NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester 1 (2003/2004) MA4253 Mathematical Programming Tutorial 9

Q1. Find a complementary basic feasible solution to the system $w - Mz = q$, $w^T z = 0$, and $w, z \geq 0$ by using Lemke's complementary pivoting algorithm. Here

$$M = \begin{bmatrix} 1 & 1 & 3 & 4 \\ 5 & 3 & 1 & 1 \\ 2 & 1 & 2 & 2 \\ 1 & 4 & 1 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} -1 \\ 2 \\ 1 \\ -3 \end{bmatrix}.$$

Q2. Consider the linear complementarity problem (LCP) to find (w, z) such that $w - Mz = q$, $w^T z = 0$, and $w, z \geq 0$, where

$$M = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 1 & -1 & -2 \\ 1 & 0 & 1 & -21 \end{bmatrix}, \quad q = \begin{bmatrix} -1 \\ 3 \\ -2 \\ -4 \end{bmatrix}.$$

(i) Is the matrix M copositive-plus?

(ii) Apply Lemke's complementary pivoting algorithm to the above problem.

Q3. Consider the linear programming

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \\ & x \geq 0\,, \end{aligned}$$

Show that if every almost complementary basic feasible solution to the LCP system obtained from the above linear programming is nondegenerate, one can solve problem (1) by applying Lemke's pivoting method to the LCP.

Q4. Use the complementary pivoting algorithm to solve the KKT system of the following problem:

$$\begin{aligned} \min \quad & -x_1 - 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 6 \\ & -x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0\,. \end{aligned}$$

Q5. Use the complementary pivoting algorithm to solve the following quadratic programming problem:

$$\begin{aligned} \max \quad & 2x_1 - 3x_2 - 2x_1^2 - 3x_1x_2 - 2x_2^2 \\ \text{s.t.} \quad & x_1 + 2x_2 \le 6 \\ & -x_1 + 2x_2 \le 4 \\ & x_1, x_2 \ge 0 . \end{aligned}$$

Q6. Consider the function $\theta : \Re^2 \to \Re$ defined by

$$\theta(a,b) = \sqrt{a^2+b^2} - (a+b) .$$

(i) Show that θ is a convex function.

(ii) Show that

$$\theta(a,b) = 0 \Longleftrightarrow a \ge 0, b \ge 0 \ \& \ ab = 0 .$$

(iii) Give the subdifferential of θ at $(0,0)$ and $(1,1)$, respectively.

(iv) Use part (ii) to reformulate the linear complementarity problem as a system of equations.

Q7. Consider the function $\theta : \Re^2 \to \Re$ defined by

$$\theta(a,b) = \max\{\sqrt{a^2+b^2+14},\, a^2+b^2+a+b\} .$$

(i) Show that θ is a convex function.

(ii) Give the subdifferential of θ at $(0,0)$ and $(1,1)$, respectively.

Q8. Consider the function θ defined by the following linear programming problem:

$$\begin{aligned} \theta(u) = \min \quad & c^T x + u^T(Ax - b) \\ \text{s.t.} \quad & x \in X , \end{aligned}$$

where X is a bounded polyhedral set.

(i) Show that θ is a concave function.

(ii) Is θ continuously differentiable? Justify your answer.

(iii) Give a subgradient of θ at any given point u. [Hint: you may use the Minkowski Theorem]

Q9. Consider the function $\theta : \Re^2 \to \Re$ defined by the following optimization problem:

$$\begin{aligned} \theta(u_1,u_2) = \min \quad & x_1(1-u_1) + x_2(1-u_2) \\ \text{s.t.} \quad & x_1^2 + x_2^2 \le 1 . \end{aligned}$$

(i) Show that θ is a concave function.

(ii) Evaluate θ at the point $(1,1)$.

(iii) Find the subdifferential of θ at $(1,1)$.