HM2 Serie 2 Aufgabe 1

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a)

$$f(x_1, x_2) = \begin{pmatrix} 5x_1 x_2 \\ x_1^2 x_2^2 + x_1 + 2x_2 \end{pmatrix}$$

Erstellung der Matrix:

$$Df(x_1, x_2) = \begin{pmatrix} 5x_2 & 5x_1 \\ 2x_1 x_2^2 + 1 & 2x_2 x_1^2 + 2 \end{pmatrix}$$

Linearisierung an der Stelle $x^{(0)} = (1,2)^T$:

$$g(x) = f(x^{(0)}) + Df(x^{(0)}) \cdot (x - x^{(0)})$$

$$g(x) = \begin{pmatrix} 10 \\ 9 \end{pmatrix} + \begin{pmatrix} 10 & 5 \\ 9 & 6 \end{pmatrix} \cdot \begin{pmatrix} x_1 - 1 \\ x_2 - 2 \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10 + 10(x_1 - 1) + 5(x_2 - 1) \\ 9 + 9(x_1 - 2) + 6(x_2 - 2) \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10 + 10x_1 - 10 + 5x_2 - 5 \\ 9 + 9x_1 - 18 + 6x_2 - 12 \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10x_1 + 5x_2 - 5 \\ 9x_1 + 6x_2 - 21 \end{pmatrix}$$

b)

$$f(x_1, x_2, x_3) = \begin{pmatrix} \ln(x_1^2 + x_2^2) + x_3^2 \\ \exp(x_2^2 + x_3^2) + x_1^2 \\ \frac{1}{(x_3^2 + x_1^2)} + x_2^2 \end{pmatrix}$$

Erstellung der Matrix:

$$Df(x_1, x_2, x_3) = \begin{pmatrix} \frac{2x_1}{x_1^2 + x_2^2} & \frac{2x_2}{x_1^2 + x_2^2} & 2x_3 \\ 2x_1 & 2x_2 \exp(x_2^2 + x_3^2) & 2x_3 \exp(x_2^2 + x_3^2) \\ -\frac{2x_1}{(x_3^2 + x_1^2)^2} & 2x_2 & -\frac{2x_3}{(x_3^2 + x_1^2)^2} \end{pmatrix}$$

Linearisierung an der Stelle $x^{(0)} = (1,2,3)^T$

$$g(x) = f(x^{(0)}) + Df(x^{(0)}) \cdot (x - x^{(0)})$$

$$g(x) = \begin{pmatrix} 10.61 \\ 442414.39 \\ 4.1 \end{pmatrix} + \begin{pmatrix} 1 & 6 & -5 \\ 0 & 0.33 & 0.64 \\ 0 & 0 & -18.75 \end{pmatrix} \cdot \begin{pmatrix} x_1 - 1 \\ x_2 - 2 \\ x_3 - 3 \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10.61 \\ 442414.39 \\ 4.1 \end{pmatrix} + \begin{pmatrix} x_1 - 1 + 6(x_2 - 2) - 5(x_3 - 3) \\ 0.33(x_2 - 2) + 0.64(x_3 - 3) \\ -18.75(x_3 - 3) \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10.61 + x_1 - 1 + 6(x_2 - 2) - 5(x_3 - 3) \\ 442414.39 + 0.33(x_2 - 2) + 0.64(x_3 - 3) \\ 4.1 - 18.75(x_3 - 3) \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 10.61 + x_1 - 1 + 6x_2 - 12 - 5x_3 - 15 \\ 442414.39 + 0.33x_2 - 0.66 + 0.64x_3 - 1.92) \\ 4.1 - 18.75x_3 - 56.25 \end{pmatrix}$$

$$g(x) = \begin{pmatrix} x_1 + 6x_2 - 5x_3 - 17.39 \\ 0.33x_2 + 0.64x_3 + 442411.81) \\ 18.75x_3 - 52.15 \end{pmatrix}$$