NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester 1 (2003/2004) MA4253 Mathematical Programming Tutorial 3

Solution to selected questions

Q1. Let $x_i := x_i - 2$, $i = 1, 2, 3$. Then the original problem becomes

$$\begin{array}{ll} \min & -4x_1 - x_2 - 6x_3 - 22 \\ \text{s.t.} & 3x_1 + 2x_2 + 4x_3 = 16 \\ & 0 \le x_1 \le 2 \\ & 0 \le x_2 \le 2 \\ & 0 \le x_3 \le 2\,. \end{array}$$

Start with any basic feasible solution, say $x^0 = (2, 2, 1.5)^T$, we can solve the above problem by using the simplex method with bounded variables technique.

Q2. The following problem

$$\begin{array}{rl} \min & c^T x \\ \text{s.t.} & Ax \le b \\ & l \le x \le u, \end{array}$$

can be written equivalently as

$$\begin{array}{rl} -\max & (-c)^T x \\ \text{s.t.} & Ax \le b \\ & -x \le -l \\ & x \le u\,, \end{array}$$

i.e.,

$$\begin{array}{rl} -\max & (-c)^T x \\ \text{s.t.} & Mx \le d\,, \end{array} \tag{1}$$

where

$$M = \begin{bmatrix} A \\ -I \\ I \end{bmatrix} \quad \text{and} \quad d = \begin{pmatrix} b \\ -l \\ u \end{pmatrix}.$$

Then the dual of problem (1) is

$$\begin{array}{rl} -\min & d^T y \\ \text{s.t.} & M^T y = -c \\ & y \ge 0\,. \end{array} \tag{2}$$

Using $(p, q, s) \in \Re^m \times \Re^n \times \Re^n$ to substitute y in (2), we obtain the desired dual problem

$$\begin{array}{rl} -\min & b^T p \,-\, l^T q \,+\, u^T s \\ \text{s.t.} & A^T p \,-\, q \,+\, s \,=\, -c \\ & p,\, q,\, s \,\geq\, 0\,. \end{array} \tag{3}$$

Q3. (i) Making use of the solution to **Q2**, we obtain the dual problem as

$$\begin{array}{rl} \min & 9p_1 \,+\, 8p_2 \,-\, q_1 \,-\, 0q_2 \,-\, 0q_3 \,+\, 3s_1 \,+\, 5s_2 \,+\, 2s_3 \\ \text{s.t.} & 4p_1 \,+\, p_2 \,-\, q_1 \,+\, s_1 \,=\, 4 \\ & p_1 \,-\, p_2 \,-\, q_2 \,+\, s_2 \,=\, 2 \\ & 2p_2 \,-\, q_3 \,+\, s_3 \,=\, 6 \\ & p,\, q,\, s \,\geq\, 0\,. \end{array} \tag{4}$$

(ii) Use the simplex method with the bounded variables technique to solve

$$\begin{array}{rl} \max & 4x_1 \,+\, 2x_2 \,+\, 6x_3 \\ \text{s.t.} & 4x_1 \,+\, x_2 \,\leq\, 9 \\ & x_1 \,-\, x_2 \,+\, 2x_3 \,\leq\, 8 \\ & -x_1 \,\leq\, -1 \\ & x_1 \,\leq\, 3 \\ & -x_2 \,\leq\, -0 \\ & x_2 \,\leq\, 5 \\ & -x_3 \leq\, 0 \\ & x_3 \,\leq\, 2 \end{array} \tag{5}$$

to get (see T2)

$$x^* \,=\, (1,\, 5,\, 2)^T\,.$$

Problem (5) has four active constraints at x^*, namely

$$I(x^*) \,=\, \{\,1,\, 3,\, 6,\, 8\,\}\,.$$

(a) If we take the basis matrix

$$B^T \,=\, \begin{bmatrix} 4 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then from

$$B(p_1^*,\, q_1^*,\, s_3^*)^T \,=\, (4,\, 2,\, 6)^T$$

we get

$$p_1^* \,=\, \frac{1}{2},\; q_1^* \,=\, -2,\; s_3^* \,=\, 6\,.$$

Therefore, an optimal solution to (b) is not found in this case.

(b) If we take the basis matrix

$$B^T = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then from

$$B(p_1^*,\, s_2^*,\, s_3^*)^T = (4,\, 2,\, 6)^T$$

we get

$$p_1^* = 1,\; s_2^* = 1,\; s_3^* = 6\,.$$

Therefore, an optimal solution to (4) is

$$p_1^* = 1,\; s_2^* = 1,\; s_3^* = 6\,,\; p_2^* = p_3^* = q_1^* = q_2^* = q_3^* = s_1^* = 0\,.$$

(c) If we take the basis matrix

$$B^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then from

$$B(q_1^*,\, s_2^*,\, s_3^*)^T = (4,\, 2,\, 6)^T$$

we get

$$q_1^* = -4,\; s_2^* = 2,\; s_3^* = 6\,.$$

Therefore, an optimal solution to (4) is not found in this case.

Q4. (Omit)