HM2 Serie 11 Aufgabe 2

Leo Rudin

$\frac{dy}{dx} = \frac{x^2}{y}$

$y'(x) = \frac{x^2}{y(x)}$

Interval: $0 \leq x \leq 1.4$

Startwert: $y(0) = 2$

a)

$h = 0.7$

$x_0 = 0$

$y_0 = 2$

1. Schritt:

$x_1 = x_0 + h = 0 + 0.7 = 0.7$

$y_1 = y_0 + h \cdot f(x_0, y_0) = 2 + 0.7 \cdot \frac{0^2}{2} = 2$

2. Schritt:

$x_2 = x_1 + h = 0.7 + 0.7 = 1.4$

$y_2 = y_1 + h \cdot f(x_1, y_1) = 2 + 0.7 \cdot \frac{0.7^2}{2} = 2.1715$

b)

$h = 0.7$

$x_0 = 0$

$y_0 = 2$

1. Schritt:

$x_{h/2} = x_0 + \frac{h}{2} = 0 + \frac{0.7}{2} = 0.35$

$y_{h/2} = y_0 + \frac{h}{2} \cdot f(x_0, y_0) = 2 + \frac{0.7}{2} \cdot \frac{0^2}{2} = 2$

$x_1 = x_0 + h = 0 + 0.7 = 0.7$

$y_1 = y_0 + h \cdot f(x_{h/2}, y_{h/2}) = 2 + 0.7 \cdot \frac{0.35^2}{2} = 2.04$

2. Schritt:

$x_{h/2} = x_1 + \frac{h}{2} = 0.7 + \frac{0.7}{2} = 1.05$

$y_{h/2} = y_1 + \frac{h}{2} \cdot f(x_1, y_1) = 2.04 + \frac{0.7}{2} \cdot \frac{0.7^2}{2.04} = 2.12$

$x_2 = x_1 + h = 0.7 + 0.7 = 1.4$

$y_2 = y_1 + h \cdot f(x_{h/2}, y_{h/2}) = 2.04 + 0.7 \cdot \frac{1.05^2}{2.12} = 2.40$

c)

$h = 0.7$

$x_0 = 0$

$y_0 = 2$

1. Schritt:

$x_1 = x_0 + h = 0 + 0.7 = 0.7$

$y_1^{Euler} = y_0 + h \cdot f(x_0, y_0) = 2 + 0.7 \cdot \frac{0^2}{2} = 2$

$k_1 = f(x_0, y_0) = \frac{0^2}{2} = 0$

$k_2 = f(x_1, y_1^{Euler}) = \frac{0.7^2}{2} = 0.245$

$y_1 = y_0 + h \cdot \frac{k_1 + k_2}{2} = 2 + 0.7 \cdot \frac{0 + 0.245}{2} = 2.08$

2. Schritt:

$x_2 = x_1 + h = 0.7 + 0.7 = 1.4$

$y_2^{Euler} = y_1 + h \cdot f(x_1, y_1) = 2.08 + 0.7 \cdot \frac{0.7^2}{2.08} = 2.24$

$k_1 = f(x_1, y_1) = \frac{0.7^2}{2.08} = 0.24$

$k_2 = f(x_2, y_2^{Euler}) = \frac{1.4^2}{2.24} = 0.875$

$y_2 = y_1 + h \cdot \frac{k1 + k2}{2} = 2.08 + 0.7 \cdot \frac{0.24 + 0.875}{2} = 2.47$