

HM2 Serie 9 Aufgabe 2

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$$h_0 = \frac{\pi}{2^0}$$

$$h_1 = \frac{\pi}{2^1}$$

$$h_2 = \frac{\pi}{2^2}$$

$$h_3 = \frac{\pi}{2^3}$$

$$h_4 = \frac{\pi}{2^4}$$

0. Exterapolation:

$$T_{00} = h_0 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^0 f(x_i) \right) = \pi \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} \right) = 0.153$$

$$T_{10} = h_1 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^1 f(x_i) \right) = \frac{\pi}{2^1} \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left((0+1 \cdot \frac{\pi}{2^1})^2\right) \right) = -1.151$$

$$T_{20} = h_2 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^3 f(x_i) \right) = \frac{\pi}{2^2} \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left((0+1 \cdot \frac{\pi}{2^2})^2\right) + \cos\left((0+2 \cdot \frac{\pi}{2^2})^2\right) + \cos\left((0+3 \cdot \frac{\pi}{2^2})^2\right) \right) = 0.650$$

$$T_{30} = h_3 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^7 f(x_i) \right) = \frac{\pi}{2^3} \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left((0+1 \cdot \frac{\pi}{2^3})^2\right) + \dots + \cos\left((0+7 \cdot \frac{\pi}{2^3})^2\right) \right) = 0.603$$

$$T_{40} = h_4 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{15} f(x_i) \right) = \frac{\pi}{2^4} \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left((0+1 \cdot \frac{\pi}{2^4})^2\right) + \dots + \cos\left((0+15 \cdot \frac{\pi}{2^4})^2\right) \right) = 0.575$$

1. Extropolation:

$$T_{01} = \frac{4T_{10}-T_{00}}{3} = \frac{4 \cdot (-1.151) - 0.153}{3} = -1.586$$

$$T_{11} = \frac{4T_{20}-T_{10}}{3} = \frac{4 \cdot 0.650 - (-1.151)}{3} = 1.250$$

$$T_{21} = \frac{4T_{30}-T_{20}}{3} = \frac{4 \cdot 0.603 - 0.650}{3} = 0.587$$

$$T_{31} = \frac{4T_{40}-T_{30}}{3} = \frac{4 \cdot 0.575 - 0.603}{3} = 0.565$$

2. Extropolation:

$$T_{02} = \frac{16T_{11}-T_{01}}{15} = \frac{16 \cdot 1.250 - (-1.586)}{15} = 1.439$$

$$T_{12} = \frac{16T_{21}-T_{11}}{15} = \frac{16 \cdot 0.587 - 1.250}{15} = 0.543$$

$$T_{22} = \frac{16T_{31}-T_{21}}{15} = \frac{16 \cdot 0.565 - 0.587}{15} = 0.564$$

3. Extropolation:

$$T_{03} = \frac{64T_{12}-T_{02}}{63} = \frac{64 \cdot 0.543 - 1.439}{63} = 0.528$$

$$T_{13} = \frac{64T_{22}-T_{12}}{63} = \frac{64 \cdot 0.564 - 0.543}{63} = 0.564$$

4. Extropolation:

$$T_{04} = \frac{256T_{13}-T_{03}}{255} = \frac{256 \cdot 0.564 - 0.528}{255} = 0.564$$