

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
Semester I (2009/2010) MA4254 Discrete Optimization Tutorial 5

Q1. A digraph $G = (V, E)$ has arc lengths given by the following matrix

$$C = \begin{bmatrix} 0 & 2 & 6 & \infty & \infty & \infty \\ \infty & 0 & \infty & 1 & 3 & \infty \\ \infty & \infty & 0 & \infty & \infty & 2 \\ \infty & \infty & 1 & 0 & 1 & \infty \\ \infty & \infty & \infty & \infty & 0 & 1 \\ \infty & \infty & \infty & 2 & \infty & 0 \end{bmatrix}.$$

- (1) Formulate the problem of finding a shortest path from node 1 to node 6 as a linear programming (LP).
- (2) Give the dual form of the LP obtained in part (1).
- (3) Use Dijkstra's algorithm to find a shortest path from node 1 to node 6.

Q2. Prove that for Dijkstra's Algorithm, at every iteration the following properties hold, prior to Step 1.

- (a) If $j \in P$, then u_j is the length of the shortest path from 1 to j ;
- (b) If $j \in T$, then $u_j = \min_{i \in P} \{u_i + c_{ij}\}$.

Q3. Use an appropriate method to solve the shortest path problem of finding a shortest path from node 1 to all other nodes, where arc lengths are given by the following matrix:

$$C = \begin{bmatrix} 0 & 1 & 3 & -6 & \infty & \infty \\ \infty & 0 & 2 & 3 & \infty & \infty \\ \infty & \infty & 0 & -4 & 9 & 6 \\ \infty & \infty & \infty & 0 & 1 & 2 \\ \infty & \infty & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix}.$$

Q4. Use an appropriate method to solve the longest path problem of finding longest paths from node 1 to all other nodes, where arc lengths are given by the matrix in Q3.

Q5. Use the Bellman-Ford method to solve the shortest path problem with arc lengths given by

$$C = \begin{bmatrix} 0 & \infty & 4 & 10 & 3 & \infty & \infty \\ \infty & 0 & -1 & -3 & 2 & 11 & 0 \\ \infty & 9 & 0 & 8 & 3 & 2 & 1 \\ \infty & 4 & 0 & 0 & 8 & 6 & 3 \\ \infty & 0 & 1 & 2 & 0 & 3 & -1 \\ \infty & -1 & -1 & 3 & 2 & 0 & 0 \\ \infty & 4 & 3 & \infty & \infty & 2 & 0 \end{bmatrix}.$$

Q6. Use the Floyd-Warshall method to solve the shortest path problem with arc lengths given by

$$\begin{bmatrix} 0 & -2 & \infty \\ 3 & 0 & 1 \\ 1 & \infty & 0 \end{bmatrix}.$$

Q7. Use the Floyd-Warshall method to solve the shortest path problem with arc lengths given by

$$\begin{bmatrix} 0 & 1 & \infty \\ \infty & 0 & -1 \\ -1 & \infty & 0 \end{bmatrix}.$$

What happens with diagonal elements of $U^{(3)}$?