

Une théorie locale des polylogarithmes.

Local continuation of Li.

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Collaboration at various stages of the work
and in the framework of the Project

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Introduction

The aim of this quick talk is to explain how to extend polylogarithms

$$\mathrm{Li}(s_1, \dots, s_r) = \sum_{n_1 > n_2 > \dots > n_r > 0} \frac{z^{n_1}}{n_1^{s_1} \dots n_r^{s_r}} \quad (1)$$

They are a priori coded by lists (s_1, \dots, s_r) but, when $s_i \in \mathbb{N}_+$, they admit an *iterated integral representation* and are better coded by words with letters in $X = \{x_0, x_1\}$. We will use the one-to-one correspondences.

$$(\mathbf{s}_1, \dots, \mathbf{s}_r) \in \mathbb{N}_+^r \leftrightarrow x_0^{\mathbf{s}_1-1} x_1 \dots x_0^{\mathbf{s}_r-1} x_1 \in X^* x_1 \leftrightarrow y_{\mathbf{s}_1} \dots y_{\mathbf{s}_r} \in Y^* \quad (2)$$

- $\mathrm{Li}(s)[z]$ is Jonquière and, for $\Re(s) > 1$, one has $\mathrm{Li}(s)[1] = \zeta(s)$
- Completed by $Li(x_0^n) = \frac{\log^n(z)}{n!}$ this provides a family of independent functions admitting an analytic continuation on the cleft plane $\mathbb{C} \setminus (]-\infty, 0] \cup [1, +\infty[)$ or $\mathbb{C} \setminus \widetilde{\{0, 1\}}$.

Explicit construction of Li

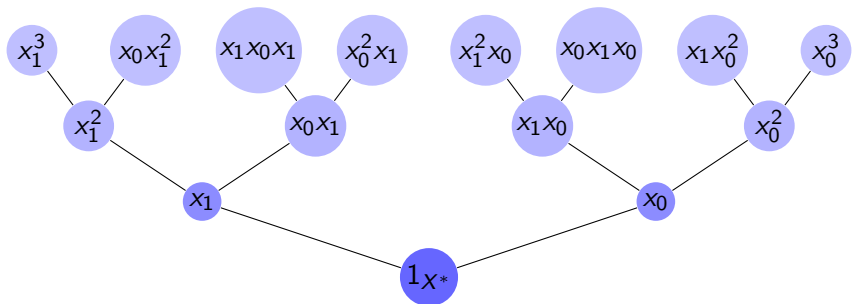
Given a word w , we note $|w|_{x_1}$ the number of occurrences of x_1 within w

$$\alpha_0^z(w) = \begin{cases} 1_\Omega & \text{if } w = 1_{X^*} \\ \int_0^z \alpha_0^s(u) \frac{ds}{1-s} & \text{if } w = x_1 u \\ \int_1^z \alpha_0^s(u) \frac{ds}{s} & \text{if } w = x_0 u \text{ and } |u|_{x_1} = 0 \\ \int_0^z \alpha_0^s(u) \frac{ds}{s} & \text{if } w = x_0 u \text{ and } |u|_{x_1} > 0. \end{cases} \quad (3)$$

Of course, the third line of this recursion implies

$$\alpha_0^z(x_0^n) = \frac{\log(z)^n}{n!}$$

one can check that (a) all the integrals (although improper for the fourth line) are well defined (b) the series $S = \sum_{w \in X^*} \alpha_0^z(w) w$ satisfies (4). We then have $\alpha_0^z = \text{Li}$.



As an example, we compute some coefficients

$$\begin{aligned}
 \langle \text{Li} \mid x_0^n \rangle &= \frac{\log(z)^n}{n!} & ; & \quad \langle \text{Li} \mid x_1^n \rangle = \frac{(-\log(1-z))^n}{n!} \\
 \langle \text{Li} \mid x_0 x_1 \rangle &= \text{Li}_2(z) = \sum_{n \geq 1} \frac{z^n}{n^2} & ; & \quad \langle \text{Li} \mid x_1 x_0 \rangle = \langle \text{Li} \mid x_1 \sqcup x_0 - x_0 x_1 \rangle(z) \\
 \langle \text{Li} \mid x_0^2 x_1 \rangle &= \text{Li}_3(z) = \sum_{n \geq 1} \frac{z^n}{n^3} & ; & \quad \langle \text{Li} \mid x_1 x_0 \rangle = (-\log(1-z))\log(z) - \text{Li}_2(z) \\
 \langle \text{Li} \mid x_0^{r-1} x_1 \rangle &= \text{Li}_r(z) = \sum_{n \geq 1} \frac{z^n}{n^r} & ; & \quad \langle \text{Li} \mid x_1^2 x_0 \rangle = \langle \text{Li} \mid \frac{1}{2}(x_1 \sqcup x_1 \sqcup x_0) - (x_1 \sqcup x_0 x_1) + x_0 x_1^2 \rangle
 \end{aligned}$$

Li From Noncommutative Diff. Eq.

The generating series $S = \sum_{w \in X^*} Li(w)$ satisfies (and is unique to do so)

$$\left\{ \begin{array}{l} \mathbf{d}(S) = \left(\frac{x_0}{z} + \frac{x_1}{1-z} \right) \cdot S \\ \lim_{\substack{z \rightarrow 0 \\ z \in \Omega}} S(z) e^{-x_0 \log(z)} = 1_{\mathcal{H}(\Omega) \langle\langle X \rangle\rangle} \end{array} \right. \quad (4)$$

with $X = \{x_0, x_1\}$. This is, up to the sign of x_1 , the solution G_0 of Drinfel'd [2] for KZ3. We define this unique solution as Li . All Li_w are \mathbb{C} - and even $\mathbb{C}(z)$ -linearly independant (see CAP 17 *Linear independance without monodromy*).

Domain of Li (definition)

In order to extend Li to series, we define $Dom(Li; \Omega)$ (or $Dom(Li)$) if the context is clear) as the set of series $S = \sum_{n \geq 0} S_n$ (decomposition by homogeneous components) such that $\sum_{n \geq 0} Li_{S_n}(z)$ converges for the compact convergence in Ω . One sets

$$Li_S(z) := \sum_{n \geq 0} Li_{S_n}(z) \quad (5)$$

Starting the ladder

$$\begin{array}{ccc} (\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*}) & \xhookrightarrow{\text{Li} \bullet} & \mathbb{C}\{Li_w\}_{w \in X^*} \\ \downarrow & & \downarrow \\ (\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*})[x_0^*, (-x_0)^*, x_1^*] & \xrightarrow{\text{Li}^{(1)} \bullet} & \mathbb{C}_{\mathbb{Z}}\{Li_w\}_{w \in X^*} \end{array}$$

Examples

$$Li_{x_0^*}(z) = z, \quad Li_{x_1^*}(z) = (1 - z)^{-1}, \quad Li_{\alpha x_0^* + \beta x_1^*}(z) = z^\alpha (1 - z)^{-\beta}$$

Properties of the extended Li

Proposition

With this definition, we have

- 1 $Dom(Li)$ is a shuffle subalgebra of $\mathbb{C}\langle\langle X \rangle\rangle$ and then so is $Dom^{rat}(Li) := Dom(Li) \cap \mathbb{C}^{rat}\langle\langle X \rangle\rangle$
- 2 For $S, T \in Dom(Li)$, we have

$$Li_{S \sqcup T} = Li_S \cdot Li_T$$

Examples and counterexamples

For $|t| < 1$, one has $(tx_0)^*x_1 \in Dom(Li, D)$ (D being the open unit slit disc and $Dom(Li, D)$ defined similarly), whereas $x_0^*x_1 \notin Dom(Li, D)$. Indeed, we have to examine the convergence of $\sum_{n \geq 0} Li_{x_0^n x_1}(z)$, but, for $z \in]0, 1[$, one has $0 < z < Li_{x_0^n x_1}(z) \in \mathbb{R}$ and therefore, for these values $\sum_{n \geq 0} Li_{x_0^n x_1}(z) = +\infty$. One can show that, for $|t| < 1$

$$Li_{(tx_0)^*x_1}(z) = \sum_{n \geq 1} \frac{z^n}{t-n}$$

Passing to harmonic sums H_w , $w \in Y^*$

Polylogarithms having a removable singularity at zero

The following proposition helps us characterize their indices.

Proposition

Let $f(z) = \langle L \mid P \rangle = \sum_{w \in X^} \langle P \mid w \rangle \text{Li}_w$. The following conditions are equivalent*

- i) *f can be analytically extended around zero*
- ii) *$P \in \mathbb{C}\langle X \rangle_{X_1} \oplus \mathbb{C}.1_{X^*}$*

TODO recall the formula with $\frac{\text{Li}(z)}{1-z}$

Global and local domains

This proposition and the lemma lead us to the following definitions.

① *Global domains.*–

Let $\emptyset \neq \Omega \subset \tilde{B}$, we define $Dom_{\Omega}(Li) \subset \mathbb{C}\langle\langle X \rangle\rangle$ to be the set of series $S = \sum_{n \geq 0} S_n$ (with $S_n = \sum_{|w|=n} \langle S | w \rangle w$ each homogeneous component) such that $\sum_{n \in \mathbb{N}} Li_{S_n}$ is unconditionally convergent for the compact convergence (UCC) [4].

As examples, we have Ω_1 , the doubly cleft plane then

$$Dom(Li) := Dom_{\Omega_1}(Li) \text{ or } \Omega_2 = \tilde{B}$$

② *Local domains around zero (fit with H-theory).*–

Here, we consider series $S \in (\mathbb{C}\langle\langle X \rangle\rangle_{x_1} \oplus \mathbb{C}1_{X^*})$ (i.e. $supp(S) \cap X_{x_0} = \emptyset$).

We consider radii $0 < R \leq 1$, the corresponding open discs

$D_R = \{z \in \mathbb{C} \mid |z| < R\}$ and define

$$Dom_R(Li) := \{S = \sum_{n \geq 0} S_n \in (\mathbb{C}\langle\langle X \rangle\rangle_{x_1} \oplus \mathbb{C}1_{\Omega}) \mid \sum_{n \in \mathbb{N}} Li_{S_n} \text{ (UCC) in } D_R\}$$

$$Dom_{loc}(Li) := \bigcup_{0 < R \leq 1} Dom_R(Li). \quad (6)$$

Properties of the domains

Theorem A

- 1 For all $\emptyset \neq \Omega \subset \tilde{B}$, $Dom_\Omega(Li)$ is a shuffle subalgebra of $\mathbb{C}\langle\langle X \rangle\rangle$ and so are the $Dom_R(Li)$.
- 2 $R \mapsto Dom_R(Li)$ is strictly decreasing for $R \in]0, 1]$.
- 3 All $Dom_R(Li)$ and $Dom_{loc}(Li)$ are shuffle subalgebras of $\mathbb{C}\langle\langle X \rangle\rangle$ and $\pi_Y(Dom_{loc}(Li))$ is a stuffle subalgebra of $\mathbb{C}\langle\langle Y \rangle\rangle$.
- 4 Let $T(z) = \sum_{N \geq 0} a_N z^N$ be a Taylor series i.e. such that $\limsup_{N \rightarrow +\infty} |a_N|^{1/n} = B < +\infty$, then the series

$$S = \sum_{N \geq 0} a_N (-(-x_1)^+)^{\sqcup N} \quad (7)$$

is summable in $\mathbb{C}\langle\langle X \rangle\rangle$ (with sum in $\mathbb{C}\langle\langle x_1 \rangle\rangle$) and $S \in Dom_R(Li)$ with $R = \frac{1}{B+1}$ and $Li S = T(z)$.

Theorem A/2

- 5 Let $S \in \text{Dom}_R(\text{Li})$ and $S = \sum_{n \geq 0} S_n$ (homogeneous decomposition), we define $N \mapsto H_{\pi_Y(S)}(N)$ by^a

$$\frac{\text{Li}_S(z)}{1-z} = \sum_{N \geq 0} H_{\pi_Y(S)}(N) z^N. \quad (8)$$

Moreover, for all $r \in]0, R[$,

$$\sum_{n, N \geq 0} |H_{\pi_Y(S_n)} r^N| < +\infty, \quad (9)$$

in particular, for all $N \in \mathbb{N}$ the series (of complex numbers) $\sum_{n \geq 0} H_{\pi_Y(S_n)}(N)$ converges absolutely to $H_{\pi_Y(S)}(N)$.

^aThis definition is compatible with the old one when S is a polynomial.

Theorem A/3

- ⑥ Conversely, let $Q \in \mathbb{C}\langle\langle Y \rangle\rangle$ with $Q = \sum_{n \geq 0} Q_n$ (decomposition by weights), we suppose that it exists $r \in]0, 1]$ such that

$$\sum_{n, N \geq 0} |H_{Q_n}(N) r^N| < +\infty. \quad (10)$$

in particular, for all $N \in \mathbb{N}$, $\sum_{n \geq 0} H_{Q_n}(N) = \ell(N) \in \mathbb{C}$ unconditionally.

Under such circumstances, $\pi_X(Q) \in \text{Dom}_r(\text{Li})$ and, for all $|z| \leq r$

$$\frac{\text{Li}_S(z)}{1-z} = \sum_{N \geq 0} \ell(N) z^N, \quad (11)$$

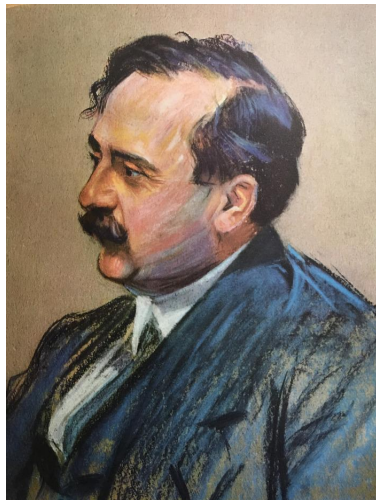


Figure: Jacques Hadamard and Paul Montel.

Continuing the ladder

$$\begin{array}{ccc}
 (\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*}) & \xrightarrow{\text{Li}_\bullet} & \mathbb{C}\{\text{Li}_w\}_{w \in X^*} \\
 \downarrow & & \downarrow \\
 (\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*})[x_0^*, (-x_0)^*, x_1^*] & \xrightarrow{\text{Li}_\bullet^{(1)}} & \mathbb{C}_{\mathbb{Z}}\{\text{Li}_w\}_{w \in X^*} \\
 \downarrow & & \downarrow \\
 \mathbb{C}\langle X \rangle \sqcup \mathbb{C}^{\text{rat}}\langle\langle x_0 \rangle\rangle \sqcup \mathbb{C}^{\text{rat}}\langle\langle x_1 \rangle\rangle & \xrightarrow{\text{Li}_\bullet^{(2)}} & \mathbb{C}_{\mathbb{C}}\{\text{Li}_w\}_{w \in X^*} \\
 \uparrow & \nearrow \text{---} & \\
 \mathbb{C}\langle X \rangle \otimes_{\mathbb{C}} \mathbb{C}^{\text{rat}}\langle\langle x_0 \rangle\rangle \otimes_{\mathbb{C}} \mathbb{C}^{\text{rat}}\langle\langle x_1 \rangle\rangle & &
 \end{array}$$

We have, after a theorem by Leopold Kronecker,

$$\mathbb{C}^{\text{rat}}\langle\langle x \rangle\rangle = \left\{ \frac{P}{Q} \right\}_{\substack{P, Q \in \mathbb{C}[x] \\ Q(0) \neq 0}} \quad (12)$$

On the right: freeness without monodromy

Theorem (Deneufchâtel, GHED, Minh & Solomon, 2011 [1])

Let (\mathcal{A}, ∂) be a k -commutative associative differential algebra with unit and \mathcal{C} be a differential subfield of \mathcal{A} (i.e. $\partial(\mathcal{C}) \subset \mathcal{C}$). We suppose that $k = \ker(\partial)$ and that $S \in \mathcal{A}\langle\langle X \rangle\rangle$ is a solution of the differential equation

$$\mathbf{d}(S) = MS ; \langle S \mid 1 \rangle = 1 \text{ with } M = \sum_{x \in X} u_x x \in \mathcal{C}\langle\langle X \rangle\rangle \quad (13)$$

(i.e. M is a homogeneous series of degree 1)

The following conditions are equivalent :

- ❶ The family $(\langle S \mid w \rangle)_{w \in X^*}$ of coefficients of S is (linearly) free over \mathcal{C} .
- ❷ The family of coefficients $(\langle S \mid x \rangle)_{x \in X \cup \{1_{X^*}\}}$ is (linearly) free over \mathcal{C} .
- ❸ The family $(u_x)_{x \in X}$ is such that, for $f \in \mathcal{C}$ et $\alpha_x \in k$

$$\partial(f) = \sum_{x \in X} \alpha_x u_x \implies (\forall x \in X)(\alpha_x = 0).$$

A useful property

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Independence of characters with respect to polynomials



I came across the following property :

5

Let \mathfrak{g} be a Lie algebra over a ring k without zero divisors, $\mathcal{U} = \mathcal{U}(\mathfrak{g})$ be its enveloping algebra. As such, \mathcal{U} is a Hopf algebra and ϵ , its counit, is the only character of $\mathcal{U} \rightarrow k$ which vanishes on \mathfrak{g} .



Set $\mathcal{U}_+ = \ker(\epsilon)$. We build the following filtrations ($N \geq 1$)

1

$$\mathcal{U}_N = \mathcal{U}_+^N = \underbrace{\mathcal{U}_+ \dots \mathcal{U}_+}_{N \text{ times}} \quad (1)$$

and

$$\mathcal{U}_N^* = \mathcal{U}_{N+1}^\perp = \{f \in \mathcal{U}^* \mid (\forall u \in \mathcal{U}_{N+1})(f(u) = 0)\} \quad (2)$$

the first one is decreasing and the second one increasing. One shows easily that (with \diamond as the convolution product)

$$\mathcal{U}_p^* \diamond \mathcal{U}_q^* \subset \mathcal{U}_{p+q}^*$$

so that $\mathcal{U}_\infty^* = \bigcup_{n \geq 1} \mathcal{U}_n^*$ is a convolution subalgebra of \mathcal{U}^* .

Now, we can state the

Theorem : The set of characters of $(\mathcal{U}, \cdot, 1_{\mathcal{U}})$ is linearly free w.r.t. \mathcal{U}_∞^* .

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17/24

Left and then right: the arrow $\text{Li}_{\bullet}^{(1)}$

Proposition

- i. The family $\{x_0^*, x_1^*\}$ is algebraically independent over $(\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*})$ within $(\mathbb{C}\langle\langle X \rangle\rangle^{\text{rat}}, \sqcup, 1_{X^*})$.
- ii. $(\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*})[x_0^*, x_1^*, (-x_0)^*]$ is a free module over $\mathbb{C}\langle X \rangle$, the family $\{(x_0^*)^{\sqcup k} \sqcup (x_1^*)^{\sqcup l}\}_{(k,l) \in \mathbb{Z} \times \mathbb{N}}$ is a $\mathbb{C}\langle X \rangle$ -basis of it.
- iii. As a consequence, $\{w \sqcup (x_0^*)^{\sqcup k} \sqcup (x_1^*)^{\sqcup l}\}_{\substack{w \in X^* \\ (k,l) \in \mathbb{Z} \times \mathbb{N}}}$ is a \mathbb{C} -basis of it.
- iv. $\text{Li}_{\bullet}^{(1)}$ is the unique morphism from $(\mathbb{C}\langle X \rangle, \sqcup, 1_{X^*})[x_0^*, (-x_0)^*, x_1^*]$ to $\mathcal{H}(\Omega)$ such that

$$x_0^* \rightarrow z, \quad (-x_0)^* \rightarrow z^{-1} \quad \text{and} \quad x_1^* \rightarrow (1 - z)^{-1}$$

- v. $\text{Im}(\text{Li}_{\bullet}^{(1)}) = \mathcal{C}_{\mathbb{Z}}\{\text{Li}_w\}_{w \in X^*}$.
- vi. $\ker(\text{Li}_{\bullet}^{(1)})$ is the (shuffle) ideal generated by $x_0^* \sqcup x_1^* - x_1^* + 1_{X^*}$.

Sketch of the proof (pictorial)

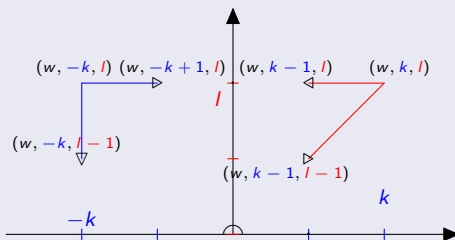


Figure: Rewriting mod \mathcal{J} of $\{w \sqcup (x_0^*) \sqcup k \sqcup (x_1^*) \sqcup l\}_{k \in \mathbb{Z}, l \in \mathbb{N}, w \in X^*}$.

Concluding remarks

- Extending the domain of polylogarithms to (some) rational series permits the projection of rational identities. Such as

$$(\alpha x)^* \sqcup (\beta y)^* = (\alpha x + \beta y)^*$$

- The theory developed here allows to pursue, for the Harmonic sums, this investigation such as

$$(\alpha y_i)^* \sqcup (\beta y_j)^* = (\alpha y_i + \beta y_j + \alpha \beta y_{i+j})^*$$

- More in Minh's talk.

- [1] M. Deneufchâtel, GHED, Hoang Ngoc Minh, A. I. Solomon.— *Independence of hyperlogarithms over function fields via algebraic combinatorics*, Lecture Notes in Computer Science (2011), Volume 6742 (2011), 127-139.
- [2] V.G. Drinfel'd, *On quasitriangular quasi-hopf algebra and a group closely connected with $\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$* , Leningrad Math. J., 4, 829-860, 1991.
- [3] J. Hadamard, *Théorème sur les séries entières*, Acta Math., Vol 22 (1899), 55-63.
- [4] P. Montel.— *Leçons sur les familles normales de fonctions analytiques et leurs applications*, Gauthier-Villars (1927)

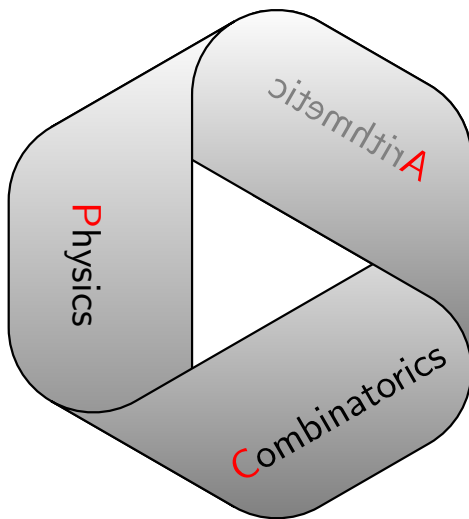


Figure: ... and a lot of (machine) computations.

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