Side channel protections for CSIDH

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based on joint work with
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Slides online at `https://defeo.lu/docet`

Why isogenies?

Six families still in NIST post-quantum competition:

Lattices	9 encryption	3 signature
Codes	7 encryption	
Multivariate		4 signature
Isogenies	1 encryption	
Hash-based		1 signature
MPC		1 signature

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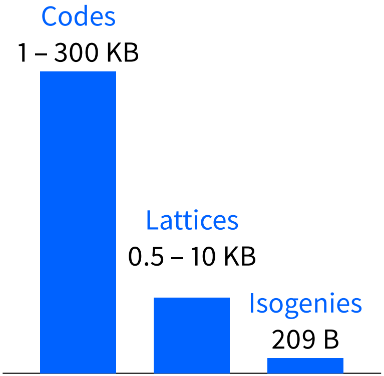

Public key size

NIST-1 level (AES128)

(not to scale)

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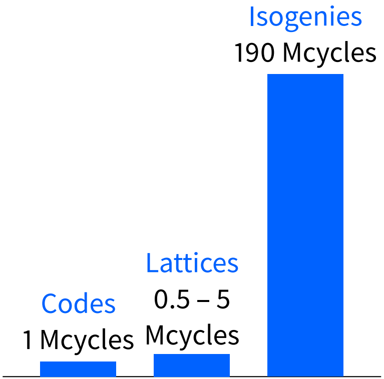

Encryption performance

NIST-1 level (AES128)

(not to scale)

Iso-what?!

Keywords

- An isogeny is a map between two elliptic curves;

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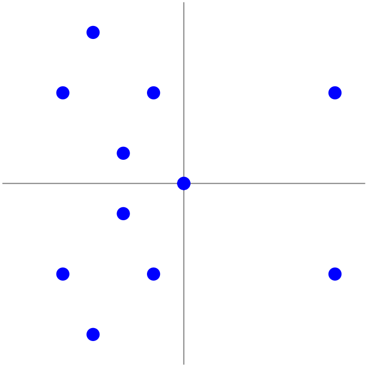
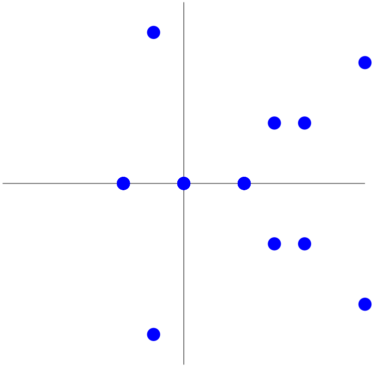
- It is entirely determined by its kernel (i.e., by a single point);
- Isogeny degree $=$ size of the kernel $=$ order of kernel generator $\approx$ size of the polynomials;

Isogenies: an example over $\mathbb{F}_{11}$

$$E \;:\; y^2 = x^3 + x$$



$$E' \;:\; y^2 = x^3 - 4x$$



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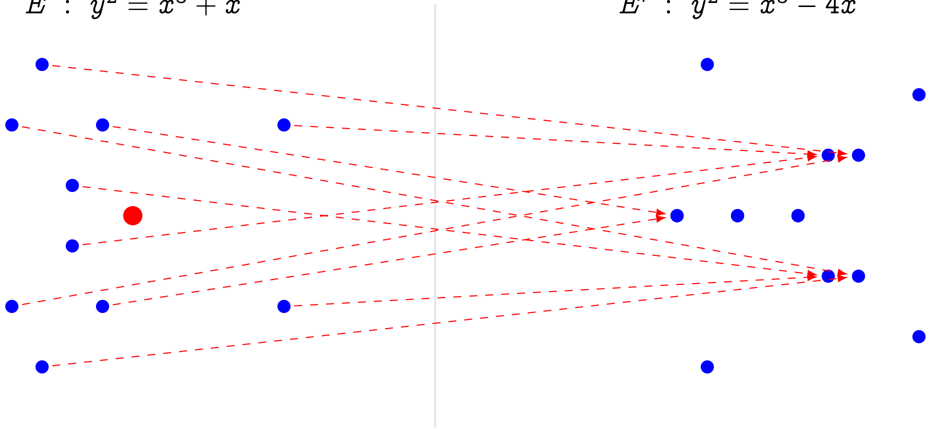
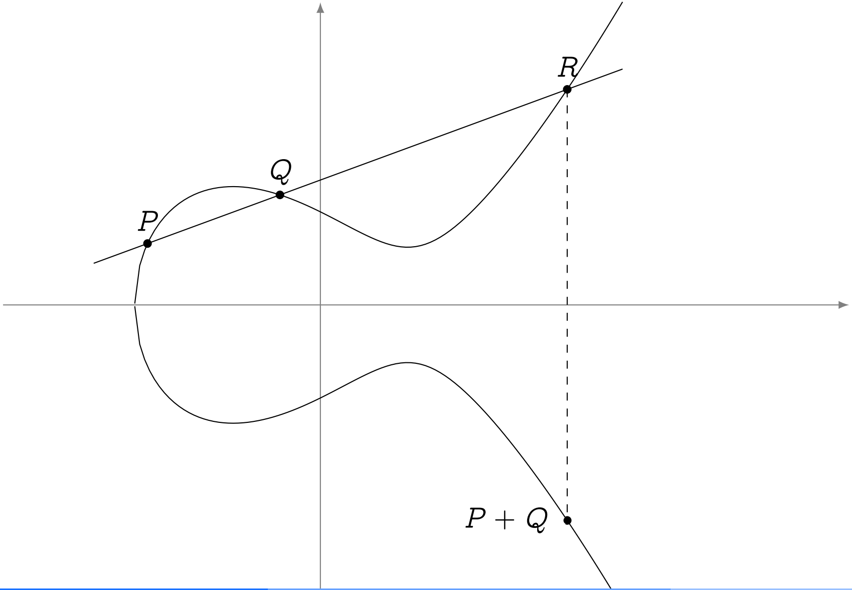


$$\phi(x, y) = \left(\frac{x^2 + 1}{x}, \quad y\frac{x^2 - 1}{x^2} \right)$$

- Kernel generator in red.
- This is a degree 2 map.
- Analogous to $x \mapsto x^2$ in $\mathbb{F}_q^*$.

Isogeny graphs



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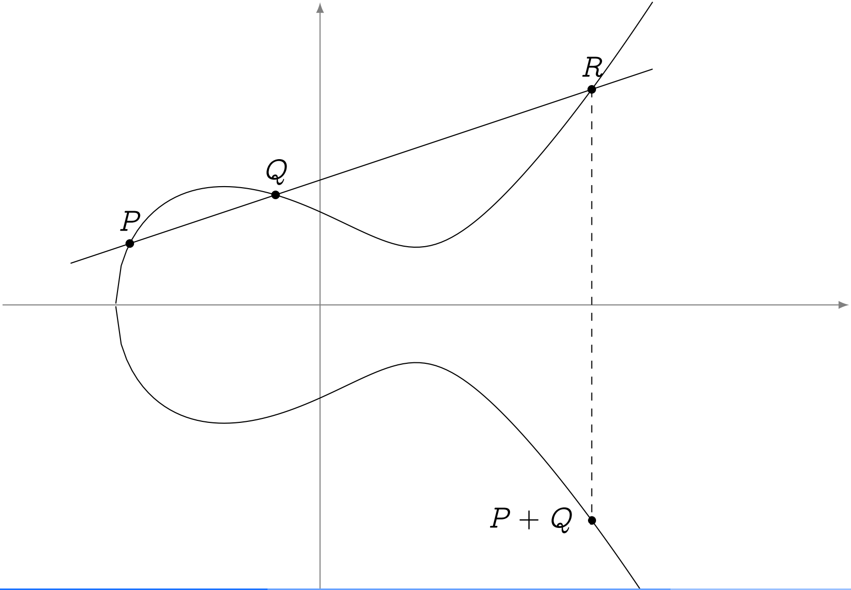

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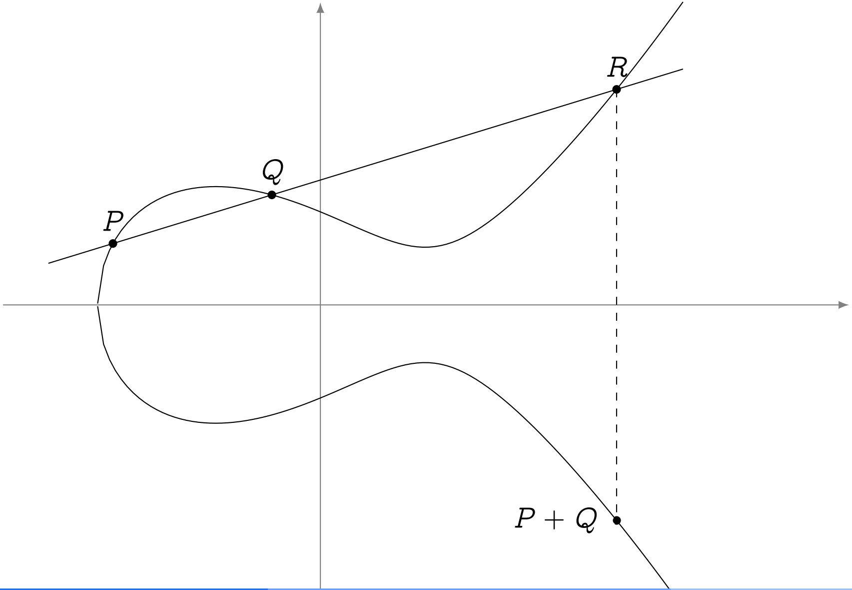

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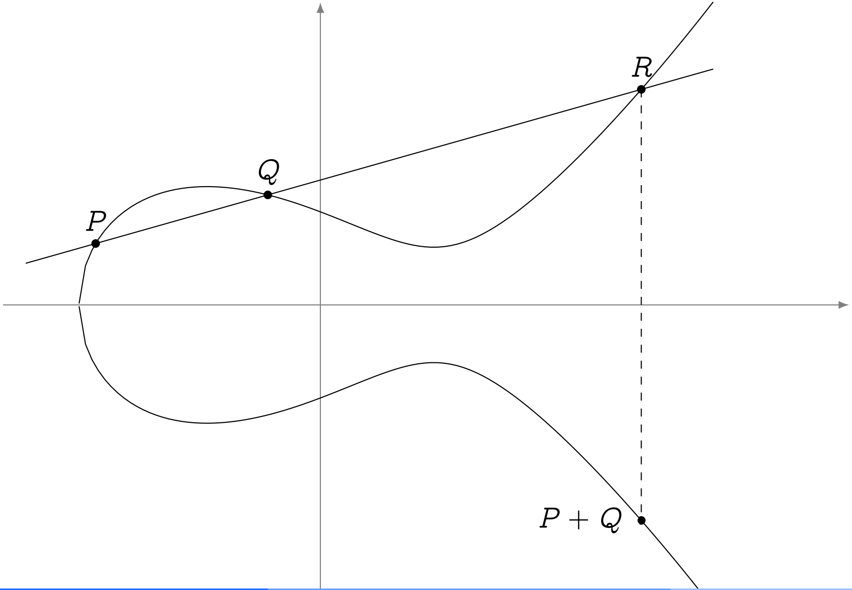

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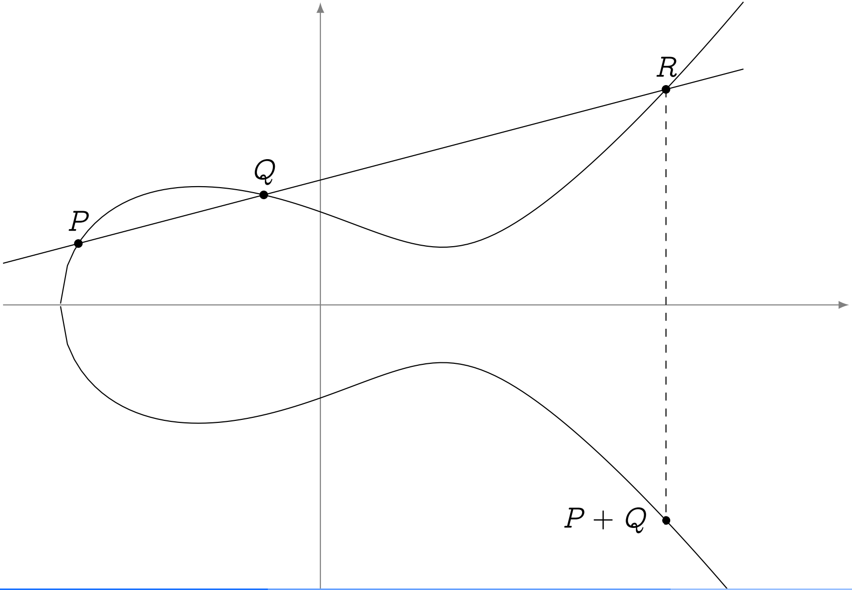

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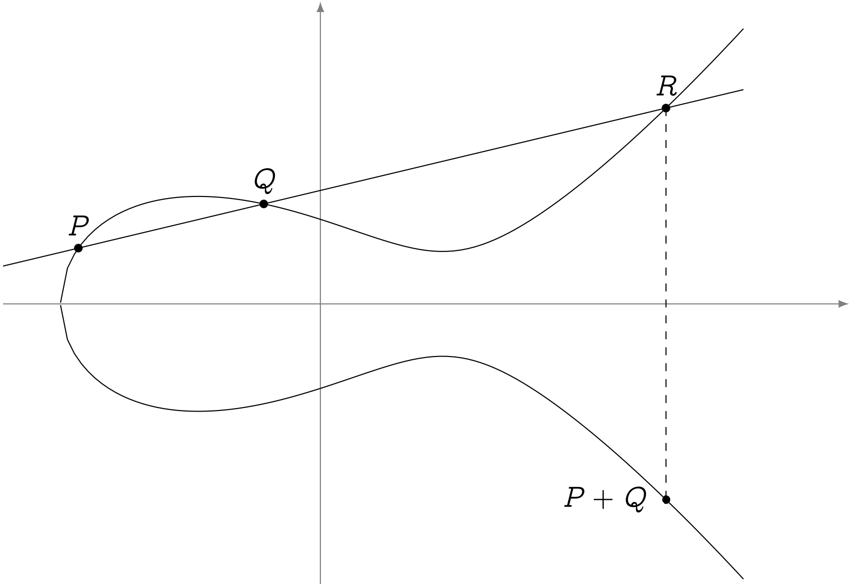

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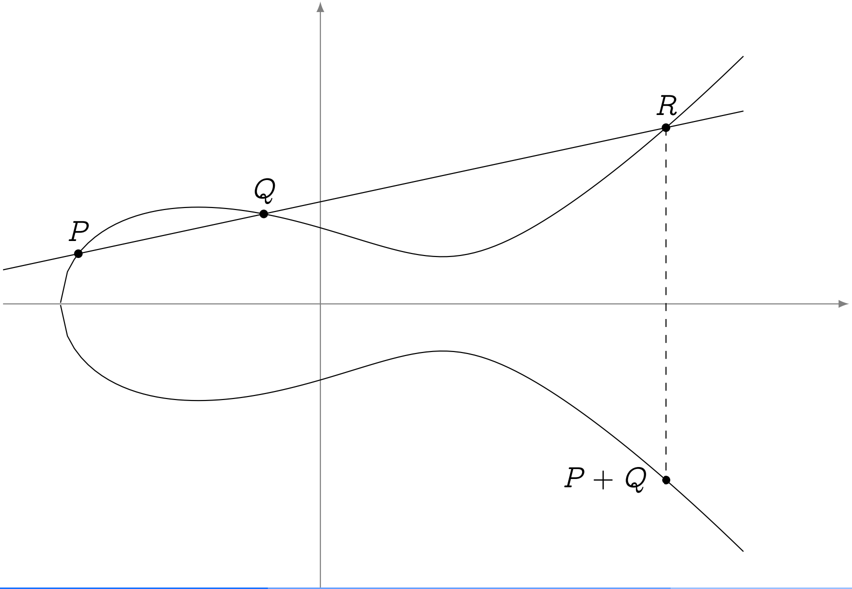

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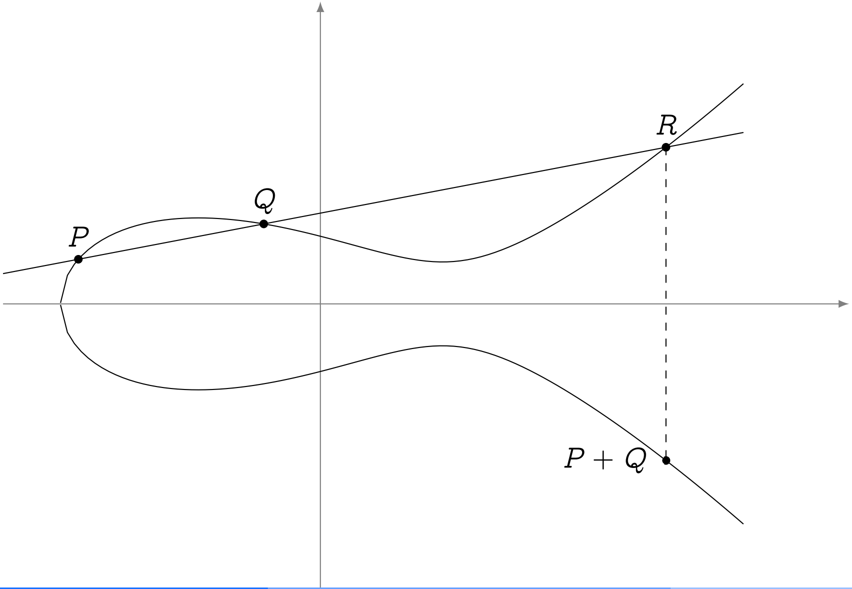

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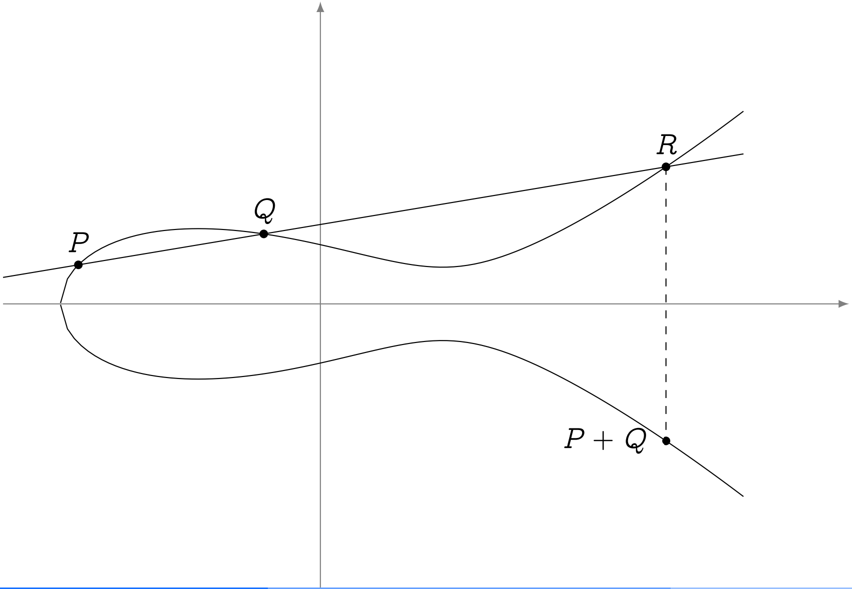

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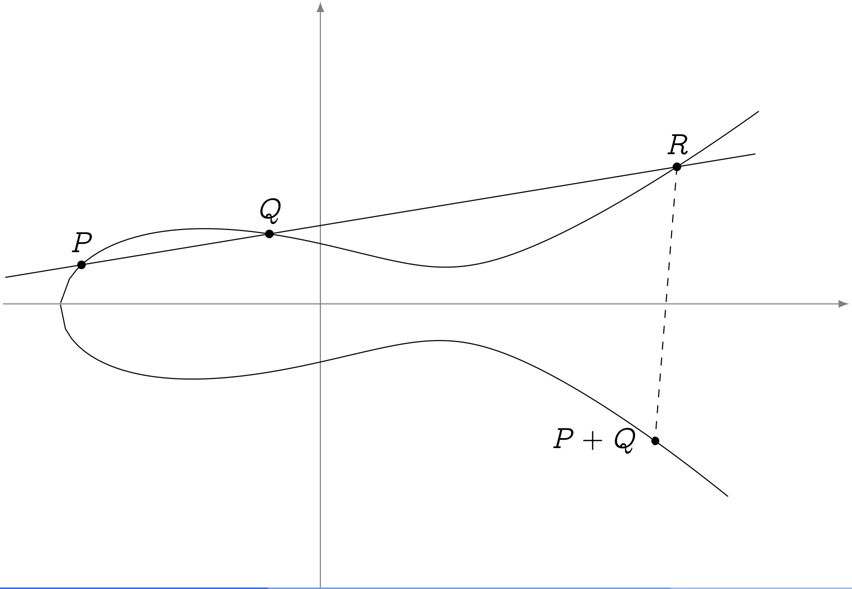

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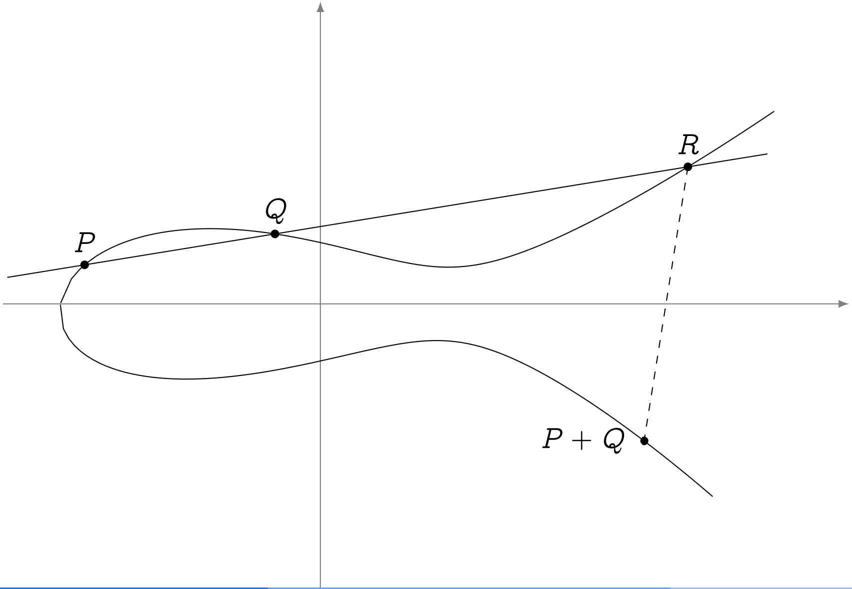

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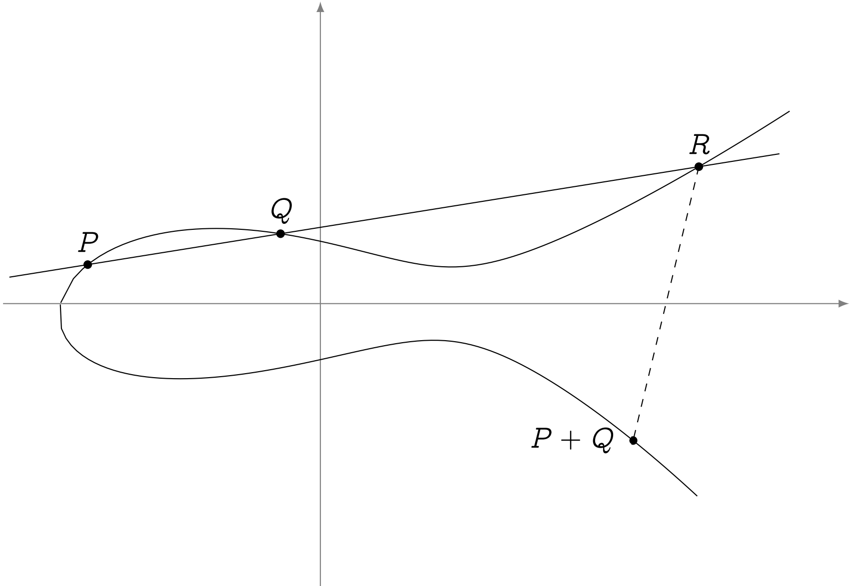

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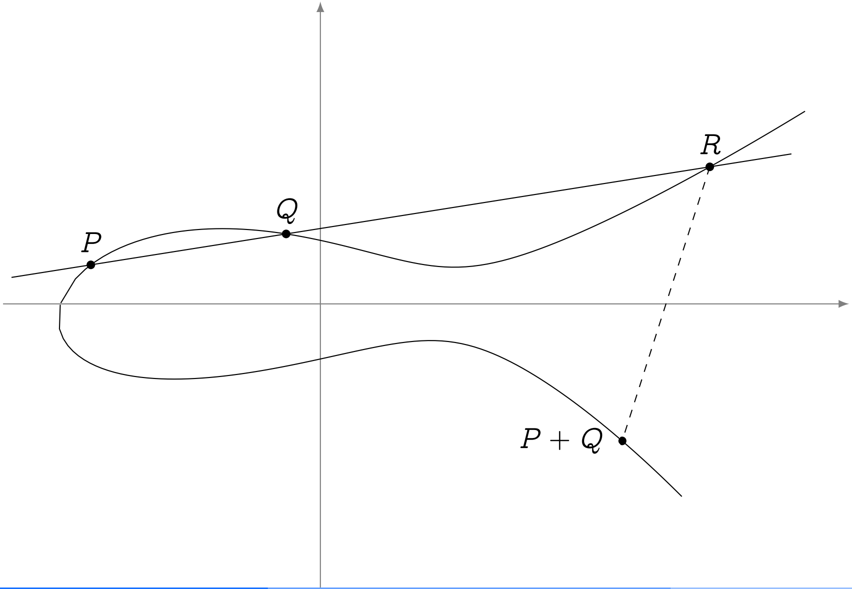

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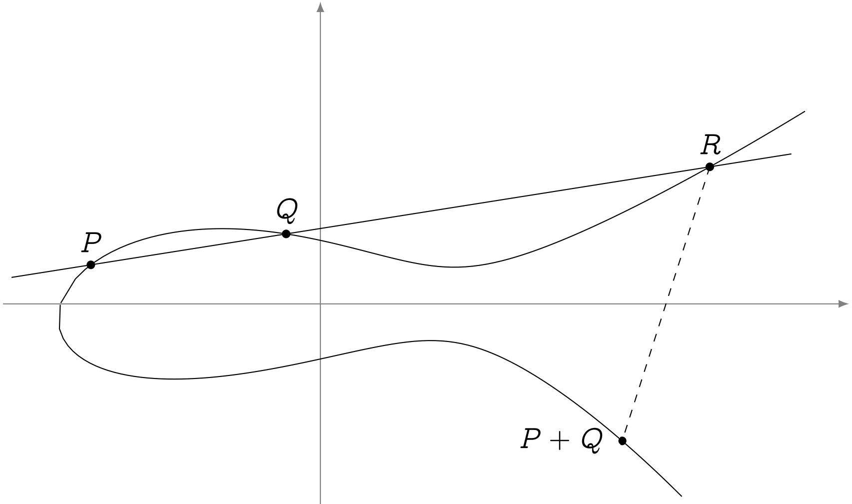
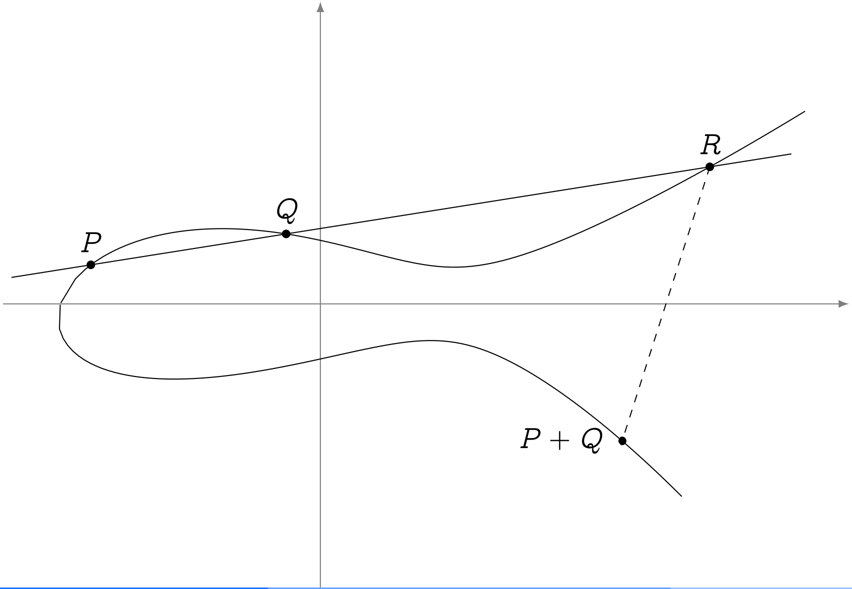


$$y^2 = x^3 + ax + b \quad \longrightarrow \quad j = 1728\frac{4a^3}{4a^3+27b^2}$$

Isogeny graphs



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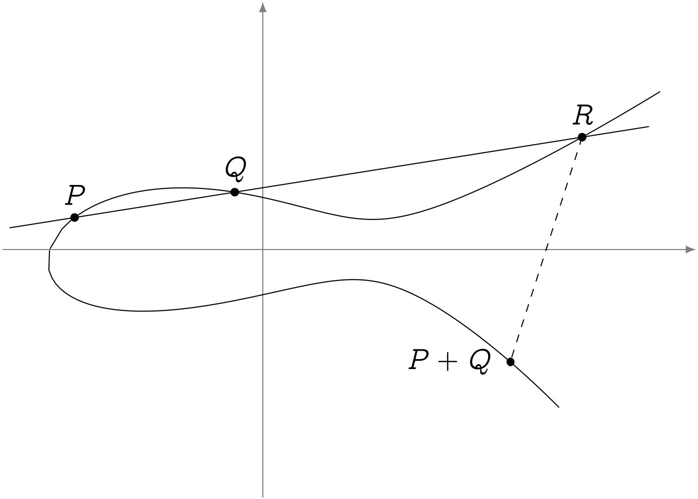

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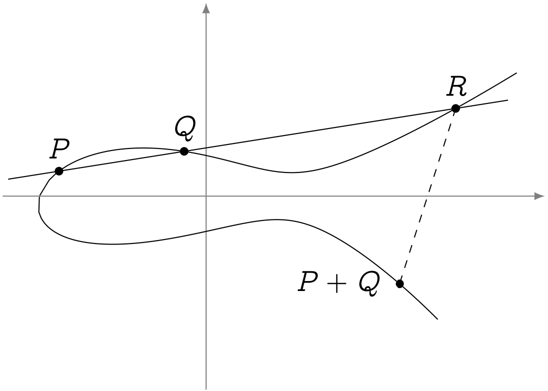

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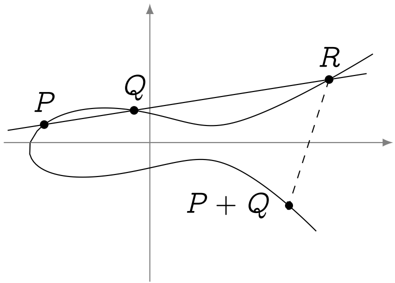

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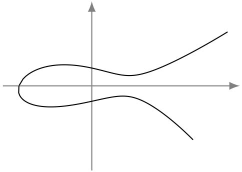

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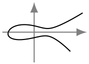

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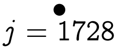

Isogeny graphs

$$j = 1728 \xrightarrow{\phi} j = 287496$$

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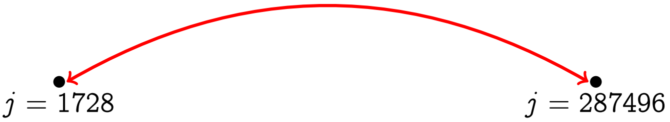
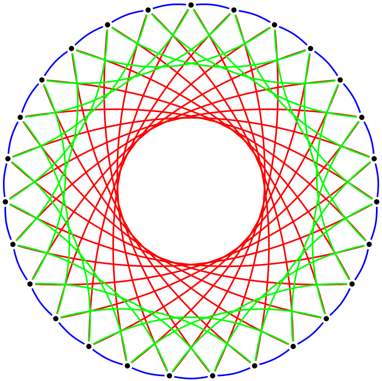
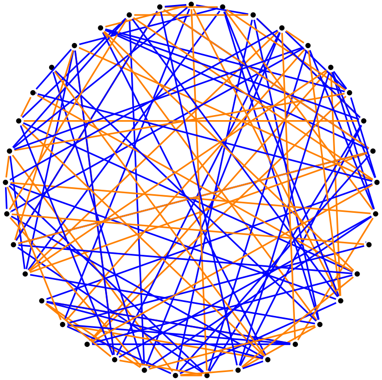


The beauty and the beast (credit: Lorenz Panny)

Components of particular isogeny graphs look like this:





Which of these is good for crypto?

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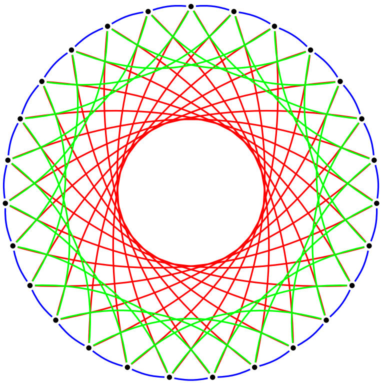

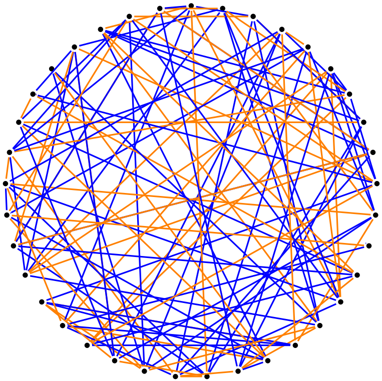

Which of these is good for crypto? **Both.**

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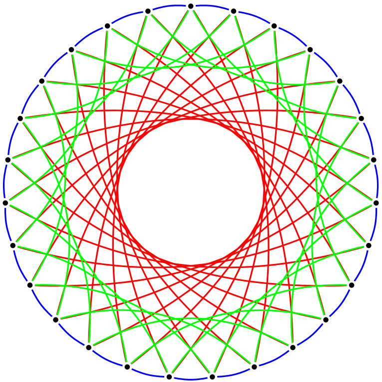
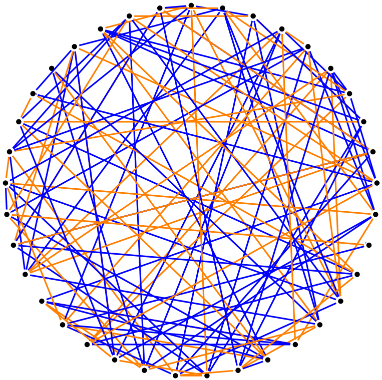
At this time, there are two distinct families of systems:



$\mathbb{F}_p$

CSIDH [pron.: sea-side]

https://csidh.isogeny.org



$\mathbb{F}_{p^2}$

SIDH

https://sike.org

CSIDH vs SIDH

	CSIDH	SIDH
Speed (on x64 arch., NIST 1)	$\sim$ 70ms	$\sim$ 6ms
Public key size (NIST 1)	64B	346B
Key compression		
↳ speed		$\sim$ 11ms
↳ size		209B
Submitted to NIST	no	yes
TRL	4	6
Best classical attack	$p^{1/4}$	$p^{1/4}$ $(p^{3/8})$
Best quantum attack	$\tilde{O}\left(3^{\sqrt{\log_3 p}}\right)$	$p^{1/6}$ $(p^{3/8})$
Key size scales	quadratically	linearly
CPA security	yes	yes
CCA security	yes	Fujisaki-Okamoto
Constant time	it's complicated	yes
Non-interactive key exchange	yes	no
Signatures	short but (slow | do not scale)	big and slow

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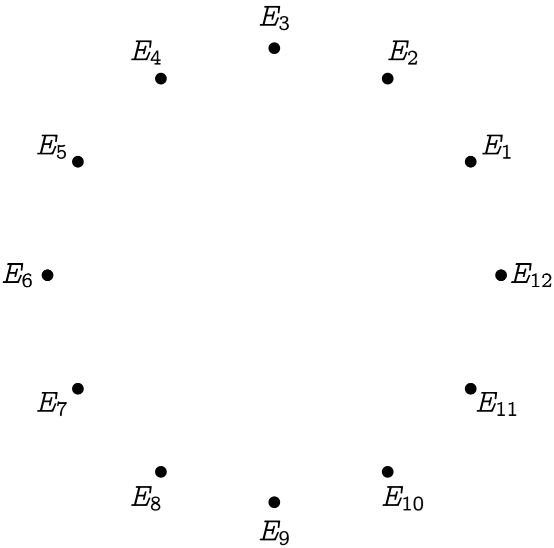

Vertices are supersingular elliptic curves over $\mathbb{F}_p$.

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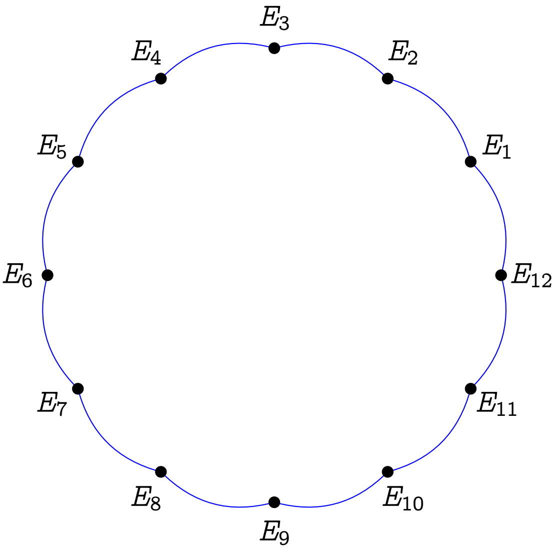

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Edges are isogenies of bounded prime degree.

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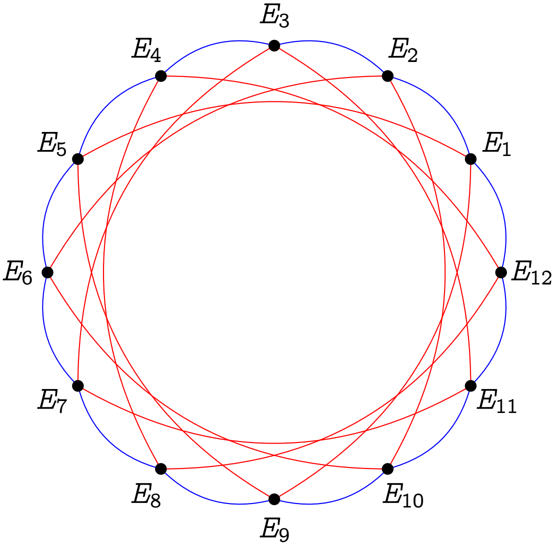

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— degree 5

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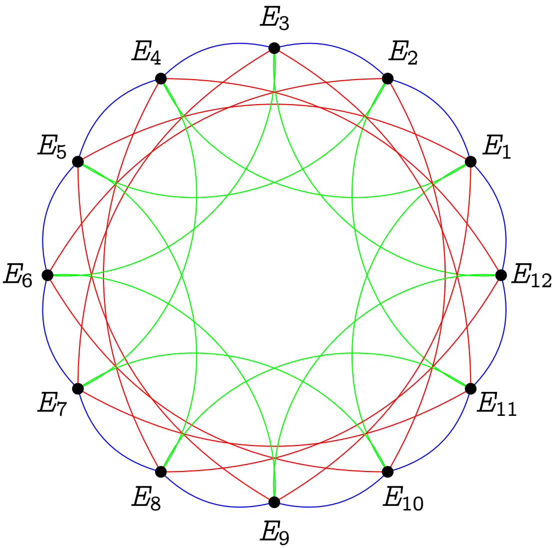

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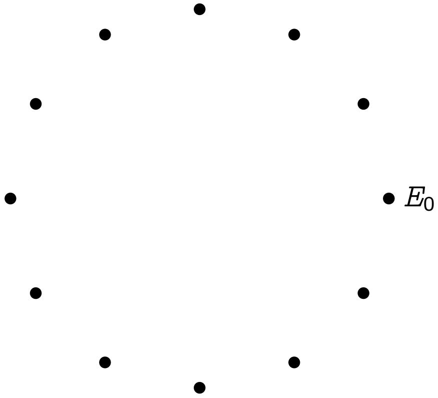
— degree 5

— degree 7

CSIDH key exchange



Public parameters:

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- A set of small prime degree isogenies.

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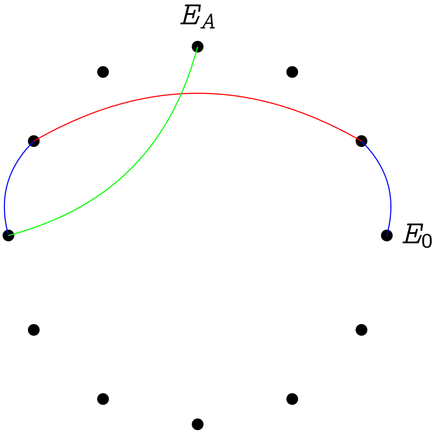

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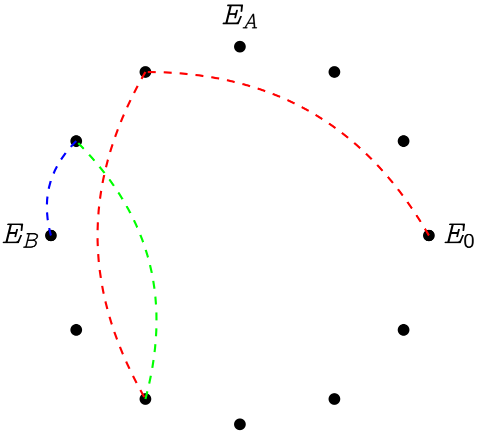

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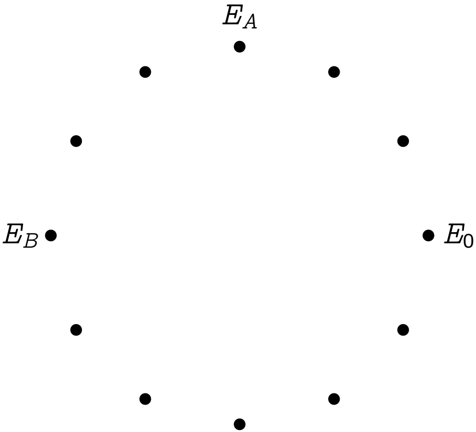

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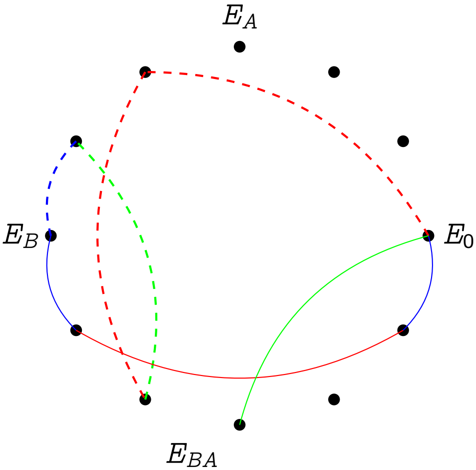

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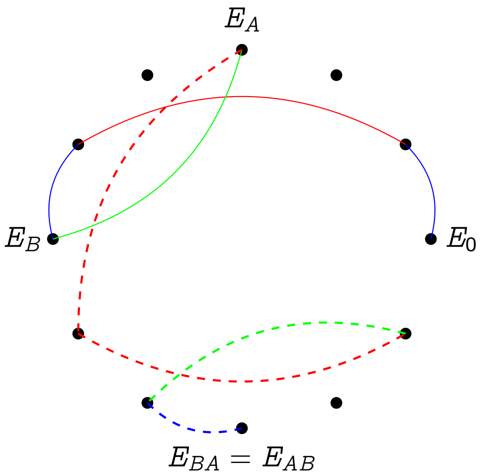
CSIDH key exchange



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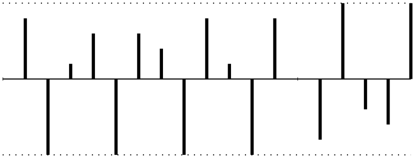
CSIDH data flow

Your secret: a vector of number of isogeny steps for each degree

$$(5, 1, -4, \ldots)$$



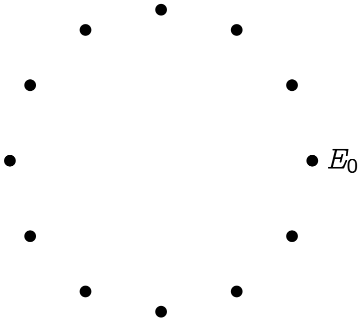
Your public key: (the j-invariant of) a supersingular elliptic curve

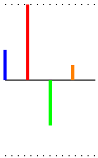
$$j = \texttt{0x23baf75419531a44f3b97cc9d8291a275047fcdae0c9a0c0ebb993964f821f2}$$
$$\texttt{0c11058a4200ff38c4a85e208345300033b0d3119ff4a7c1be0acd62a622002a9}$$

Isogeny evaluation

Repeat:

- Take a random point $P \in E(\mathbb{F}_p)$;
- Set $Q = [c]P$, where c is an appropriate cofactor, so that $N = \#\langle Q \rangle$ contains only useful prime factors;
- Advance by the degree N isogeny of kernel $\langle Q \rangle$.





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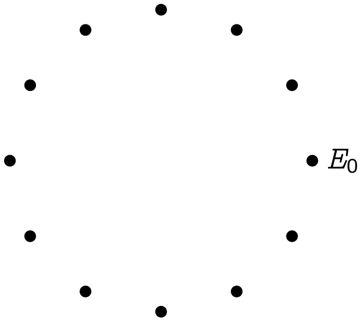
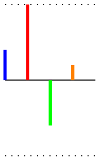


$$\#\langle P\rangle = 3 \cdot 5 \cdot 7 \cdot 11$$



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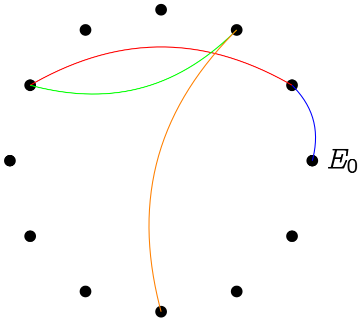

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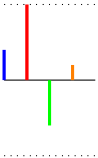

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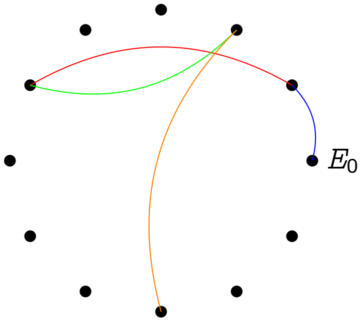

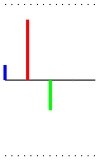

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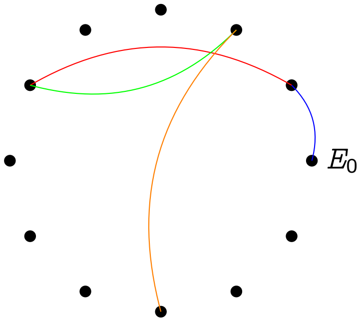

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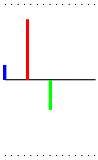

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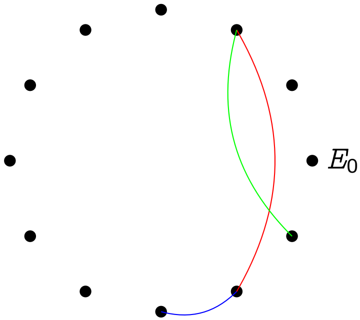

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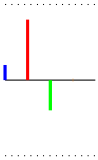

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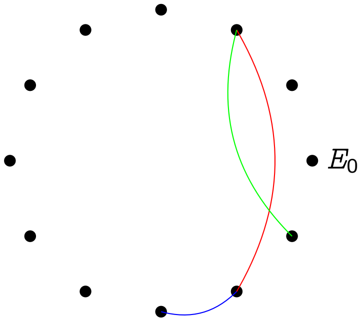

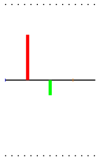

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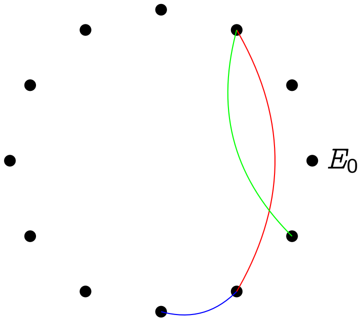

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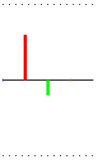

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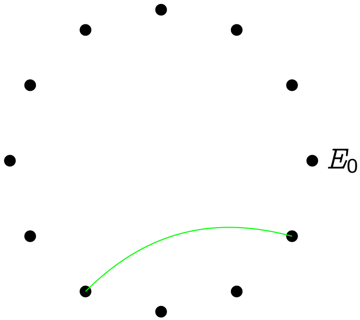

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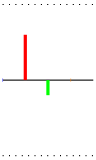

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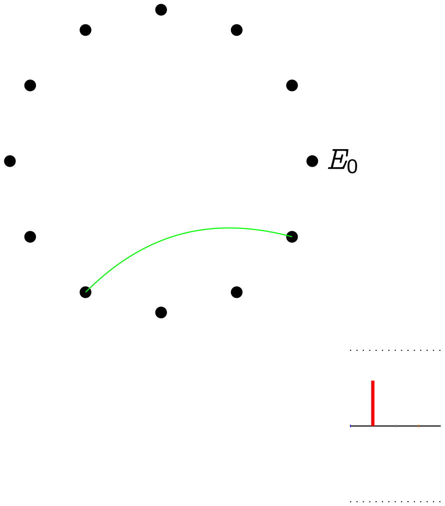

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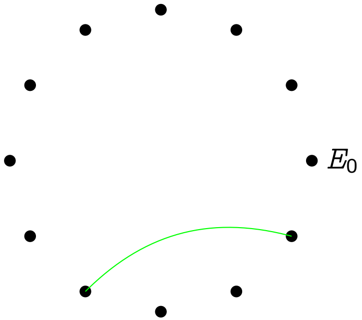

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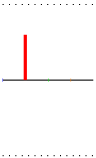

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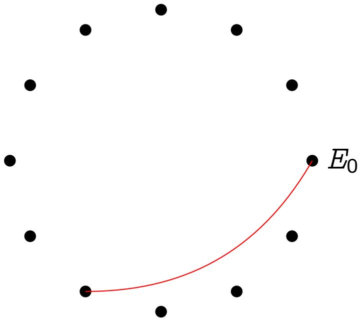

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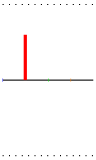

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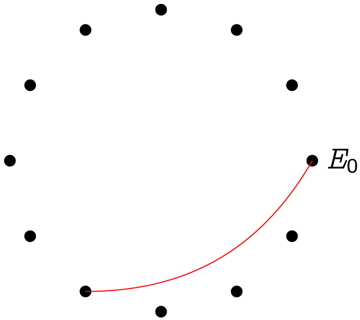

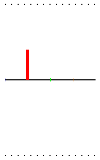

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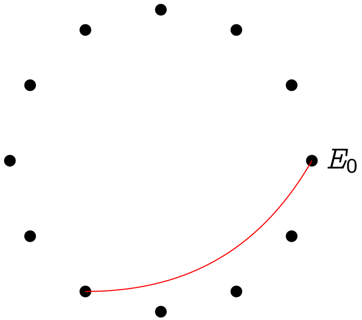

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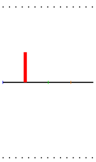

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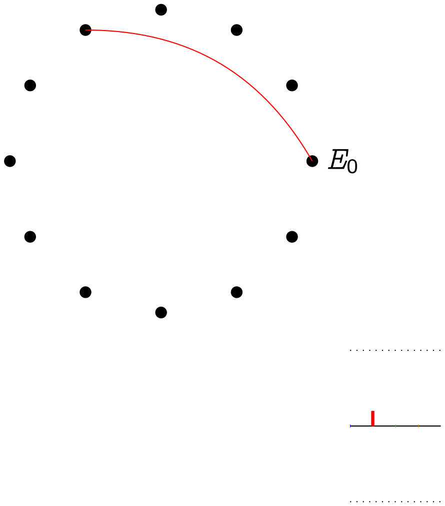

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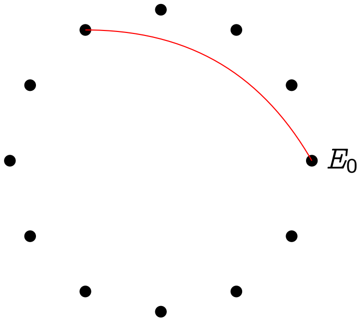

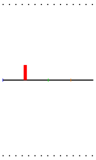

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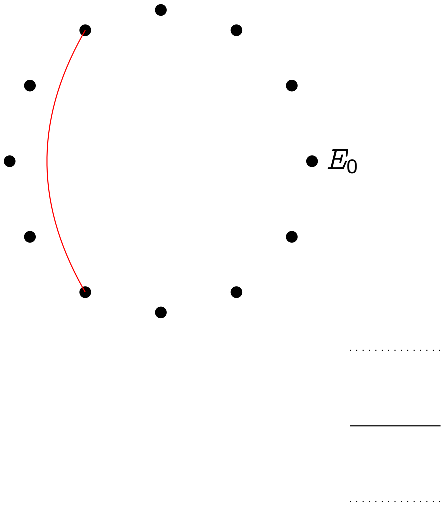

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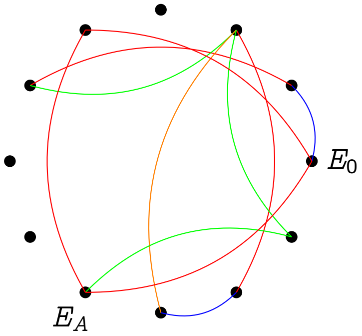

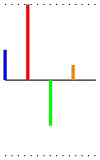

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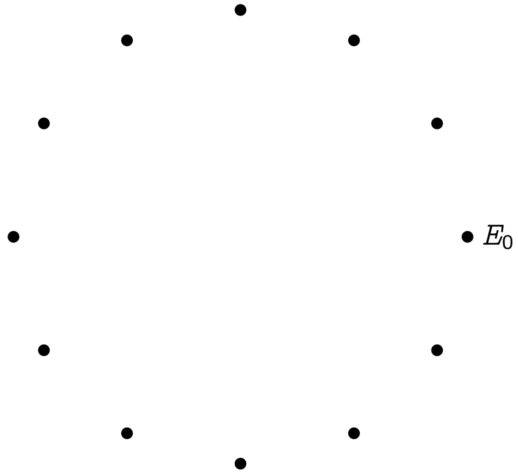

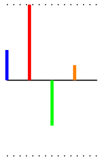

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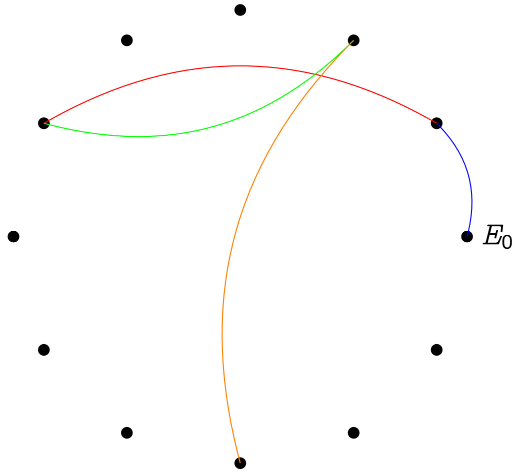

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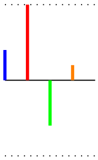

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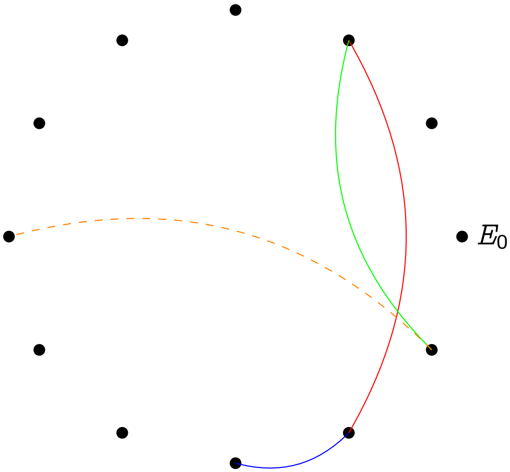

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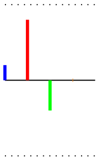

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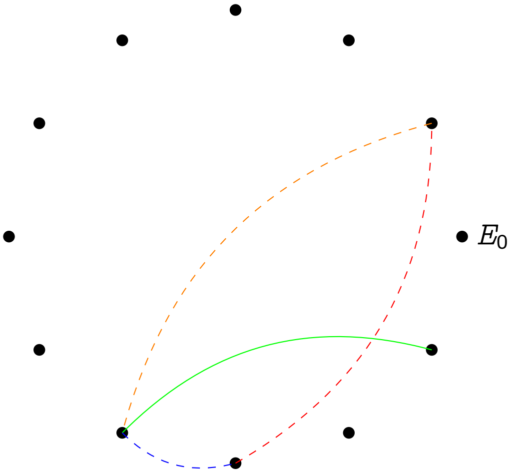

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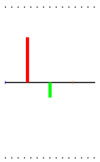

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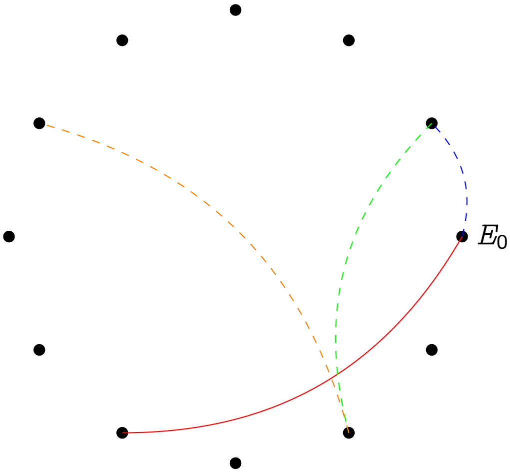

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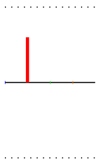

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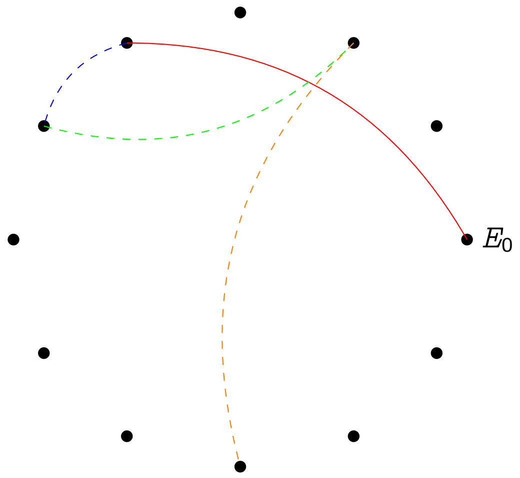

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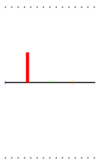

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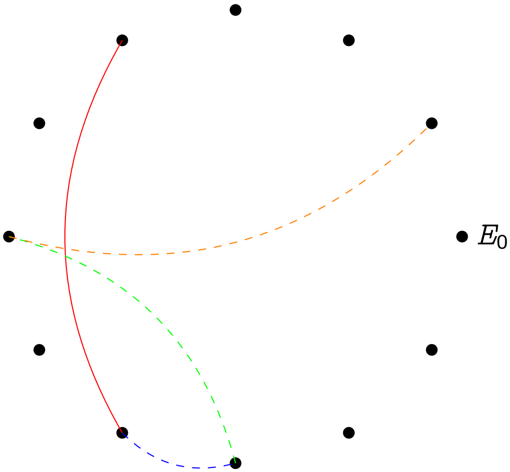

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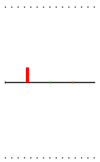

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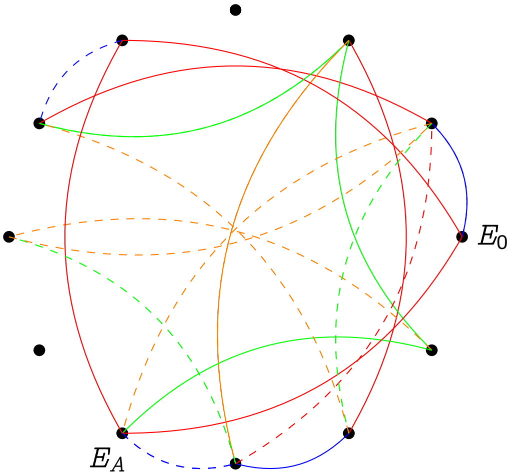

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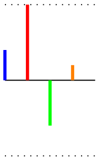

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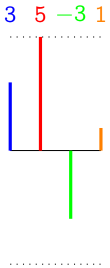
- Initiated study of fully constant time variant (very expensive, though).

Avoiding dummies

We change the format of the secret key:

Original: vectors with coefficients in $[-B, B]$.

Modified: vectors with odd[1] coefficients in $[-B, B]$.



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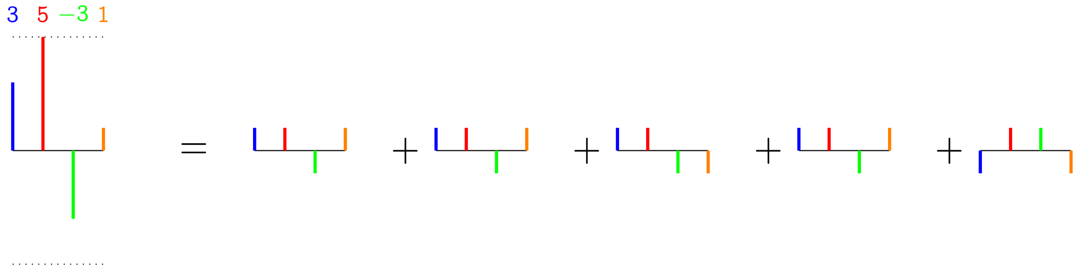
- Translate vector to sum of ± 1 vectors;
- Each vector costs exactly one isogeny evaluation per degree.



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Summary

- Repeat with me: I need isogeny-based crypto!
- CSIDH is the new Diffie–Hellman:
  Very short keys, easy key validation, ...
- Implementing isogeny-based crypto efficiently is challenging,
  even more so with side-channel protections.

Thank you

https://defeo.lu/

@luca_defeo