CPSC 335 Project 1

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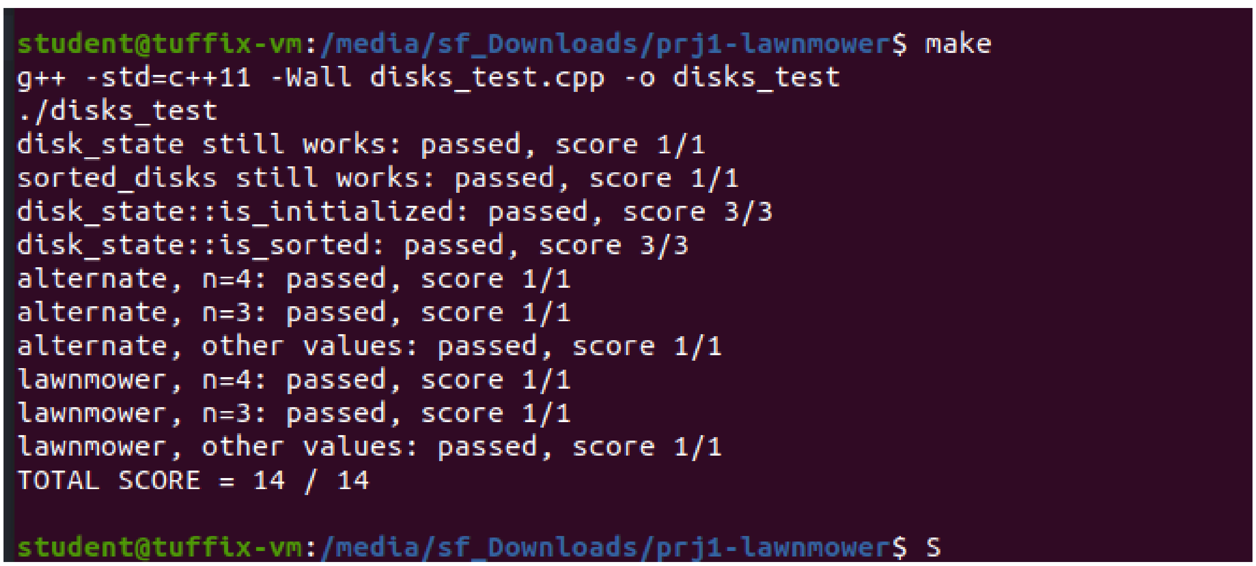

The lawnmower algorithm

Algorithm Design:

Pseudo Code

Input: a positive integer **n** and a list of **2n** disks of alternating colors -light-disk
Output: List of 2n disks, m for the number of swaps

```
 for i = 0 to n/2 do

    for j = 0 to 2n-1 do
       if list[j] > list[j+1]
          swap
       end
    end for

     for j = 2n-1 to 0 do
       if list[j] <  list[j-1]
    swap
       end
    end for
 end for
```

Step Count

```
                              // SC outer loop =
for i = 0 to n/2 do               // (n+1)/2 times
                                  // SC forward loop
   for j = 0 to 2n-1 do            // 2n times
      if list[j] > list[j+1]       // 1 TU
         Swap                      // 3 TU
         m++                       // 1 TU
      end
   end for
                                // SC reverse loop
   for j = 2n-1 to 0 do         // 2n times
      if list[j] <  list[j-1]      // 1 TU
         Swap                      // 3 TU
         M++                       // 1 TU

      end
   end for
end for
```

$$S.C_{forward} = 2n(1 + max(4, 0))$$
$$= 2n(1 + 4)$$
$$= 2n(5)$$
$$= 10n$$
$$S.C_{reverse} = 2n(1 + max(4, 0))$$
$$= 2n(1 + 4)$$
$$= 2n(5)$$
$$= 10n$$

$$S.C. = S.C_{outer} * (S.C_{forward} + S.C_{reverse})$$
$$= (\frac{n+1}{2})(10n + 10n)$$
$$= (\frac{n+1}{2})(20n)$$
$$= (\frac{20n^2+20n}{2})$$
$$= (10n^2 + 10n)$$

Mathematical Analysis

-Big-O efficiency class: O(n^2)

-Definition:

$$10n^2 + 10n \in O(n^2)$$
$$10n^2 + 10n \leq C \cdot n^2$$

$$C = 10 + 10 = 20,\ no = 1$$
$$10n^2 + 10n \leq 20n^2$$
$$10 + 10 \leq 20$$
$$20 \leq 20 \rightarrow true$$

By Definition, we can say that $10n^2 + 10n \in O(n^2)$.

Limit Theorem:

Prove that $10n^2 + 10n \in O(n^2)$ using the limit theorem

$$lim_{n\to\infty} \frac{10n^2+10n}{n^2}$$

$$= \ lim_{n\to\infty} \frac{10n^2}{n^2} + \frac{10n}{n^2}$$

$$= lim_{n\to\infty} 10 + \ lim_{n\to\infty} \frac{10}{n}$$

$$lim_{n\to\infty} \ = 10 \geq 0,$$

By Limit theorem, we can say that $10n^2 + 10n \in O(n^2)$.

The Alternating Disk Algorithm

Algorithm Design:

Pseudo Code

```
For i = 0 to N+1 step
   if (i % 2 == 0)
      for j = 0 to N-1 step + 2
         if (list[j]  > list[j + 1])
            Swap disks
            m++;
         end if
      end for
   else do
      for j = 1 to N-2 step + 2
         if (list[j]  > list[j + 1])
            Swap disks
            m++;
         end if
      end for
   end if
end for
```

Step Count

```
For i = 0 to N                  // (n-0)+1/1= n+1
   if (i % 2 == 0)                     // 2 TU
      for j = 0 to N-1 step + 2        // (n-1-0+1)/2 = n/2
         if (list[j]  > list[j + 1])  // 1 TU
            Swap disks                 // 3 TU
            M++;                       // 1 TU
         end
      end
   else
      for j = 1 to N-2 step + 2        // ((n-2)-1+1)/2= n-2/2
         if (list[j]  > list[j + 1])  // 1 TU
            Swap disks                // 3 TU
            M++;                       // 1 TU
         end
       end
  End
```

$$S.C. = S.C_{outer} * (S.C_{if\ block})$$

$$S.C_{outer\ loop} = n + 1$$

$$S.C_{if\ block} = 2 + max(S.C_{even\ loop}, S.C_{odd\ loop}))$$

$$S.C_{even\ loop} = \frac{n}{2}(1 + max(4, 0))$$
$$= \frac{n}{2}(1 + 4)$$
$$= \frac{n}{2}(5)$$
$$= \frac{5n}{2}$$

$$S.C_{odd\ loop} = \frac{n-2}{2}(1 + max(4, 0))$$
$$= \frac{n-2}{2}(1 + 4)$$
$$= \frac{n-2}{2}(5)$$
$$= \frac{5n-10}{2}$$

$$S.C \quad = (n + 1)(2 + max(\frac{5n}{2}, \frac{5n-10}{2}))$$

$$= (n + 1)(2 + \frac{5n}{2})$$

$$= \frac{5n^2}{2} + \frac{5n}{2} + 2n + 2$$

$$= \frac{5n^2}{2} + \frac{9n}{2} + 2$$

Mathematical Analysis

-Big-O efficiency class:

-Definition:

$\frac{5n^2}{2} + \frac{9n}{2} + 2 \in O(n^2)$

$\frac{5n^2}{2} + \frac{9n}{2} + 2 \leq C \cdot n^2$

$C = \frac{5}{2} + \frac{9}{2} + 2 = 9\,,\ no = 1$

$\frac{5n^2}{2} + \frac{9n}{2} + 2 \leq 9n^2$

$\frac{5}{2} + \frac{9}{2} + 2 \leq 9$

$9 \leq 9 \rightarrow true$

By definition, we can say that $\frac{5n^2}{2} + \frac{9n}{2} + 2 \in O(n^2)$.

Limit Theorem:

Prove that $\frac{5n^2}{2} + \frac{9n}{2} + 2 \in O(n^2)$ using the limit theorem

$lim_{n\rightarrow\infty} \frac{\frac{5}{2}n^2 + \frac{9}{2}n + 2}{n^2}$

$= lim_{n\rightarrow\infty} \frac{\frac{5}{2}n^2}{n^2} + \frac{\frac{9}{2}n}{n^2} + \frac{2}{n^2}$

$= lim_{n\rightarrow\infty} \frac{5}{2} + lim_{n\rightarrow\infty} \frac{\frac{9}{2}}{n} + lim_{n\rightarrow\infty} \frac{2}{n^2}$

$= \frac{5}{2} + 0 + 0$

$= \frac{5}{2}$

$lim_{n\rightarrow\infty} = \frac{5}{2} \geq 0,$

By Limit Theorem, we can say that $\frac{5n^2}{2} + \frac{9n}{2} + 2 \in O(n^2)$.