EEE 321-Signals and Systems

-Lab 03-

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Section 001

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DTMF Transmitter

Part 1.1

The code for the sound:

```
Number=[0 5 5 5 5 5 5 5 5 5 5];

x = DTMFTRA(Number);
soundsc([x],8192)

function [x]= DTMFTRA(Number)
inf= [[941 1336] %0
[697 1209] %1
[697 1336] %2
[697 1477] %3
[770 1209] %4
[770 1336] %5
[770 1477] %6
[852 1209] %7
[852 1336] %8
[852 1477]]; %9
ss= 8192;
t= linspace(0,length(Number)*0.25,length(Number)*0.25*ss);
x= zeros(1,length(t));
for i = 1:length(Number)
if i == 1
sub_t= linspace(0,0.25,0.25*ss);
digit= int32(Number(i));
x(1:2048)= cos(2*pi*inf(digit+1)*sub_t)+cos(2*pi*inf(digit+1,2)*sub_t);
else
sub_t= linspace((i-1)*0.25,0.25*i,0.25*ss);
digit= int32(Number(i));
x(((i-1)*2048):(2048*i-1))=
cos(2*pi*inf(digit+1)*sub_t)+cos(2*pi*inf(digit+1,2)*sub_t);
end
end
end
```

The sound produced by the code sounds very familiar to the ones that we enter the cell one number of someone on our phones. For the code the display time of each sound is fixed as 0.25 second this resembles to taping the number on the phone then pulling the finger back.
Since the sound consists of the addition of two cosine waves at different frequencies the sound of each digit is unique and sounds different. As an example the signals for number 0 and number 9 can be seen below.

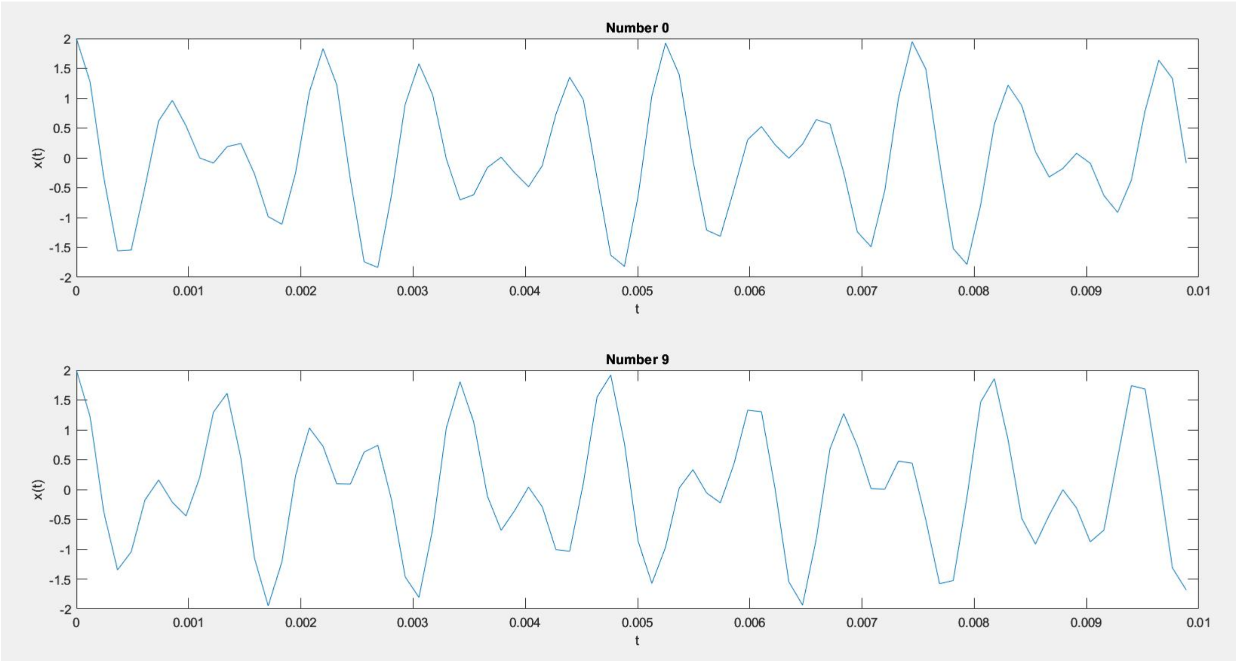


Fig.1 Different number sounds represented as a sum of cosines

Part 1.2 Reciever

Here are the answers to the questions asked in the lab instructions:

a) $x(t) = e^{j2\pi f_0 t}$

the inverse transform of $\delta(\omega-\omega_0) \Rightarrow$ $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega_0 t} d\omega$

$$y(t) = \frac{e^{j\omega_0 t}}{2\pi} \underbrace{\int_{-\infty}^{\infty} \delta(\omega-\omega_0) d\omega}_{1} = \frac{e^{j\omega_0 t}}{2\pi}$$

So we can say that $F^{-1}\{\delta(\omega-\omega_0)\} = \frac{e^{j\omega_0 t}}{2\pi} \xrightarrow{\text{using}}$ duality $\rightarrow$ $\underline{\underline{F\{e^{j\omega_0 t}\} = 2\pi\,\delta(\omega-\omega_0)}}$ $\hookrightarrow 2\pi f_0$

b) $x(t) = \cos(2\pi f_0 t)$

$$\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \Rightarrow F\{\cos(2\pi f_0 t)\} = \frac{1}{2} \underbrace{F\{e^{j2\pi f_0 t}\}}_{2\pi\delta(\omega-\omega_0)} + \frac{1}{2} \underbrace{F\{e^{-j2\pi f_0 t}\}}_{2\pi\delta(\omega-\omega_0)}$$

from part a

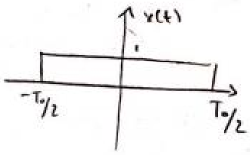
$$\underline{F\{\cos(2\pi f_0 t)\} = \pi\,\delta(\omega-2\pi f_0) + \pi\,\delta(\omega+2\pi f_0)} \quad //$$

c) $x(t) = \sin(2\pi f_0 t)$

$$\sin(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} \Rightarrow F\{\sin(2\pi f_0 t)\} = \frac{1}{2j} \underbrace{F\{e^{j2\pi f_0 t}\}}_{2\pi\delta(\omega-\omega_0)} - \frac{1}{2j} \underbrace{F\{e^{-j2\pi f_0 t}\}}_{2\pi\delta(\omega+\omega_0)}$$

$$\underline{F\{\sin(2\pi f_0 t)\} = \frac{\pi}{j}\delta(\omega-2\pi f_0) - \frac{\pi}{j}\delta(\omega+2\pi f_0)}$$

d) $x(t) = \text{rect}\left(\frac{t}{T_0}\right) \Rightarrow \begin{cases} 1, & -T_0/2 < t < T_0/2 \\ 0, & \text{otherwise} \end{cases} \Rightarrow$



$$X(\omega) = \langle x(t), e^{j\omega t}\rangle = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-T_0/2}^{T_0/2} e^{-j\omega t} dt$$

$$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-T_0/2}^{T_0/2} = \frac{2}{\omega}\left(\frac{e^{j\omega T_0/2} - e^{-j\omega T_0/2}}{2j}\right) = \frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) = T_0 \frac{\sin\left(\frac{\omega T_0}{2}\right)}{\left(\frac{\omega T_0}{2}\right)}$$

$$= \underline{\underline{T_0 \,\text{sinc}\left(\frac{\omega T_0}{2}\right)}}$$

e) $x(t) = \underbrace{e^{j2\pi f_0 t}}_{e^{j\omega_0 t}} \cdot rect\left(\frac{t}{T_0}\right) \Rightarrow x(t) = x_1(t) \cdot x_2(t)$ using convolution property $X(\omega) = \frac{X_1(\omega) * X_2(\omega)}{2\pi}$

$X_1(\omega) = F\{e^{j\omega_0 t}\} = 2\pi\,\delta(\omega - \omega_0)$

$X_2(\omega) = F\{rect(\frac{t}{T_0})\} = T_0 \, sinc\left(\frac{\omega T_0}{2}\right)$

$$X(\omega) = \frac{2\pi\,\delta(\omega-\omega_0) * T_0\, sinc\left(\frac{\omega T_0}{2}\right)}{2\pi}$$

$\hookrightarrow$ convolution with a shifted impulse shifts the frequency

$$X(\omega) = \frac{T_0}{2\pi} sinc\left(\frac{(\omega-\omega_0)T_0}{2}\right)$$

f) $x(t) = \cos(\overbrace{2\pi f_0}^{\omega_0} t)\, rec\left(\frac{t}{T_0}\right)$ using the same method above:

$X_1(\omega) = F\{\cos(\omega_0 t)\} = \pi\,\delta(\omega - \omega_0) + \pi\,\delta(\omega + \omega_0)$

$X_2(\omega) = F\{rect(\frac{t}{T_0})\} = T_0\, sinc\left(\frac{\omega T_0}{2}\right)$

$$X(\omega) = \frac{\left[\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)\right] * T_0 sinc\left(\frac{\omega T_0}{2}\right)}{2\pi}$$

$$= \frac{1}{2}\left[T_0\, sinc\left(\frac{(\omega-\omega_0)T_0}{2}\right) + T_0\, sinc\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right]$$

we have 2 impulses with frequencies shifted to right & left hence, we will have two signals shifted to right & left for each impulse

g) $x(t) = rect\left(\frac{t-t_0}{T_0}\right)$

$* \quad x(t) \to x(t-t_0)$

$X(\omega) \to e^{-j\omega t_0} X(\omega)$ Hence

$X(\omega) = F\{rect\left(\frac{t}{T_0}\right)\} = T_0\, sinc\left(\frac{\omega T_0}{2}\right)$

$e^{-j\omega t_0} X(\omega) = e^{-j\omega t_0}\, T_0\, sinc\left(\frac{\omega T_0}{2}\right)$

h) $x(t) = \underbrace{e^{j2\pi f_0 t}}_{e^{j2\omega_0 t}}\, rect\left(\frac{t-t_0}{T_0}\right) \Rightarrow$ Both shifting frequency content and time content.

shifting frequencies

$X(\omega) = T_0 sinc\left(\frac{(\omega-\omega_0)T_0}{2}\right) \cdot \underbrace{e^{-j(\omega-\omega_0)t_0}}_{\text{shifting in time}}$

c) $x(t) = \underbrace{\cos(2\pi f_0 t)}_{\cos(w_0 t)} \, rect\left(\frac{t-t_0}{T_0}\right)$

$$X(\omega) = \frac{\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)] * e^{-j\omega t_0} T_0 \, sinc\left(\frac{\omega T_0}{2}\right)}{2\pi}$$

Again using shifting feature of the impulse within a convolution,

$$= \frac{1}{2}\left[e^{-j(\omega-\omega_0)t} T_0 \, sinc\left(\frac{(\omega-\omega_0)T_0}{2}\right) + e^{-j(\omega+\omega_0)t_0} T_0 \, sinc\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right)$$

$$= \frac{e^{-j\omega t_0}}{2} T_0 \left[sinc\left(\frac{(\omega-\omega_0)T_0}{2}\right) e^{j\omega_0 t_0} + sinc\left(\frac{(\omega+\omega_0)T_0}{2}\right) e^{-j\omega_0 t_0}\right.$$

//

My number for this part: [7 6 1 2 1]

The frequency information of the signal plotted as follows.

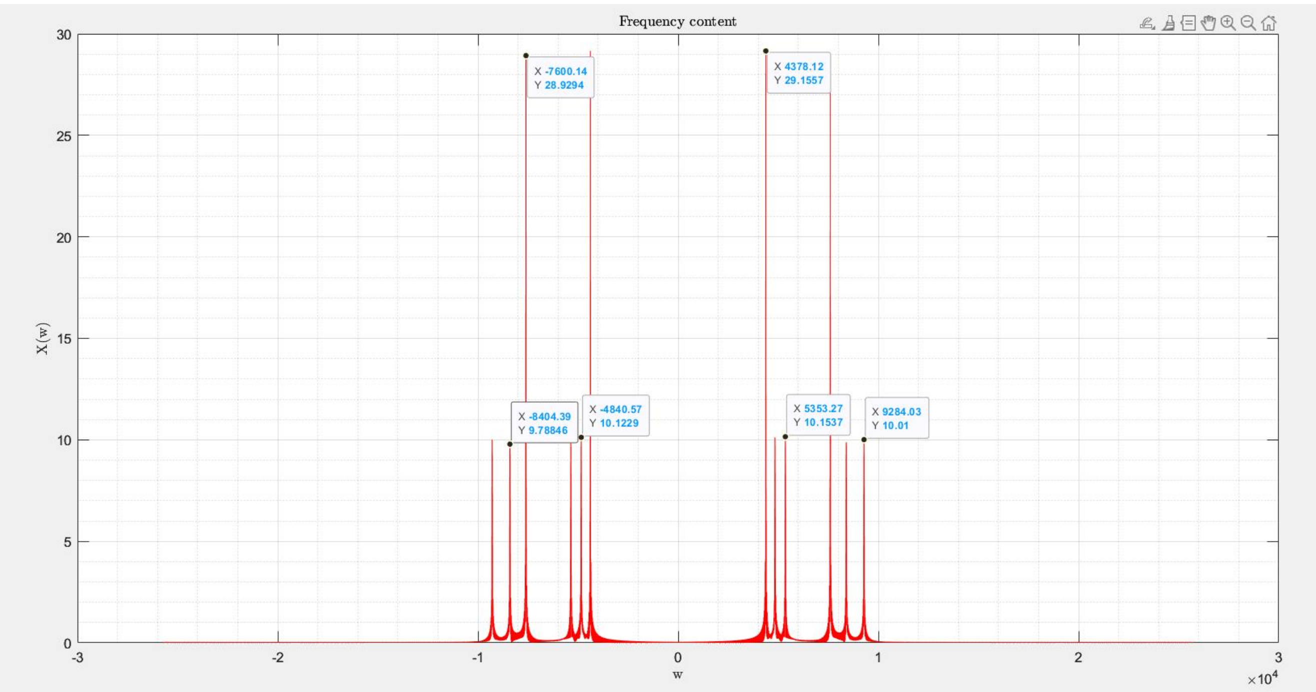


Fig.2 Magnitude of the fourier transform of x

From the plot above we can know the magnitude corresponding each angular frequency. The peaks show that the contribution of those frequencies to the fourier transform of x is higher than the other ones. They are the “dominant ones”. Hence by investigating them we can get an idea about the different cosines that produced the signal. Using the relation (1) we can calculate the frequency values .

$$f = \frac{w}{2\pi} \qquad (1)$$

W (rad/sec) at peaks	9284.03	8404.39	7600.14	5353.27	4840.57	4378.12
f (Hz)	**1477.59**	**1337.60**	**1209.59**	**851.99**	**770.40**	**696.79**

Table 1. Frequency values at peaks

	1209 Hz	1336 Hz	1477 Hz	1633 Hz
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

Table 2. DTMF frequencies

Comparing these values to the values written in the table of DTMF frequencies, it can said that the peak values are very close to the DTMF frequencies that codes different numbers. However only knowing the higher magnitude frequencies does not give us enough information for determining the numbers expressed by the signal. For instance, the highest magnitude occurs at the frequency value 696Hz. This value is used at the same time for digits 1,2 and 3. Just by knowing 696Hz we cannot tell whether it is used for 1,2 or 3 and we also cannot say how many times the digits are used for obtaining the main number.

The following code investigates the frequency content as parts of 0.25 seconds iterating on the signal using a square wave.

```
Number=[7 6 1 2 1];
x = DTMFTRA(Number);
ss= 8192;
t= linspace(0,length(Number)*0.25,length(Number)*0.25*ss);
sq= zeros(1,length(t));
for i = 1:2048
sq(i)=1;
end

x1=sq.*x;
X1=FT(x1);
omega=linspace(-8192*pi,8192*pi,10241);
omega=omega(1:10240);
figure
```

```
plot (omega,abs(X1),'r');
ylabel ({'X1(w)'},'Interpreter','latex');
xlabel ({'w'},'Interpreter','latex');
box on
grid, grid minor
title({'Frequency content'},'Interpreter','latex')
```

Here are the results for each division:

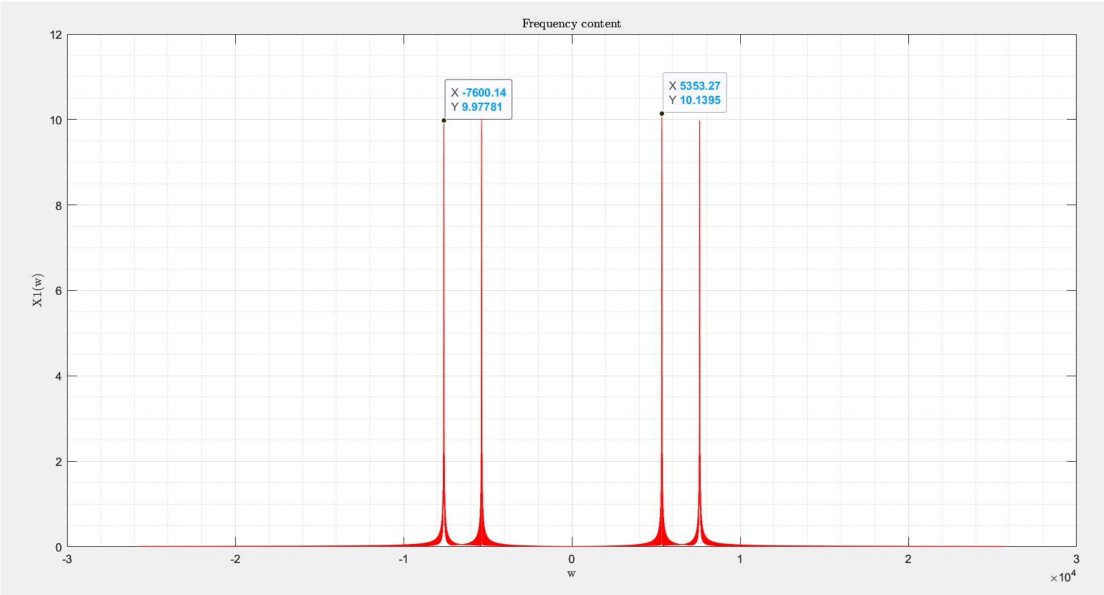


Fig.3 Frequency content for 0<t<0.25

W (rad/sec) at peaks	5353.27	7600.14
f (Hz)	**851.99**	**1209.59**

*852 Hz + 1209 Hz : **7**

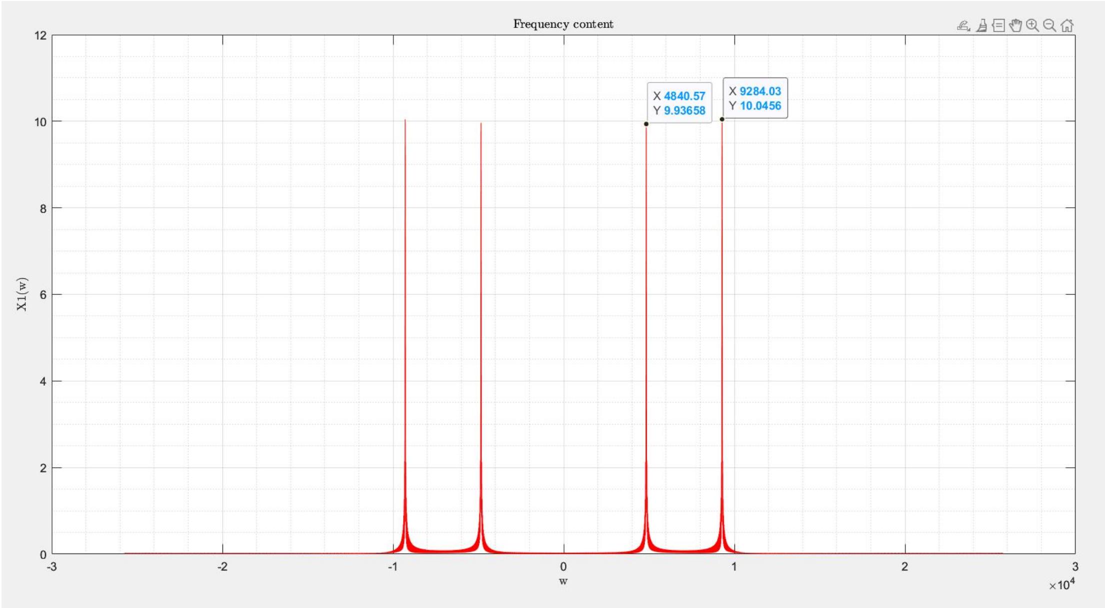


Fig.4 Frequency content for 0.25<t<0.50

W (rad/sec) at peaks	4840.57	9284.03
f (Hz)	**770.40**	**1477.59**

*770 Hz + 1477 Hz : **6**

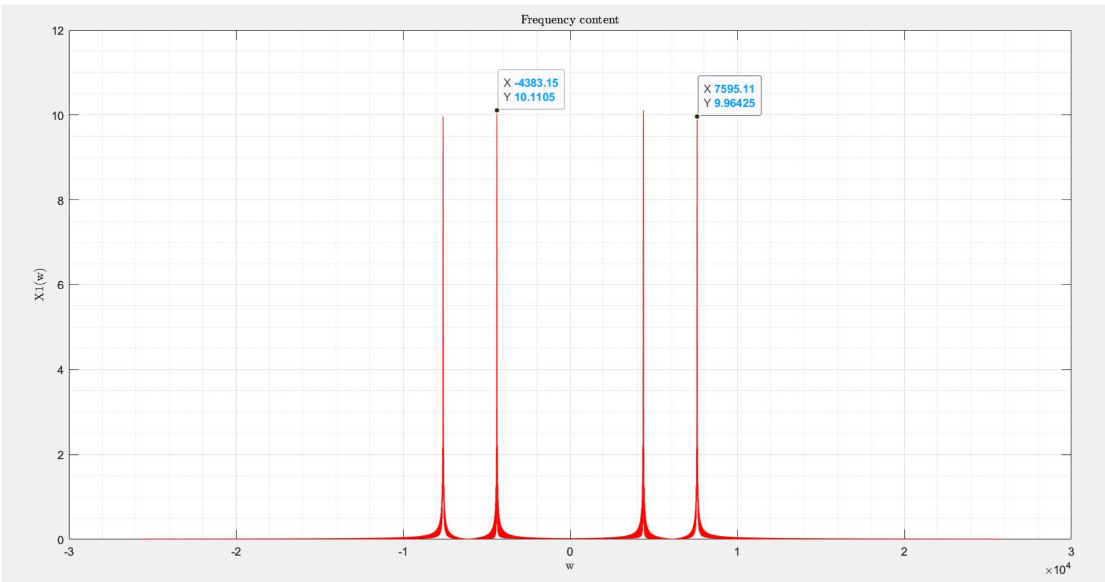


Fig.5 Frequency content for 0.50<t<0.75

W (rad/sec) at peaks	4383.15	7595.11
f (Hz)	**697.59**	**1208.79**

*697Hz + 1209 Hz : **1**

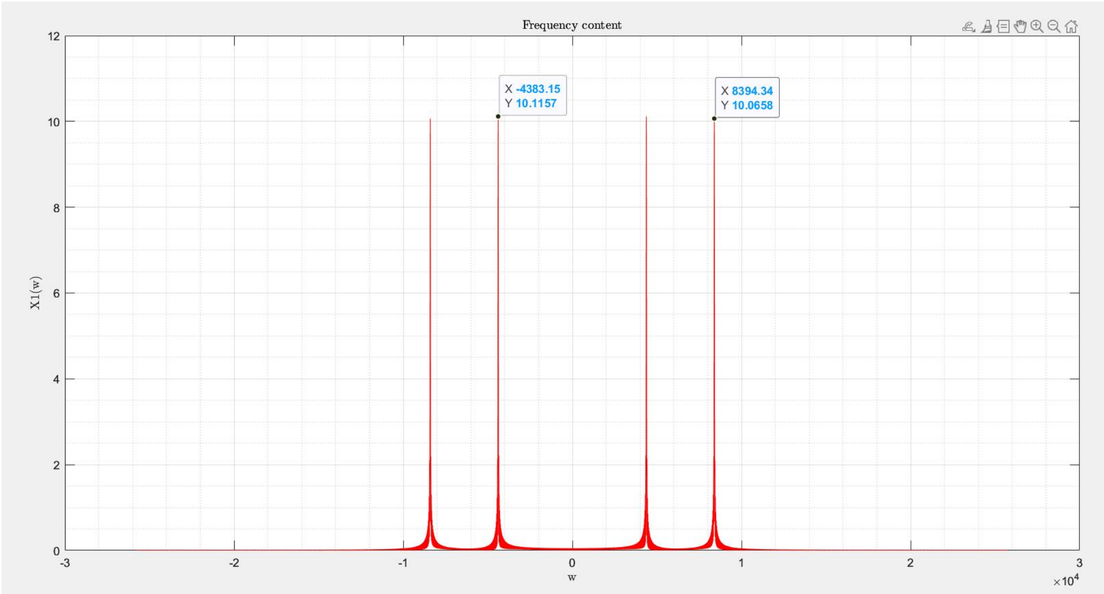


Fig.6 Frequency content for 0.75<t<1

W (rad/sec) at peaks	4383.15	8394.34
f (Hz)	**697.59**	**1336.00**

*697Hz + 1336 Hz : **2**

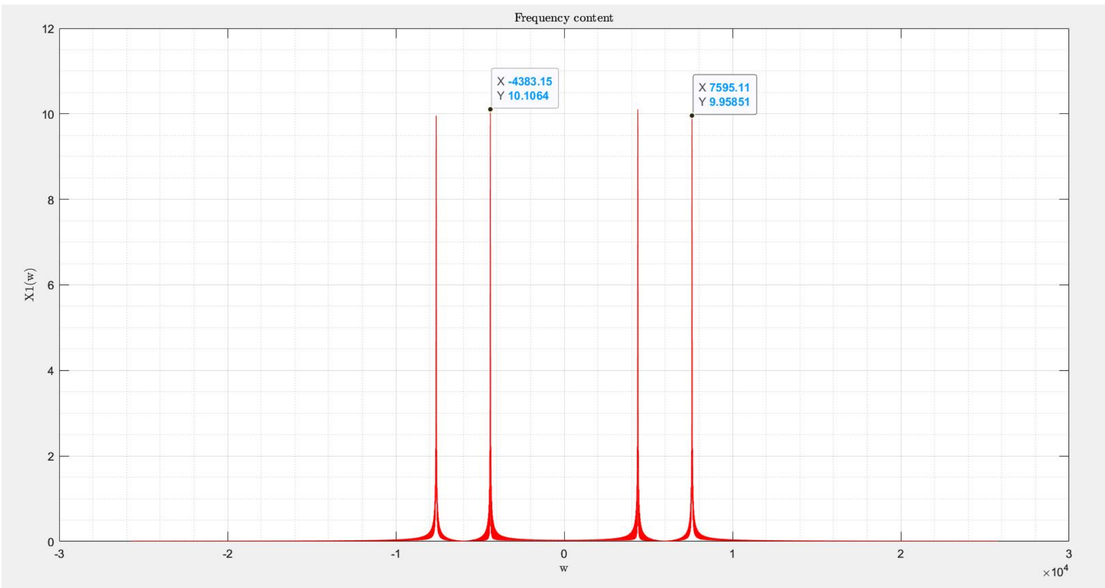


Fig.5 Frequency content for 1<t<1.25

W (rad/sec) at peaks	4383.15	7595.11
f (Hz)	**697.59**	**1208.79**

*697Hz + 1209 Hz : **1**

As indicated in the tables below the figures; each figure shows the frequency content of a single number. When we do a general Fourier analysis over the whole time interval, as explained in the previous section numbers with the same frequency values concatenate and one single frequency peak appears for all those numbers. However with this method, by partitioning the signal using square waves, we make sure that for each interval only one number is being played therefore it is for sure that the frequency peaks in the figures show the frequency content of a single number displayed only one time. If we add up all these separated plots, we get the general frequency response of x shown in Fig.2.

Part 2

Code for voice recording:

```
% Define the sampling rate (8192 samples per second)
fs = 8192;

% Define the duration of the recording (10 seconds)
recording_duration = 10;

recorder = audiorecorder(fs, 16, 1);

% Record audio
disp('Start speaking.');
recordblocking(recorder, recording_duration);
disp('Recording finished.');

% Get the recorded audio data
voice = getaudiodata(recorder);desired_length = 98304;
if length(voice) < desired_length
voice(end+1:desired_length) = 0; % Zero-padding
else
voice = voice(1:desired_length); % Crop if longer
end

% Crop the array to the desired length (98304 samples)
voice = voice(1:98304);
```

I said :

"If you notice this notice, you will

notice that this notice

Is not worth noticing."

Here are the answers to the asked questions:

a) impulse

$$x(t) \longrightarrow \boxed{h(t)}_{\,LTI} \longrightarrow y(t) = x(t) * h(t)$$

we know that $y(t) = x(t) + \sum_{i=1}^{m} A_i x(t-t_i)$

Using the convolution property

$$y(t) = x(t) * h(t) \xrightarrow{FT} Y(\omega) = X(\omega) H(\omega)$$

$$Y(j\omega) = X(j\omega) + \sum_{i=1}^{m} A_i X(j\omega) e^{-j\omega t_i} = X(\omega) H(\omega)$$

$F\{\delta(t)\} = 1$

frequency response of $\delta(t)$

$$H(\omega) = \frac{\cancel{X(j\omega)} + \sum_{i=1}^{m} A_i \cancel{X(j\omega)} e^{-j\omega t_i}}{\cancel{X(j\omega)}} = 1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i}$$

b) $$\boxed{H(\omega) = 1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i} \Longrightarrow h(t) = \delta(t) + \sum_{i=1}^{m} A_i \delta(t-t_i)}$$

c) $Y(\omega) = X(\omega) H(\omega)$ since $y(t) = h(t) * x(t)$ this results in multiplication in frequency domain

d) using this property we can find $X(\omega) = \dfrac{Y(\omega)}{H(\omega)}$ //

After recording, the speech signal looks like this:

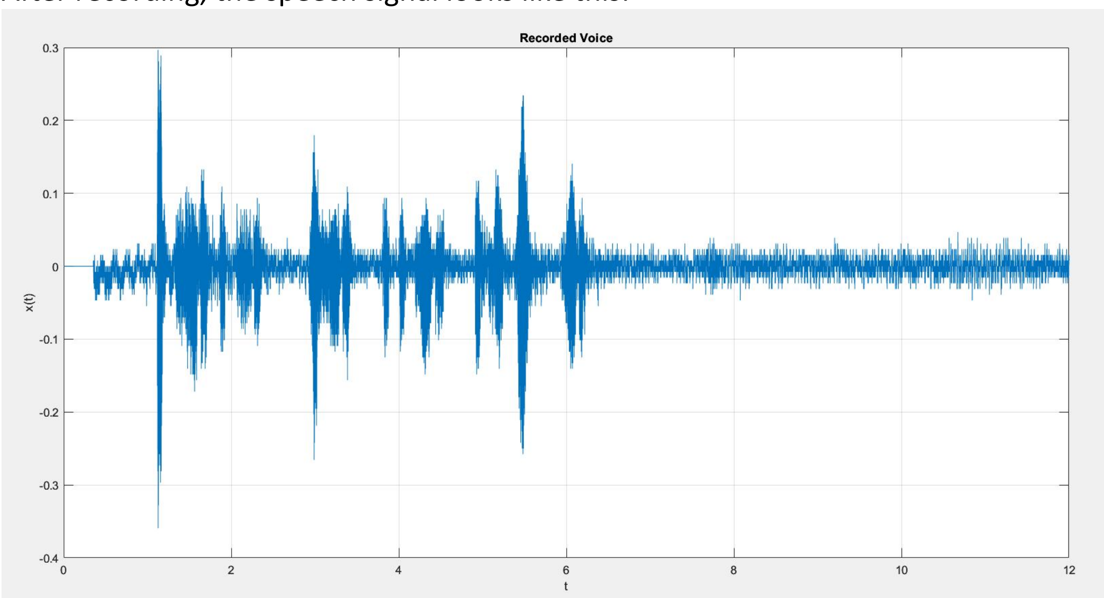


Fig.6. Speech signal x(t)

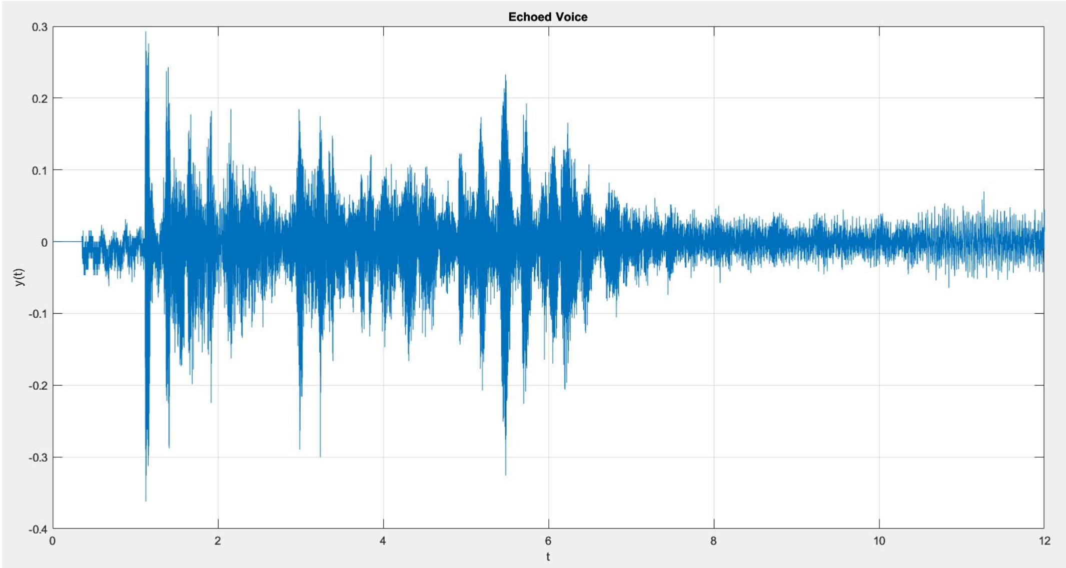


Fig.7. Echoed speech signal y(t)

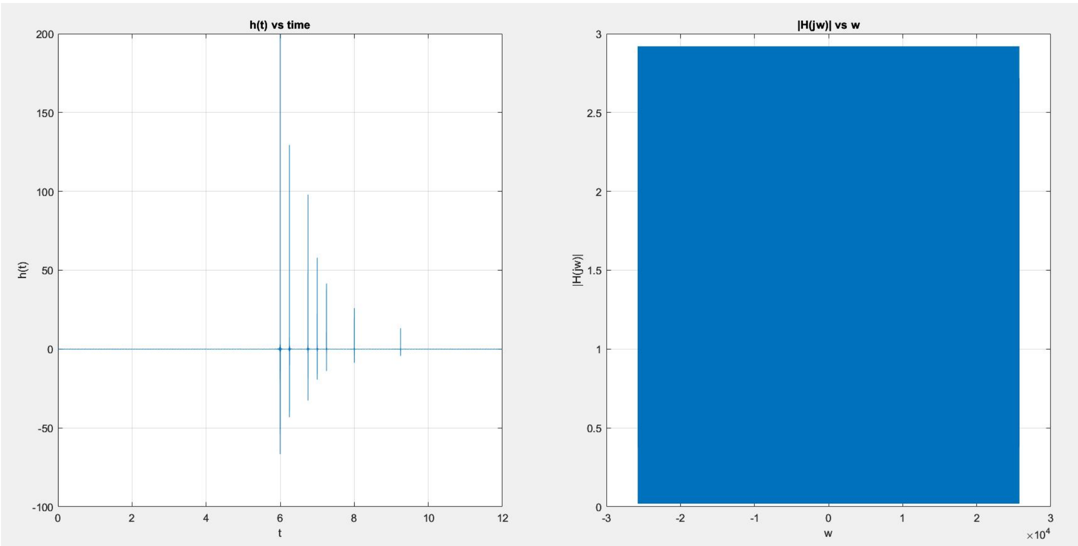


Fig.8. Impulse response and transfer function

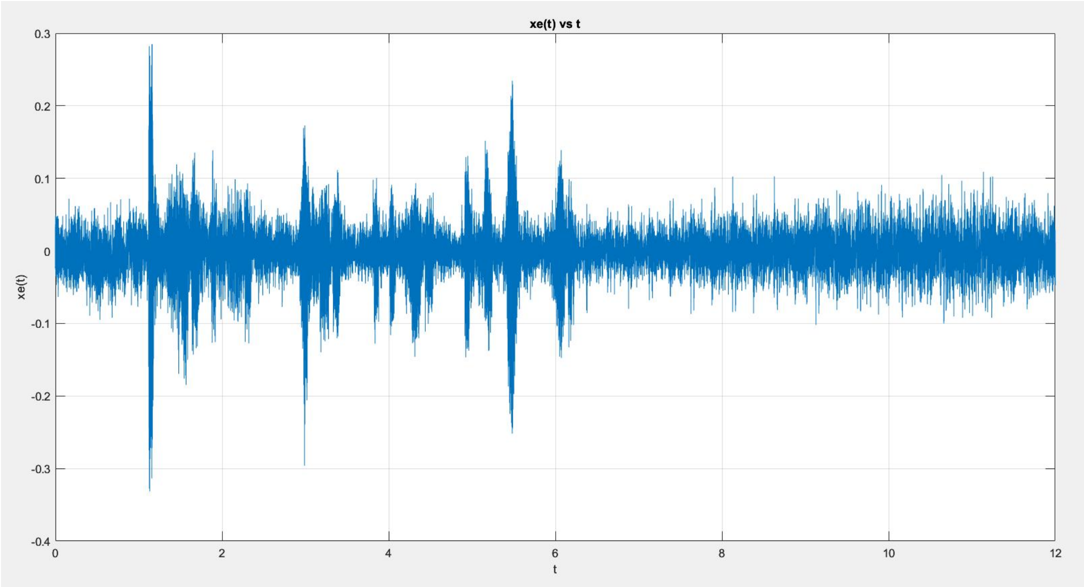


Fig.9. Estimated voice signal

For the echoing process the output signal can be expressed as a convolution sum:

$$y(t) = x(t) * h(t)$$

Where h(t) is the unit impulse response. Using the convolution property of FT we know that

$$Y(w) = X(w)H(w)$$

We can calculate Y(w) and H(w) and therefore we can find xe(t) by doing IFT . However the calculated xe(t) is not exactly same with the initial x(t) signal it contains more noise. Although the two signal have differences when they are compared to the echoed one (y(t)), it is obvious that the echoes are mostly eliminated in xe(t).