Übung 11
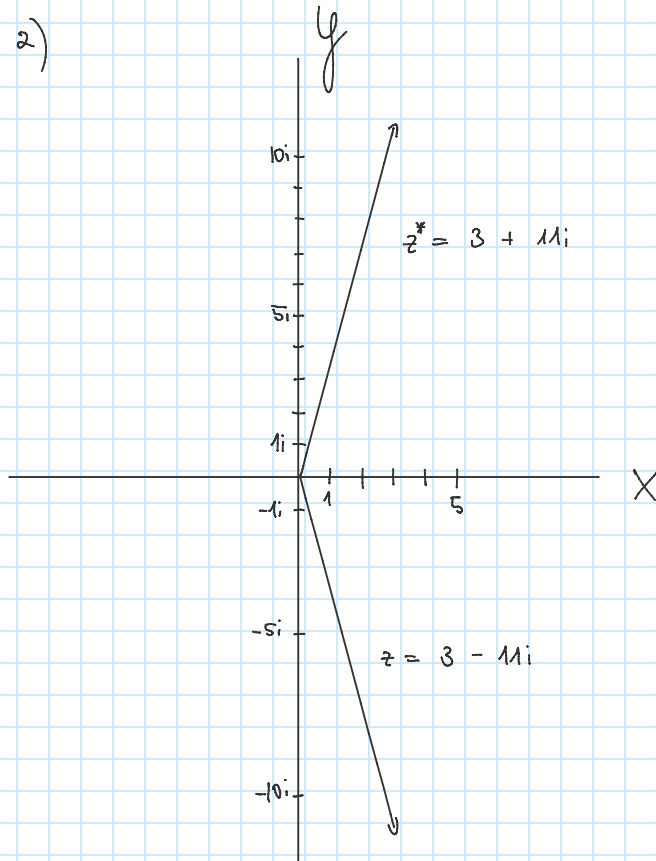
Aufgabe 1

2)

$z = 3 - 11i$

$|z| = \sqrt{3^2 + (-11)^2} = 11.402 = r$

$\varphi = 2\pi - \tan^{-1}\left(\frac{11}{3}\right) = 4.979$

$z = 11.402 \left(\cos(4.979) + i \sin(4.979) \right) \rightarrow$ Polarform

$z = 11.402 \, e^{i \, 4.979} \longrightarrow$ Exponenzialform

$z^* = 3 + 11i$

$|z^*| = \sqrt{3^2 + 11^2} = 11.402 = r^*$

$\varphi^* = \tan^{-1}\left(\frac{3}{11}\right) = 1.305$

$z^* = 11.402 \left(\cos(1.305) + i \cdot \sin(1.305) \right) \rightarrow$ Polarform

$z^* = 11.402 \cdot e^{i 1.305} \rightarrow$ Exponenzialform

b)

$$z = 4\left(\cos(-40°) + i \cdot \sin(-40°)\right) + 2e^{i30°} - 3 + 1.5i$$

$$360° - 40° = 320°$$

$$4\left(\cos(320°) + i\sin(320°)\right)$$

$$3.064 - 2.571i$$

$$2e^{i30°}$$

$$2\left(\cos(30°) + i \cdot \sin(30°)\right)$$

$$1.732 + 0.5i$$

$$\left(3.064 - 2.571i\right) + \left(1.732 + 0.5i\right) + \left(-3 + 1.5i\right)$$

$$\left(3.064 + 1.732 - 3\right) + \left(-2.571i + 0.5i + 1.5i\right)$$

$$\underline{\underline{1.796 - 0.571i}} \qquad \text{und} \qquad z^* = \underline{\underline{1.796 + 0.571i}}$$

c)

$$z_1^* = \frac{2 - i}{1 + 2i} \cdot \frac{(1 - 2i)}{(1 - 2i)}$$

$$= \frac{2 - 4i - i + 2i^2}{1 - 2i + 2i - 4i^2}$$

$$= \frac{-5i}{5} \qquad = \qquad -i$$

$$z_2 = 2e^{-i\frac{\pi}{3}}$$

$$= 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

$$= 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$$

$$= 1 - 1.732i$$

$$z_3 = 4\left(\cos(30°) + i\sin(30°)\right)$$

$$30° \cdot 2\pi / 360° = 0.524$$

$$= 4\left(\cos(0.524) + i\sin(0.524)\right)$$

$$= 3.463 + 2.002i$$

$$\frac{z_1^* \cdot z_3}{0.5\, z_2}$$

$$= \frac{(0-i)(3.463 + 2.002i)}{0.5(1 - 1.732i)}$$

$$= \frac{-3.463i + 2.002 \cdot i \cdot (-i)}{0.5 - 0.866i}$$

$$= \frac{2.002 - 3.463i}{0.5 - 0.866i} \cdot \frac{(0.5 + 0.866i)}{(0.5 + 0.866i)}$$

$$= \frac{1.001 + 1.734i - 1.7315i + 3}{0.25 + 0.75}$$

$$= \underline{\underline{4.001 + 0.0025i}}$$

d)

Exponenzialform:

$$r = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3} = 1.732$$

$$\varphi = \tan^{-1}(\sqrt{2}) = 0.955$$

$$\left(1.732\, e^{i0.955}\right)^3$$

$$\underline{\underline{5.196\, e^{i2.865}}}$$

Aufgabe 2

$$z^4 + 4z^2 + 16 = 0$$

$$u = z^2$$

$$u^2 + 4u + 16 = 0$$

$$u_{1,2} = \frac{-4 \pm \sqrt{(-4)^2 - (4 \cdot 1 \cdot 16)}}{2 \cdot 1}$$

$$= \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm \left(i\sqrt{48} \right)}{2}$$

$$u_1 = \frac{-4 + i\sqrt{48}}{2} = -2 + i\frac{\sqrt{48}}{2} = 4\left(\cos(2.094) + i\sin(2.094) \right)$$

$$u_2 = \frac{-4 - i\sqrt{48}}{2} = -2 - i\frac{\sqrt{48}}{2} = 4\left(\cos(5.236) + i\sin(5.236) \right)$$

$$u = z^2$$
$$z = \sqrt{u}$$

$$z_1 = \sqrt{u_1} = \sqrt{-2 + i\frac{\sqrt{48}}{2}}$$

$$(p + iq)$$

$$p = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(-2)^2 + \left(\frac{\sqrt{48}}{2}\right)^2} - 2}$$

$$q = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(-2)^2 + \left(\frac{\sqrt{48}}{2}\right)^2} - -2}$$

$$z_1 = \left(1 + 1.732i \right)$$

$$z_2 = \sqrt{u_2} = \sqrt{-2 - i\frac{\sqrt{48}}{2}}$$

$$(p + iq)$$

$$p = \frac{1}{\sqrt{2}}\sqrt{\sqrt{(-2)^2 + \left(-\frac{\sqrt{48}}{2}\right)^2} - 2}$$

$$q = -\frac{\sqrt{\sqrt{(-2)^2 + \left(-\frac{\sqrt{48}}{2}\right)^2} - -2}}{\sqrt{2}}$$

$$z_2 = (1 - 1.732i)$$