

# TLA+

**An introduction to writing “exhaustively testable pseudocode”  
[newcombe2015aws]**

**Seminar Advanced Programming Tools WS 22/23  
By Daniel Stachnik, 31.01.2023**

**“For the human makers of things, the incompletenesses and inconsistencies of our ideas, become clear only during implementation.”**

**- *Fred Brooks, The Mythical Man-Month, p. 15 [brooks1982month]***

# How to verify behavior & properties?

	Design*	Implementation
Building a bridge	CAD	Manual Checks
Building software	?	Testing (automated, manual, ...)

*\* designing = conceptualization and framing of a system  
[freeman2004design]*

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# How to verify behavior & properties?

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Building a bridge	CAD	Manual Checks
Building software	TLA+ \in Formal Methods	Testing (automated, manual, ...)

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**“Going deeper, most software people are just trying to do FAB, and most of the tools are FAB tools — there is very little CAD and even less SIM in “software engineering”. To my old eyes, this doesn’t look or feel like real engineering process.”**

Alan Kay, <https://www.quora.com/How-does-one-get-as-close-as-possible-to-mastering-software-engineering/answer/Alan-Kay-11?share=1>

**“For the human makers of things, the incompletenesses and inconsistencies of our ideas, become clear only during implementation.”**

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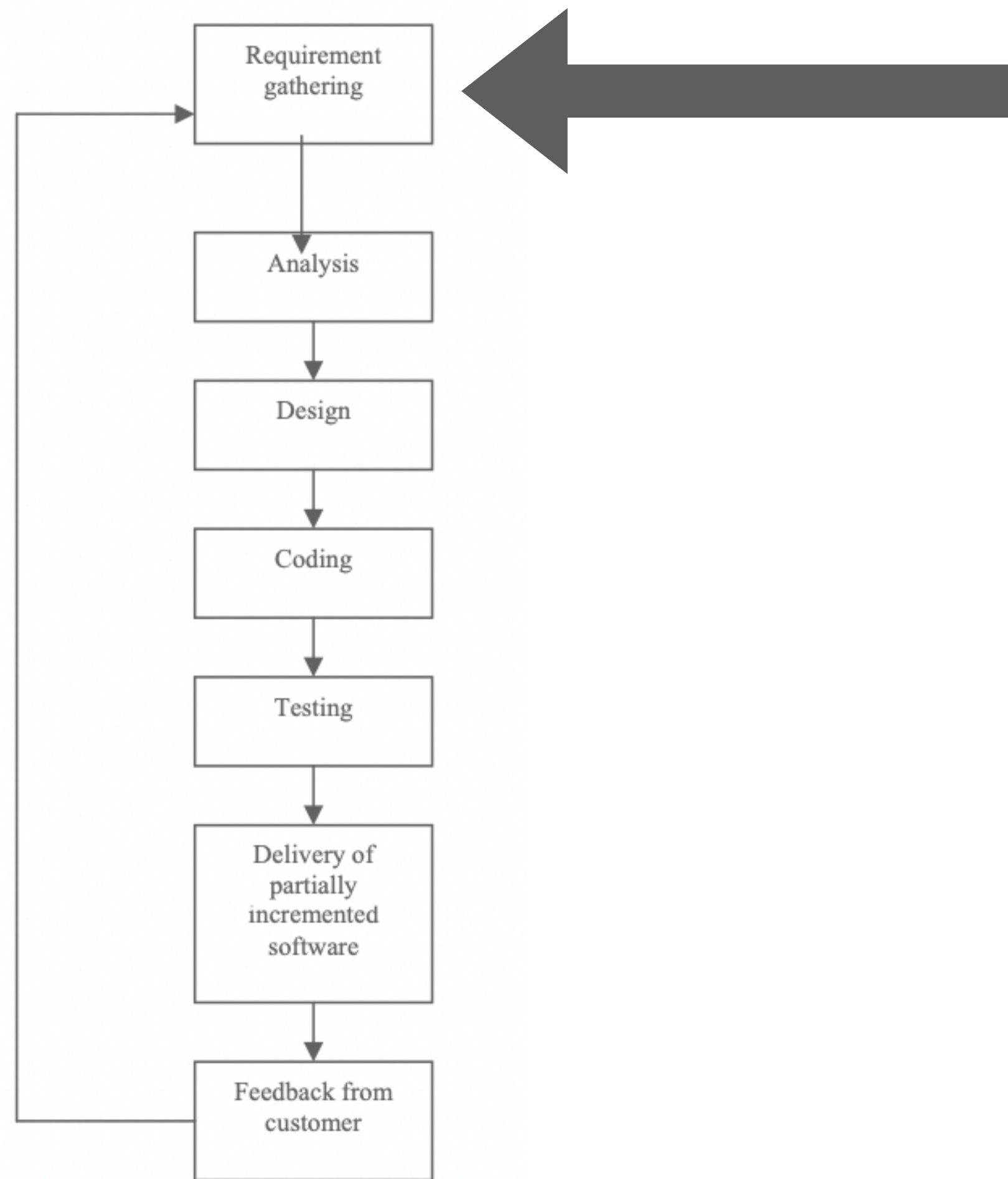
**➡ TLA+ and other lightweight formal methods try to lower “incompletenesses” and “inconsistencies” by formalizing the design part**

# Overview TLA+

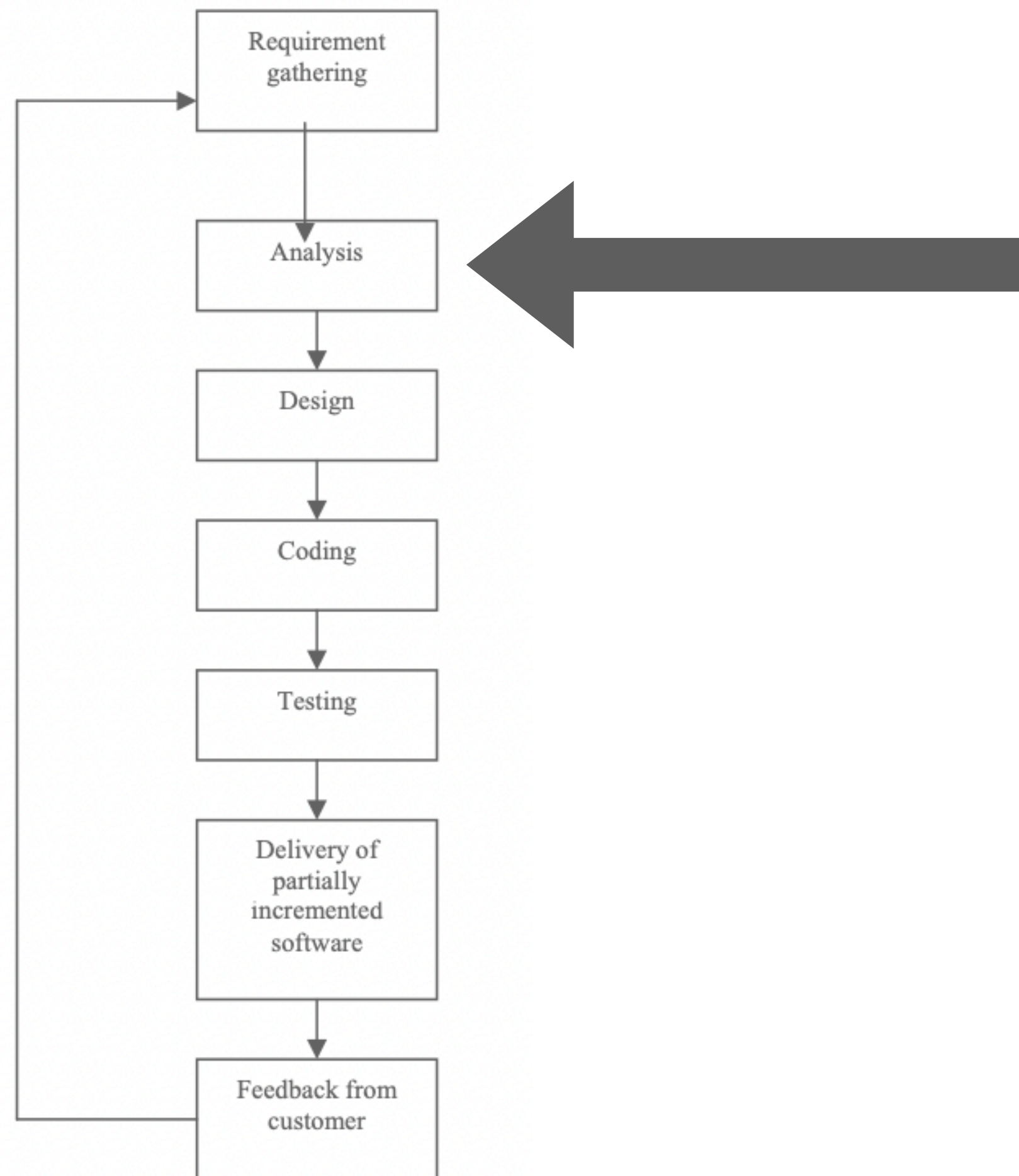


# TLA+ Workflow

*C: “I want my files to synchronize between all my devices”*



# TLA+ Workflow



- 1. SW supports multiple client devices connected over internet*
- 2. Changes at one device will be propagated to other nodes*

# TLA+ Workflow

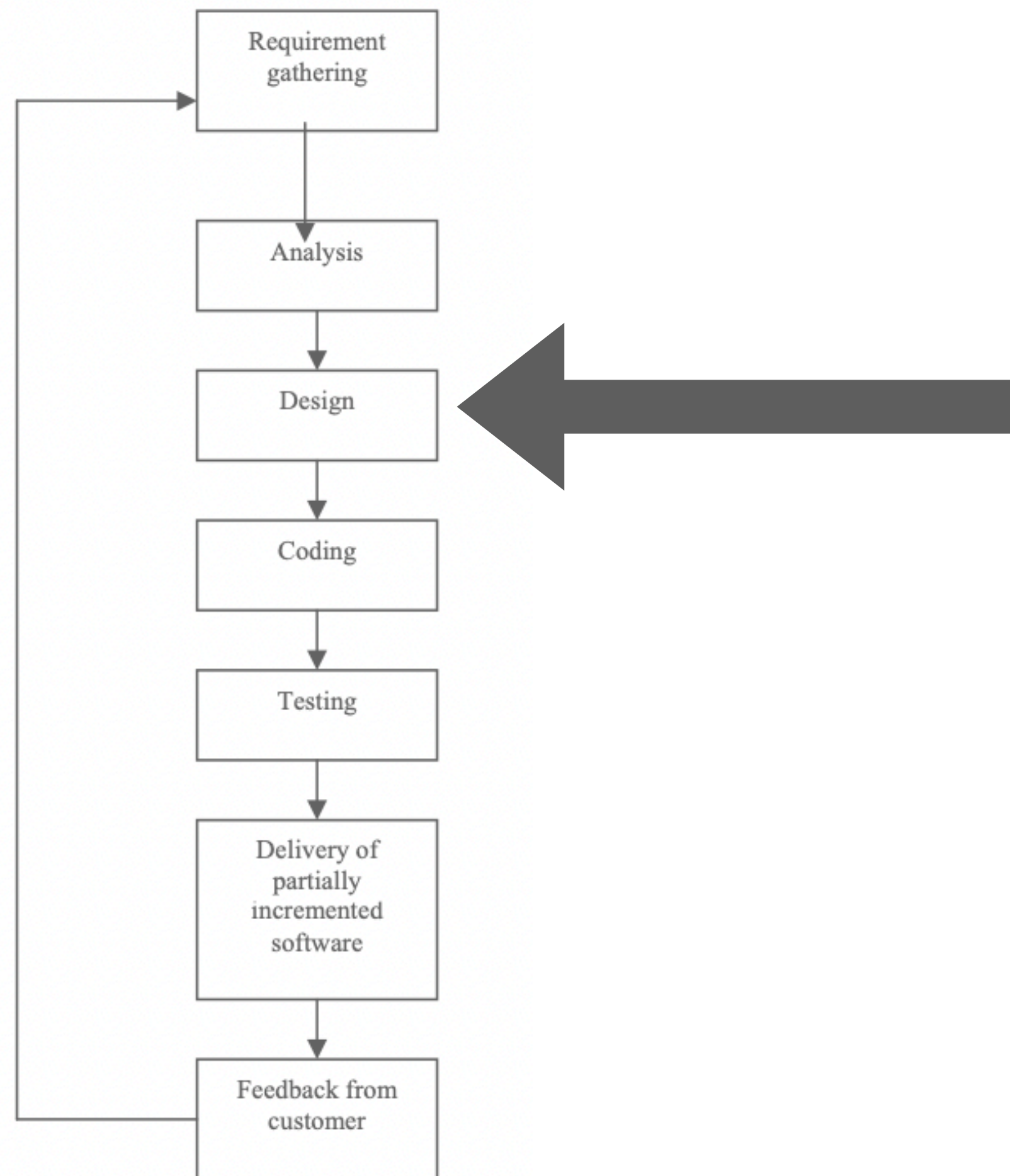
## ***Sketch idea***

*if madeChanges:*

*Commit all changes to other active nodes*

*If all confirm: commit on own device*

*If any didn't confirm: retry after  
exponentially increasing time-out*





# TLA+ Workflow

## *Sketch idea*

*if madeChanges:*

*Commit all changes to other active nodes*

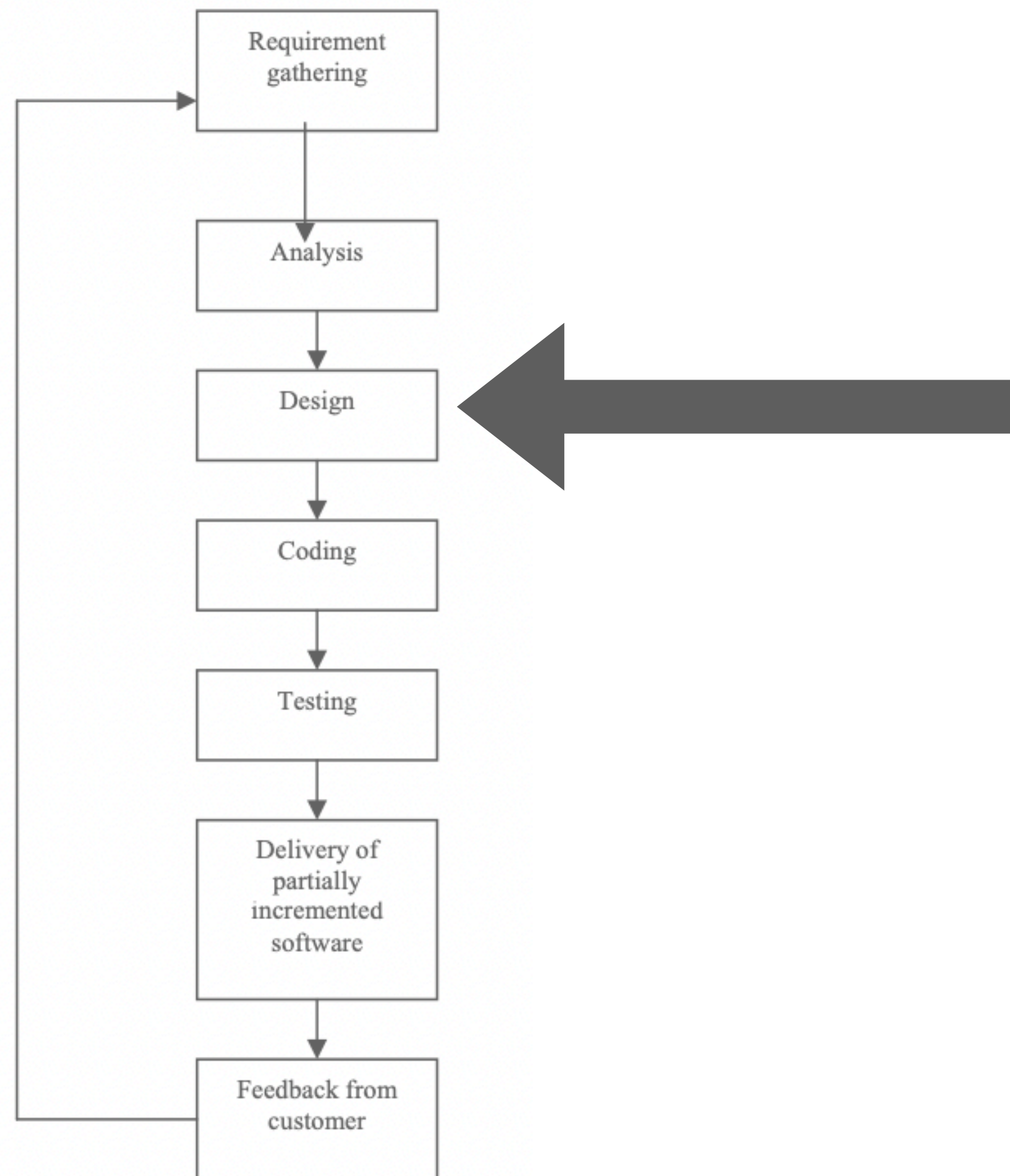
*If all confirm: commit on own device*

*If any didn't confirm: retry after  
exponentially increasing time-out*

**Write TLA+-spec (~20 loc)**

**Model-check spec**

*→ Fails because of deadlock: two nodes  
can commit at same time and wait forever*

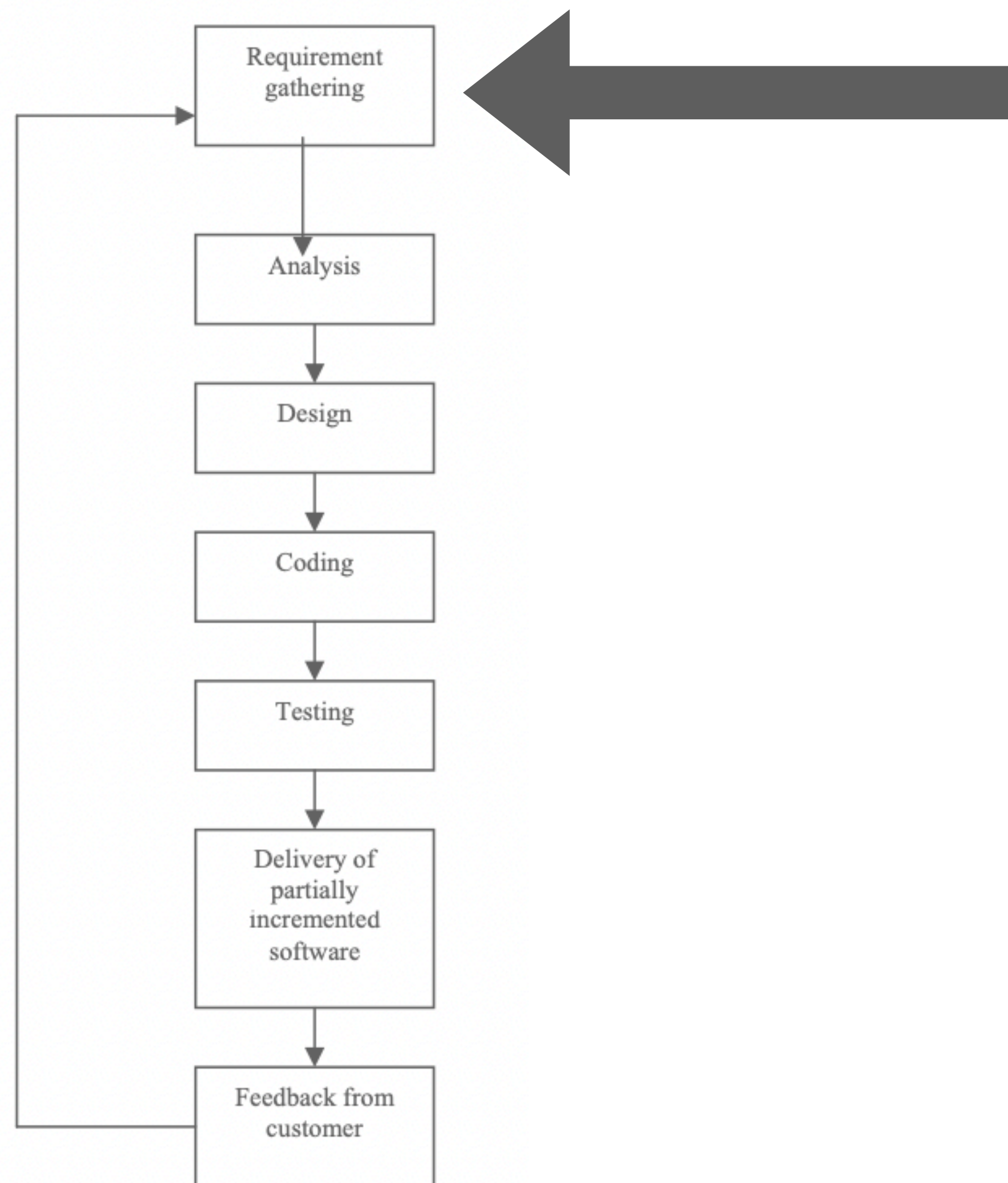


# TLA+ Workflow

*C: “I want my files to synchronize between all my devices”*

*You: “Support file changes on multiple devices at same time?”*

*C: “Yes”*



# TLA+ Workflow

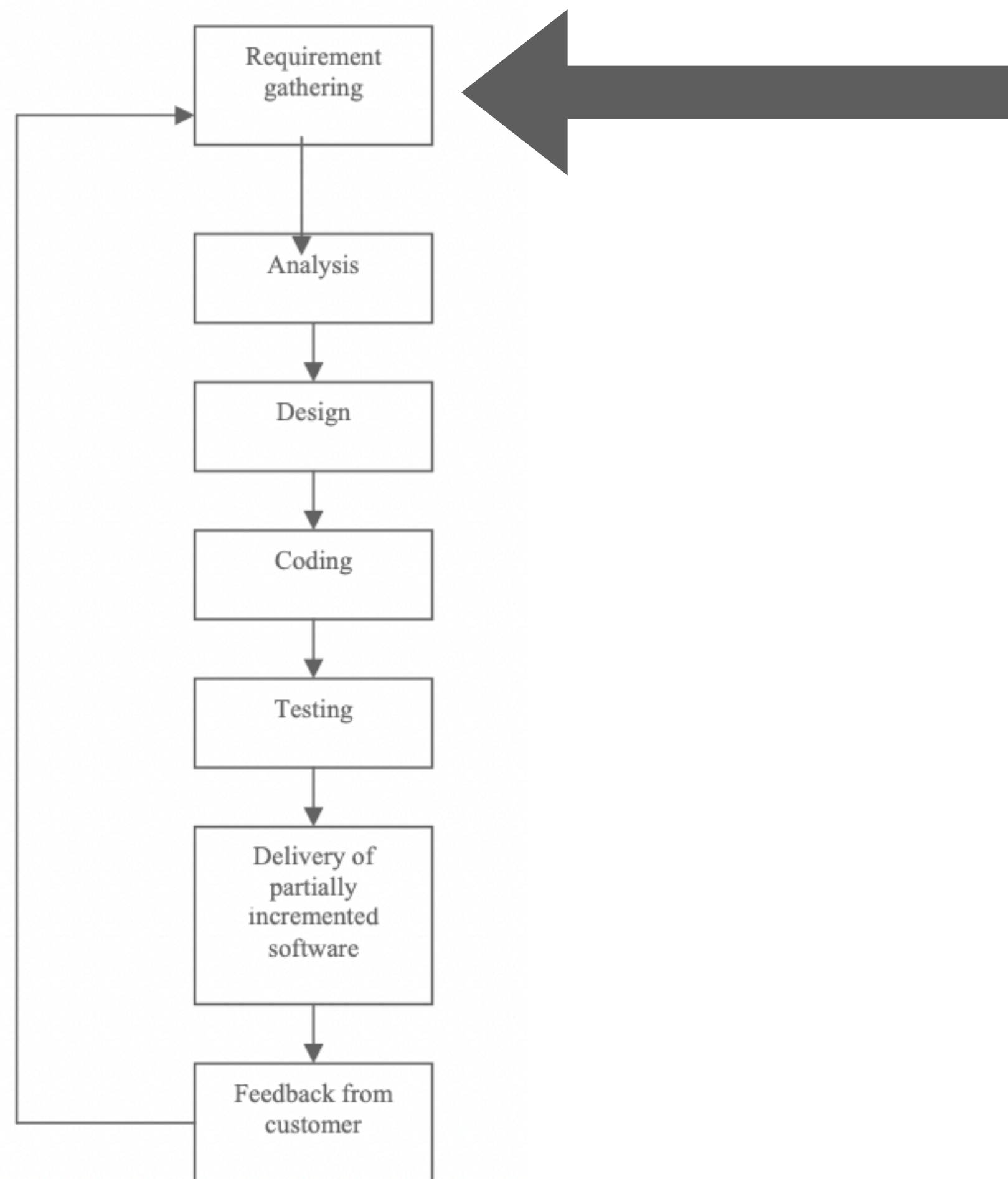
*C: “I want my files to synchronize between all my devices”*

*You: “Support file changes on multiple devices at same time?”*

*C: “Yes”*

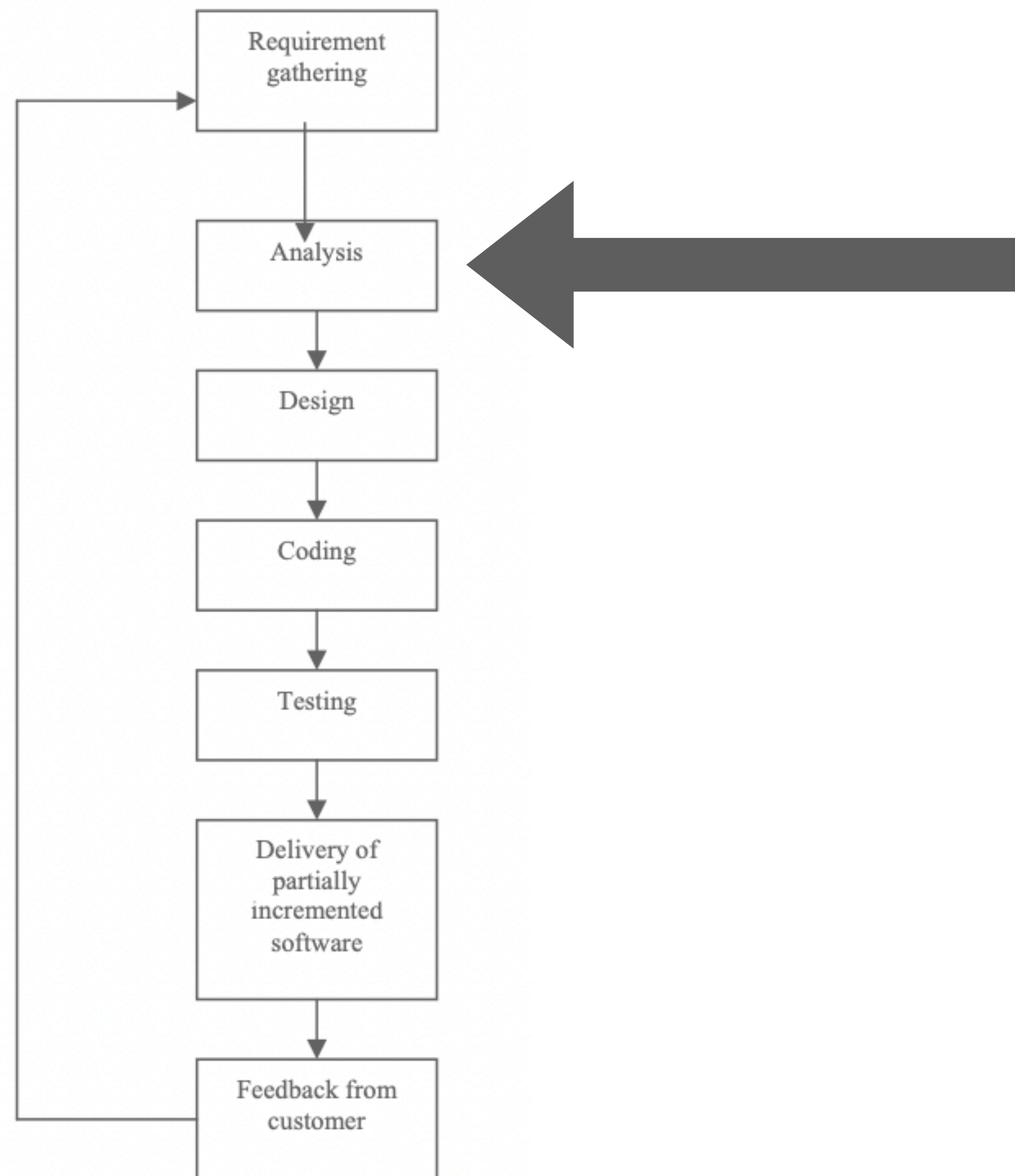
*You: “That will cost extra”*

*C: “No worries, I know software design is hard”*



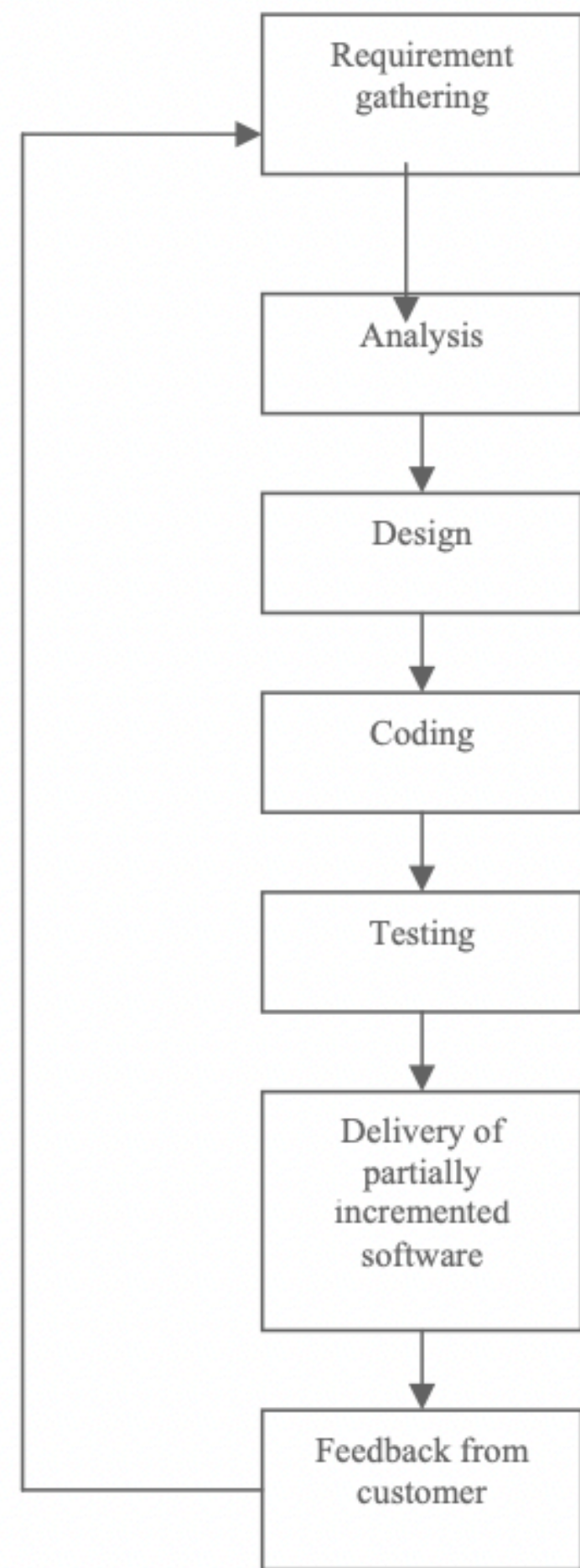


# TLA+ Workflow



1. *SW supports multiple client devices connected over internet*
2. *Changes at one device will be propagated to other nodes*
3. *Multiple changes may be done at the same time*

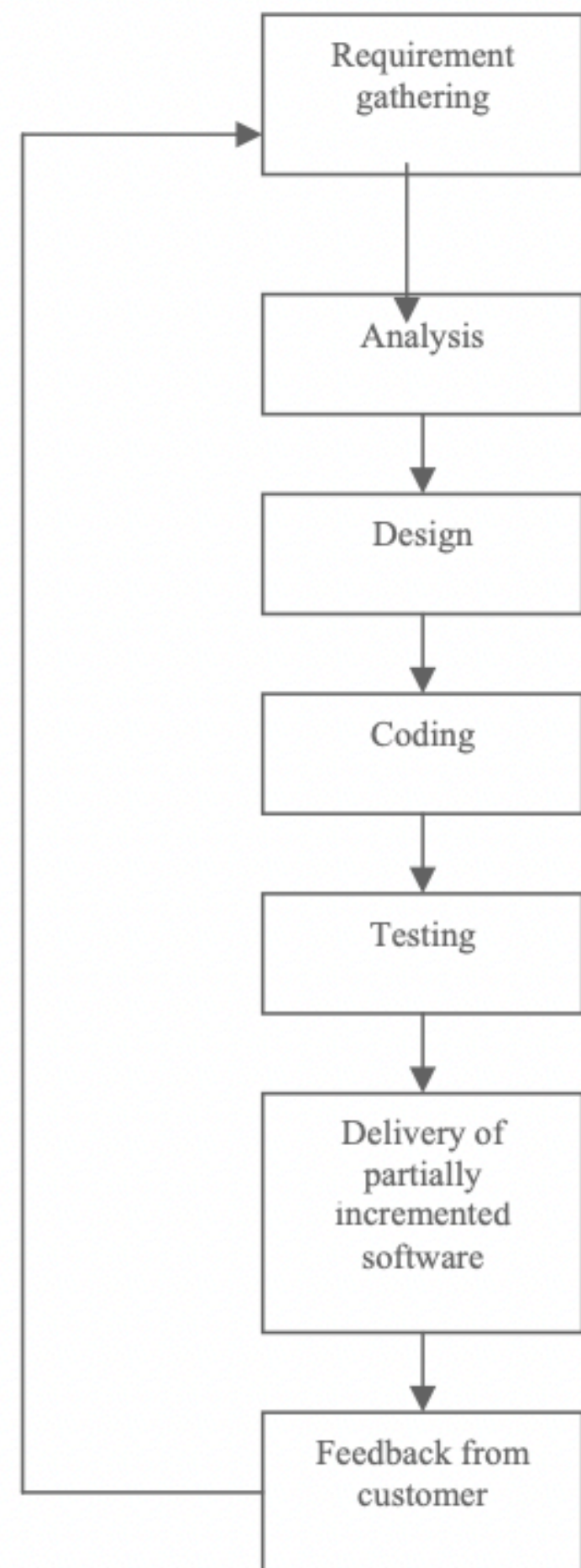
# TLA+ Workflow



***Sketch → Write Spec → Model-check → ...***



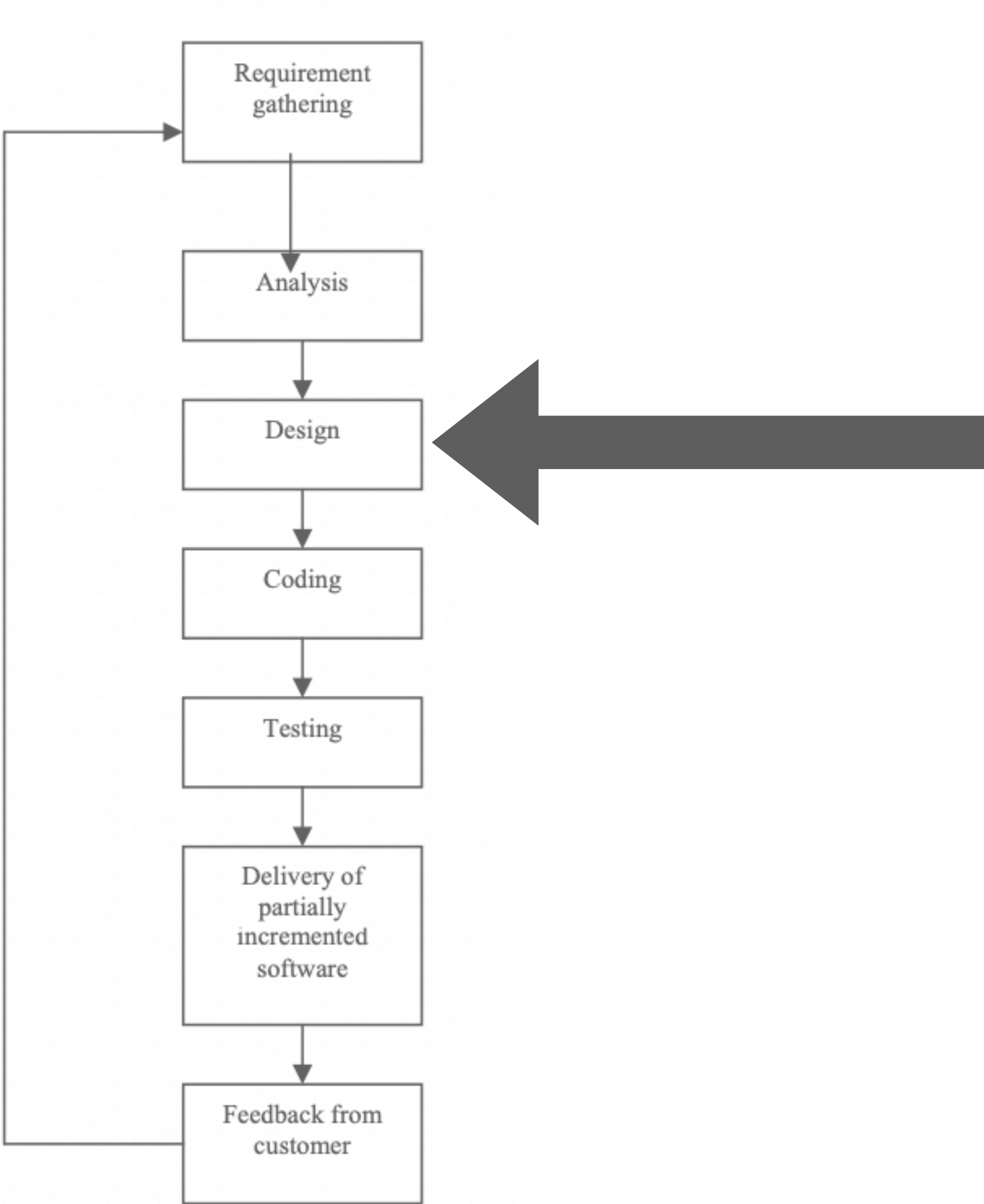
# TLA+ Workflow



Agile Dev Cycle  
[sharma2012agile]

**How much time would you have needed, hadn't you used TLA+?**

# TLA+ Workflow



Agile Dev Cycle  
[sharma2012agile]

## 1. Write specification

```
1 ----- MODULE Untitled -----
2 EXTENDS Integers, Sequences, TLC
3
4 CONSTANT seq
5
6 (*--algorithm duplicateChecker {
7   variables seen = {};
8   i = 1;
9   isUnique = TRUE;
10
11   {
12     while (i <= Len(seq)) {
13       if (seq[i] \in seen) {
14         isUnique := FALSE;
15       };
16       seen := {seq[i]} \cup seen;
17       i := i + 1;
18     };
19   }
20 }*)
```

Model\_1 x TLC Errors

Model Overview Model Checking Results x

Model Checking Results

General

Start: 20:27:39 (Jan 7) End: 20:27:40 (Jan 7) Not running

1 Error

Statistics

State space progress (click column header for graph) Sub-actions of next-state (at 00:00:01)

Time	Diamete	States Foun	Distinct States	Queue Size	Module	Action	Location	States Foun	Distinct Sta
00:00:01	5	6	5	0	Untitled	Terminating	line 45, col 1 to lin...	1	0
00:00:01	0	1	1	1	Untitled	Init	line 26, col 1 to lin...	2	2
					Untitled	Lbl_1	line 32, col 1 to lin...	9	4

## 2. Check model



# The TLA+ Tools

- TLA+ = Temporal Logic of Actions Plus
- TLA+ is a specification language (not a programming language)
- Developed by Leslie Lamport to model/specify concurrent systems
- Relevant tools that work on TLA specs:
  - TLC Model Checker
  - TLAPS Proof System

Sources: [lamport2002systems], [learntla]

```

95
96 ⊢ THEOREM Spec ⇒ []Correct
97 ⊢ <1>1. Init ⇒ Inv
98 ⊢ <2> SUFFICES ASSUME Init
99 ⊢ PROVE Inv
100 ⊢ OBVIOUS
101 ⊢ <2> USE DEF Init
102 ⊢ <2>1. TypeOK
103 ⊢ BY DEF TypeOK,max
104 ⊢ <2>2. ∀E prefix \in Seq({c}) : output = prefix \o s
105 ⊢ <3>1. output = << >> \o s
106 ⊢ OBVIOUS
107 ⊢ <3>2. << >> \in Seq({c})
108 ⊢ BY DEF Seq
109 ⊢ <3>3. QED BY <3>1, <3>2
110 ⊢ <2>3. QED
111 ⊢ BY <2>1, <2>2 DEF Inv,Correct
112 ⊢ <1>2. Inv ∧ [Next]_vars ⇒ Inv'
113 ⊢ <2> SUFFICES ASSUME Inv,
114 ⊢ [Next]_vars
115 ⊢ PROVE Inv'
116 ⊢ OBVIOUS
117 ⊢ <2> USE DEF Inv
118 ⊢ <2>1. CASE a
119 ⊢ <3> USE DEF a,TypeOK
120 ⊢ <3>1. TypeOK'
121 ⊢ BY <2>1
122 ⊢ <3>2. (∀E prefix \in Seq({c}) : output = prefix \o s)'
123 ⊢ <4>1. IF i<pad THEN output' = <<c>> \o output ELSE UNCHANGED output
124 ⊢ BY <2>1
125 ⊢ <4>2. CASE i<pad
126 ⊢ <5>1. ∀E prefix \in Seq({c}) : <<c>> \o output = <<c>> \o prefix \o s
127 ⊢ OBVIOUS
128 ⊢ <5>2. ∀A p \in Seq({c}) : <<c>> \o p \in Seq({c})
129 ⊢ OBVIOUS
130 ⊢ <5>3. QED
131 ⊢ BY <2>1,<4>1,<4>2,<5>1,<5>2
132 ⊢ <4>3. CASE ~(i<pad)
133 ⊢ BY <2>1,<4>1,<4>3
134 ⊢ <4>4. QED BY <4>1,<4>2,<4>3
135 ⊢ <3>3. (Len(output) = Len(s) ∨ Len(output) ≤ n)'
136 ⊢ BY <2>1 DEF Inv,Next,vars,max
137 ⊢ <3>4. (Len(output) = Len(s) + i)'
138 ⊢ BY <2>1 DEF Inv
139 ⊢ <3>5. (i ≥ 0)'
140 ⊢ BY <2>1
141 ⊢ <3>6. (pad = max(n - Len(s), 0))'
142 ⊢ BY <2>1
143 ⊢ <3>7. Correct'
144 ⊢ BY <2>1,<3>2 DEF max,Inv,Correct
145 ⊢ <3>8. QED
146 ⊢ BY <3>1, <3>2, <3>3, <3>4, <3>5, <3>6, <3>7 DEF Inv
147 ⊢ <2>2. CASE pc = "Done" ∧ UNCHANGED vars
148 ⊢ BY <2>2 DEF vars,TypeOK,Correct,Inv
149 ⊢ <2>3. CASE UNCHANGED vars
150 ⊢ BY <2>3 DEF vars,TypeOK,Correct,Inv
151 ⊢ <2>4. QED
152 ⊢ BY <2>1, <2>2, <2>3 DEF Next
153 ⊢ <1>3. Inv ⇒ Correct
154 ⊢ BY DEF Inv
155 ⊢ <1>4. QED
156 ⊢ BY <1>1, <1>2, <1>3, PTL DEF Spec
157

```

TLAPS spec, proving “Leftpad” [merz2020leftpad]

# Story 1: The Amazon Story

# The Amazon Story

## Background

- Amazon Web Services (AWS) provide a range of fault-tolerant distributed systems
- Critical systems are highly tested using conventional automated tests
  - → this is “inadequate as a method for finding subtle errors in design, as the number of reachable states of the code is astronomical.”
- Engineers check formal methods and choose TLA+

# The Amazon Story

## Usage

Source: [newcombe2015aws]

System	Components	Line Count (Excluding Comments)	Benefit
S3	Fault-tolerant, low-level network algorithm	804 PlusCal	Found two bugs, then others in proposed optimizations
	Background redistribution of data	645 PlusCal	Found one bug, then another in the first proposed fix
DynamoDB	Replication and group-membership system	939 TLA+	Found three bugs requiring traces of up to 35 steps
EBS	Volume management	102 PlusCal	Found three bugs
Internal distributed lock manager	Lock-free data structure	223 PlusCal	Improved confidence though failed to find a liveness bug, as liveness not checked
	Fault-tolerant replication-and-reconfiguration algorithm	318 TLA+	Found one bug and verified an aggressive optimization

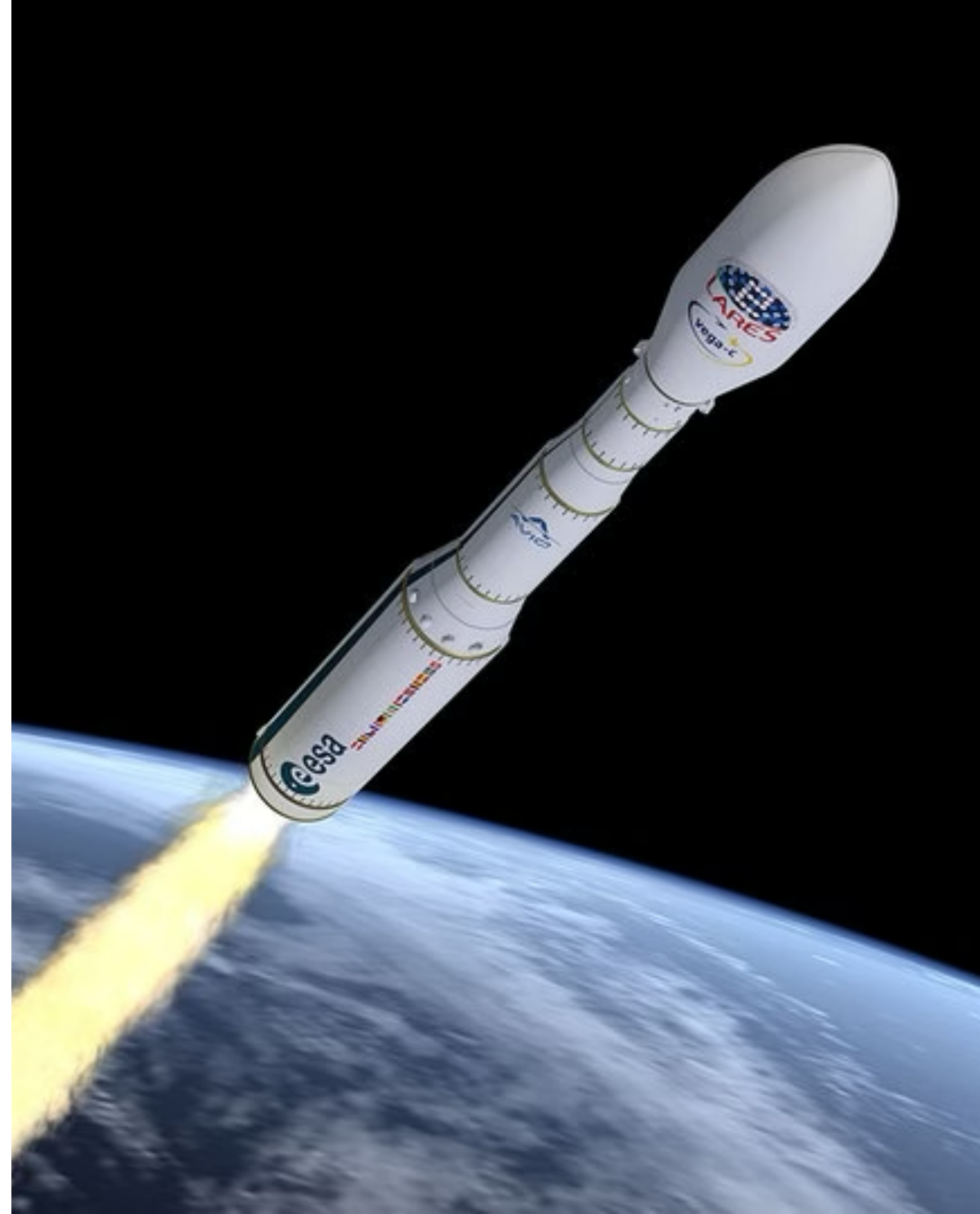
# The Amazon Story

## Benefits of using TLA+

- Higher correctness
- Seen as “what-if”-tool for system designs → facilitates big + safe refactoring
- Precise Documentation
- Better design by “stating precisely ‘what needs to go right.’”



# Story 2: The ESA Story





# The ESA Story

## Background

- ESA uses real-time operating systems (RTOS) for embedded systems
- RTOS considered “complex and difficult”
- Engineers wanted to develop it from ground-up to improve architecture and reduce complexities
- RTOS has many concurrent algorithms, needs to be “safe, correct and performing”
- Decided to use TLA+ at all abstraction levels (system-level to algorithm-level)

# The ESA Story

## Example

Requirement: All Packets in the Semaphore Waiting List must be sorted, according to their priority, in decreasing order.

7. All Semaphore requests are sorted in order of the priority of the issuing Tasks:

$$\begin{aligned}
 &\forall p \in \text{SemaphoreId} : \\
 &\quad \forall i, j \in 1..Len(\text{Semaphore WL}[p]) : \\
 &\quad \quad (i \leq j) \implies (\text{PreallocatedPacket}[\text{Semaphore WL}[p][i]].prio \leq \\
 &\quad \quad \text{PreallocatedPacket}[\text{Semaphore WL}[p][j]].prio)
 \end{aligned}
 \tag{5.28}$$

[verhulst2011rtos], p. 101

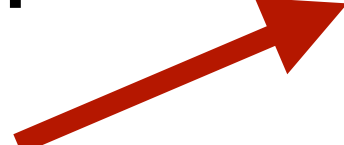
# The ESA Story

## Benefits of using TLA+

- TLA+ caused engineers to “formulate their thoughts clearly while exposing the hidden assumptions”
- “[*The TLA+*] abstraction helped a lot in coming to a much cleaner architecture (we witnessed first hand the brain washing done by years of C programming).” [lampport2019tla]
- → Clearer communication within team
- → Code size 5-10 times smaller than predecessor system
- → Required no “extra resources and time”

# The ESA Story

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  - → Clearer communication within team
  - → Code size 5-10 times smaller than predecessor system
  - → Required no “extra resources and time”
- Beware of implied causation: reimplemented OS  
+ used different architecture
- 

# OpenComRTOS Architecture

“An OpenComRTOS program is an implementation model of a more abstract architectural model that is composed of entities and interactions. In OpenComRTOS, we mainly have two types of entities: Tasks and Hubs.” [verhulst2011rtos], p. 105

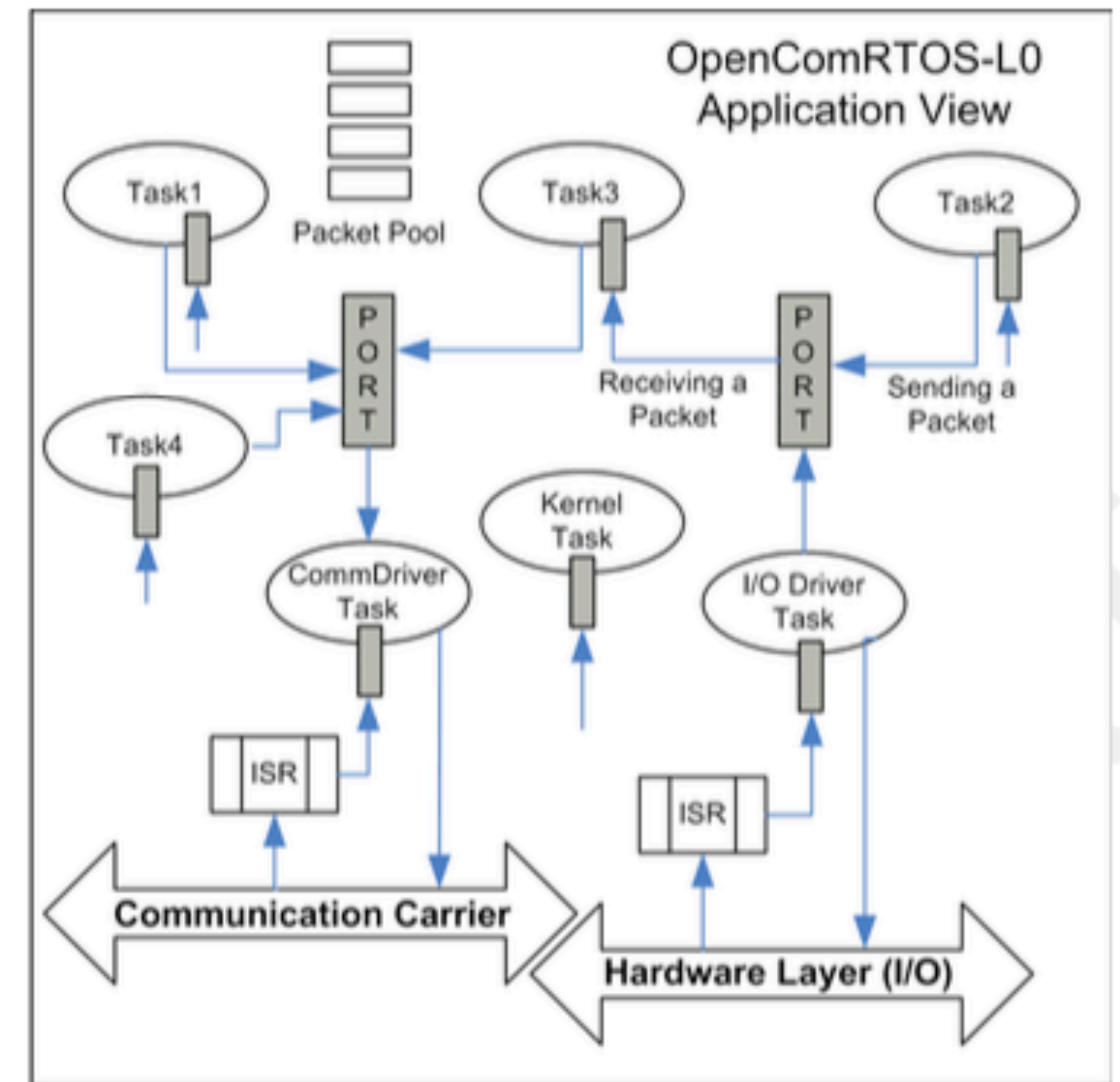


Fig. 6.1 OpenComRTOS application

# Whorfian Syndrome

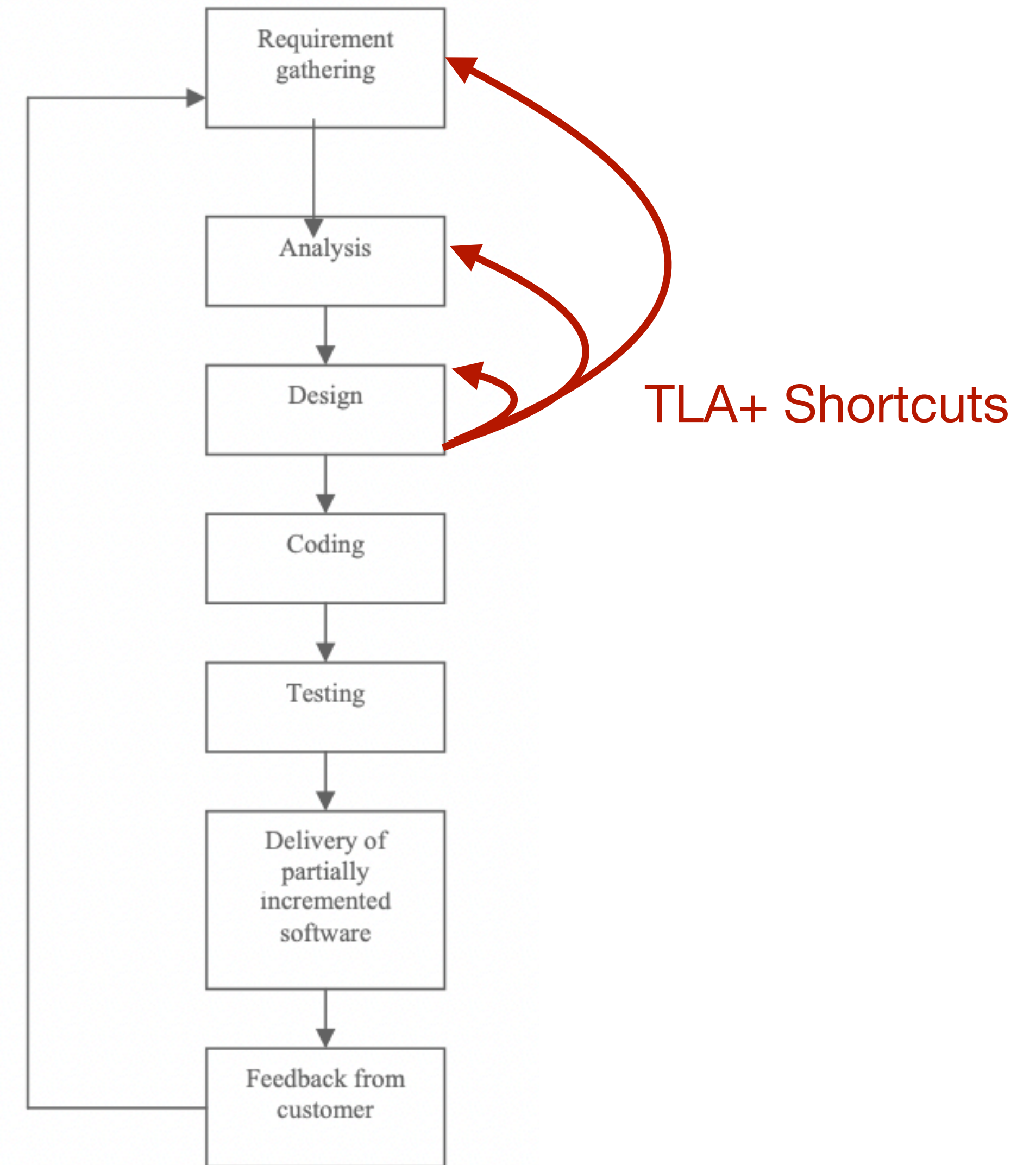
**“Computer Science should be about concepts, not languages. [...] I believe that the best way to get better programs is to teach programmers how to think better. [...] But how does one teach concepts without getting distracted by the language in which these concepts are expressed? My answer is to use the same language as every other branch of science and engineering – namely, mathematics”**

Leslie Lamport, <https://www.youtube.com/watch?v=15uy9Ga-14I>



# TLA+ can provide...

- Higher Correctness
- Precise high-level overview  
revealing implicit assumptions in  
mental model  
→ Cleaner Design
- Shorter (design) feedback loop



# Agile Dev Cycle

[sharma2012agile], adapted  
to TLA+

**“TLA+ is the most valuable thing that I've learned in my professional career. [...] It has changed how I think, by giving me a framework for constructing new kinds of mental-models, by revealing the precise relationship between correctness properties and system designs, and by allowing me to move from ‘plausible prose’ to precise statements much earlier in the software development process.”**

- Chris Newcombe, then Principal Engineer at Amazon, now at Oracle, [newcombe2012groups]



# Introduction to TLA+

Programming	Logic	TLA+
and, &&	$\wedge$	$/\backslash$
or,	$\vee$	$\backslash/$
not, !, ~	$\neg$	$\sim$
!=, <>, ~=	$\neq$	$\#, /=$
all()	$\forall$	$\backslash A$
any()	$\exists$	$\backslash E$
Set union	$\cup$	$\backslash cup, \backslash union$
Set intersection	$\cap$	$\backslash cap, \backslash intersect$
	$\Rightarrow$	$"=>"$

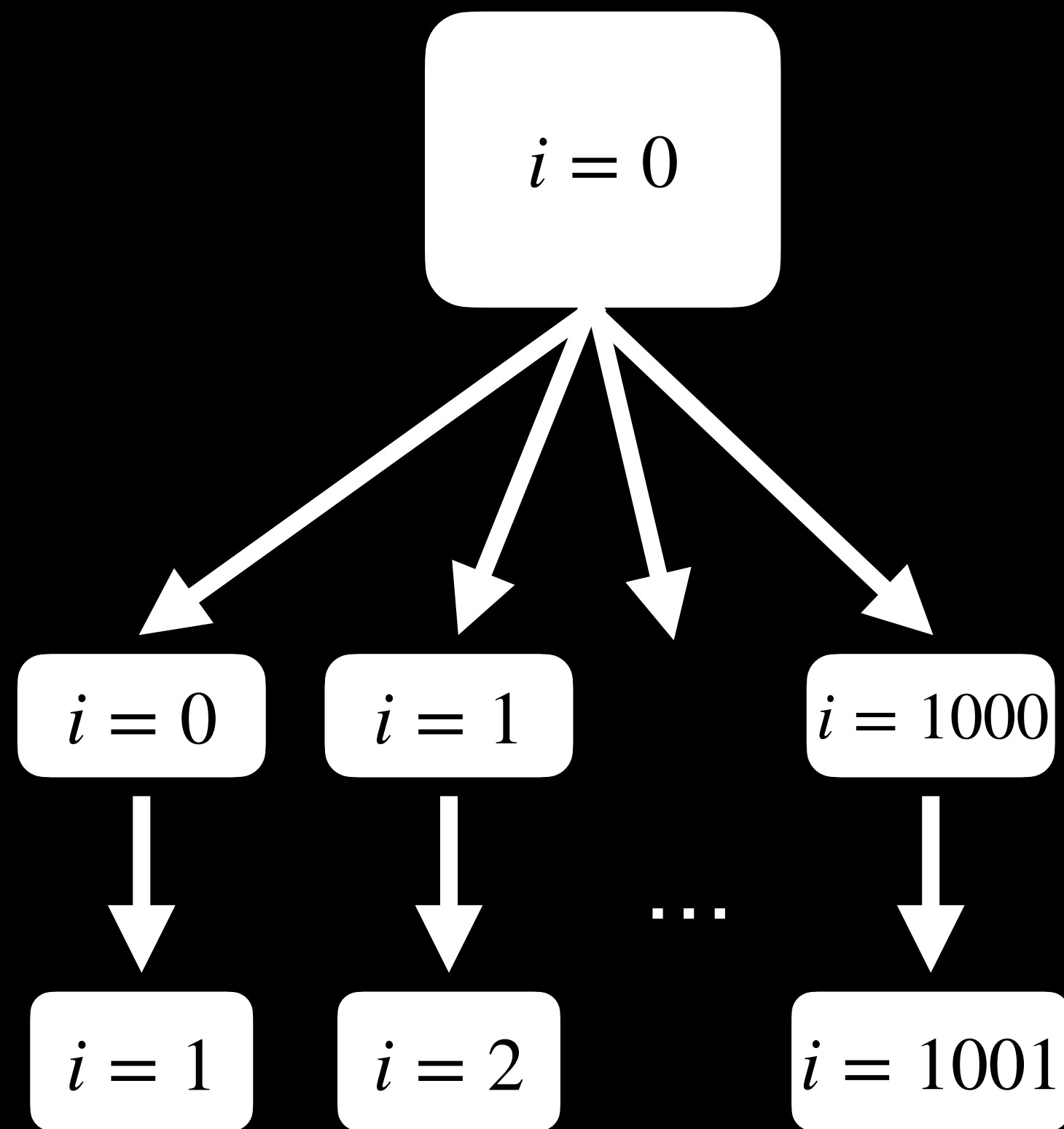
# Math Refresher

```
int i = 0;

void main() {
    i = pickInRange(0, 1000);
    i += 1;
}
```

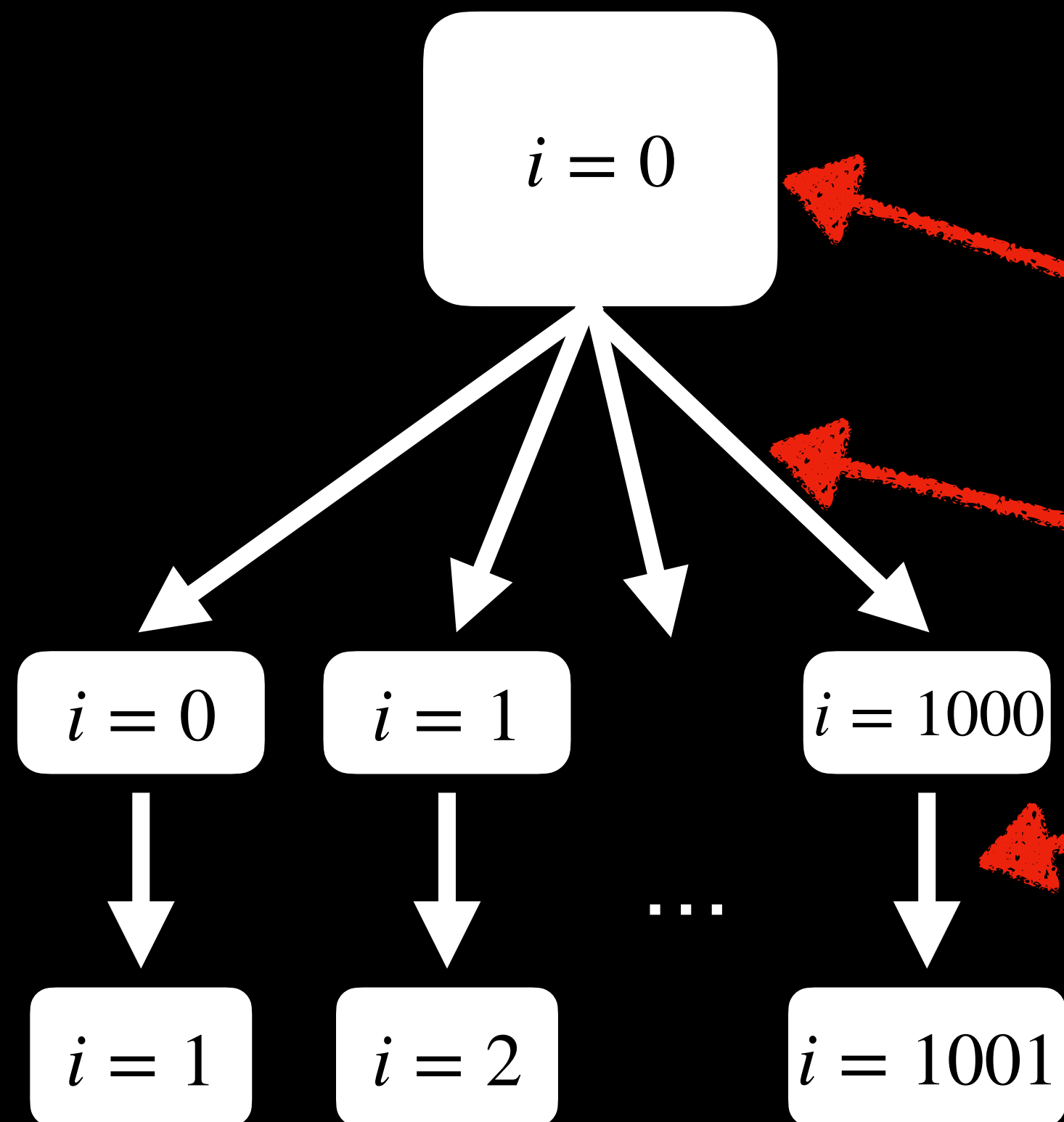
# Some Code

# TLA+ models Programs as State Machines



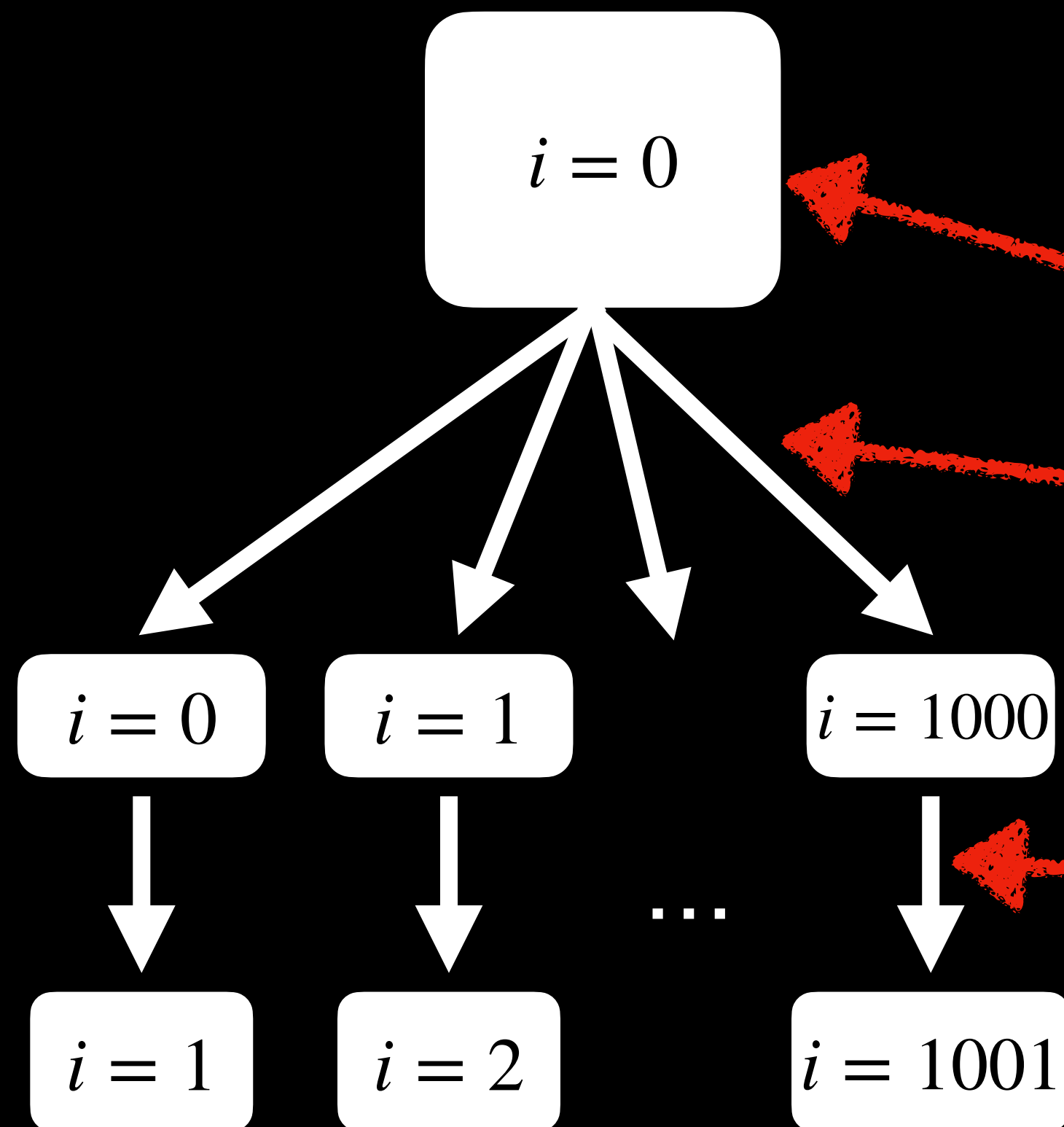
- TLA+ follows mathematical convention of defining state machines using
  - *Init* as initial state
  - *Next* as state transition function for all states

# TLA+ models Programs as State Machines



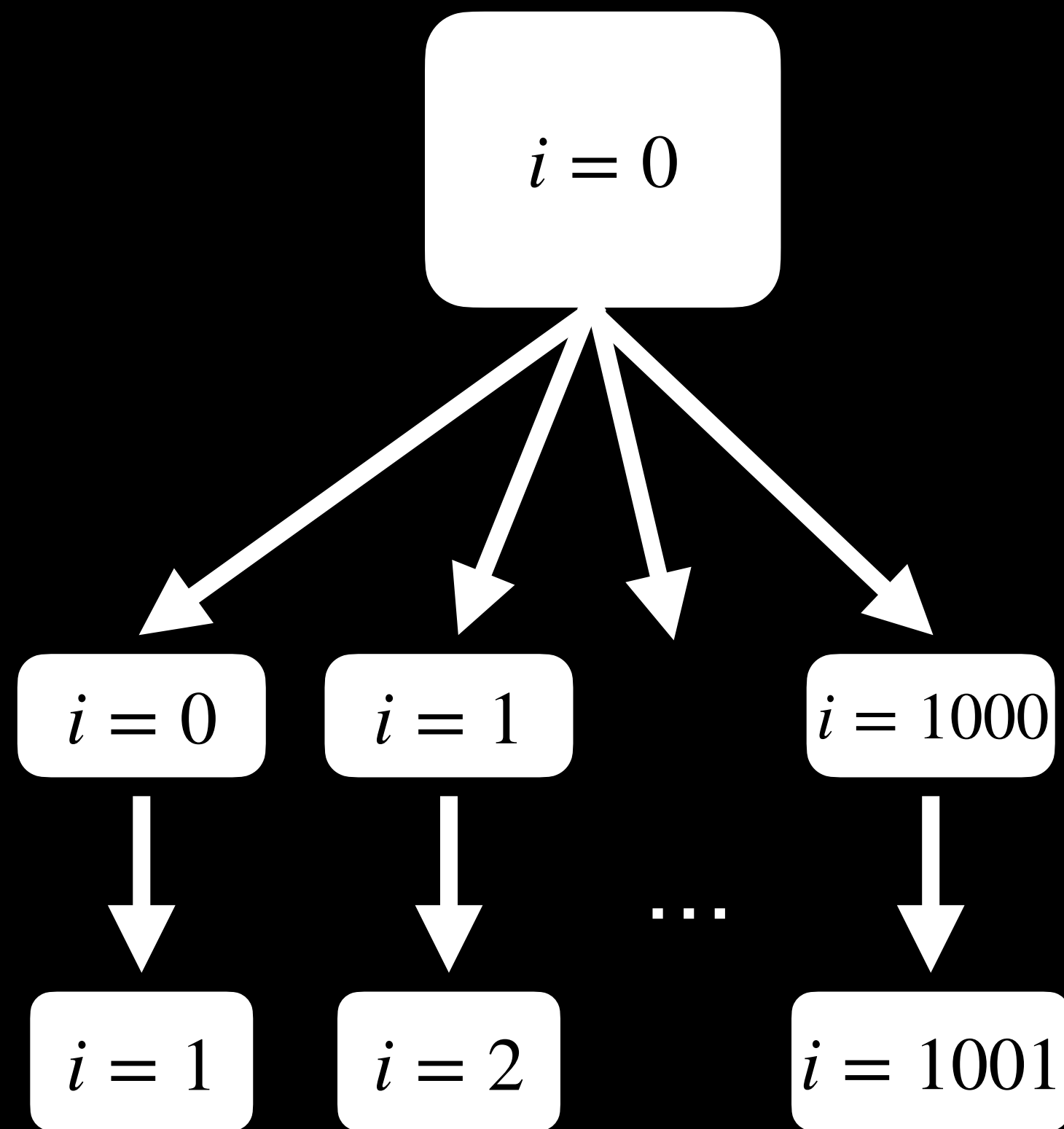
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# TLA+ models Programs as State Machines



- TLA+ follows mathematical convention of defining state machines using
- $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$
- $\text{Next} == \vee (\text{pc} = \text{"start"} \wedge i' \in 0..1000 \wedge \text{pc}' = \text{"middle"})$   
 $\vee (\text{pc} = \text{"middle"} \wedge i' = i + 1 \wedge \text{pc}' = \text{"done"})$

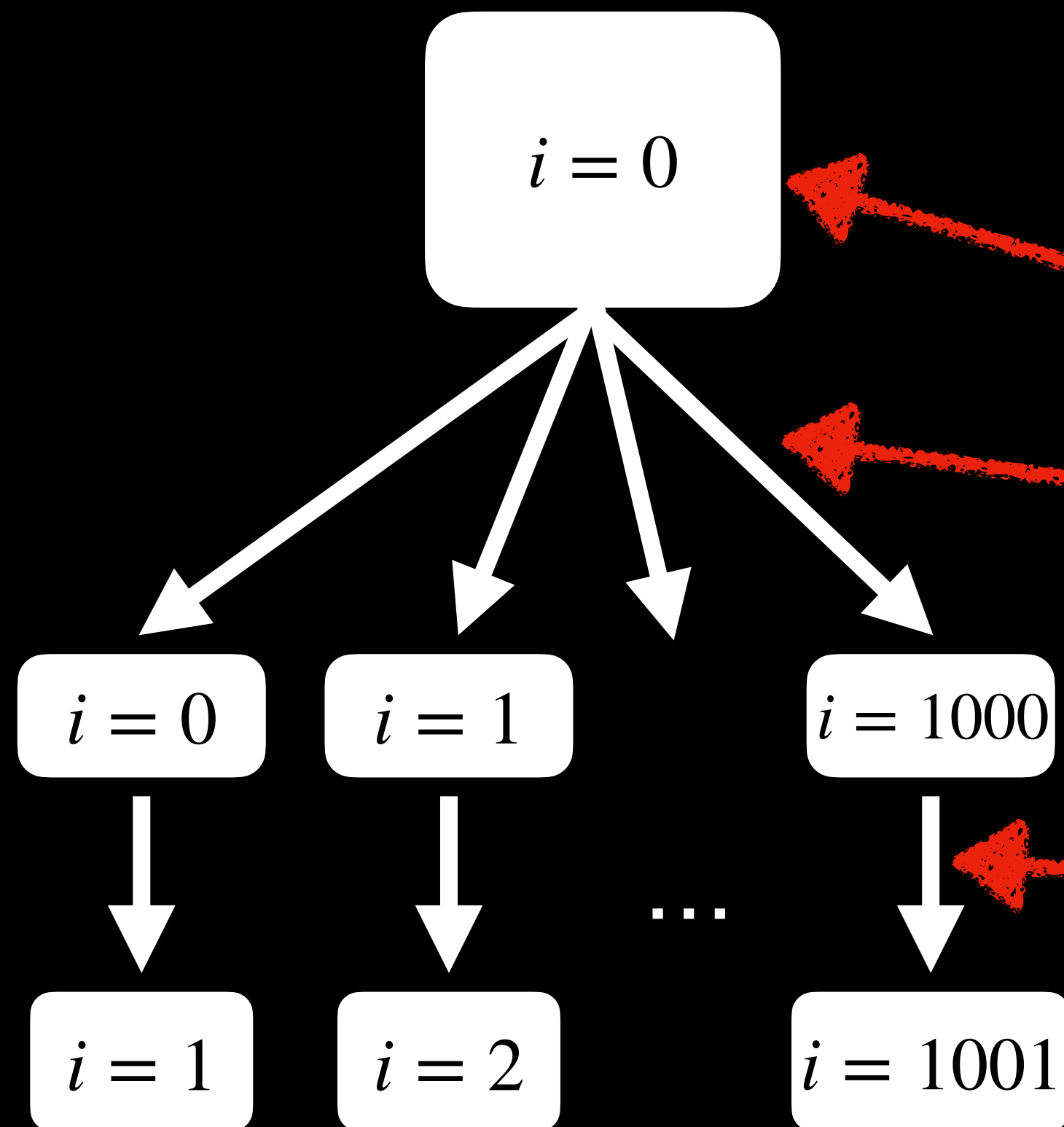
# TLA+ models Programs as State Machines



Implicitly define  
next state variables  
using ‘

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# TLA+ models Programs as State Machines



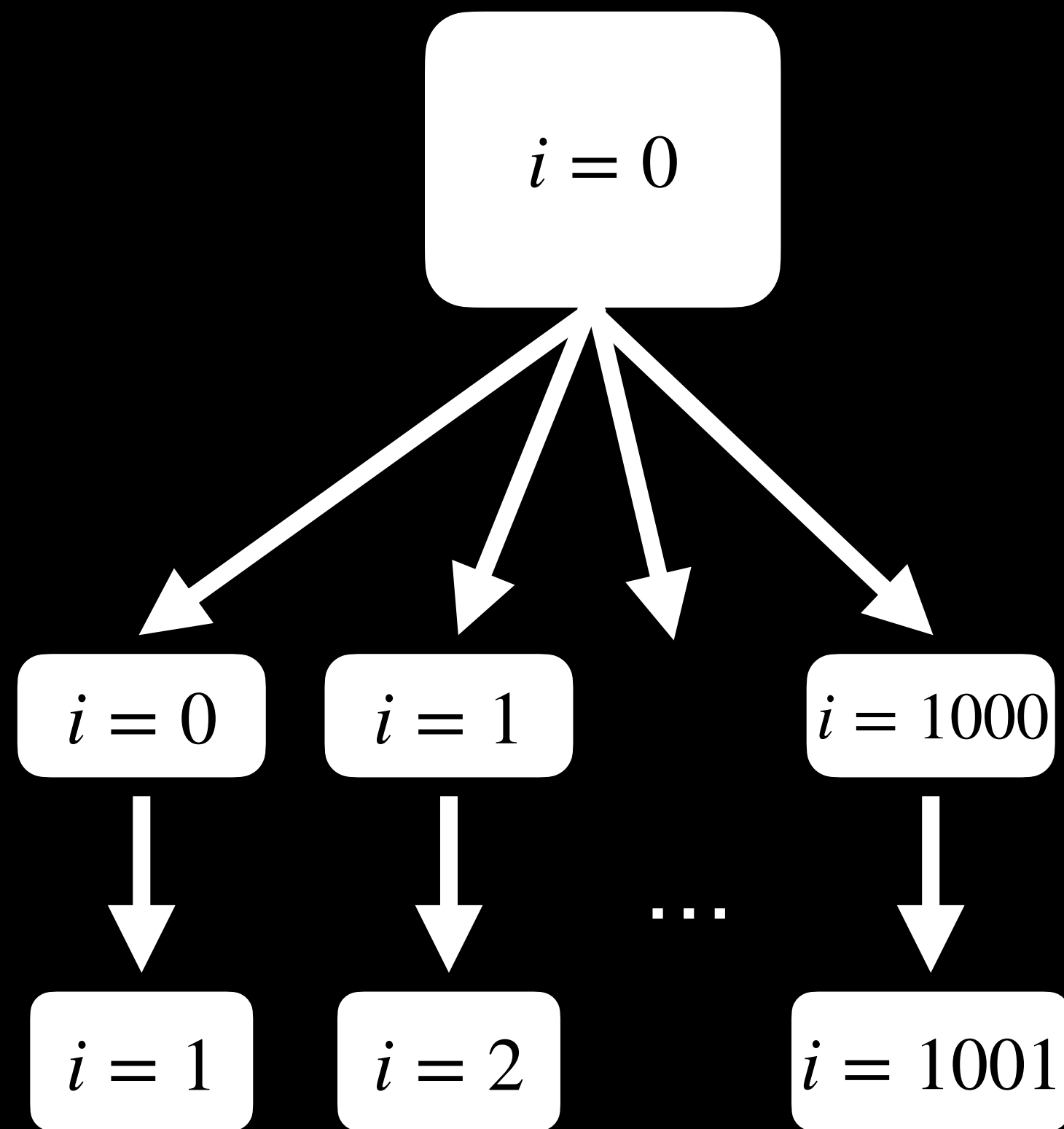
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- $\text{Next} == \vee \begin{array}{l} \wedge \text{pc} = \text{"start"} \\ \wedge i' \in 0..1000 \\ \wedge \text{pc}' = \text{"middle"} \end{array} \vee \begin{array}{l} \wedge \text{pc} = \text{"middle"} \\ \wedge i' = i + 1 \\ \wedge \text{pc}' = \text{"done"} \end{array}$



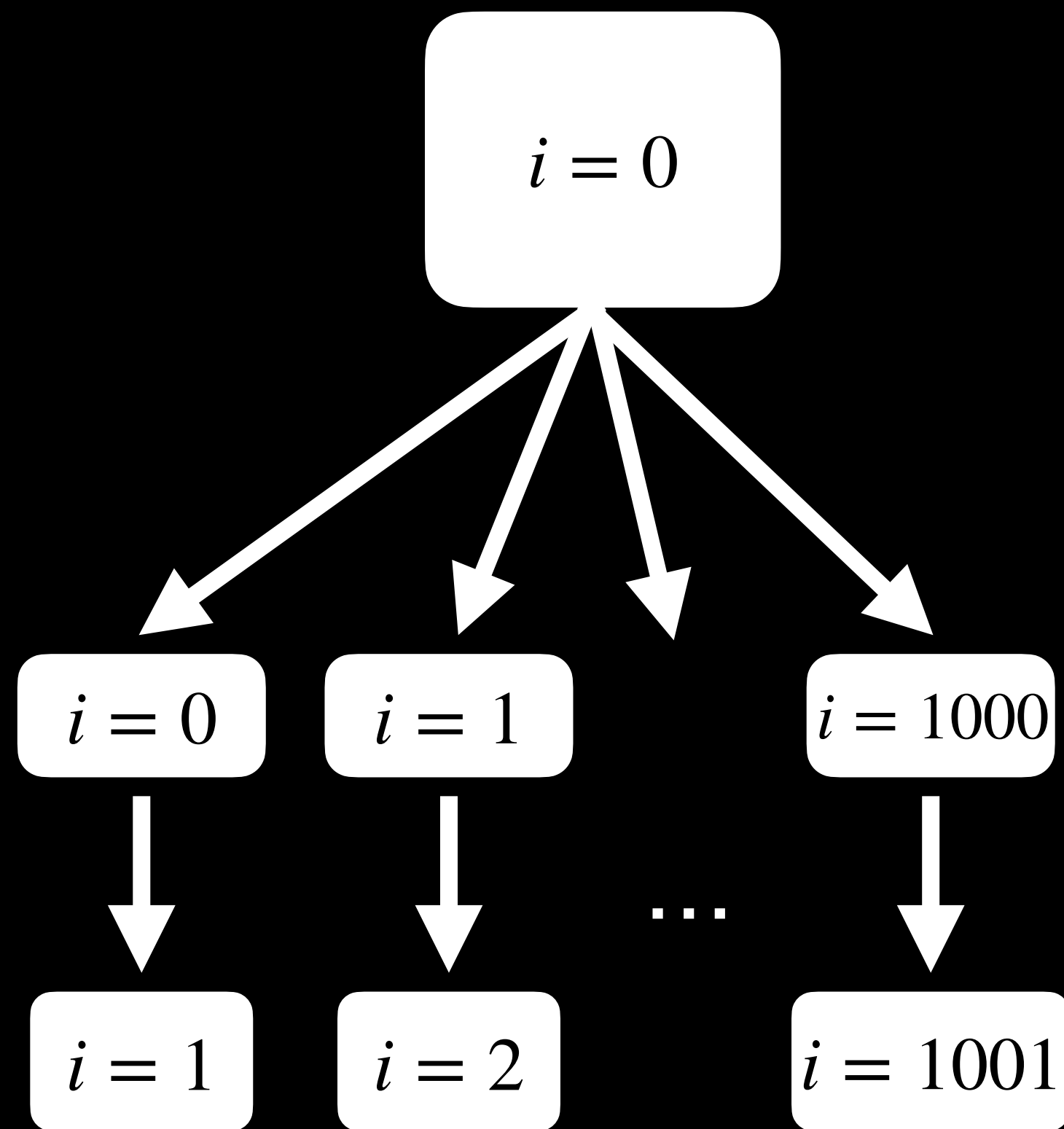
# TLA+ models Programs as State Machines



What happens if multiple clauses match?

- TLA+ follows mathematical convention of defining state machines using
  - $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$
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# TLA+ models Programs as State Machines



- TLA+ follows mathematical convention of defining state machines using

- $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$

- $\text{Next} == \bigvee \begin{array}{l} \wedge \text{pc} = \text{"start"} \\ \wedge i' \in 0..1000 \\ \wedge \text{pc}' = \text{"middle"} \end{array} \bigvee \begin{array}{l} \wedge \text{pc} = \text{"middle"} \\ \wedge i' = i + 1 \\ \wedge \text{pc}' = \text{"done"} \end{array}$

What happens if multiple  
clauses match?  
All states are checked!

# TLA+ checks Programs using Assertions and Invariants

- *Given: set  $P$ , integer  $n$ , ensure that set  $P$  contains only elements smaller than  $n$*
- TLA+ uses invariants and assertions to check correctness
- E.g. in math could specify invariant as:  $\forall p \in P : p < n$   
(in TLA+:  $\forall p \in P : p < n$ )
- This ensures **safety** properties (“what mustn’t fail”)

# TLA+ checks Programs using Assertions and Invariants

- TLA+ is build on temporal logic
- Temporal logic allows reasoning over time, i.e. reasoning about concurrent programs
- This ensures liveness properties (“Good things happen eventually”)
  - In practice, this covers only a minor part of TLA+ specs

## In English

Eventually, there will be no elements  
smaller than  $n$

Forever, there will be no elements smaller  
than  $n$

At some time, there will be no elements  
smaller than  $n$  ever again

## In TLA+

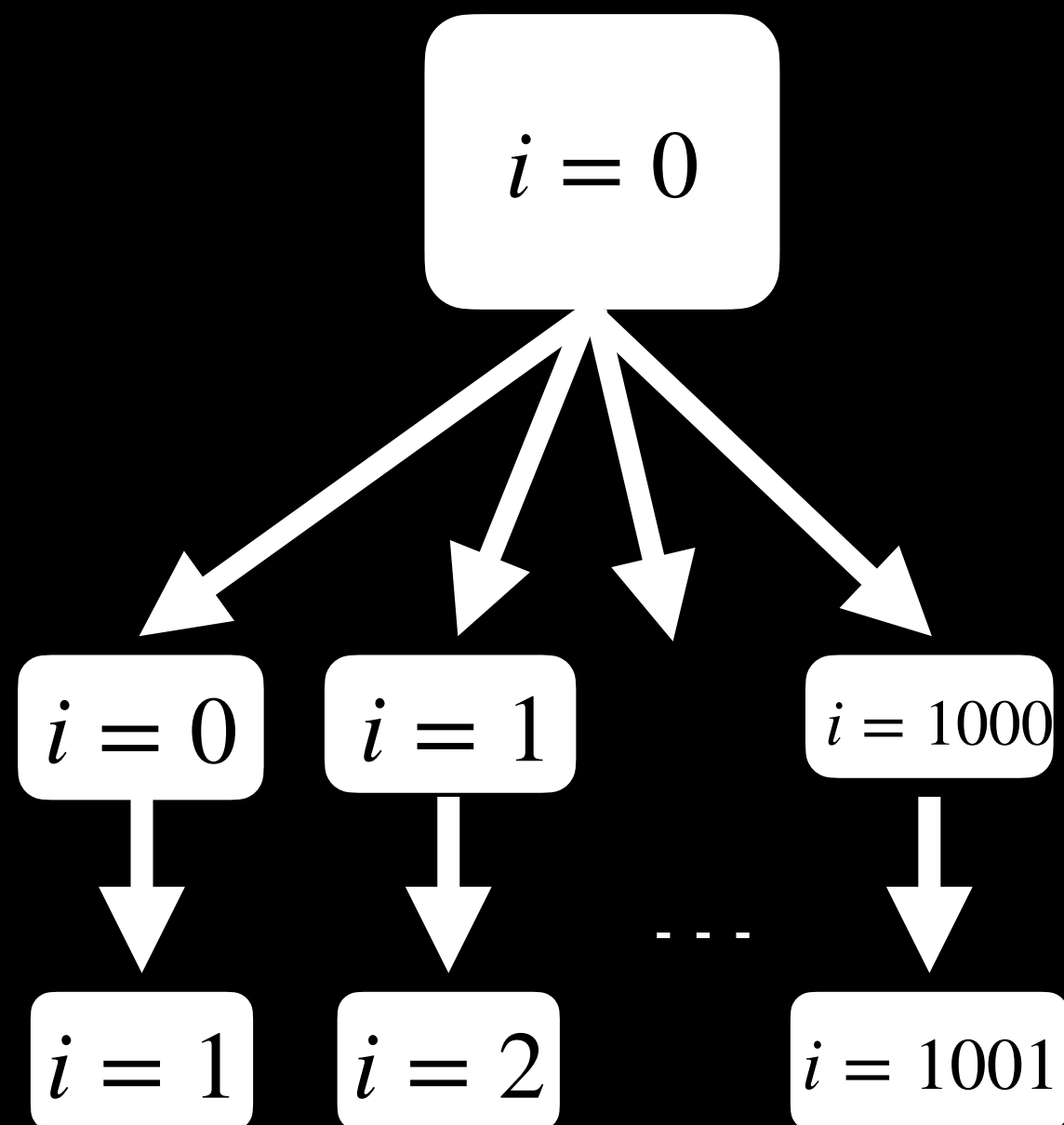
$\langle \rangle (\forall p \in P : p < n)$

$[] (\forall p \in P : p < n)$

$\langle \rangle [] (\forall p \in P : p < n)$

**Too unfamiliar?**

# PlusCal is a DSL in TLA+ that is more similar to code



- $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$
- $\text{Next} == \vee \wedge \text{pc} = \text{"start"} \wedge i' \in 0..1000 \wedge \text{pc}' = \text{"middle"} \vee \wedge \text{pc} = \text{"middle"} \wedge i' = i + 1 \wedge \text{pc}' = \text{"done"}$

```

(**--algorithm SomeName {
  variable i = 0;

  {
    with (n \in 0..1000) {
      i := n;
    };
    i := i + 1;
  }
}**)

```

# Euclids Algorithm

```
--fair algorithm Euclids {
  (* declaration of global variables *)
  variables x = M, y = N;
  (* operator definitions *)
  define {
    p | q == \E d \in 1..q : q = d * p

    Divisors(q) == { d \in 1..q : d | q }

    Maximum(S) == CHOOSE n \in S : (\A i \in S: i <= n)

    GCD(p, q) == Maximum(Divisors(p) \cap Divisors(q))
  }
  (* algorithm body or process declarations *)
  {
    while (x # y) {
      if (x < y) {
        y := y - x;
      } else {
        x := x - y;
      }
    };
    print GCD(M,N);
    assert x = GCD(M,N) /\ y = GCD(M,N);
  }
}
```

```
p | q == \E d \in 1..q : q = d * p

Divisors(q) == { d \in 1..q : d | q }

Maximum(S) == CHOOSE n \in S : (\A i \in S: i <= n)

GCD(p, q) == Maximum(Divisors(p) \cap Divisors(q))

vars == << x, y, pc >>

Init == (* Global variables *)
  /\ x = M
  /\ y = N
  /\ pc = "Step"

Step == /\ pc = "Step"
  /\ IF x # y
    THEN /\ IF x < y
      THEN /\ y' = y - x
            /\ x' = x
            ELSE /\ x' = x - y
                  /\ y' = y
      /\ pc' = "Step"
    ELSE /\ PrintT(GCD(M,N))
          /\ Assert(x = GCD(M,N) /\ y = GCD(M,N),
            "Failure of assertion at line 31, column 9.")
          /\ pc' = "Done"
          /\ UNCHANGED << x, y >>

(* Allow infinite stuttering to prevent deadlock on termination. *)
Terminating == pc = "Done" /\ UNCHANGED vars

Next == Step \/ Terminating
```

# Demo

“Consider a bare minimum carbon credit trading platform:

Every credit has an owner.

An owner may offer the credit to a different user. The recipient user can accept the offer, in which case ownership of the credit transfers to them, or reject it, in which case nothing happens.

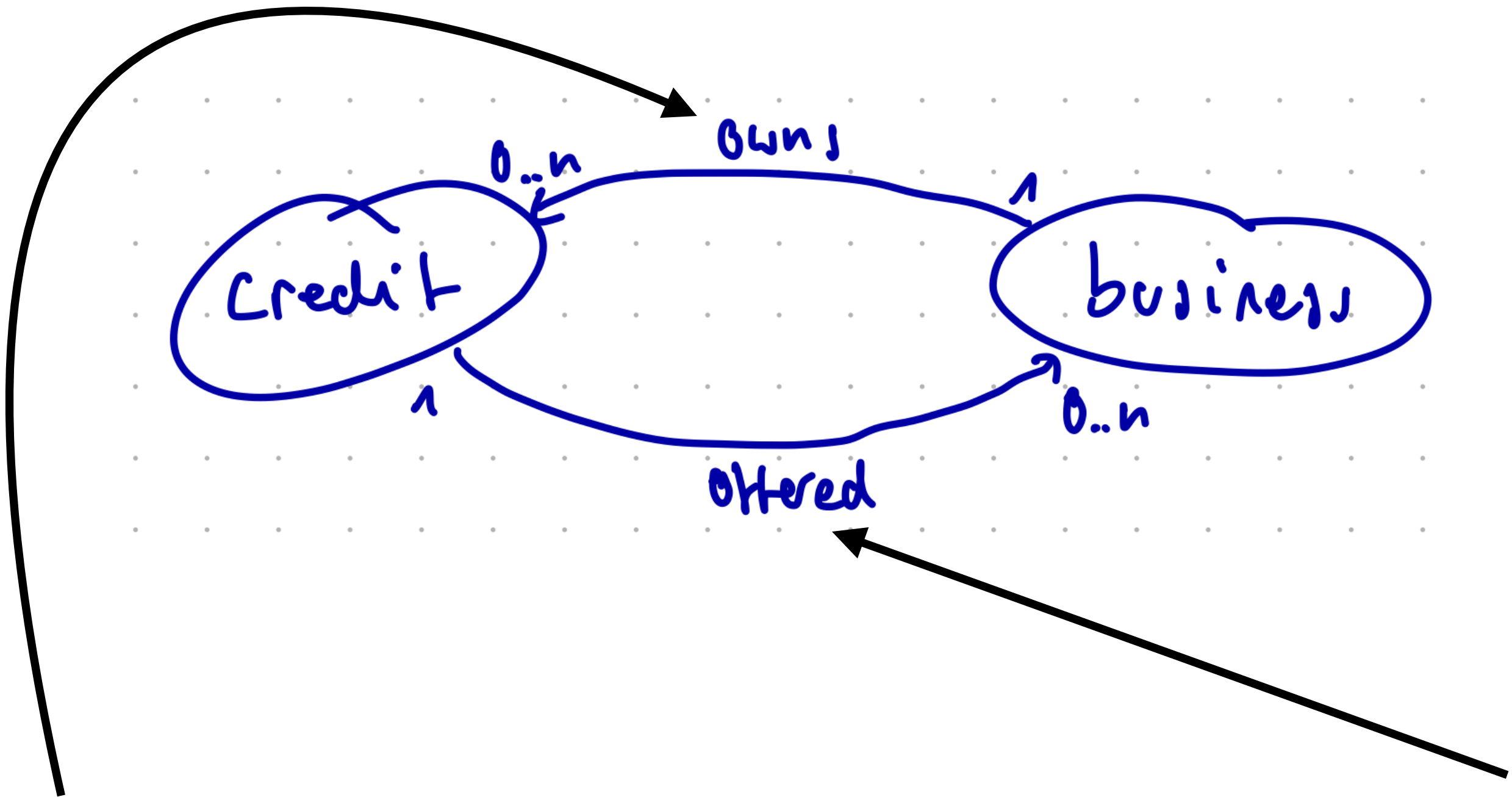
The accept/reject is asynchronous: you can offer the same credit to multiple people (to scaffold out trades), and the person may wait a day before accepting or rejecting an offer.”

(Credit: <https://www.hillelwayne.com/post/business-case-formal-methods/>)



# Demo

## Design



owners

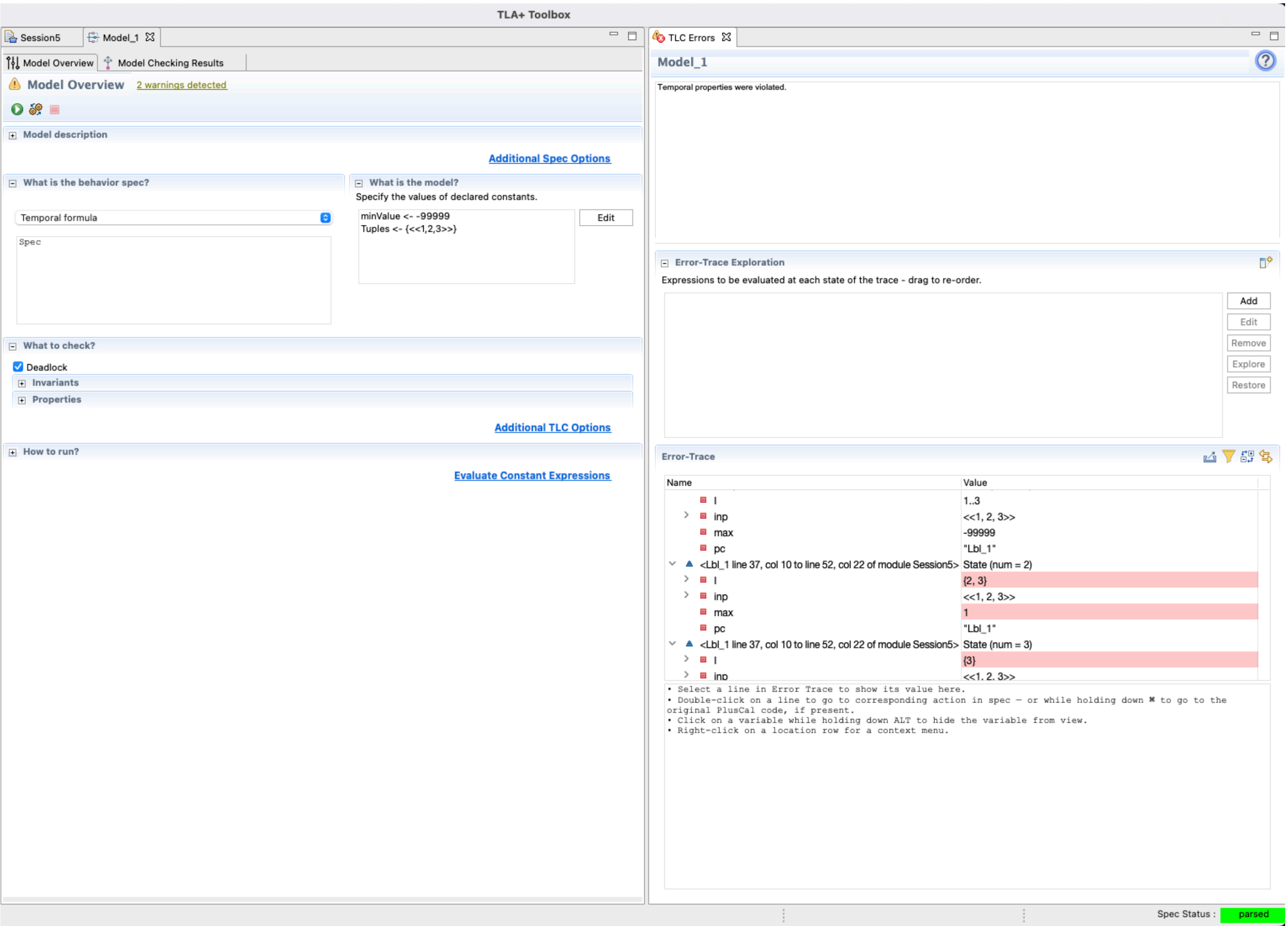
credit	Owner
"C1"	"HPI"
...	...

offers

from	to	credit
"HPI"	"MIT"	"C1"
...	...	...

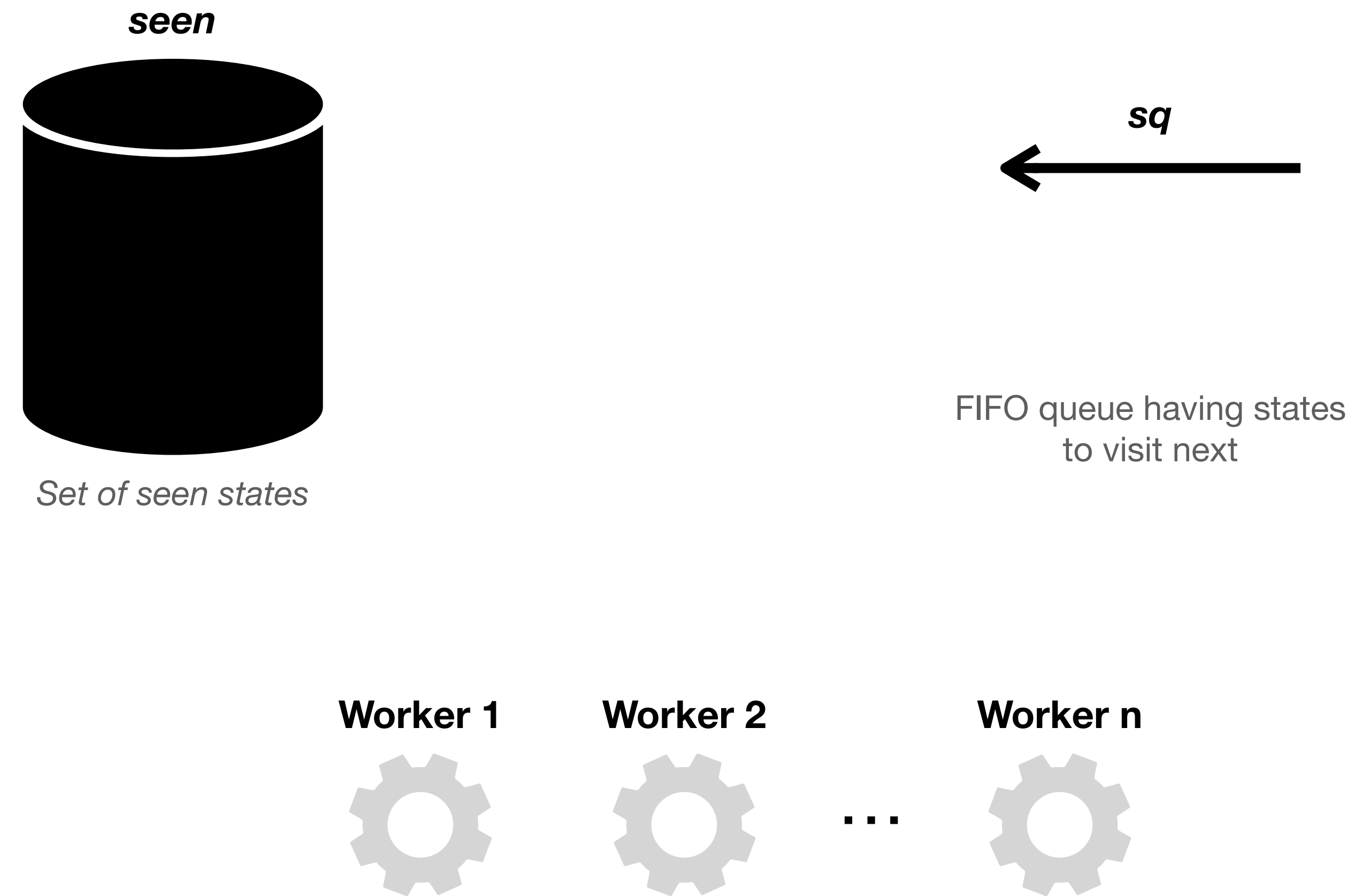
# TLC

## TLA+’s model checker



# How TLC's model checker works

Init

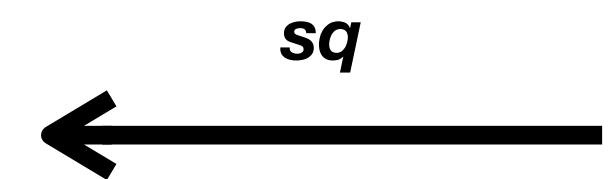
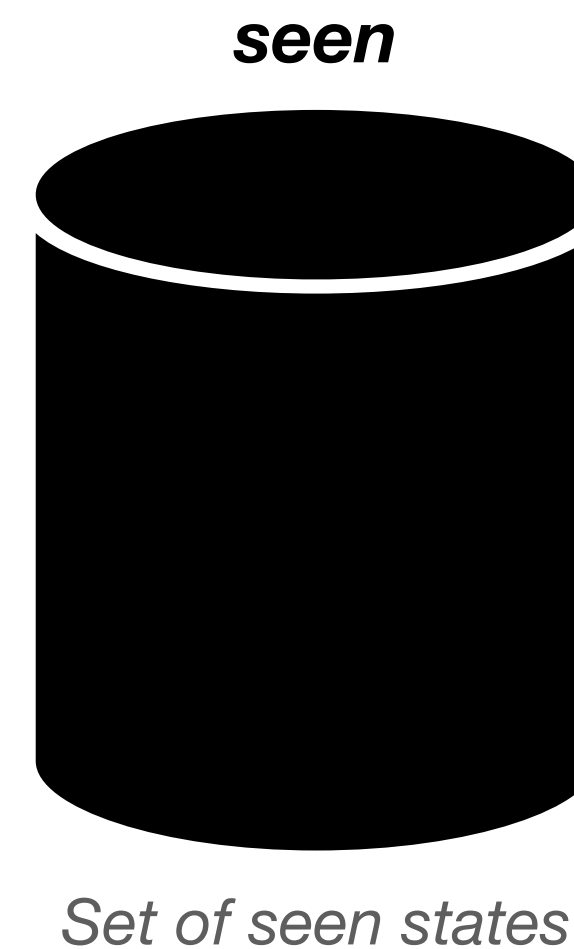


# How TLC's model checker works

## Init

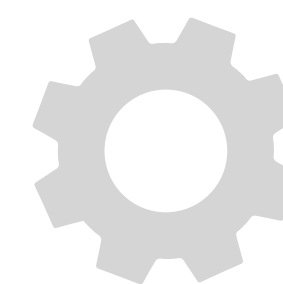
- $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$

- $\text{Next} == \bigvee \bigwedge \text{pc} = \text{"start"} \wedge i' \in 0..1000 \wedge \text{pc}' = \text{"middle"} \vee \bigwedge \text{pc} = \text{"middle"} \wedge i' = i + 1 \wedge \text{pc}' = \text{"done"}$

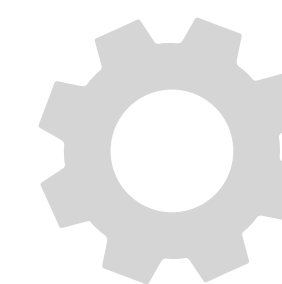


FIFO queue having states  
to visit next

Worker 1

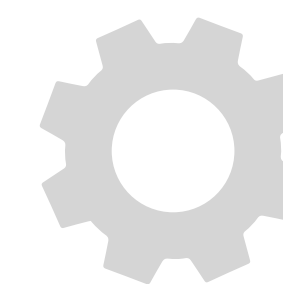


Worker 2



...

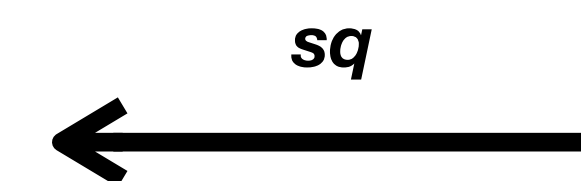
Worker n



# How TLC's model checker works

## Init

1. Add all possible init states to *seen* and *sq*

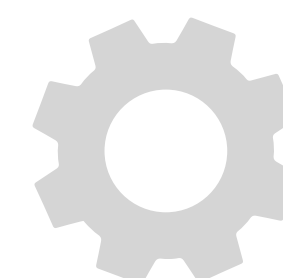


pc = "start" ∧ i = 0

FIFO queue having states  
to visit next

- Init == (pc = "start") ∧ (i = 0)
- Next ==  $\bigvee \bigwedge$  pc = "start"  
 $\bigwedge i' \in 0..1000$   
 $\bigwedge$  pc' = "middle"  
 $\bigvee \bigwedge$  pc = "middle"  
 $\bigwedge i' = i + 1$   
 $\bigwedge$  pc' = "done"

**Worker 1**

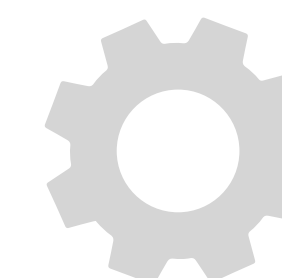


**Worker 2**



...

**Worker n**

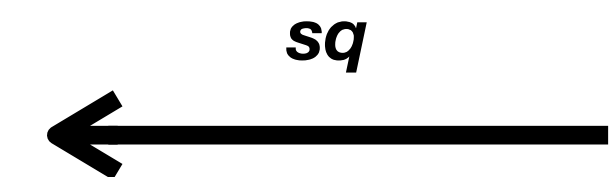


# How TLC's model checker works

```

1 in each worker:
2 → s = sq.pop()
3   for each possible next state t of s:
4       if (t not in seen) and (assertInvariant(t)):
5           seen.add(t)
6           sq.push(t)

```

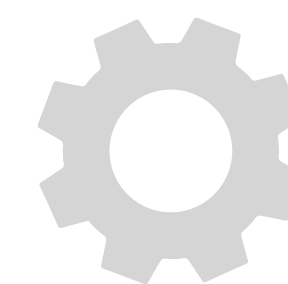


pc = "start" ∧ i = 0

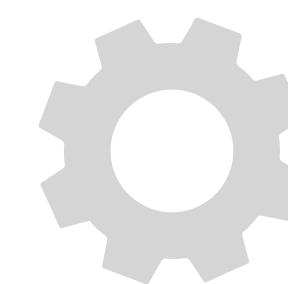
FIFO queue having states  
to visit next

- Init == (pc = "start") ∧ (i = 0)
- Next ==  $\bigvee \bigwedge$  pc = "start"  
 $\bigwedge i' \setminus \text{in } 0..1000$   
 $\bigwedge$  pc' = "middle"  
 $\bigvee \bigwedge$  pc = "middle"  
 $\bigwedge i' = i + 1$   
 $\bigwedge$  pc' = "done"

**Worker 1**

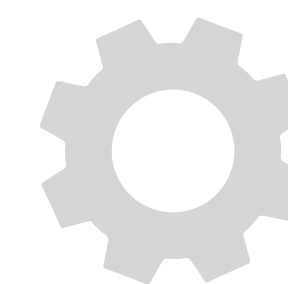


**Worker 2**



...

**Worker n**

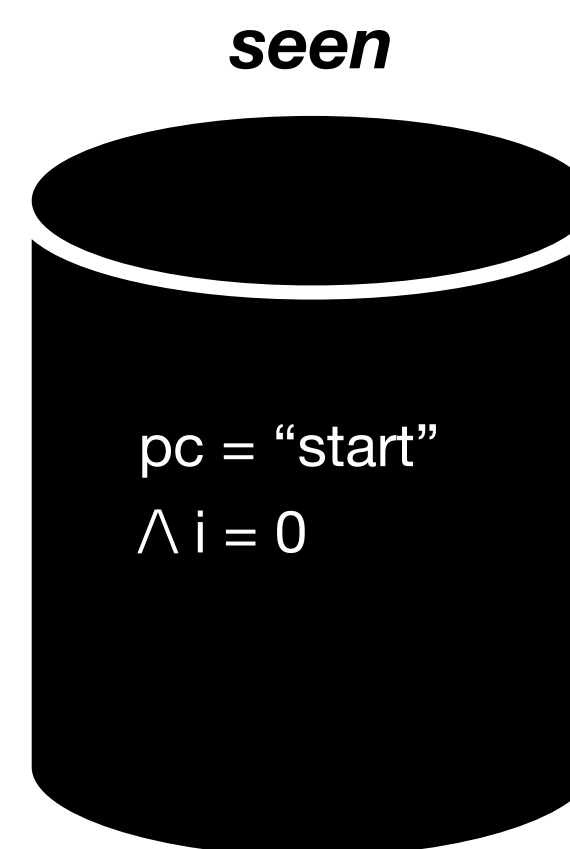


# How TLC's model checker works

```

1 in each worker:
2   s = sq.pop()
3   for each possible next state t of s:
4     if (t not in seen) and (assertInvariant(t)):
5       seen.add(t)
6       sq.push(t)

```



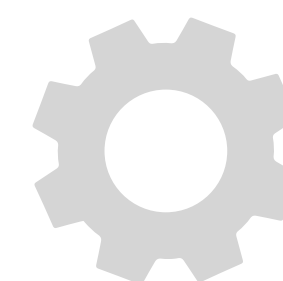
*Set of seen states*



FIFO queue having states  
to visit next

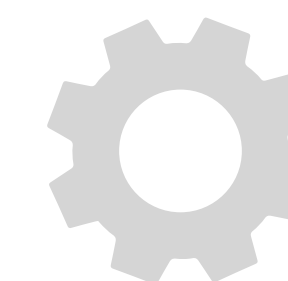
- Init == (pc = "start") ∧ (i = 0)
- Next ==  $\bigvee \bigwedge$  pc = "start"  
 $\bigwedge i' \in 0..1000$   
 $\bigwedge$  pc' = "middle"  
 $\bigvee \bigwedge$  pc = "middle"  
 $\bigwedge i' = i + 1$   
 $\bigwedge$  pc' = "done"

**Worker 1**



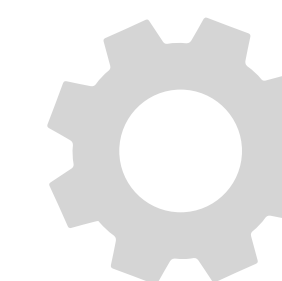
pc = "start" ∧ i = 0

**Worker 2**



...

**Worker n**



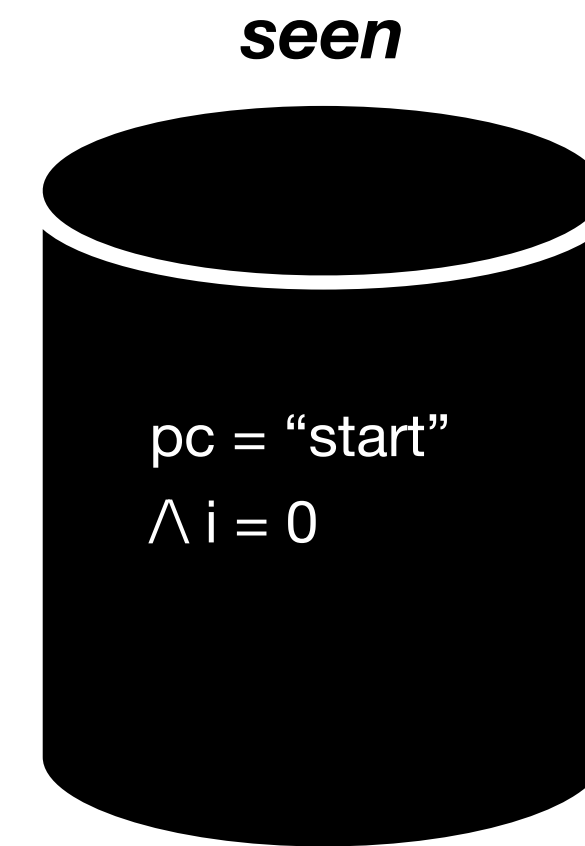
# How TLC's model checker works

## Lines 3-6

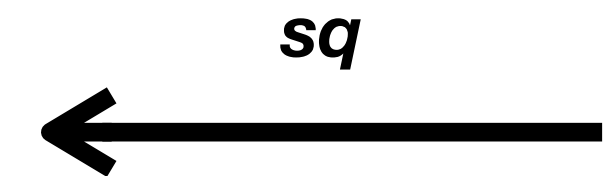
```

1 in each worker:
2   s = sq.pop()
3   for each possible next state t of s:
4     if (t not in seen) and (assertInvariant(t)):
5       seen.add(t)
6       sq.push(t)

```



*Set of seen states*

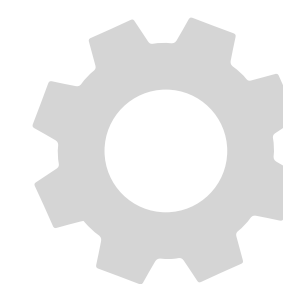


FIFO queue having states  
to visit next

- Init == (pc = "start") ∧ (i = 0)

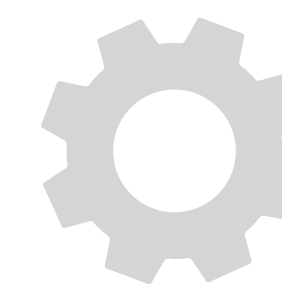
- Next ==  $\bigvee \bigwedge$  pc = "start"  
 $\bigwedge i' \setminus \text{in } 0..1000$   
 $\bigwedge$  pc' = "middle"  
 $\bigvee \bigwedge$  pc = "middle"  
 $\bigwedge i' = i + 1$   
 $\bigwedge$  pc' = "done"

**Worker 1**



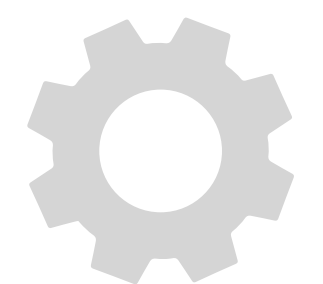
pc = "start" ∧ i = 0

**Worker 2**



...

**Worker n**





# How TLC's model checker works

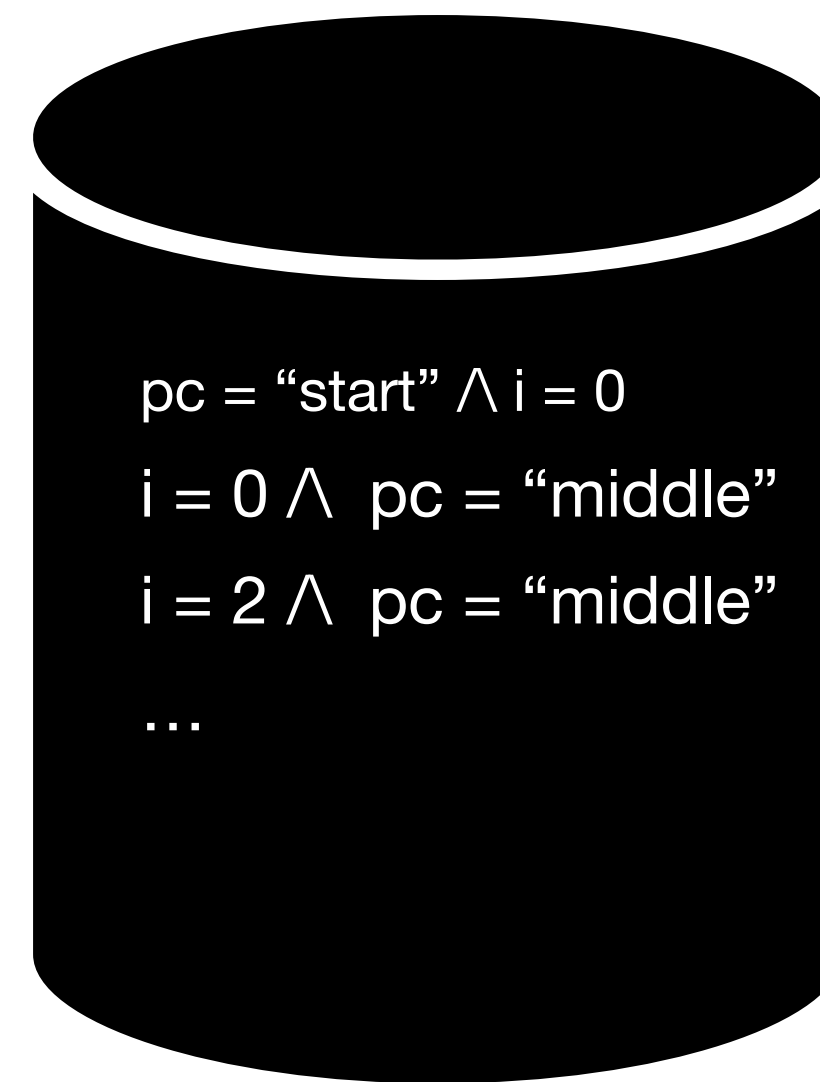
## Init

```

1 in each worker:
2   s = sq.pop()
3   for each possible next state t of s:
4     if (t not in seen) and (assertInvariant(t)):
5       seen.add(t)
6       sq.push(t)

```

*seen*



*Set of seen states*

*sq*



$i = 0 \wedge pc = \text{"middle"}$

$i = 1 \wedge pc = \text{"middle"}$

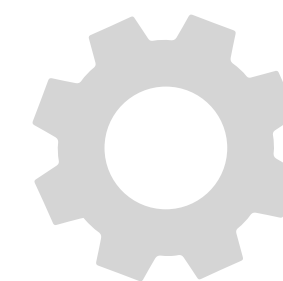
$i = 2 \wedge pc = \text{"middle"}$

...

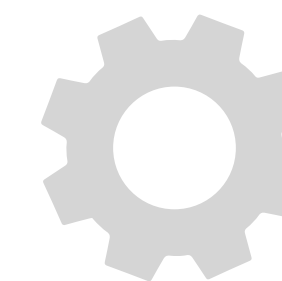
FIFO queue having states  
to visit next

- $\text{Init} == (pc = \text{"start"}) \wedge (i = 0)$
- $\text{Next} == \bigvee \bigwedge pc = \text{"start"} \wedge i' \in 0..1000 \wedge pc' = \text{"middle"}$   
 $\bigvee \bigwedge pc = \text{"middle"} \wedge i' = i + 1 \wedge pc' = \text{"done"}$

**Worker 1**

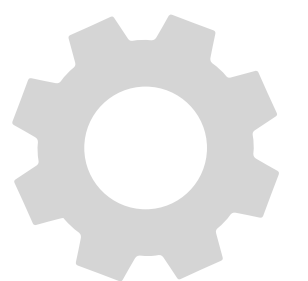


**Worker 2**



...

**Worker n**

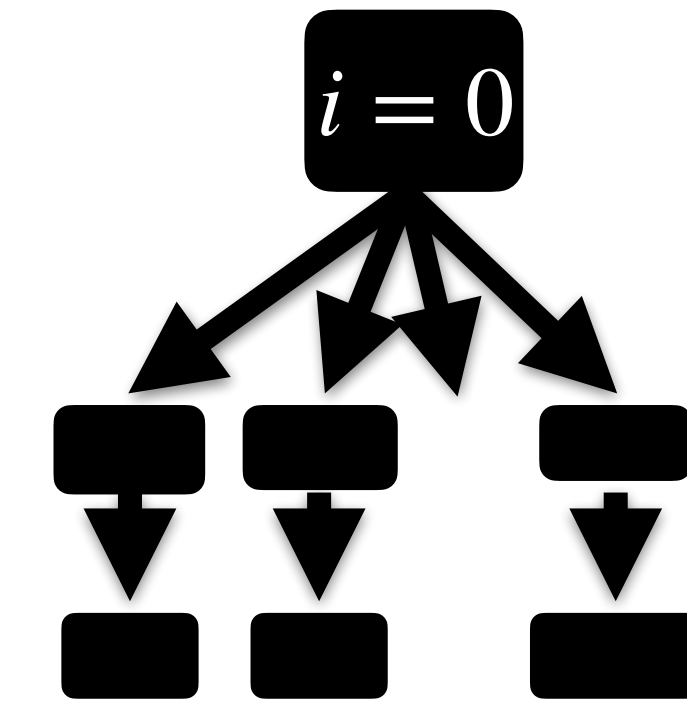


# Limitations

# Theoretical Limitations

- No floats
- No support for probabilistic systems
- No chars
- Untyped
- No modeling of real-time properties
- Implementation Gap

## State machine



## Mathematical simplicity

Set theory, predicate logic  
[lamport1999types]

## Temporal Logic

$\langle \rangle [] (\forall p \in P: p < n)$

# Practical Limitations

- Learning curve
  - Amazon estimates 1 month [newcombe2015aws]
  - Intel estimates 3 months [batson2003intel]
- State explosion must be considered and gives upper bound for complexity of TLA+ specs
- Aged Tooling (no code completion, minimal linting, older eclipse-version of toolbox)
- Fragmentation between PlusCal (supports so-called C and P syntax) / TLA+ makes problem searching harder

```
(**--algorithm Init
  variable i

  begin
    with (n \in 0..1000) begin
      i := n
    end with
    i := i + 1
  end algorithm
*)
```

PlusCal P-syntax

```
(**--algorithm SomeName {
  variable i = 0;

  {
    with (n \in 0..1000) {
      i := n;
    };
    i := i + 1;
  }
}**)
```

PlusCal C-syntax

- $\text{Init} == (\text{pc} = \text{"start"}) \wedge (i = 0)$
- $\text{Next} == \bigvee \bigwedge \text{pc} = \text{"start"} \wedge i' \in 0..1000 \wedge \text{pc}' = \text{"middle"}$   
 $\bigvee \bigwedge \text{pc} = \text{"middle"} \wedge i' = i + 1 \wedge \text{pc}' = \text{"done"}$

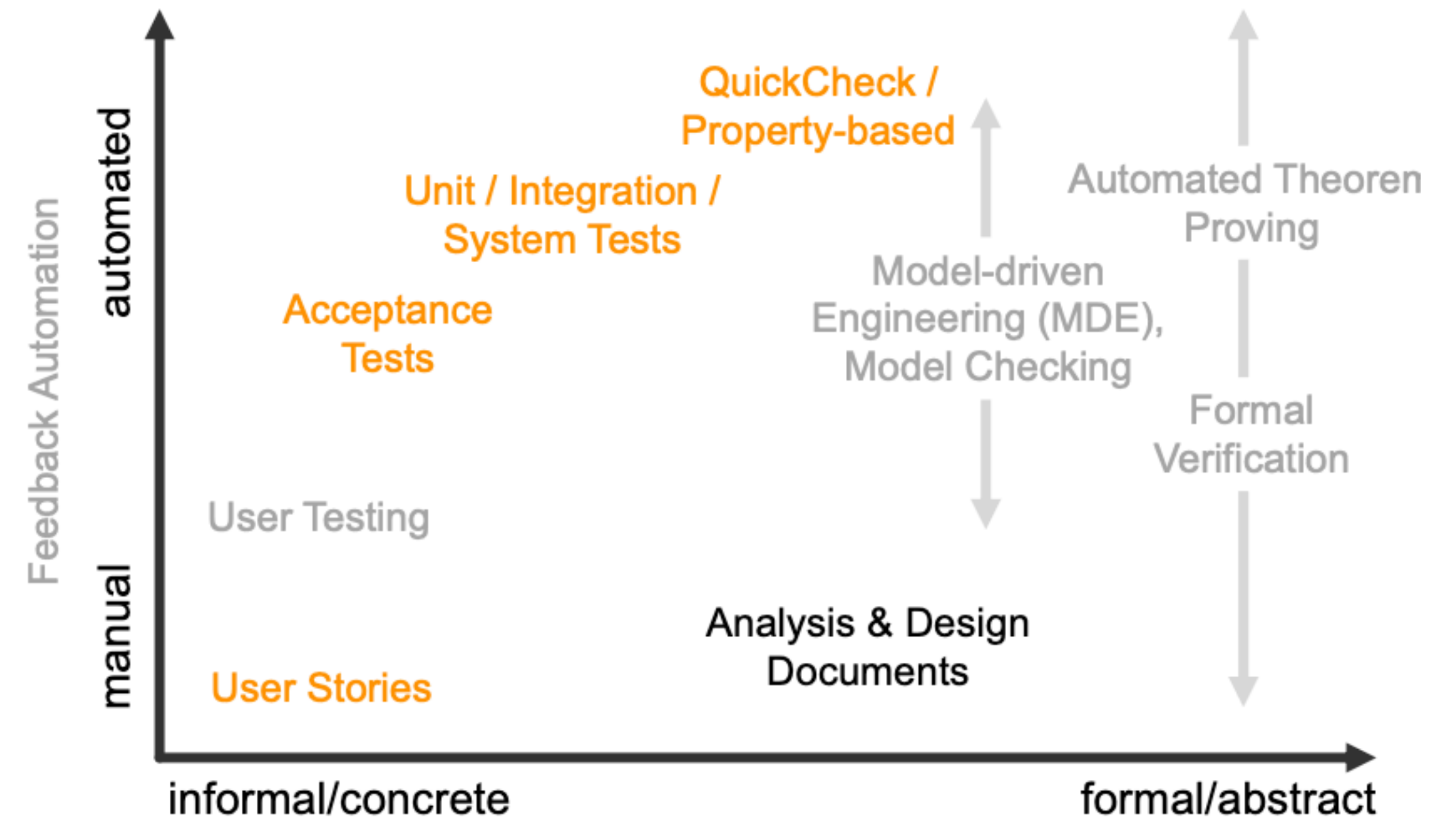
TLA+ syntax

# Alternatives

# TLA+ vs Testing

- Tests check implementation, TLA+ checks design
- Example from [learntla]:
  - “*our microservice architecture never submits the same payment twice, even if services go down*”
- Very hard to test thoroughly
- Easy to find concurrency-related bugs in TLA+

## Levels of (Executable) Specification



Source: SWA: Vorlesung Test-driven Development (TDD)

## TLA+ vs other formal methods

# TLA+ vs Alloy

- Alloy defines a spec, like TLA+
- Alloy was designed to describe the structure of a system
- TLA+ was build for modeling concurrent systems
- Pro Alloy: generally easier to use
- But: Concurrency is more difficult to model

```

sig Time {}

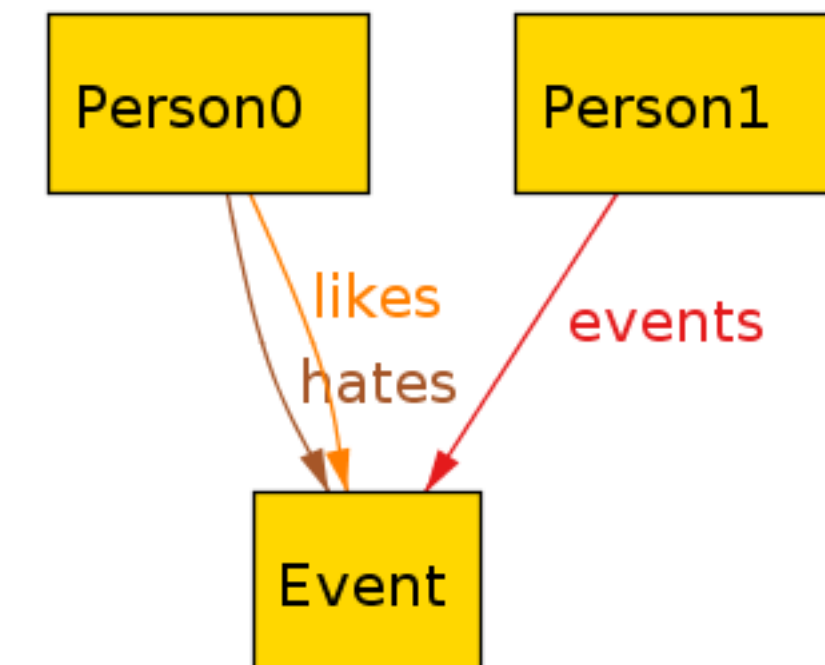
sig Key {}

sig Room {
  keys: set Key,
  current: keys one -> Time }

fact DisjointKeys {
  keys in Room lone -> Key }

```

Source: [cunha2016alloy]

Source: <https://www.hillelwayne.com/post/formally-modeling-migrations/>

```

/* e.g. KEYS = {1, 2, 3}, ROOMS = {r1, r2, r3}
CONSTANT KEYS, ROOMS

VARIABLES rooms, keys

TypeInv == /\ rooms \in SUBSET ROOMS
           /\ keys \in [rooms -> SUBSET Key]
           /\ \A r1, r2 \in rooms: (keys[r1] \cap keys[r2]) /= {} => r1 = r2

```

Equivalent TLA+ as seen in [cunha2016alloy]



## TLA+ vs other formal methods

# TLA+ vs Spark

- Spark is a programming language offering capabilities for proving program correctness
- Pro Spark: No implementation gap because it works directly on code
- But: Formally verifying code increases development time drastically (according to [klein2018fm] ~3.3 times of traditional development)

```
package Padding with SPARK_Mode is

-- add [Pad_Char] characters to the left of the string [S] so that its
-- length becomes [Len] if greater than [S'Length]
function Left_Pad (S : String; Pad_Char : Character; Len : Natural)
    return String
with Contract_Cases =>
    -- if the string is shorter than the desired length ...
    (Len > S'Length =>
        -- length of the result is [Len]
        Left_Pad'Result'Length = Len and then
        -- and looking at all characters in the result ...
        (for all I in Left_Pad'Result'Range =>
            Left_Pad'Result (I) =
                -- characters before those from [S] are equal to [Pad_Char]
                (if I <= Len - S'Length then
                    Pad_Char
                -- remaining characters are those from [S]
                else
                    S (I - (Len - S'Length + 1) + S'First))),
        -- if not, the result is equal to the input string [S]
        others => Left_Pad'Result = S);

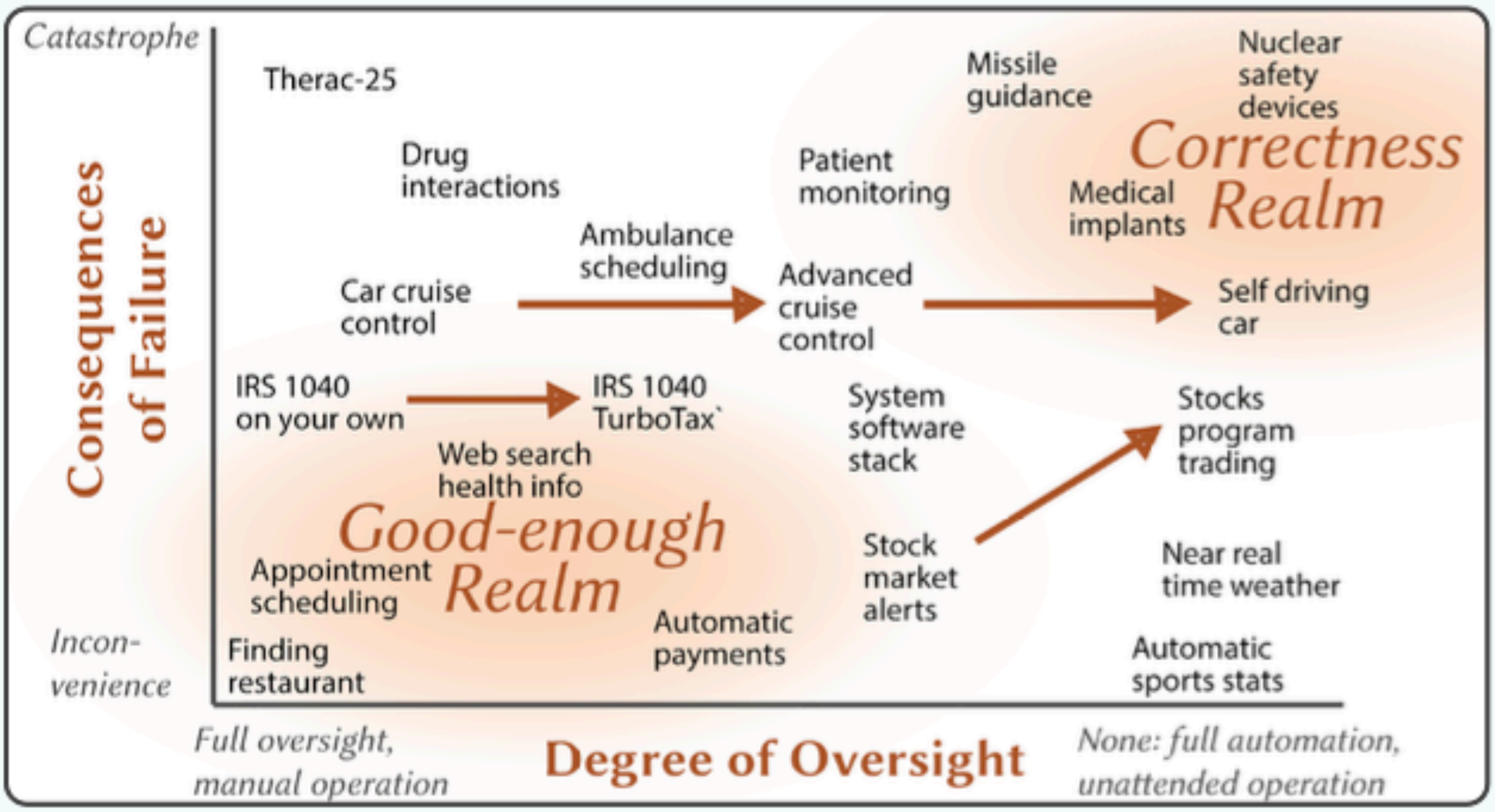
end Padding;
```

Source: [kanig2019spark]

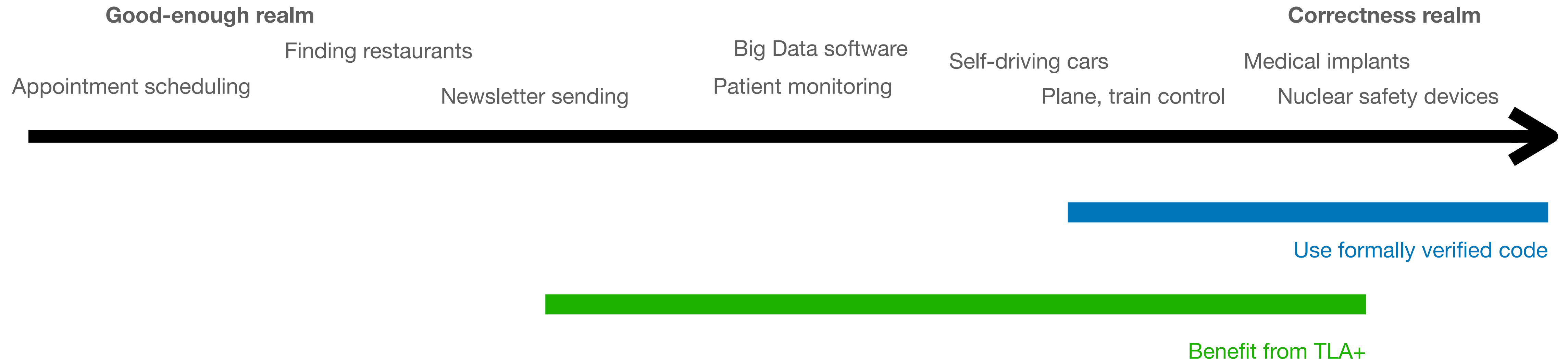


# When should you use TLA+?

“Good Enough is Often Good Enough”  
[shaw2022myths]



[shaw2022myths], p 22



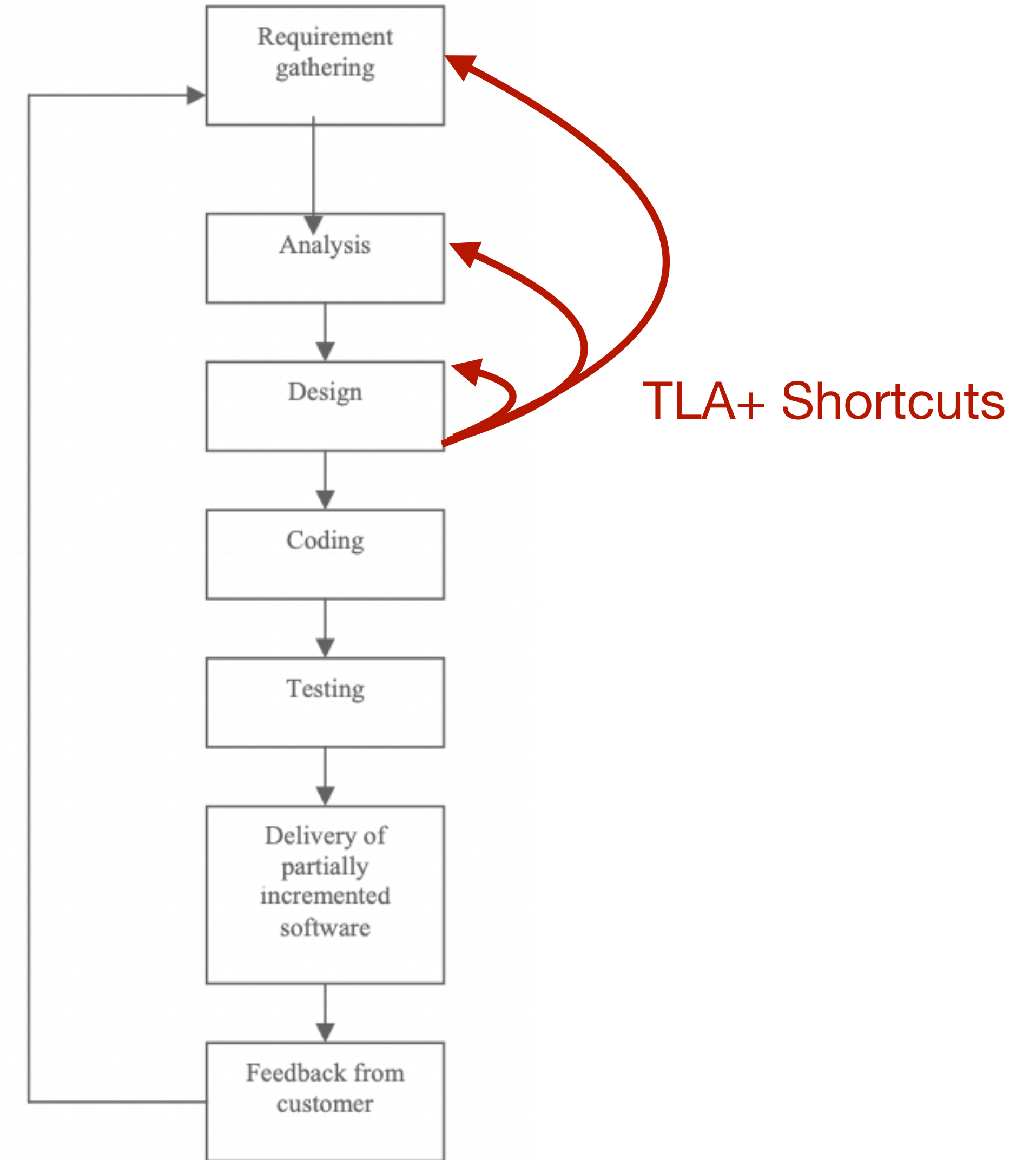
Characterized by:

- Software failure would lead to physical harm or sizable financial costs
- You need assurance that your high-level design works
- You are working on a concurrent system

**TLA+ is a tradeoff between formally verifying code and development speed**

# Summary

1. Formal specification languages like TLA+ provide a relatively easy way to check high-level concepts
2. It does that earlier in the development cycle than would otherwise be possible
3. As the model checker checks all states of specific cases exhaustively, you gain confidence in the correctness of your idea
4. My recommendation: Use lightweight formal methods like TLA+ whenever you are designing a system which is non-trivial



Agile Dev Cycle [sharma2012agile], adapted to TLA+



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# Appendix

# ***IMHO: How could this develop?***

**“The best way to predict the future is to invent it” :-)**

- Assumption:
  - Moore’s Law will make state explosions for many complex cases acceptable
  - Engineering is CAD  $\leftrightarrow$  SIM  $\leftrightarrow$  FAB & software engineering today is mostly FAB
- Formal methods give us insight into correctness and liveness properties
- Could they also inform implementation choices? Could they tell us what sorting algorithm would be most suitable? And maybe even give us an implementation?
- Could they detect what we are trying to do, and point us to better concepts? E.g. if “multiple processes change one variable”  $\rightarrow$  recommend appropriate async strategy (this would be proper CAD)

**“Weeks of programming can save hours of planning”**

**- Unknown**

“In other words, it's hard for a human being to look at the relatively short proofs used in standard mathematics and be able to determine whether they are good or not. So we can well imagine that if we had something wonderful that we don't currently have -- namely a great specification (meta)language -- that we might [...] know if the specification we write really specifies what we are after.

[...]

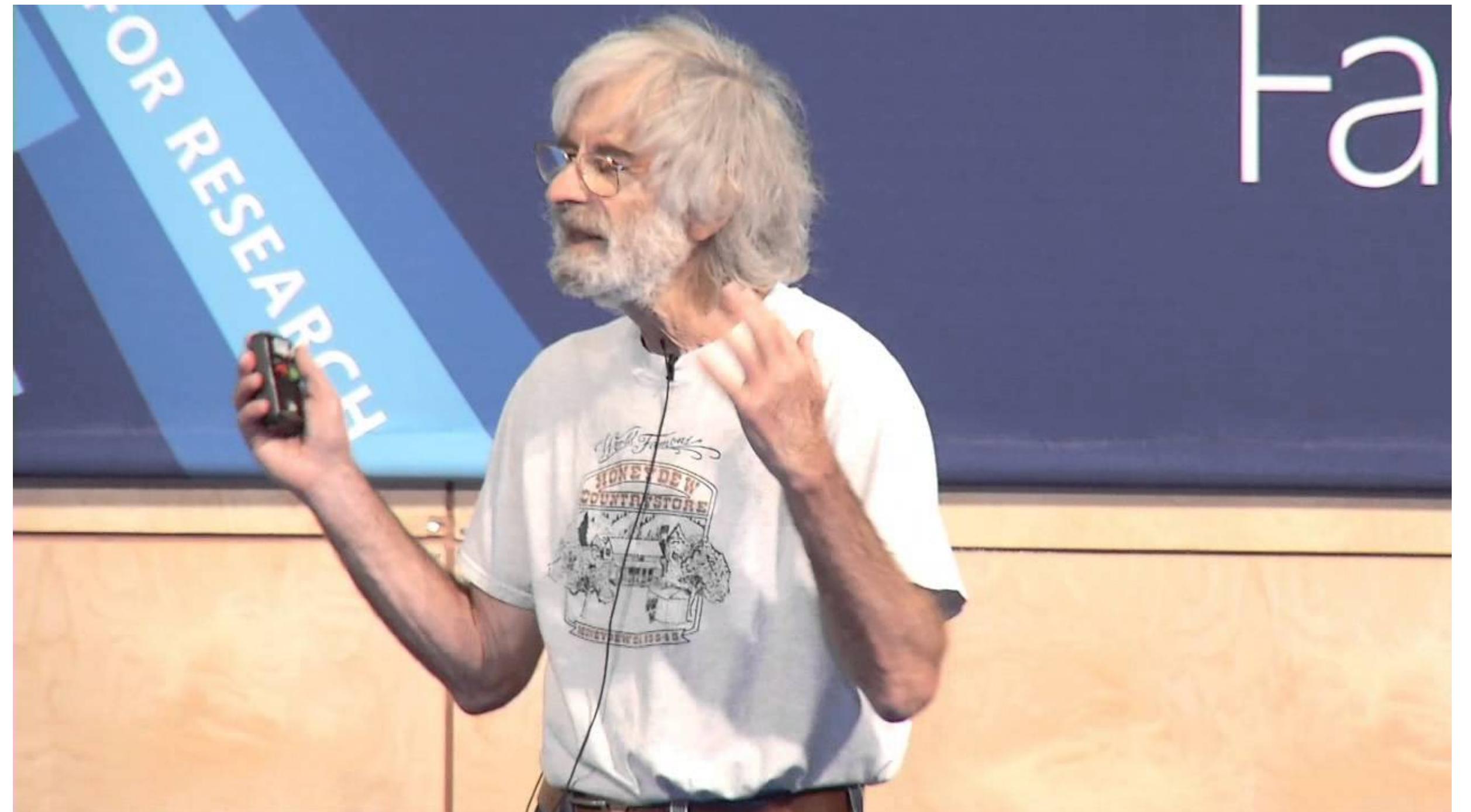
We can now think of the specification system as a kind of "active math" in that it is trying to be more like regular math than a standard programming language, but still has to be run and debugged to test it.”

## **Alan Kay, on formal methods in programming**

<https://www.quora.com/Is-using-classical-math-in-computing-a-big-mistake-Is-this-what-most-of-functional-programming-is-all-about>

# Lamport's take on Specification by Example

52:40



**Lamport's take on  
“Coding nowadays is  
mostly connecting  
existing software”  
55:18**

