NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester I (2009/2010) MA4254 Discrete Optimization Tutorial 1

Q1. Suppose that there are n people and n jobs. Each job must be assigned to exactly one person, and each person does only one job. The cost of person j doing job i is c_{ij}. Formulate the above assignment problem as an integer linear programming problem.

Q2. Formulate the Set Covering and Set Packing problems as zero-one integer linear programming problems. You need to indicate what the coefficients are.

Q3. A vessel has to be loaded with batches of n items, $n \geq 1$. Each unit of item i has a weight w_i and a value v_i, $i = 1, \ldots, n$. The maximum cargo weight is W. It is required to determine the most valuable cargo load without exceeding the maximum weight W. Formulate this problem as an integer linear programming problem.

Q4. Draw the following hyperplanes:

(a) $\{x \in \Re^3 \,|\, x_1 + x_2 + x_3 = 1\}$.

(b) $\{x \in \Re^3 \,|\, x_1 - x_2 = 0\}$.

(c) $\{x \in \Re^3 \,|\, x_1 = 0\}$.

(d) $\{x \in \Re^3 \,|\, 2x_1 + x_3 = 3\}$.

Q5. Which of the following sets are polyhedral?

(a) $S = \{y_1 a_1 + y_2 a_2 \,|\, -1 \leq y_1 \leq 1, -1 \leq y_2 \leq 1\}$, where $a_1, a_2 \in \Re^n$.

(b) $S = \{x \in \Re^2 \,|\, x \geq 0, x^T y \leq 1 \;\forall\; y \text{ with } \| y \| = 1\}$.

(c) $S = \{x \in \Re^2 \,|\, x \geq 0, x^T y \leq 1 \;\forall\; y \text{ with } |y_1| + |y_2| = 1\}$.

(d) $S = \{x \in \Re^2 \,|\, x \geq 0\,,\; x^T y \leq 1 \;\forall\; y \text{ with } \sqrt{y_1^2 + y_2^2} \geq 1\}$.

Can you figure out the solutions for (b)-(c) when $\Re^2$ is replaced by $\Re^n$?

Q6. Find all extreme points of the following problem:

$$\begin{array}{ll} \min & 2x_1 + 4x_2 + 7x_3 \\ \text{s.t.} & 2x_1 + x_2 + 6x_3 \geq 5 \\ & 4x_1 - 6x_2 + 5x_3 \geq 8 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

Q7. Let $P = \{x \in \Re^3 \mid 2x_3 \geq 1,\ 4x_1 \leq 3,\ x_1 + x_2 + x_3 = 1,\ x \geq 0\}$. Find all basic feasible solutions of P.