Simple conditions for the intersection with polydisks

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An important class of systems in control theory is the class of multidimensional systems in which information propagates in more than one independent direction. At the core of the study of such systems is the effective computation over the ring of rational fractions which have no poles in the closed unit polydisk $\overline{\mathbb{U}}^n = \{z = (z_1, \ldots, z_n) \in \mathbb{C}^n \mid |z_1| \leq 1, \ldots, |z_n| \leq 1\}$. In this context, one important question related to the stability of these systems is to decide whether a polynomial $D(z_1, \ldots, z_n) \in \mathbb{Q}[z_1, \ldots, z_n]$ has zeros that lie in the unit polydisk $\overline{\mathbb{U}}^n$

$$V_{\mathbb{C}}(D(z_1, \ldots, z_n)) \cap \overline{\mathbb{U}}^n \neq \emptyset \tag{1}$$

In [1], M. Strintzis states a fundamtental theorem showing that condition (1), i.e., the existence of complex zeros of D in $\overline{\mathbb{U}}^n$ is equivalent to the existence of complex zeros of D at one face of the polydisk, i.e., $\mathbb{F}_i = \overline{\mathbb{U}}^n \cap \{z = (z_1, \ldots, z_n), z_1 = 1, \ldots, z_n = 1, |z_i| \leq 1\}$, or at the poly-circle i.e., $\mathbb{D}^n = \{z = (z_1, \ldots, z_n) \in \mathbb{C}^n \mid |z_1| = 1, \ldots, |z_n| = 1\}$.

In this presentation, we extend the fundamental result of Strintzis to some specific algebraic varieties, namely varieties of dimension one in $\mathbb{C}^n$ (algebraic curves). As a consequence, an importante question about the stabilizability of multidimensional systems can be answered using a simple algorithm based on computer algebra techniques.

Références

[1] Strintzis, M. Tests of stability of multidimensional filters. *IEEE Transactions on Circuits and Systems*, 24(8) :432–437, 1977.

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