Confronting Data with the Theory

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What we have done so far?

- We have introduced various sources of data
- Ways to understand the data:
  - Data visualization: use graphics to convey messages
  - Econometrics models: OLS, RE, FE, Diff-in-Diff, Synthetic controls.
  - Machine learning methods (more suitable for prediction): regularized regressions (LASSO, Ridge, hybrid...), regression trees, neural nets, text analysis methods...
- Software tools to analyze the data:
  - STATA: suitable for econometric analysis, especially running regressions
  - Matlab: linear algebra and computation
  - Python: data visualization and open-source economic programs (like machine learning)
  - other popular softwares like R and Julia are not covered

Still not clear...

- How come we have such a model written like $y_{it} = \mathbf{X}'_{it}\beta + \varepsilon_{it}$?
- What variables should we consider for $\mathbf{X}_{it}$?
- How should we interpret β?
- Can we explain the sign/value of $\hat{\beta}$?
- What's the welfare implication of the estimated value for β?

We want to be more rigorous

- We seek for theories that can provide an consistent explanation for the empirical results
  - Qualitative theory: signs of the marginal effect, $\partial y_{it} / \partial x_{it} > or < 0$.
  - Quantitative theory: signs and magnitudes of the marginal effect, $\partial y_{it} / \partial x_{it} = \beta$, β is used for further estimation and welfare evaluation
- Why theory?
  - add disciplines or guide regressions: e.g., production function estimation
  - gain deeper insights into the problem
  - obtain estimates for unknown variables: welfare, hidden costs, production efficiency, etc.
  - counterfactual analysis (comparative statics)

Qualitative Theory: Melitz (2003, ECMA)

- Empirical Backgrounds:
  - Starting from late 90s, a growing body of empirical analysis using firm-level data point to the "premium" of exporting firms
  - Exporting firms are found to be more productive and larger than firms only sell domestically
  - Resources reallocation is also correlated with exposure to trade: exposure to trade forces the least productive firms to exit
  - These empirical facts call for a consistent theoretical framework
- **Melitz model**: embedding firm productivity heterogeneity a lá Hopenhayn (1992) within Krugman's model of monopolistic competition and increasing returns

Model Setup

Demand

- C.E.S utility function (Dixit and Stiglitz, 1977)

$$U \equiv Q = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma}{\sigma-1}} \, d\omega \right]^{\frac{\sigma-1}{\sigma}}$$

The associated price index is

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}} \tag{1}$$

- The optimal consumption choice yields:

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma} \tag{2}$$

$$r(\omega) = p(\omega) q(\omega) = R \left[\frac{p(\omega)}{P} \right]^{1-\sigma} \tag{3}$$

where $R \equiv PQ = \int_{\omega \in \Omega} r(\omega) \, d\omega$.

Model Setup

Production

- There is a continuum of firms, each choosing variety ω; production only requires labor, with total amount L
- The production technology is given by

$$l = f + \frac{q}{\phi} \tag{4}$$

  - f: fixed cost of production
  - ϕ: labor productivity levels, or quality levels
- Pricing rule for monopolistic competition:

$$\max_{p} \left\{ R\left(\frac{p}{P}\right)^{1-\sigma} - w\left(f + q/\phi\right) \right\}$$

$$\Rightarrow p(\phi) = \frac{\sigma}{\sigma - 1}\frac{w}{\phi} \tag{5}$$

where $\frac{\sigma}{\sigma-1}$ is the markup, w is normalized to be one.

- The firm profit is

$$\pi(\phi) = r(\phi) - l(\phi) = \frac{r(\phi)}{\sigma} - f \tag{6}$$

  - revenue $r(\phi) = R\left(P\frac{\sigma-1}{\sigma}\phi\right)^{\sigma-1}$
  - profits $\pi(\phi) = \frac{R}{\sigma}\left(P\frac{\sigma-1}{\sigma}\phi\right)^{\sigma-1} - f \equiv \frac{r(\phi)}{\sigma} - f$

- Ranking of firms:

$$\begin{aligned} \frac{q(\phi_1)}{q(\phi_2)} &= \left(\frac{\phi_1}{\phi_2}\right)^{\sigma} \\ \frac{r(\phi_1)}{r(\phi_2)} &= \left(\frac{\phi_1}{\phi_2}\right)^{\sigma-1} \end{aligned} \tag{7}$$

- **Predictions**: a more productive firm will be bigger, charge lower price, and earn higher profits than less productive firms

Model Setup

Aggregation

- Equilibrium: a mass M of firms, a productivity distribution $\mu(\phi)$ over $\mathbb{R}^+$
- The aggregate price is

$$P = \left[\int_0^\infty p(\phi)^{1-\sigma} M\mu(\phi)\, d\phi\right]^{\frac{1}{1-\sigma}} \tag{8}$$

- Define $\tilde{\phi}$ as the aggregate productivity:

$$\tilde{\phi} = \left[\int_0^\infty \phi^{\sigma-1}\mu(\phi)\, d\phi\right]^{\frac{1}{\sigma-1}} \tag{9}$$

- Aggregate variables:

$$P = M^{\frac{1}{1-\sigma}} p\left(\tilde{\phi}\right), R = PQ = Mr\left(\tilde{\phi}\right)$$
$$Q = M^{\frac{\sigma}{\sigma-1}} q\left(\tilde{\phi}\right), \Pi = M\pi\left(\tilde{\phi}\right)$$

Firm Entry and Exit

- Timeline for entry decision:
  1. Prospective entrants pay a fixed entry cost $f_e > 0$ and draw productivity ϕ from a distribution $g(\phi)$ (CDF $G(\phi)$)
  2. Depending on realized ϕ, firm chooses to exit or produce
  3. If produce, firm face a bad shock in every period
- Melitz considers a steady state equilibria in which the aggregate variables remain constant over time
- ϕ, f_e are constant over time; each firm's value function is

$$v(\phi) = \max\left\{0, \frac{1}{\delta}\pi(\phi)\right\}$$

- Cutoff productivity for entry:

$$\frac{1}{\delta}\pi(\phi^*) = \frac{1}{\delta}\left[\frac{r(\phi^*)}{\sigma} - f\right] = 0$$

firms exit whenever $\phi < \phi^*$

- The actual distribution of firm productivity is a truncation of the ex-ante distribution G:

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi^*)} & if\ \phi \geq \phi^* \\ 0 & otherwise \end{cases} \tag{10}$$

$p_e = 1 - G(\phi^*)$ is the ex-ante probability of successful entry.

- The aggregate productivity level is thus

$$\tilde{\phi}(\phi^*) = \left[\frac{1}{1-G(\phi^*)}\int_{\phi^*}^{\infty} \phi^{\sigma-1} g(\phi)\, d\phi\right]^{\frac{1}{\sigma-1}} \tag{11}$$

- Zero Cutoff Profit Condition (ZCP):

$$\frac{r(\tilde{\phi})}{r(\phi^*)} = \left[\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right]^{\sigma-1}, \pi(\tilde{\phi}) = \bar{\pi} = \left[\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right]^{\sigma-1} \frac{r(\phi^*)}{\sigma} - f$$

$$\pi(\phi^*) = 0 \Leftrightarrow r(\phi^*) = \sigma f \Leftrightarrow \bar{\pi} = f\left[\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right]^{\sigma-1} - f$$

- Free Entry condition the expected net value of entry is zero:

$$v_e = p_e \bar{v} - f_e = \frac{1 - G(\phi^*)}{\delta} \bar{\pi} - f_e = 0$$

  - entry stops when $v_e = 0$

- **Prediction:** new entrants will have lower productivity and a higher probability of exit than incumbents

Equilibrium in a Closed Economy

- Free entry (FE) and zero cutoff profit (ZCP) conditions:

$$ZCP : \bar{\pi} = f \left\{ \left[\frac{\tilde{\phi}(\phi^*)}{\phi^*} \right]^{\sigma - 1} - 1 \right\} \tag{12}$$

$$FE : \bar{\pi} = \frac{\delta f_e}{1 - G(\phi^*)} \tag{13}$$

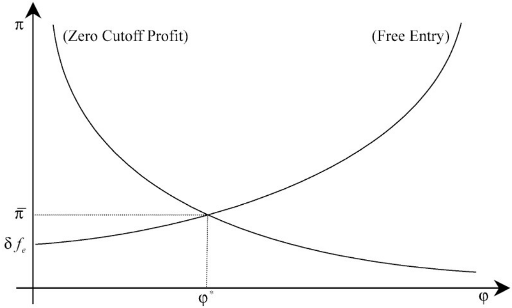


FIGURE 1.—Determination of the equilibrium cutoff φ^* and average profit $\bar{\pi}$.

- In the stationary equilibrium, the mass of entrants is

$$
\begin{aligned}
\chi M_e &= \delta M \\
L_e &= M_e f_e \\
&= \frac{\delta M}{\chi} f_e = M\bar{\pi} = \Pi
\end{aligned}
$$

  - production labor: $L_p = R - \Pi = R - L_e$
  - aggregate revenue: $R = L_p + L_e$
- The mass of producing firms is

$$
M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)}
$$

- The price index is

$$
P = M^{1/(1-\sigma)} p\left(\tilde{\phi}\right) = M^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \frac{1}{\tilde{\phi}}
$$

Equilibrium Analysis

- $\{\phi^*, \tilde{\phi}, \bar{\pi}, \bar{r}\}$ are independent of the country size L
- The mass of firms $M \propto L$
- Welfare per worker is

$$W = P^{-1} = M^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma} \tilde{\phi}$$

  - larger countries have more varieties (Krugman, 1980)
  - $\bar{\pi}$ and $\tilde{\phi}$ are endogenously determined
  - aggregate productivity can be changed through reallocations

Open Economy

Additional assumptions

- Firms must pay a fixed cost to export after the firm's productivity is revealed
- The per-unit trade costs are standard iceberg costs: $\tau > 1$ units of a good must be shipped for 1 unit to arrive at destination
- The economy can trade with $n \geq 1$ other countries; entry into each market requires a fixed investment cost f_x

Open-Economy Equilibrium

- Pricing rules and revenues in domestic and foreign market:

$$p_d(\phi) = \frac{\sigma}{\sigma - 1}\frac{1}{\phi},\ r_d(\phi) = R\left(\frac{\sigma - 1}{\sigma}P\phi\right)^{\sigma - 1}$$
$$p_x(\phi) = \frac{\sigma}{\sigma - 1}\frac{\tau}{\phi},\ r_x(\phi) = \tau^{1-\sigma} r_d(\phi)$$

- The combined revenue of a firm is

$$r(\phi) = \begin{cases} r_d(\phi) & \text{non-exporter} \\ r_d(\phi) + n r_x(\phi) = \left(1 + n\tau^{1-\sigma}\right) r_d(\phi) & \text{exporters} \end{cases} \tag{14}$$

Firm Entry, Exit, and Export Status

- Given that export cost is equal across countries, a firm will either export to ALL countries or never export
  - An extension with unequal export costs would imply an increasing relationship between a firm's productivity and the number of its export destinations
- Firm profits

$$\pi_d(\phi) = \frac{r_d(\phi)}{\sigma} - f, \; \pi_x(\phi) = \frac{r_x(\phi)}{\sigma} - f_x$$
$$\pi(\phi) = \pi_d(\phi) + \pi_x(\phi)$$

- Cutoff productivity levels:

$$\text{entry:} \phi^* = \inf\{\phi : v(\phi) > 0\} \Leftrightarrow \pi_d(\phi^*) = 0$$
$$\text{export:} \phi_x^* = \inf\{\phi : \phi \geq \phi^* \text{ and } \pi_x(\phi) > 0\} \Leftrightarrow \pi_x(\phi_x^* | \phi_x^* \geq \phi^*) = 0$$

  - If $\phi_x^* = \phi^*$: all firms export
  - If $\phi_x^* > \phi^*$: some firms export. This requires that

$$\tau^{\sigma-1} f_x > f$$

- The probabilities of entry:

$$p_e = 1 - G(\phi^*)$$
$$p_x = \frac{1 - G(\phi_x^*)}{1 - G(\phi^*)}$$

- The equilibrium mass of incumbent firms

$$M_x = p_x M$$
$$M_t = M + nM_x$$

M_t is the total mass of varieties (or total mass of firms)

Aggregation

- Average productivity levels: $\tilde{\phi} = \tilde{\phi}(\phi^*)$ and $\tilde{\phi}_x = \tilde{\phi}(\phi_x^*)$; Let $\tilde{\phi}_t$ be the weighted productivity average reflecting the combined production efficiency

$$\tilde{\phi}_t = \left\{ \frac{1}{M_t} \left[M\tilde{\phi}^{\sigma-1} + nM_x \left(\frac{\tilde{\phi}_x}{\tau} \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}$$

- $\tilde{\phi}_t$ summarizes aggregate variables:

$$P = M_t^{\frac{1}{1-\sigma}} p\left(\tilde{\phi}_t\right) = M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{1}{\tilde{\phi}_t}, \; R = M_t r_d\left(\tilde{\phi}_t\right)$$

$$W = \frac{R}{LP} = \frac{R}{L} M_t^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \tilde{\phi}_t$$

- Average revenues and profits:

$$\bar{r} = r_d\left(\tilde{\phi}\right) + p_x n r_x\left(\tilde{\phi}_x\right)$$

$$\bar{\pi} = \pi_d\left(\tilde{\phi}\right) + p_x n \pi_x\left(\tilde{\phi}_x\right)$$

Equilibrium Conditions

- The zero cutoff profit condition implies

$$\frac{r_x(\phi_x^*)}{r_d(\phi^*)} = \tau^{1-\sigma}\left(\frac{\phi_x^*}{\phi^*}\right)^{\sigma-1} = \frac{f_x}{f} \Leftrightarrow \frac{\phi_x^*}{\phi^*} = \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$$

- The average profit is

$$\begin{aligned}(\text{ZCP}):\ \bar{\pi} &= \pi_d(\tilde{\phi}) + p_x n \pi_x(\tilde{\phi}_x) \\ &= f\left\{\left[\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right]^{\sigma-1} - 1\right\} + p_x n f_x\left\{\left[\frac{\tilde{\phi}(\phi_x^*)}{\phi_x^*}\right]^{\sigma-1} - 1\right\}\end{aligned}$$

- The free-entry condition entails

$$(\text{FE}):\ v_e = p_e\frac{\bar{\pi}}{\delta} - f_e = 0 \Leftrightarrow \bar{\pi} = \frac{\delta f_e}{p_x}$$

- The average revenue is determined by ZCP and FE conditions:

$$\bar{r} = r_d\left(\tilde{\phi}\right) + p_x n r_x\left(\tilde{\phi}_x\right) = \sigma\left(\bar{\pi} + f + p_x n f_x\right)$$

- Aggregate variables:

$$\text{mass of incumbent firms:} M = \frac{R}{\bar{r}} = \frac{L}{\sigma\left(\bar{\pi} + f + p_x n f_x\right)}$$

$$\text{mass of variety:} M_t = \left(1 + n p_x\right) M$$

$$\text{price index:} P = M_t^{\frac{1}{1-\sigma}} \frac{\sigma - 1}{\sigma} \frac{1}{\tilde{\phi}}_t$$

The Impact of Trade

- Benefits of a general equilibrium model:
  - The range of firm productivity levels
  - Do all firms benefit from trade?
  - How is aggregate productivity and welfare affected by trade?
- These questions are answered by analyzing the comparative statics of the steady states
  - Notations: domestic (foreign) revenue in open economy $r_d(\phi)$ ($r_x(\phi)$), revenue in autarky, $r_a(\phi)$
- Revenue (or equivalently, market shares) inequalities:

$$r_d(\phi) < r_a(\phi) < r_d(\phi) + nr_x(\phi)$$

  - *A firm who exports increases its market share*
  - *A firm who does not export loses market share, with the least productive firms exit*
- Change in the profit:$\Delta\pi(\phi) = \pi(\phi) - \pi_a(\phi) = \phi^{\sigma-1} f \left[\frac{1+n\tau^{1-\sigma}}{\phi^{*\sigma-1}} - \phi_a^{*1-\sigma}\right] - nf_x$

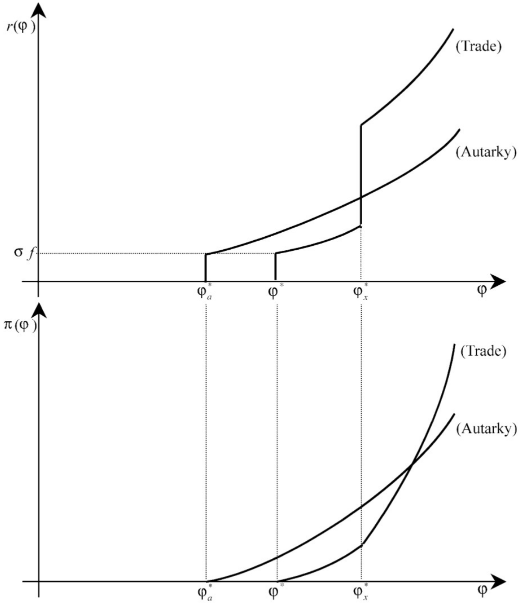


FIGURE 2.—The reallocation of market shares and profits.

Underlying mechanism

- Two potential channels through which trade affect the distribution of surviving firms:
  1. Competition effect: firms face more competitors
  2. Labor market effect: firms compete to hire labor
- With CES utility function, only the *second* channel operates
- In extension with variable markups, the first effect can arise

Gradual trade liberalization

- Increasing the number of trading partners: ZCP curve shifts up, $\phi^* \uparrow$, $\phi_x^* \uparrow$, $r_d(\phi) \downarrow$
  - The most productive firms enjoy an increase in profits; both market shares and profits are reallocated towards the more efficient firms
  - Welfare and aggregate productivity increases
- Decrease in trade costs and decrease in fixed market entry cost f_x have similar effects except that $\phi_x^* \downarrow$

A tractable version of Melitz model with gravity equation

- Chaney (2008) analyzes the impact of σ on bilateral trade flows:

$$\text{Exports}_{ni} = Constant \times \frac{GDP_A \times GDP_B}{(Trade\, brarriers_{AB})^{\varepsilon(\sigma)}}$$

- He shows that $\varepsilon'(\sigma) < 0$
- The novelty is a parameterizatio of $G(\phi)$:

$$G(\phi) = 1 - \phi^{-\gamma}, \gamma > \sigma - 1$$

  - It is also a good approximation of the firm size distribution
- There are other technical assumptions, but are not essential:
  - multiple sectors
  - entry is proportional to total income
  - asymmetric countries as in Helpman, Melitz, and Yeaple (2004)

Chaney Model

- Aggregate trade:

$$X_{ni} \propto \frac{Y_i \times Y_n}{\tau_{ni}^{\gamma}} f_{ni}^{-\frac{\gamma}{\sigma-1}+1}$$

- The trade elasticity :

$$\zeta \equiv -\frac{d \ln X_{ni}}{d \ln \tau_{ni}} = \gamma, \xi \equiv -\frac{d \ln X_{ni}}{d \ln f_{ni}} = \frac{\gamma}{\sigma-1} - 1$$

  - The substitution elasticity σ has no effect of trade flow with respect to variable trade costs
  - σ has a negative effect on trade flows with respect to trade costs

Quantitative theory: Eaton et al. (2012, ECMA)

- A step forward by taking the Melitz model to data and performing counterfactuals
- They start by examining the pattern of French exporting firms and observe several striking regularities:
- They offer an extended version of Melitz model to confront the data
- They estimate the model and perform counterfactuals
- They find that the firm's heterogeneity plays a pivotal role in explaining the exporting patterns

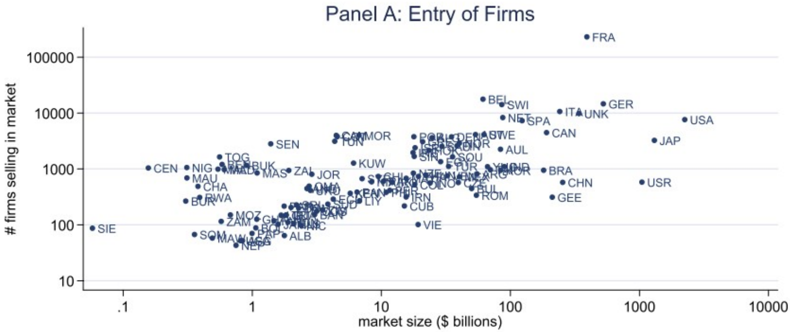
The basic model fails when taking to data

1. Firms do not enter markets according to an exact hierarchy
2. Their sales deviate from the exact correlations the basic model insists on
3. Firms that export sell too much in France
4. In the typical destination, there are too many firms selling small amounts

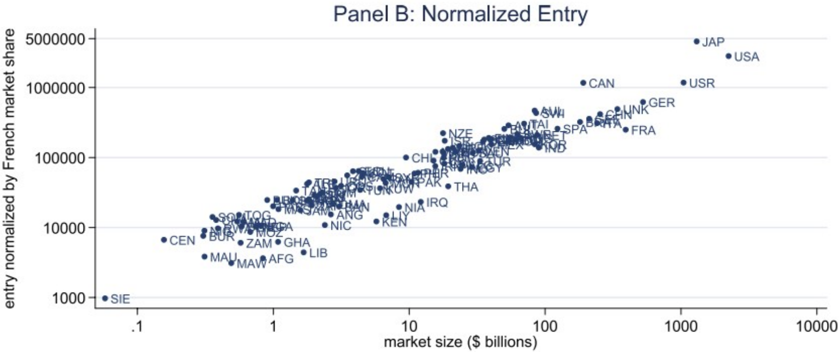
Market Entry

- Number of French manufacturing firms N_{nF}, total manufacturing absorption X_n, X_{nF} is total exports

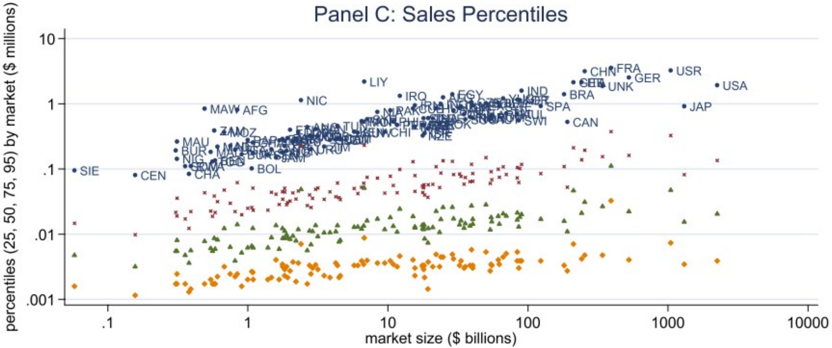
$$\pi_{nF} = \frac{X_{nF}}{X_n}$$



- Normalized entry: $\frac{\pi_{nF}}{N_{nF}}$



- Sales percentiles



Market entry

Firm entry into different sets of markets

TABLE I

FRENCH FIRMS EXPORTING TO THE SEVEN MOST POPULAR DESTINATIONS

Export Destination	Number of Exporters	Fraction of Exporters
Belgium[a] (BE)	17,699	0.520
Germany (DE)	14,579	0.428
Switzerland (CH)	14,173	0.416
Italy (IT)	10,643	0.313
United Kingdom (UK)	9752	0.287
Netherlands (NL)	8294	0.244
United States (US)	7608	0.224
Any destination (all French exporters)	34,035	

[a]Belgium includes Luxembourg.

TABLE II

FRENCH FIRMS SELLING TO STRINGS OF TOP-SEVEN COUNTRIES

	Number of French Exporters		
Export String[a]	Data	Under Independence	Model
BE[a]	3988	1700	4417
BE–DE	863	1274	912
BE–DE–CH	579	909	402
BE–DE–CH–IT	330	414	275
BE–DE–CH–IT–UK	313	166	297
BE–DE–CH–IT–UK–NL	781	54	505
BE–DE–CH–IT–UK–NL–US	2406	15	2840
Total	9260	4532	9648

[a]The string BE means selling to Belgium but no other among the top 7; BE–DE means selling to Belgium and Germany but no other, and so forth.

Sales Distribution

$$Quantiles: \; Pr\left(X_n \leq x_n^q\right) = q$$

$$1 - \left(\frac{a x_n^q}{a-1}\right)^{-a} = q$$

$$\ln\left(x_n^q\right) = \ln\left(\frac{a-1}{a}\right) - \frac{1}{a}\ln\left(1-q\right)$$

Sales Distribution

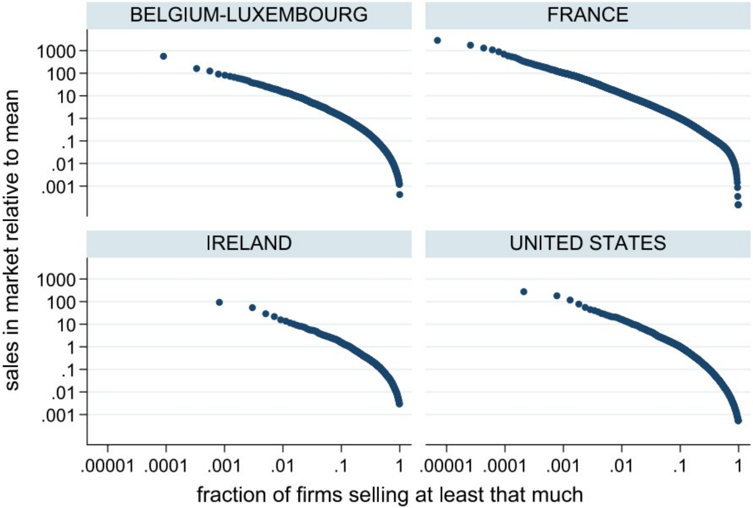


FIGURE 2.—Sales distributions of French firm: Graphs by country.

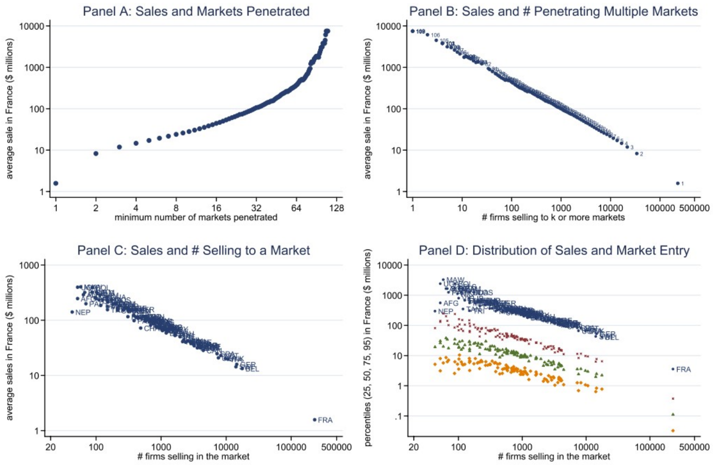
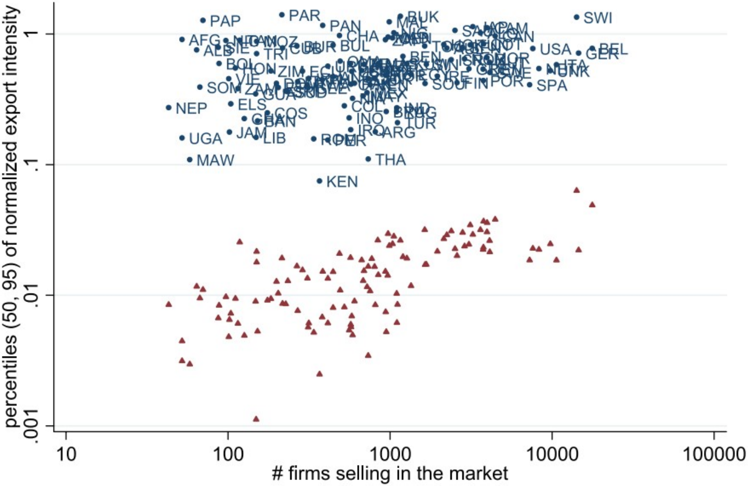


FIGURE 3.—Sales in France and market entry.

Export Intensity

- Normalized export intensity:

$$\frac{X_{nF}(j)/\bar{X}_{nF}}{X_{FF}(j)/\bar{X}_{FF}}$$



Theory[1]

Environment

- Monopolistic competition: Goods are differentiated
- Selling in a market requires a fixed cost
- Moving goods across countries incurs iceberg transport costs
- Firms are heterogeneous in (i) production efficiency (ii) other characteristics: fixed costs, taste shocks
- Market access costs as Arkolakis (2007)

[1] Subsequent notes is indebted to lecture notes by Eaton (2017 Fall)

Basics

- Potential producer of good j in country i with efficiency $z_i(j)$

- Input costs w_i (exogenous until the very end)

- Iceberg shipping cost $d_{ni} \geq 1$ units ($d_{ii} = 1$)

- Unit cost of selling in n from i

$$c_{ni}(j) = \frac{w_i d_{ni}}{z_i(j)}.$$

Producer Heterogeneity

- measure of potential producers (firms) in country i who can produce their good with efficiency at least z:

$$\mu_i^z(z) = T_i z^{-\theta} \quad z > 0$$

- hence, measure of firms that can deliver to destination n from country i at unit cost below c:

$$\mu_{ni}(c) = \mu_i^z\left(\frac{w_i d_{ni}}{c}\right) = \Phi_{ni} c^{\theta}$$

where:

$$\Phi_{ni} = T_i (w_i d_{ni})^{-\theta}$$

Entry, Demand, and Market Structure

- Entry (Melitz+Arkolakis, *JPE* 2010)
  - To sell to a fraction f of buyers producer j must spend:

    $$E_{ni}(j) = \varepsilon_n(j) E_{ni} M(f).$$

    $\varepsilon_n(j)$ fixed cost shock specific firm j in market n
  - E_{ni} faced by all sellers from i in n.
  - costs of entry (Arkolakis):

    $$M(f) = \frac{1 - (1 - f)^{1-1/\lambda}}{1 - 1/\lambda},$$

$\lambda \geq 0$ reflects increasing cost of reaching a larger fraction of potential buyers;.$\lambda \to \infty$ Melitz special case (constant cost). Same across all destinations.

– Note that $M(0) = 0,\ M'(f) > 0,\ M'(0) = 1$, $M(f)$ goes to infinity as $f \to 1$ for $\lambda \leq 1$.

- Demand for j in n (Standard CES):

$$X_n(j) = \alpha_n(j) f X_n \left(\frac{p}{P_n}\right)^{1-\sigma}$$

  - X_n total spending
  - $\alpha_n(j)$ demand shock specific to good j in market n.
  - P_n CES price index derived below.

Firm Decisions

- Profit in Market n

$$\Pi_{ni}(p,f) = \left(1 - \frac{c_n(j)}{p}\right)\alpha_n(j) f \left(\frac{p}{P_n}\right)^{1-\sigma} X_n - \varepsilon_n(j) E_{ni} \frac{1-(1-f)^{1-1/\lambda}}{1-1/\lambda}.$$

- Firm chooses p and f to maximize Π.

– Price:

$$p_n(j) = \overline{m} c_n(j)$$

where $\overline{m} = \sigma/(\sigma - 1)$

– Market share:

$$f_{ni}(j) = \max \left\{ 1 - \left[\eta_n(j) \frac{X_n}{\sigma E_{ni}} \left(\frac{\overline{m} c_n(j)}{P_n} \right)^{1-\sigma} \right]^{-\lambda}, 0 \right\}$$

where $\eta_n(j) = \alpha_n(j)/\varepsilon_n(j)$

- Enter if and only if:

$$\eta_n(j)\left(\frac{\overline{m}c_n(j)}{P_n}\right)^{1-\sigma}\frac{X_n}{\sigma}\geq E_{ni}.$$

- for a firm with α, η, c in market n
  - enter if:

$$c \leq \overline{c}_{ni}(\eta)$$

    where:

$$\overline{c}_{ni}(\eta) = \left(\eta \frac{X_n}{\sigma E_{ni}}\right)^{1/(\sigma-1)} \frac{P_n}{\overline{m}}.$$

– sell to a fraction:

$$f_{ni}(\eta, c) = 1 - \left(\frac{c}{\overline{c}_{ni}(\eta)}\right)^{\lambda(\sigma-1)}.$$

– total sales:

$$X_n(j) = \alpha f_n(\eta, c) \left(\frac{\overline{m}c}{P_n}\right)^{1-\sigma} X_n.$$

– gross profit:

$$\Pi^G(j) = X_n(j)/\sigma$$

– fixed cost:

$$
\begin{aligned}
E_{ni}(j) &= \frac{\alpha}{\eta} E_{ni} M(f_{ni}(\eta, c)) \\
&= \varepsilon E_{ni} \frac{\lambda}{\lambda - 1} \left[1 - \left(\frac{c}{\overline{c}_{ni}(\eta)} \right)^{(\sigma-1)(\lambda-1)} \right].
\end{aligned}
$$

Price Index and Entry Cutoffs

- Each buyer has an equal chance of buying any good so that:

$$\begin{aligned} P_n &= \overline{m}\left[\int\int\left(\sum_{i=1}^{N}\int_0^{\overline{c}_{ni}(\eta)}\alpha f_{ni}(\eta,c)c^{1-\sigma}d\mu_{ni}(c)\right)g(\alpha,\eta)d\alpha d\eta\right]^{-1/(\sigma-1)} \\ &= \overline{m}\,(\kappa_1\Psi_n)^{-1/\theta}\,X_n^{(1/\theta)-1/(\sigma-1)} \end{aligned}$$

where:

$$\Psi_n = \sum_{i=1}^{N}\Phi_{ni}(\sigma E_{ni})^{-[\theta-(\sigma-1)]/(\sigma-1)}$$

and:

$$\kappa_1 = \left[\frac{\theta}{\theta-(\sigma-1)} - \frac{\theta}{\theta+(\sigma-1)(\lambda-1)}\right] \cdot \int\int \alpha\eta^{[\theta-(\sigma-1)]/(\sigma-1)} g(\alpha,\eta) d\alpha d\eta.$$

- Substituting P_n into the previous expression for $\overline{c}_{ni}(\eta)$ gives:

$$\overline{c}_{ni}(\eta) = \left(\frac{\eta}{\sigma E_{ni}}\right)^{1/(\sigma-1)} \left(\frac{X_n}{\kappa_1 \Psi_n}\right)^{1/\theta}$$