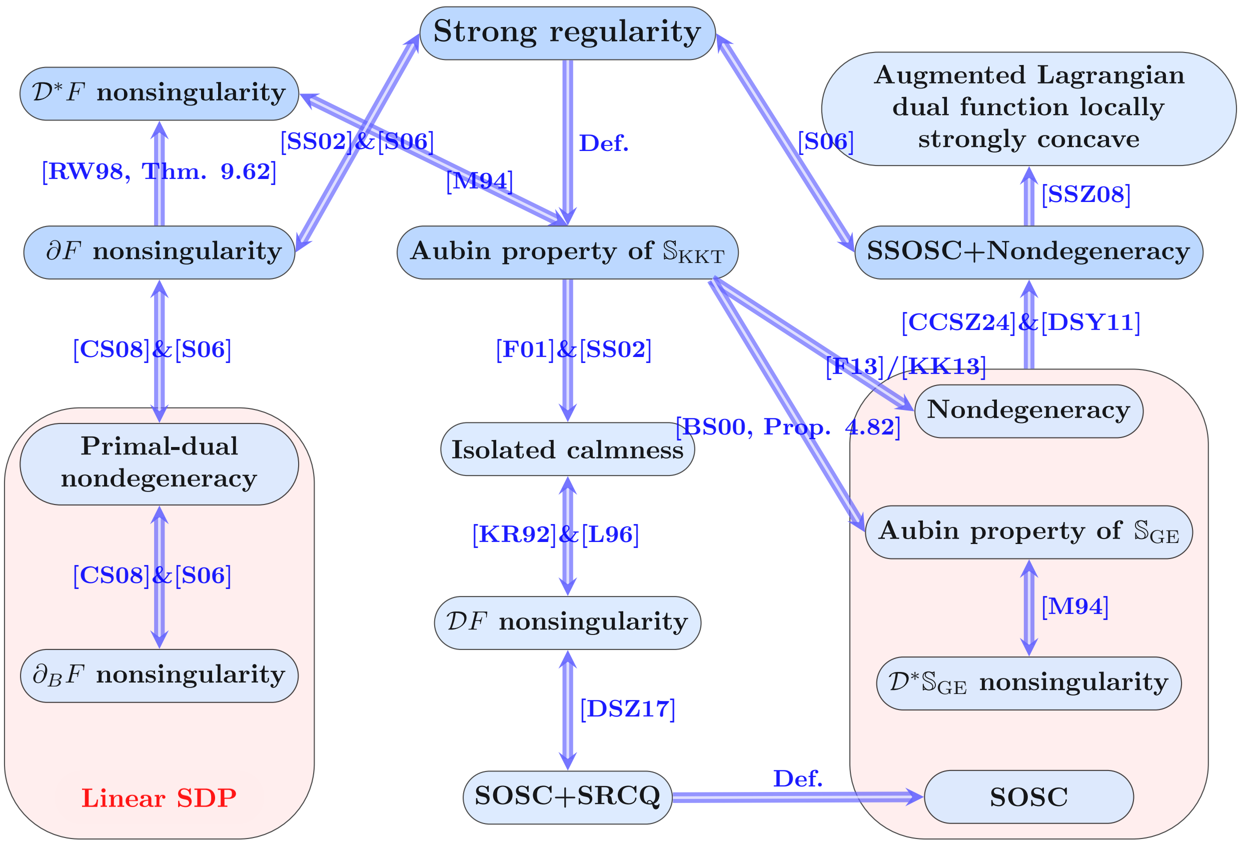
where the Lagrangian function of (1) is defined by $L(x, y) := f(x) + \langle G(x), y\rangle$. For given a, define the set-valued mapping $\mathbb{S}_{\rm GE}$ as

$$\mathbb{S}_{\rm GE}(a) := \{x \mid a \in \nabla f(x) + \nabla G(x)N_K(G(x))\}. \tag{3}$$

Define the nonsmooth mapping

$$F(x, y) := \begin{pmatrix} \nabla_x L(x, y) \\ G(x) - \Pi_K(G(x) + y) \end{pmatrix}. \tag{4}$$

The following relationships hold at a locally optimal solution of (1) which admits a multiplier.



SOSC:	second-order sufficient condition	∂_B:	Bouligand subdifferential
SSOSC:	strong second-order sufficient condition	∂:	Clarke's generalized Jacobian
SRCQ:	strict Robinson's constraint qualification	$\mathcal{D}$:	graphical derivative
		$\mathcal{D}^*$:	Mordukhovich's coderivative

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