Shift-Share Instrumental Variables (SSIV)

Based on Peter Hull's Lecture Notes

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Empirical Methods

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Roadmap

Roadmap

What is a (Linear) SSIV?

A weighted sum of a common set of shocks, with weights reflecting heterogeneous exposure shares:

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- Could be a "structural" equation or a potential outcomes model
- Could be misspecified, with heterogeneous treatment effects $\boldsymbol{\beta}_\ell$
- Could be a "reduced form" analysis, with $x_\ell = z_\ell$
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Key question: Under what assumptions does this SSIV strategy "work"?

SSIV Examples

$$\text{Instrument } z_\ell = \sum_n \underbrace{\boxed{s_{\ell n}}}^{\text{shares}} \overbrace{\boxed{g_n}}^{\text{shocks}} \quad \text{for model } y_\ell = \beta x_\ell + \gamma' w_\ell + \varepsilon_\ell$$

Bartik (1991); Blanchard and Katz (1992):

- β = inverse local labor supply elasticity
- x_ℓ and y_ℓ = employment and wage growth in region ℓ
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- $\boldsymbol{\beta}$ = inverse local labor supply elasticity
- x_ℓ and y_ℓ = employment and wage growth in region ℓ
- Need a labor demand shifter as an IV
- g_n = national growth of industry n
- $s_{\ell n}$ = lagged employment shares (of industry in a region)
- z_ℓ = predicted employment growth due to national industry trends

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"Enclave instrument", e.g., Card (2009)

- $\boldsymbol{\beta}$ = inverse elasticity of substitution between native and immigrant labor of some skill level (need a relative labor supply instrument)
- x_ℓ and y_ℓ = relative employment and wage in region ℓ
- g_n = national immigration growth from origin country n
- $s_{\ell n}$ = lagged shares of migrants from origin n in region ℓ
- z_ℓ = share of migrants predicted from enclaves & recent growth

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Hummels et al. (2014) on offshoring:

- β = effect of imports on wages
- x_ℓ = imports by Danish firm ℓ, y_ℓ = wages
- g_n = changes in transport costs by $n =$ (product, country)
- $s_{\ell n}$ = lagged import shares
- z_ℓ = predicted change in firm inputs via transport costs

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Recall IV validity condition: $E\left[\frac{1}{L}\sum_\ell z_\ell \boldsymbol{\varepsilon}_\ell\right] = 0$ for model residual $\boldsymbol{\varepsilon}_\ell$

- Looks a little different than normal because we're not assuming *i.i.d.* sampling, i.e.,

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What properties of shocks and shares make this condition hold?

- Is SSIV like a natural experiment? A diff-in-diff? Something new?
- Since z_ℓ combines multiple sources of variation, it can be difficult to think about it being randomly assigned across ℓ (unlike a lottery IV)

Roadmap

Exogenous Shocks in Industry-Level Regressions

Acemoglu-Autor-Dorn-Hanson-Price (AADHP, 2016) look at the effects of import competition with China on US manufacturing *industries*:

$$\Delta \log Emp_{nt} = \alpha + \beta \Delta IP_{nt} + \varepsilon_{nt},$$

where ΔIP_{nt} measures growth in import penetration from China in industry n, and ε_{nt} captures industry demand/productivity shocks

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Two Key Problems with OLS estimation:

1. Endogeneity of ΔIP_{nt}: OLS is not consistent for β
2. GE spillovers: β does not capture aggregate effects

Problem 1: Endogeneity of ΔIP_{nt}

$$\Delta \log Emp_{nt} = \alpha + \beta \Delta IP_{nt} + \varepsilon_{nt}$$

ΔIP_{nt} is driven by productivity shocks in China, but also potentially by productivity and demand shocks in the US

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AADHP instrument ΔIP_{nt} with ΔIPO_{nt}, measuring average Chinese import penetration growth in 8 non-US countries

- **Relevance**: both ΔIP_{nt} and ΔIPO_{nt} are driven by the same Chinese productivity shocks
- **Validity**: local productivity/demand shocks in the US are uncorrelated with those of other countries (entering ΔIPO_{nt})

Identification from a Natural Experiment

Suppose ΔIPO_{nt} is as-good-as-randomly assigned, as in a RCT:

$$E[\Delta IPO_{nt} \mid \mathbb{I}] = \boldsymbol{\mu} \qquad \text{for all } n, t$$

where $\mathbb{I} = \{\boldsymbol{\varepsilon}_{nt}, \text{pre-trends, balance variables}, \ldots\}$

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Consistent IV estimation then follows from many observations of nt, with sufficiently independent variation in ΔIPO_{nt}

Identification from a Natural Experiment

Can relax to add observables capturing systematic variation:

$$E[\Delta IPO_{nt} \mid \mathbb{I}] = q'_{nt}\boldsymbol{\mu} \qquad \text{for all } n, t$$

where q_{nt} may include:

- period FE, isolating within-period variation in the shocks
- FE of 10 broad sectors, isolating within-sector variation, etc.

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We would then just want to control for q_{nt} in the industry-level IV

Problem 2: GE Spillovers

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- When employment shrinks in industry n after a negative shock, aggregate employment may or may not respond
- In a flexible labor market, comparing wages of similar workers across industries does not make sense

Problem 2: GE Spillovers

ADH Solution: specify the outcome equation for local labor markets

- Works if local economies are isolated "islands" (simple model in Adao-Kolesar-Morales 2019; richer structure of spatial spillovers in Adao-Arkolakis-Esposito 2020)

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- Works if local economies are isolated "islands" (simple model in Adao-Kolesar-Morales 2019; richer structure of spatial spillovers in Adao-Arkolakis-Esposito 2020)

But correct specification is not the same as identification!

- **Key point**: the same industry-level natural experiment can be used to estimate a regional specification, via SSIV

Borusyak, Hull, and Jaravel (BHJ; 2022)

Consider the SSIV estimator of $y_\ell = \boldsymbol{\beta} x_\ell + \boldsymbol{\gamma}' w_\ell + \boldsymbol{\varepsilon}_\ell$ instrumented by $z_\ell = \sum_n s_{\ell n} g_n$ and, for now, $\sum_n s_{\ell n} = 1$ for all ℓ

- Reduced-form allowed: $x_\ell = z_\ell$
- Only the shift-share structure of z_ℓ matters; x_ℓ can be anything
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E.g., $g_n = \Delta IPO_n$ aggregated w/mfg employment shares $s_{\ell n}$

- Can we leverage a natural experiment in g_n, as before?

Leveraging g_n

Shift-Share Estimand

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First step: note that by the Frisch–Waugh–Lovell theorem, the estimator can be written

$$\hat{\beta} = \frac{\sum_\ell z_\ell \tilde{y}_\ell}{\sum_\ell z_\ell \tilde{x}_\ell} = \frac{\sum_\ell \sum_n s_{\ell n} g_n \tilde{y}_\ell}{\sum_\ell \sum_n s_{\ell n} g_n \tilde{x}_\ell}$$

where $\tilde{v}_\ell$ denotes sample residuals from regressing v_ℓ on w_ℓ. Proof

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BHJ Numerical Equivalence

BHJ show $\hat{\beta}$ can be obtained from a shock-level IV procedure that uses g_n to instrument for a shock-level "aggregate" of the treatment:

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where $s_n = \frac{1}{L}\sum_\ell s_{\ell n}$ are weights capturing the average importance of shock n, and $\bar{v}_n = \frac{\sum_\ell s_{\ell n} v_\ell}{\sum_\ell s_{\ell n}}$ is an exposure-weighted average of v_ℓ

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$$\hat{\beta} = \frac{\sum_n s_n g_n \bar{\bar{y}}_n}{\sum_n s_n g_n \bar{\bar{x}}_n}$$

The IV estimate from the original "location-level" IV procedure is equivalent to a "industry-level" IV regression with model $\bar{\bar{y}}_n = \alpha + \bar{\bar{x}}_n \beta + \bar{\varepsilon}_n$ instrumented by g_n with weights s_n.

The residual $\bar{\varepsilon}_n$ of this shock-level IV procedure is the average residual of observations with a high share of n

- E.g. in ADH, the average unobserved determinants of regional employment in regions most specialized in industry n

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It follows that $\hat{\beta}$ is consistent iff this shock-level IV procedure is...

BHJ Baseline Assumptions

A1 (Quasi-random shock assignment): $E[g_n \mid \bar{\boldsymbol{\varepsilon}}, s] = \boldsymbol{\mu}$, for all n

- Each shock has the same expected value, conditional on the shock-level unobservables $\bar{\boldsymbol{\varepsilon}}_n$ and average exposure s_n

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- Each shock has the same expected value, conditional on the shock-level unobservables $\bar{\varepsilon}_n$ and average exposure s_n
- Implies SSIV exogeneity, as $z_\ell = \mu + \sum_n s_{\ell n}(g_n - \mu) = \mu +$ "noise"

BHJ Baseline Assumptions

A2 (Many uncorrelated shocks):

- $E\left[\sum_n s_n^2\right] \to 0$: expected Herfindahl index of average shock exposure converges to zero (implies $N \to \infty$)
- $Cov(g_n, g_{n'} \mid \bar{\varepsilon}, s) = 0$ for all $n' \neq n$: shocks are mutually uncorrelated given the unobservables

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Both assumptions, while novel for SSIV, would be standard for a shock-level IV regression with weights s_n and instrument g_n

BHJ Extensions

Conditional Quasi-Random Assignment: $E[g_n \mid \bar{\varepsilon}, q, s] = q_n' \mu$ for some observed shock-level variables q_n

- Consistency follows when $w_\ell = \sum_n s_{\ell n} q_n$ is controlled for in the IV

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Estimated Shocks: $g_n = \sum_\ell w_{\ell n} g_{\ell n}$ proxies for an infeasible g_n^*

- Consistency may require a "leave-out" adjustment: $z_\ell = \sum_\ell s_{\ell n} \tilde{g}_{\ell n}$ for $\tilde{g}_{\ell n} = \sum_{\ell' \neq \ell} \omega_{\ell' n} g_{\ell' n}$ (akin to JIVE solution to many-IV bias)

BHJ Extensions (cont.)

Panel Data: Have $(y_{\ell t}, x_{\ell t}, s_{\ell nt}, g_{nt})$ across $\ell = 1, \ldots, L$, $t = 1, \ldots, T$

- Consistency can follow from either $N \to \infty$ or $T \to \infty$
- Unit fixed effects "de-mean" the shocks, if $s_{\ell nt}$ are time-invariant

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Heterogeneous Effects: LATE theorem logic goes through

- Under a first-stage monotonicity condition, SSIV identifies a convex weighted average of heterogeneous treatment effects

Practical Consideration 1: Incomplete Shares

The Problem

So far we have assumed a constant sum-of-shares: $S_\ell \equiv \sum_n s_{\ell n} = 1$

- But in some settings, S_ℓ varies across ℓ
- E.g. in ADH, S_ℓ is region ℓ's share of non-manufacturing emp.

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BHJ show that **A1**/**A2** are not enough for validity of z_ℓ in this case

- Now $z_\ell = \sum_n s_{\ell n}(\boldsymbol{\mu} + (g_n - \boldsymbol{\mu})) = \boldsymbol{\mu} S_\ell + \sum_n s_{\ell n}(g_n - \boldsymbol{\mu})$
- So z_ℓ is mechanically correlated with S_ℓ, which may be endogenous

E.g. in ADH, Comparing locations with larger and smaller z_ℓ could be comparing places with larger vs. smaller manufacturing employment (e.g. Midwest vs. South)

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$$z_\ell = \sum_n s_{\ell n} (\boldsymbol{\mu} + (g_n - \boldsymbol{\mu})) = \boldsymbol{\mu} S_\ell + \underbrace{\sum_n s_{\ell n}(g_n - \boldsymbol{\mu})}_{\text{Clean Shock Variation}}$$

Controlling for the sum-of-shares S_ℓ isolates clean shock variation

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Controlling for the sum-of-shares S_ℓ isolates clean shock variation

- Further controls are needed when **A1** only holds conditional on q_n; e.g. in panels, S_ℓ should be interacted with time FE:

$$z_{\ell t} = \sum_n s_{\ell n} (\mu_t + (g_{nt} - \mu_t)) = \mu_t S_\ell + \underbrace{\sum_n s_{\ell n}(g_{nt} - \mu_t)}_{\text{Clean Shock Variation}}$$

Practical Consideration 2: Exposure Clustering

The Problem

Adão, Kolesar, and Morales (2019) study a novel inference challenge when SSIV identification leverages quasi-random shocks

- Observations with similar shares $s_{\ell 1,}, \ldots, s_{\ell N}$ are likely to have correlated z_ℓ, even when observations are not "clustered" in conventional ways (e.g., by distance)

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They then derive a new CLT + SEs to address "exposure clustering"

- "Design-based": leverage *iid*ness of shocks, not observations

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BHJ use similar logic to show robust/clustered SEs can be valid when $\hat{\beta}$ is given by estimating the 'industry-level' regression

$$\bar{\bar{y}}_n = \alpha + \beta \bar{\bar{x}}_n + q_n' \tau + \bar{\varepsilon}_n^{\perp},$$

instrumenting $\bar{\bar{x}}_n$ by g_n and weighting by s_n

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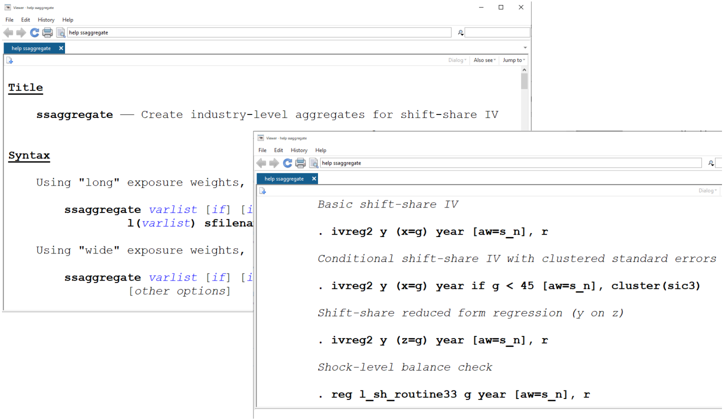
Same logic applies to performing valid balance/pre-trend tests and evaluating first-stage strength of the instrument

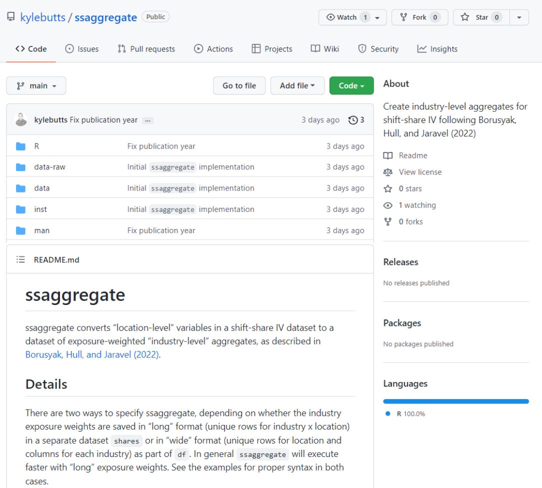
SSIV with ssaggregate

Stata package `ssaggregate` leverages the BHJ equivalence result: it translates data to the shock level, after which researchers can proceed with familiar estimation commands (install w/ `ssc install ssaggregate`)



SSIV with ssaggregate...in R!

ssaggregate is now available in R!



Download at https://github.com/kylebutts/ssaggregate

Application: “The China Shock”

ADH study the effects of rising Chinese import competition on US commuting zones, 1991-2000 and 2000-2007

- Treatment x_ℓ: local growth of Chinese imports in \$1,000/worker (slightly different from AADHP and ADHS)
- Main outcome y_ℓ: local change in manufacturing emp. share

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To address endogeneity challenge, use a SSIV $z_{\ell t} = \sum_n s_{\ell nt} g_{nt}$

- n: 397 SIC4 manufacturing industries ($\times$ 2 periods)
- g_{nt}: growth of Chinese imports in non-US economies per US worker
- $s_{\ell nt}$: lagged share of mfg. industry n in *total* emp. of location ℓ

ADH Revisited

BHJ show how ADH can be seen as leveraging quasi-random shocks

- *Ex ante* plausible: imagine random industry productivity shocks in China affecting imports in U.S. & elsewhere

ADH Revisited

*Plausability of **A1/A2***

Evaluate **A1** by regional and industry-level balance tests

- Industry shocks are uncorrelated with observables

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Check sensitivity to adjusting for potential industry-level confounders:

- Control for $w_{\ell t} = \sum_n s_{\ell nt} q_{nt}$, where q_{nt} include period FE, sector FE, the Acemoglu et al. (2016) observables, ...

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Evaluate **A2** by studying variation across industries

- Effective sample size (1/HHI of s_n weights): 58-192
- Shocks appear mutually uncorrelated across SIC3 sectors

BHJ do ADH: Shock-Level Balance

Table 3: Shock Balance Tests in the Autor et al. (2013) Setting

Balance variable	Coef.	SE
Production workers' share of employment, 1991	-0.011	(0.012)
Ratio of capital to value-added, 1991	-0.007	(0.019)
Log real wage (2007 USD), 1991	-0.005	(0.022)
Computer investment as share of total, 1990	0.750	(0.465)
High-tech equipment as share of total investment, 1990	0.532	(0.296)
# of industry-periods	794	

No significant correlations between shocks and industry observables, controlling for year fixed effects

BHJ do ADH: Manufacturing Employment

Table 4: Shift-Share IV Estimates of the Effect of Chinese Imports on Manufacturing Employment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Coefficient	-0.596	-0.489	-0.267	-0.314	-0.310	-0.290	-0.432
	(0.114)	(0.100)	(0.099)	(0.107)	(0.134)	(0.129)	(0.205)
Regional controls							
Autor et al. (2013) controls	✓	✓	✓		✓	✓	✓
Start-of-period mfg. share	✓						
Lagged mfg. share		✓	✓	✓	✓	✓	✓
Period-specific lagged mfg. share			✓	✓	✓	✓	✓
Lagged 10-sector shares					✓		✓
Local Acemoglu et al. (2016) controls						✓	
Lagged industry shares							✓
SSIV first stage F-stat.	185.6	166.7	123.6	272.4	64.6	63.3	27.6
# of region-periods	1,444	1,444	1,444	1,444	1,444	1,444	1,444
# of industry-periods	796	794	794	794	794	794	794

Roadmap

The Mariel Boatlift as a Basic SSIV

Card (1990) leverages a big migration “push” of low-skilled workers from Cuba to Miami, a Cuban-enclave.

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- Need parallel trends: regions with more/fewer Cuban workers on similar employment trends

This can be viewed as a simple shift-share instrument:

$$s_{\ell,\mathrm{Cuba}} \equiv s_{\ell,\mathrm{Cuba}} \cdot 1 + \sum_{n \neq \mathrm{Cuba}} s_{\ell n} \cdot 0$$

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If several migration origins had a push shock, we can pool them together with a more traditional SSIV...

Goldsmith-Pinkham, Sorkin, and Swift (GPSS; 2020)

GPSS view the set of n and values of g_n as fixed, so $z_\ell = \sum_n s_{\ell n} g_n$ is a linear combination of shares

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They then also establish a numerical equivalence: $\hat{\beta}$ can be obtained from an overidentified IV procedure that uses N share instruments $s_{\ell n}$ and a weight matrix based on the shocks g_n

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Sufficient identifying assumption: shares $s_{\ell n}$ are exogenous for each n (like parallel trends when $\boldsymbol{\varepsilon}_\ell$ are unobserved trends)

$$E[\boldsymbol{\varepsilon}_\ell \mid s_{\ell n}] = 0, \ \forall n$$

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Sufficient identifying assumption: shares $s_{\ell n}$ are exogenous for each n (like parallel trends when $\boldsymbol{\varepsilon}_\ell$ are unobserved trends)

$$E[\boldsymbol{\varepsilon}_\ell \mid s_{\ell n}] = 0,\ \forall n \implies E[\textstyle\sum_\ell z_\ell \boldsymbol{\varepsilon}_\ell] = \displaystyle\sum_\ell \sum_n g_n E[s_{\ell n}] E[\boldsymbol{\varepsilon}_\ell \mid s_{\ell n}] = 0$$

This is N moment conditions at the level of observations, e.g. 38 for Card and 397 for ADH (vs. just 1 in BHJ, at the level of industries)

In other words, GPSS show that the SSIV estimator can be seen as pooling many Boatlift-style diff-in-diff IVs, one for each industry

Rotemberg Weights

How does SSIV pool different diff-in-diffs?

- GPSS propose "opening the black box" of overidentified IV by deriving the weights SSIV implicitly puts on each share instrument
- Builds on Rotemberg (1983), so they call these "Rotemberg weights"

$$\hat{\beta} = \sum_n \hat{\alpha}_n \hat{\beta}_n, \text{ where } \quad \underbrace{\hat{\beta}_n = \frac{\sum_\ell s_{\ell n} \tilde{y}_\ell}{\sum_\ell s_{\ell n} \tilde{x}_\ell}}_{n\text{-specific IV estimate}} \quad \text{and } \underbrace{\hat{\alpha}_n = \frac{g_n \sum_\ell s_{\ell n} \tilde{x}_\ell}{\sum_{n'} g_{n'} \sum_\ell s_{\ell n'} \tilde{x}_\ell}}_{\text{Rotemberg weight}}$$

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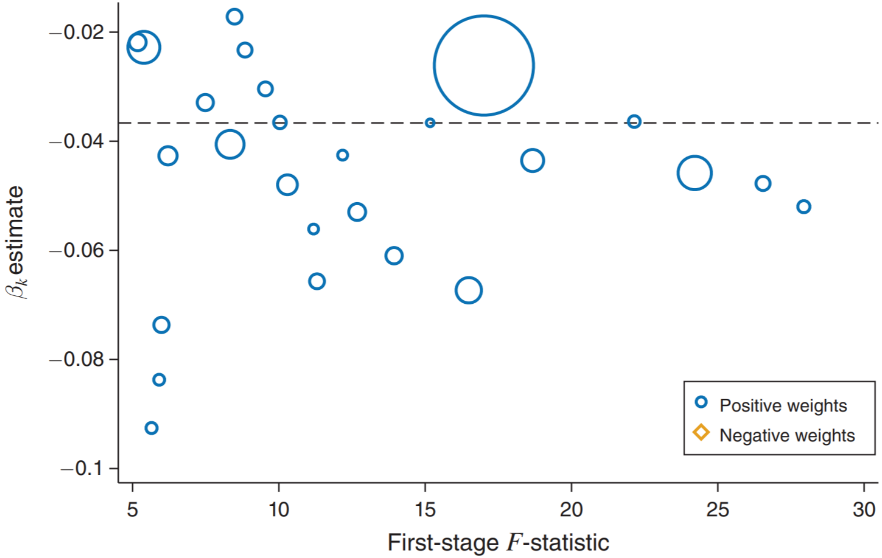
Intuitively, more weight is given to share instruments with more extreme shocks g_n and larger first stages $\sum_\ell s_{\ell n} \tilde{x}_\ell$

- Weights can be negative (potential issue w/heterogeneous effects)

Rotemberg Weights in Card (2009)



Is Share Exogeneity Plausible?

Share exogeneity assumption is **not** that “shares don’t causally respond to the residual” (they can’t: shares are pre-determined)

- It’s: “all unobservables are uncorrelated with anything about the local share distribution”

Is Share Exogeneity Plausible?

This sufficient condition is typically violated when there are *any* unobserved shocks ν_n that affect ε_ℓ via the same or correlated shares

- I.e. if $\varepsilon_\ell = \sum_n s_{\ell n}\nu_n + \tilde{\varepsilon}_\ell$, then $s_{\ell n}$ and ε_ℓ cannot be uncorrelated in large samples—even if ν_n are uncorelated with g_n
- E.g. in ADH, unobserved technology shocks across industries affect labor markets via lagged emp. shares, along with observed g_n
- Problem arises when shares are "generic" – predicting many things

Card and ADH Revisited

When share exogeneity is *ex ante* plauible, can test its assumptions *ex post* (focusing on high Rotemberg weight n):

- Balance/pre-trend tests
- Overidentification tests (under constant effects)
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GPSS find that balance/overidentification tests broadly pass for Card
... but fail badly for ADH, consistent with *ex ante* implausibility

Roadmap

A Taxonomy of SSIV Settings

Case 1 the IV is based on a set of shocks which can be thought of as an instrument (i.e. many, plausibly quasi-randomly assigned)

- BHJ shows how this identifying variation can be mapped to estimate effects at a different “level” (i.e. industries $\rightarrow$ local labor markets)

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Case 2 the researcher does not directly observe many quasi-random shocks, but can estimate them in-sample

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Case 3 the g_n cannot be naturally viewed as an instrument

- Either too few or implausibly exogenous, even given some q_n.
- Identification may (or may not) instead follow from share exogeneity

Ex Ante vs. Ex Post Validity

BHJ emphasize that the decision to pursue a “shocks” vs. “shares” identification strategy must be made *ex ante*

- Undesirable to base identifying assumptions on *ex post* tests, though balance/pre-trend tests can be used to falsify assumptions
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They suggest thinking about whether shares are "tailored" to the economic question/treatment, or are "generic"

- Generic shares (e.g. ADH): unobserved $\boldsymbol{\nu}_n$ are likely to enter $\boldsymbol{\varepsilon}_\ell$ via the same or similar shares, violating share exogeneity
- Tailored shares have a diff-in-diff feel; don't even need the shocks, except to possibly improve power or avoid many-IV bias

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Many treatments/instruments are SSIV-like: combining multiple sets of variation, w/ some as-good-as-randomly assigned, but not all:

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How can we just leverage the exogenous shocks to such z_i?

Borusyak and Hull (BH, 2023): Main Points

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   - Conventional solutions (e.g. directly instrumenting with shocks or controlling for all features of exposure) are often infeasible
3. Recentering solution also can have attractive efficiency properties
   - Leverages non-random exposure to best predict shock effects

(Some) Other Settings where these Points are Relevant

Linear shift-share IV (Autor et al. 2013, Borusyak et al. 2022)

Nonlinear shift-share IV (Boustan et al. 2013, Berman et al. 2015, Chodorow-Reich and Wieland 2020, Derenoncourt 2021)

IV based on centralized school assignment mechanisms (Abdulkadiroğlu et al. 2017, 2019, Angrist et al. 2020)

Model-implied optimal IV (Adão-Arkolakis-Esposito 2021)

Weather instruments (Gomez et al. 2007, Madestam et al. 2013)

"Free space" instruments for mass media access (Olken 2009, Yanagizawa-Drott 2014)

Example 1: Market Access Effects via RCT

Theory suggests transportation upgrades affect local outcomes (e.g. land value) of regions i by increasing their market access (MA):

$$\Delta \log V_i = \beta \Delta \log MA_i + \varepsilon_i,$$

$$\text{where } MA_{it} = \sum_j \tau(g_t, loc_i, loc_j)^{-1} pop_j,$$

for road network g_t in periods $t = 1,2$, region locations loc_j (co-determining travel cost τ), and regional population pop_j

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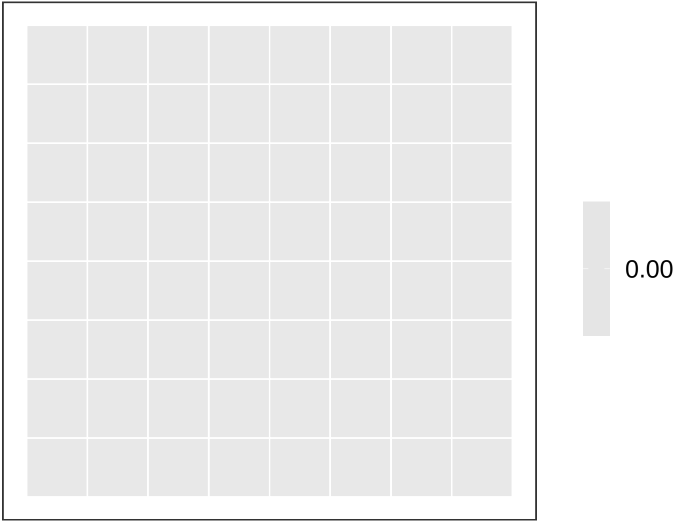
No. Randomizing roads $\not\Rightarrow$ randomizing MA due to them!

Illustration: Market Access on a Square Island

Start from no roads, assume equal population everywhere



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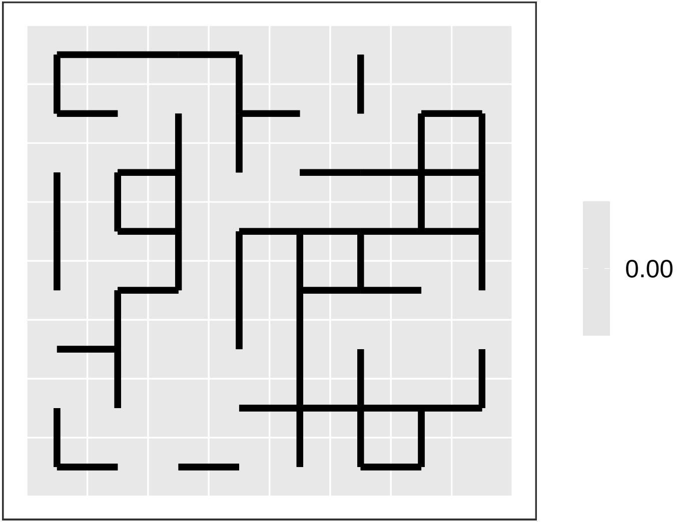

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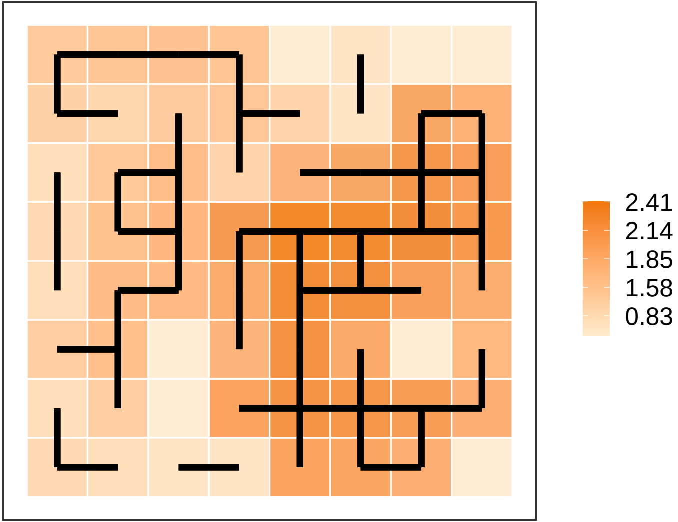
Randomly connect adjacent regions by road and compute MA growth



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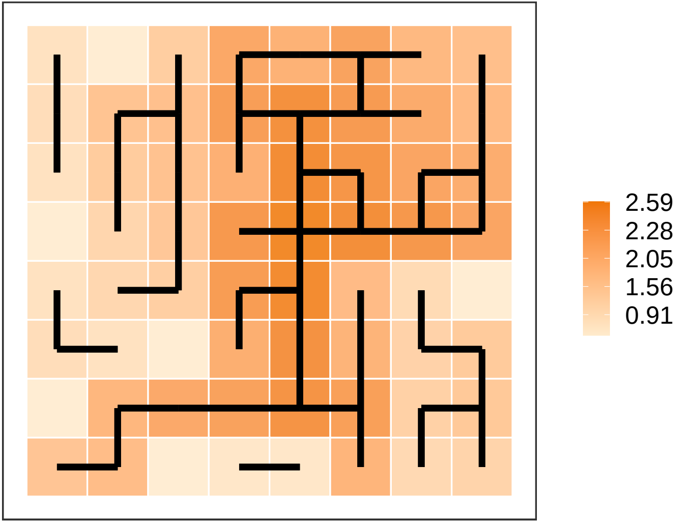

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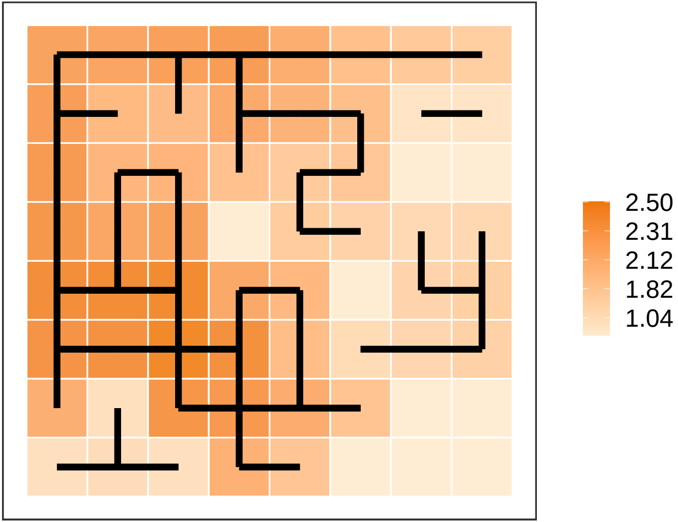


Expected Market Access Growth μ_i

Some regions get systematically more MA

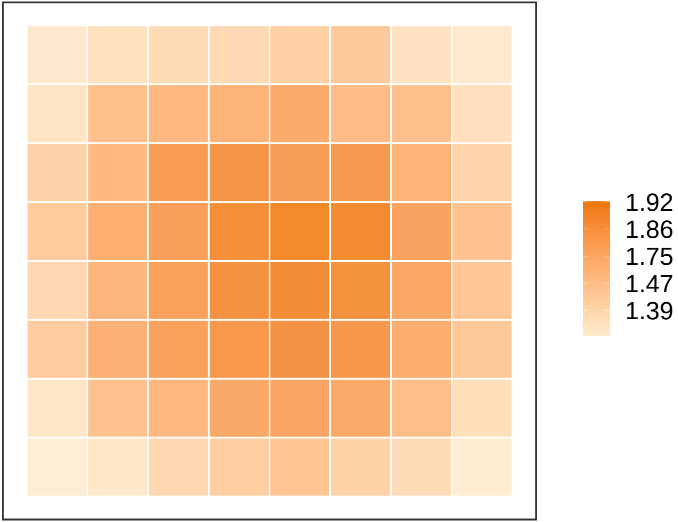


Illustration: High-Speed Rail in China

149 lines were built or planned (as of April 2019)

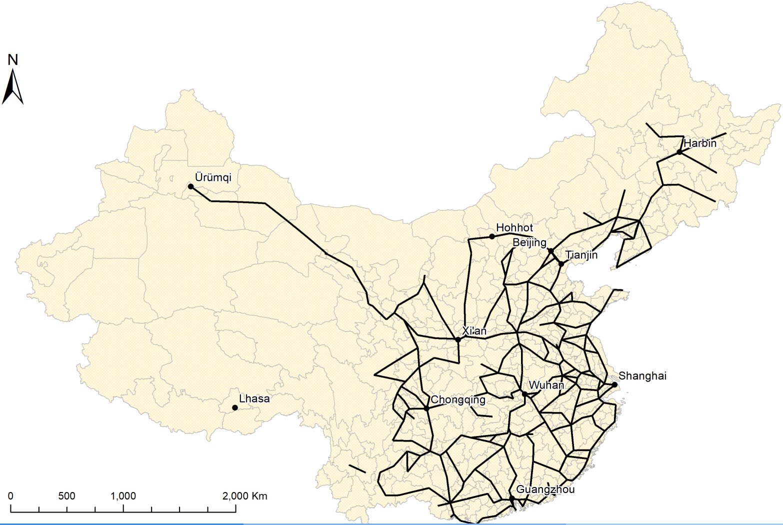
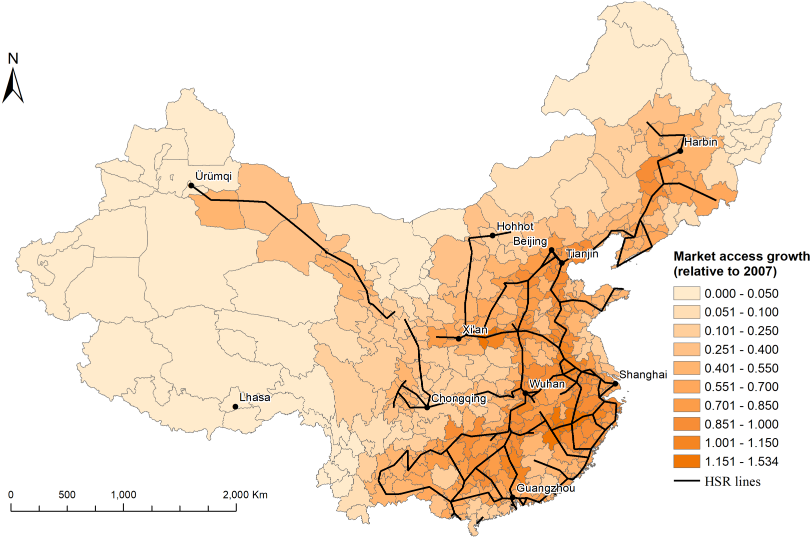


Illustration: High-Speed Rail in China

The 83 lines actually built by 2016. Suppose timing is random



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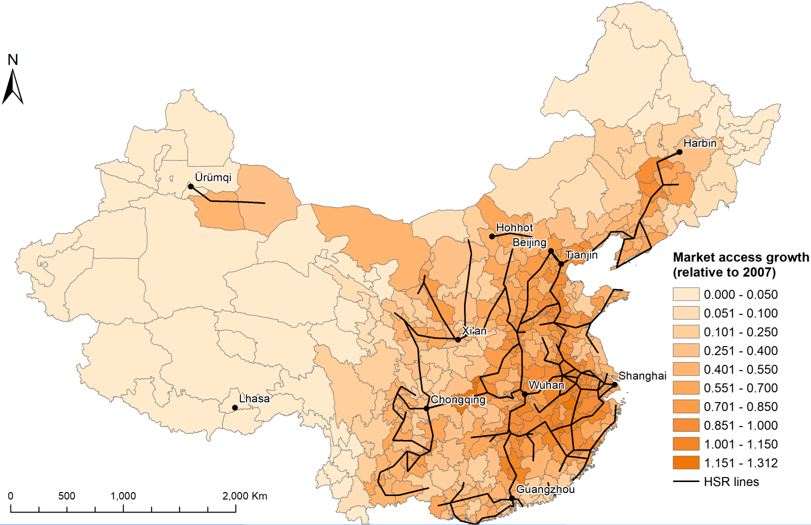
A counterfactual draw of 83 lines by 2016



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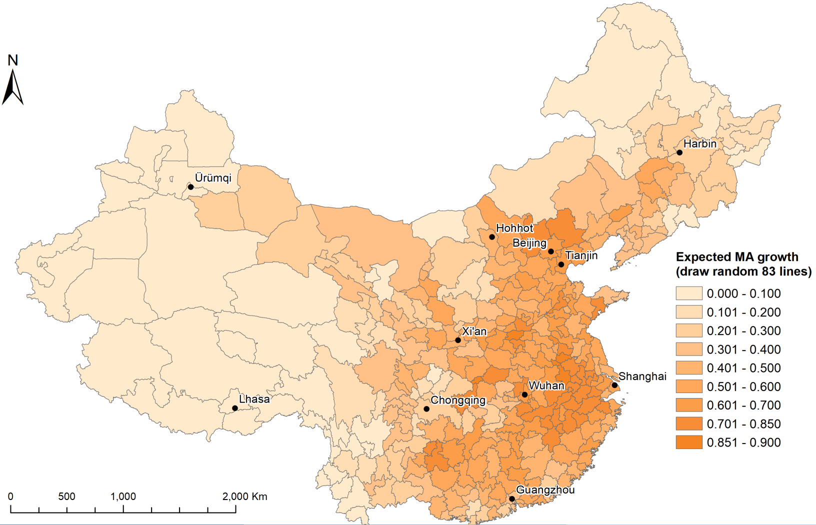
Expected MA growth, μ_i



OVB and Recentering Solution

Systematic variation in MA growth can generate OVB

- E.g. land values fall in the periphery because of rising sea levels
- More vs less developed Chinese regions may be on different trends

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- More vs less developed Chinese regions may be on different trends

Systematic variation can be removed via "recentering":

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OVB and Recentering Solution

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Recentered MA is a valid instrument for realized MA growth

- Compares MA from actual and counterfactual shocks
- As it turns out, we can also control for expected MA growth

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Classic SSIV is a special case where $z_i = \sum_n s_{in} g_n$ is linear in the exogenous shocks

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- Controlling for $\sum_n s_{in} q_n$ is enough (sound familiar?)

Example 2: Effects of Program Eligibility

Consider the effects of individual eligilibity x_i for Medicaid:

$$y_i = \beta x_i + \varepsilon_i$$

where x_i is determined by i's state policy g_{state_i} and demographics

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Standard "simulated instruments" solution (Currie and Gruber (1996)): use state-level variation (avgerage policy generosity across a "simulated" group of individuals) as a single IV for x_i

- This works, but is likely inefficient: the policy shocks likely have heterogeneous effects across individuals w/different demos

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- Yields efficiency gain by better first-stage prediction, e.g. by removing i who are always or never eligible

General Setup

We have a model of $y_i = \beta x_i + \varepsilon_i$ for a fixed population $i = 1 \dots N$

- In the paper: extensions to heterogeneous effects, other controls, multiple treatments, nonlinear outcome models, panel data...

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We have a candidate instrument $z_i = f_i(g, w)$, where g is a vector of shocks; w collects predetermined variables; $f_i(\cdot)$ are known mappings

- Applies to any z_i which can be constructed from observed data
- Nests reduced-form regressions: $x_i = z_i$
- Allows $g = (g_1, \dots, g_K)$ to vary at a different level than i

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Assumptions:

1. Shocks are exogenous: $g \perp \varepsilon \mid w$
2. Conditional distribution $G(g \mid w)$ is known (e.g. via randomization protocol or uniform across permutations of g)

Main Results

The expected instrument, $\boldsymbol{\mu}_i = E[f_i(g, w) \mid w] \equiv \int f_i(g, w) dG(g \mid w)$, is the sole confounder generating OVB:

$$E\left[\frac{1}{N}\sum_i z_i \boldsymbol{\varepsilon}_i\right] = E\left[\frac{1}{N}\sum_i \boldsymbol{\mu}_i \boldsymbol{\varepsilon}_i\right] \neq 0, \text{ in general}$$

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Regressions which control for $\boldsymbol{\mu}_i$ also identify $\boldsymbol{\beta}$ (implicitly recenter, by the FWL theorem)

Extensions

Consistency: follows when $\tilde{z}_i$ is weakly mutually dependent across i

Robustness to heterogeneous treatment effects: $\tilde{z}_i$ identifies a convex avg. of $\boldsymbol{\beta}_i$ under appropriate first-stage monotonicity

Randomization inference provides exact confidence intervals for $\boldsymbol{\beta}$ (under constant effects) and falsification tests

BH also characterize the **asy. efficient** recentered IV among all $f_i(\cdot)$

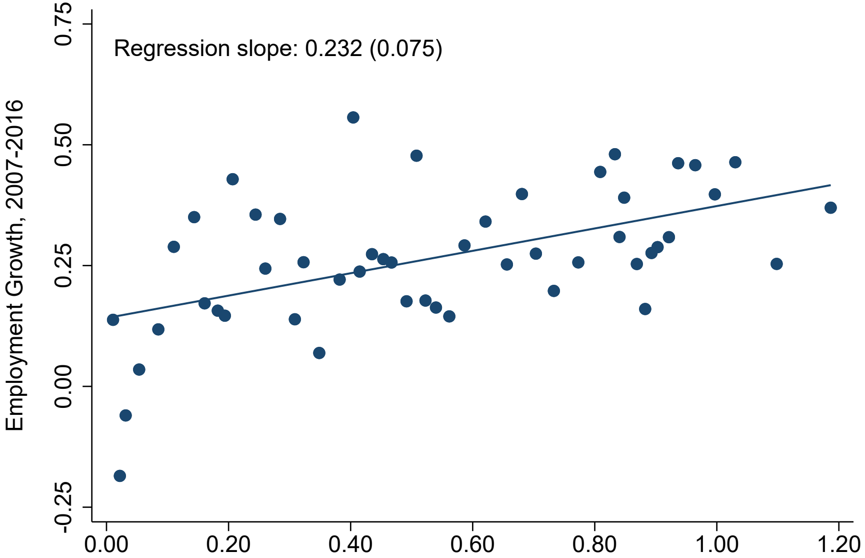
App. 1: Market Access from Chinese High-Speed Rail

BH first show how instrument recentering can address OVB when estimating the effects of market access growth

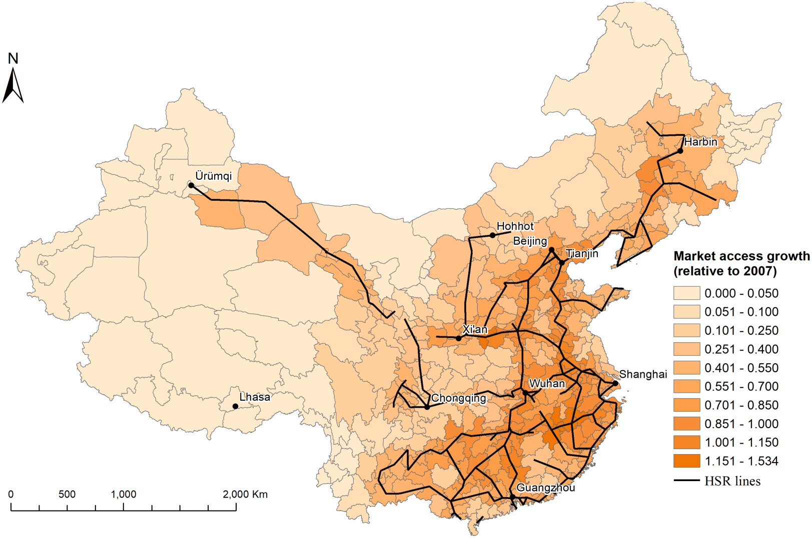
Setting: Chinese HSR; 83 lines built 2008–2016, 66 yet unbuilt

- Market access: $MA_{it} = \sum_k \exp(-0.02\tau_{ikt})\, p_{k,2000}$, where τ_{ikt} is HSR-affected travel time between prefecture capitals (Zheng and Kahn, 2013) and $p_{i,2000}$ is prefecture i's population in 2000
- Relate to employment growth in 274 prefectures, 2007-2016

Simple OLS Regressions Suggest a Large MA Effect



High vs. Low MA Growth is Not a Convincing Contrast!



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Add controls (province FE, longitude, etc...)

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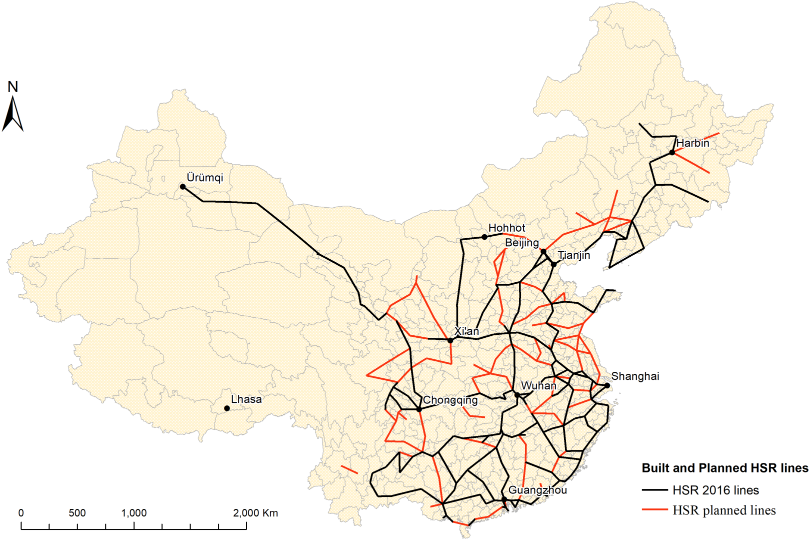
Find valid contrasts for *one* source of variation—a natural experiment

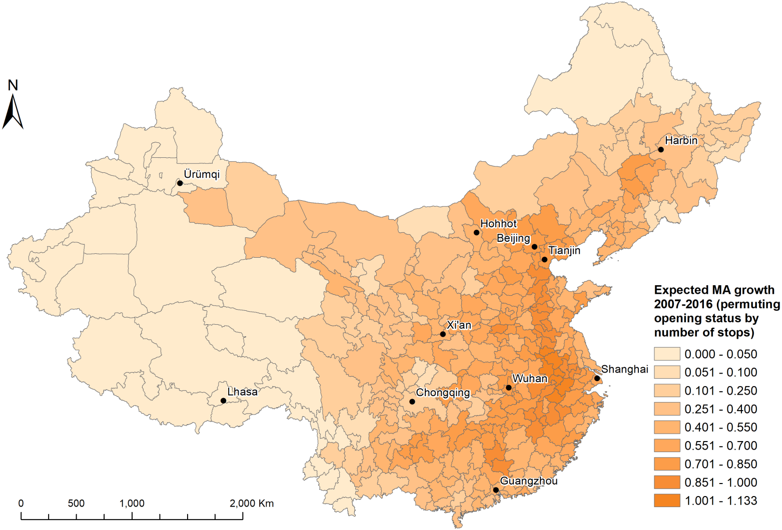
- Bartelme (2018): shocks affecting market size
- Donaldson (2018): built vs unbuilt lines
- BH application: assume random timing of observably similar lines

Built and Planned HSR Lines

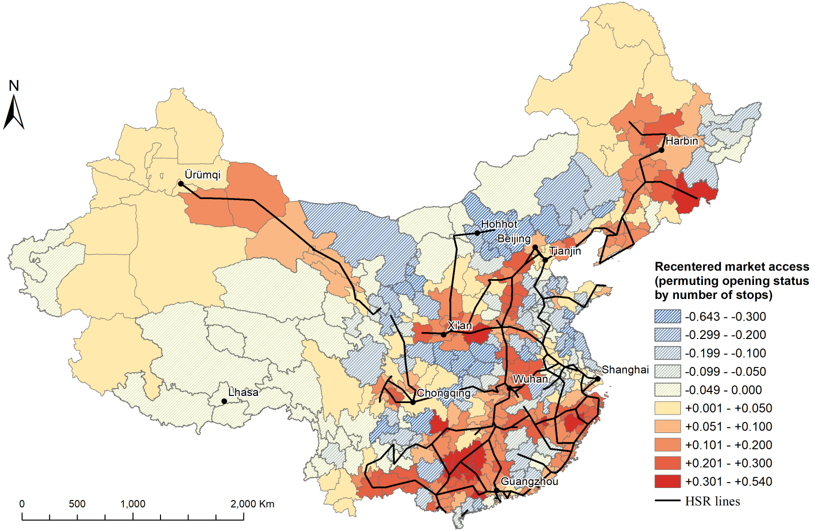
BH reshuffle built & planned lines connecting the same # of regions



Expected Market Access Growth (2007–2016), μ_i



Recentered Market Access Growth (2007–2016), $\tilde{z}_i$



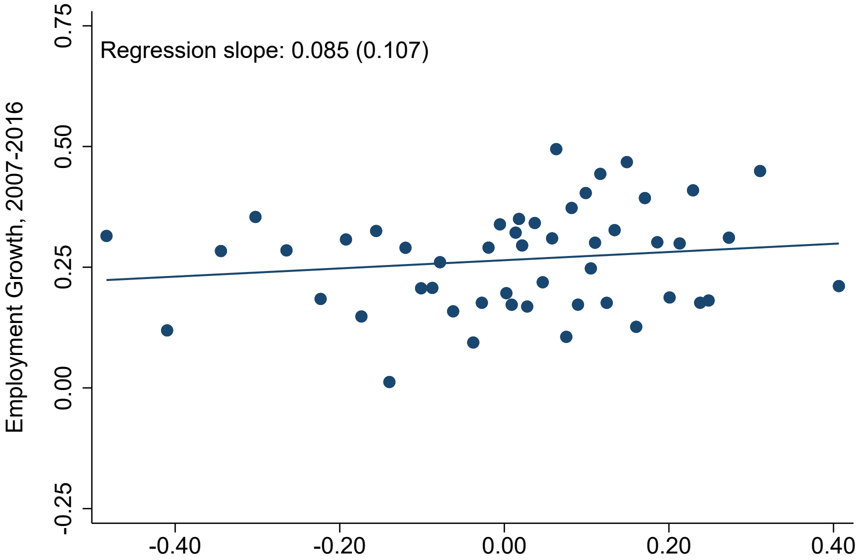
Market Access Balance Regressions

	Unadjusted	Recentered		
	(1)	(2)	(3)	(4)
Distance to Beijing	−0.292	0.069		0.089
	(0.063)	(0.040)		(0.045)
Latitude/100	−3.323	−0.325		−0.156
	(0.648)	(0.277)		(0.320)
Longitude/100	1.329	0.473		0.425
	(0.460)	(0.239)		(0.242)
Expected Market Access Growth			0.027	0.056
			(0.056)	(0.066)
Constant	0.536	0.014	0.014	0.014
	(0.030)	(0.018)	(0.020)	(0.018)
Joint RI p-value		0.489	0.807	0.536
R^2	0.823	0.079	0.007	0.082
Prefectures	274	274	274	274

Regressions of unadjusted and recentered market access growth on geographic features.

Spatial-clustered standard errors in parentheses.

Recentered MA Doesn't Predict Employment Growth!



Adjusted Estimates of Market Access Effects

	Unadjusted OLS (1)	Recentered IV (2)	Controlled OLS (3)
Panel A. No Controls			
Market Access Growth	0.232	0.081	0.069
	(0.075)	(0.098)	(0.094)
		[-0.315, 0.328]	[-0.209, 0.331]
Expected Market Access Growth			0.318
			(0.095)
Panel B. With Geography Controls			
Market Access Growth	0.132	0.055	0.045
	(0.064)	(0.089)	(0.092)
		[-0.144, 0.278]	[-0.154, 0.281]
Expected Market Access Growth			0.213
			(0.073)
Recentered	No	Yes	Yes
Prefectures	274	274	274

Regressions of log employment growth on log market access growth in 2007–2016.

Spatial-clustered standard errors in parentheses; permutation-based 95% CI in brackets

App. 2: Efficient Estimation of Medicaid Effects

Setting: U.S. Medicaid, partially expanded in 2014 under the ACA

- 19 of 43 states with low Medicaid coverage expanded to 138% FPL
- View expansion decisions as random across states with same-party governors, but not household demographics or pre-2014 policy
- Outcomes: Medicaid takeup and private insurance crowdout

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Via non-random variation, recentered IV has ≈ 3 times smaller SEs

Estimates with Simulated vs. Recentered IV

	Has Medicaid		Has Private Insurance		Has Employer-Sponsored Insurance	
	Simulated IV	Recentered IV	Simulated IV	Recentered IV	Simulated IV	Recentered IV
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Eligibility Effects						
Eligibility	0.132	0.072	−0.048	−0.023	0.009	−0.009
	(0.028)	(0.010)	(0.023)	(0.007)	(0.014)	(0.005)
	[0.080,0.216]	[0.051,0.093]	[-0.110,0.009]	[-0.040,-0.007]	[-0.034,0.052]	[-0.021,0.004]
Panel B. Enrollment Effects						
Has Medicaid			−0.361	−0.321	0.068	−0.125
			(0.165)	(0.092)	(0.111)	(0.061)
			[-0.813,0.082]	[-0.566,-0.108]	[-0.232,0.421]	[-0.263,0.070]
P-value: SIV=RIV			0.719		0.104	
Exposed Sample	N	Y	N	Y	N	Y
States	43	43	43	43	43	43
Individuals	2,397,313	421,042	2,397,313	421,042	2,397,313	421,042

1% ACS sample of non-disabled adults in 2013–14, diff-in-diff IV regressions using one of the two instruments. Controls include state and year fixed effects and an indicator for Republican governor interacted with year. State-clustered standard errors in parentheses; wild score bootstrap 95% CI in brackets

Conclusions

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Much more work to be done on the various econometrics here!

Appendix: Proof of IV Estimator using FWL Theorem

- The first-stage regression is $x_\ell = \hat{\alpha} z_\ell + \hat{\lambda}' w_\ell + \tilde{x}_\ell$. By the FWL theorem, the estimator $\hat{\alpha}$ can be expressed as

$$\hat{\alpha} = \frac{Cov(\tilde{x}_\ell, \tilde{z}_\ell)}{V(\tilde{z}_\ell)} = \frac{Cov(\tilde{x}_\ell, z_\ell)}{V(\tilde{z}_\ell)}$$

- Plugging $\hat{x}_\ell$ into the second-stage regression, we have

$$y_\ell = \beta(\hat{\alpha} z_\ell + \hat{\lambda}' w_\ell) + \gamma' w_\ell + \varepsilon_\ell$$

By the FWL theorem, the estimator $\beta\hat{\alpha}$ can be expressed as

$$\hat{\beta}\hat{\alpha} = \frac{Cov(y_\ell, \tilde{z}_\ell)}{V(\tilde{z}_\ell)} = \frac{Cov(y_\ell, z_\ell)}{V(\tilde{z}_\ell)} \Rightarrow \hat{\beta} = \frac{Cov(z_\ell, \tilde{y}_\ell)}{Cov(z_\ell, \tilde{x}_\ell)}$$

back