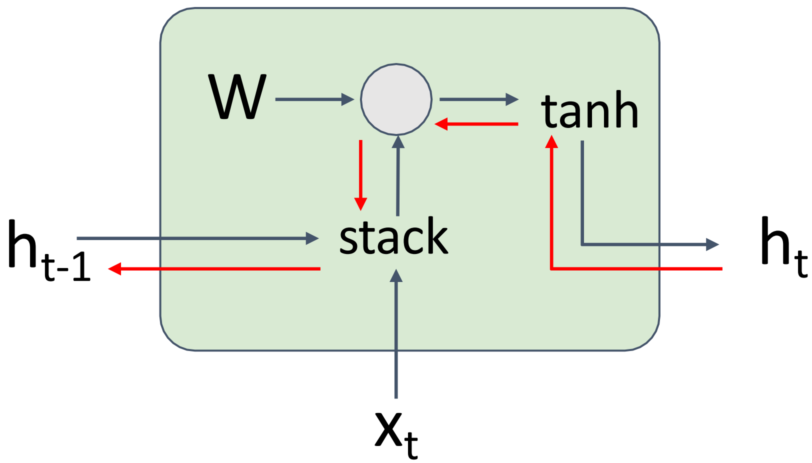
Problems learning RNNs

- In theory, should be able to propagate information over arbitrarily long contexts.
- In practice, RNNs suffer from **vanishing gradients** that decay to 0, or **exploding gradients** that increase towards infinity.

Figure: Goodfellow, Bengio and Courville. Deep Learning. MIT Press, 2016.

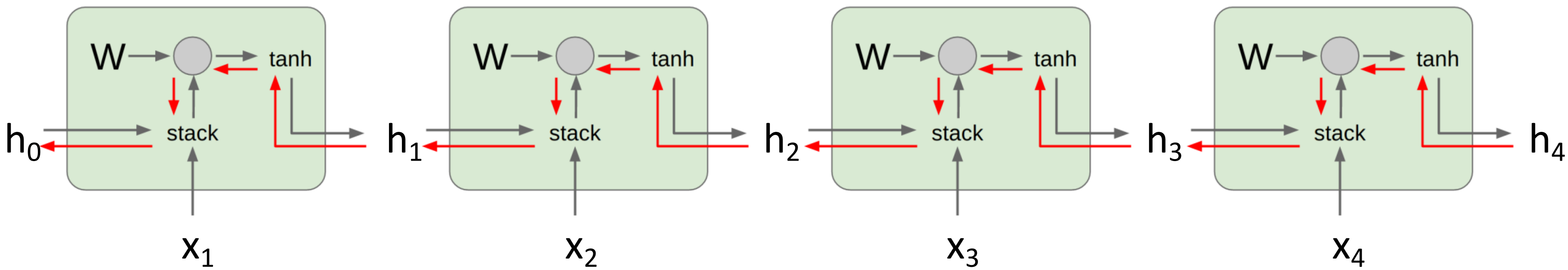
Vanilla RNN Gradient Flow

Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

Vanilla RNN Gradient Flow

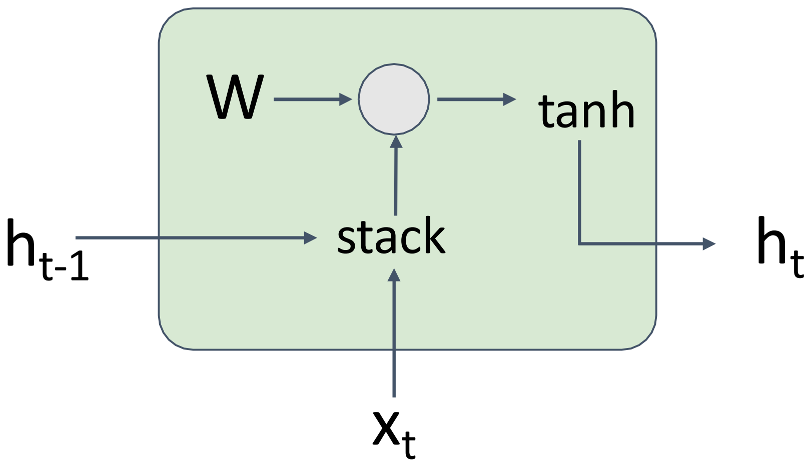


Computing gradient of h_0 involves many factors of W (and repeated tanh)

LSTM Networks

- Recurrent Neural Networks
  - ***Long Short-Term Memory (LSTM)*** networks are a variant of RNNs
  - LSTM mitigates the vanishing/exploding gradient problem
    - Solution: a Memory Cell, updated at each step in the sequence
  - Three gates control the flow of information to and from the Memory Cell
    - Input Gate: protects the current step from irrelevant inputs
    - Output Gate: prevents current step from passing irrelevant information to later steps
    - Forget Gate: limits information passed from one cell to the next
  - Most modern RNN models use either LSTM units or other more advanced types of recurrent units (e.g., GRU units)

Vanilla RNN Gradient Flow



$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\ &= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \end{aligned}$$

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Two vectors at each timestep:

Cell state

Hidden state

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

Compute four **gates** at each timestep

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

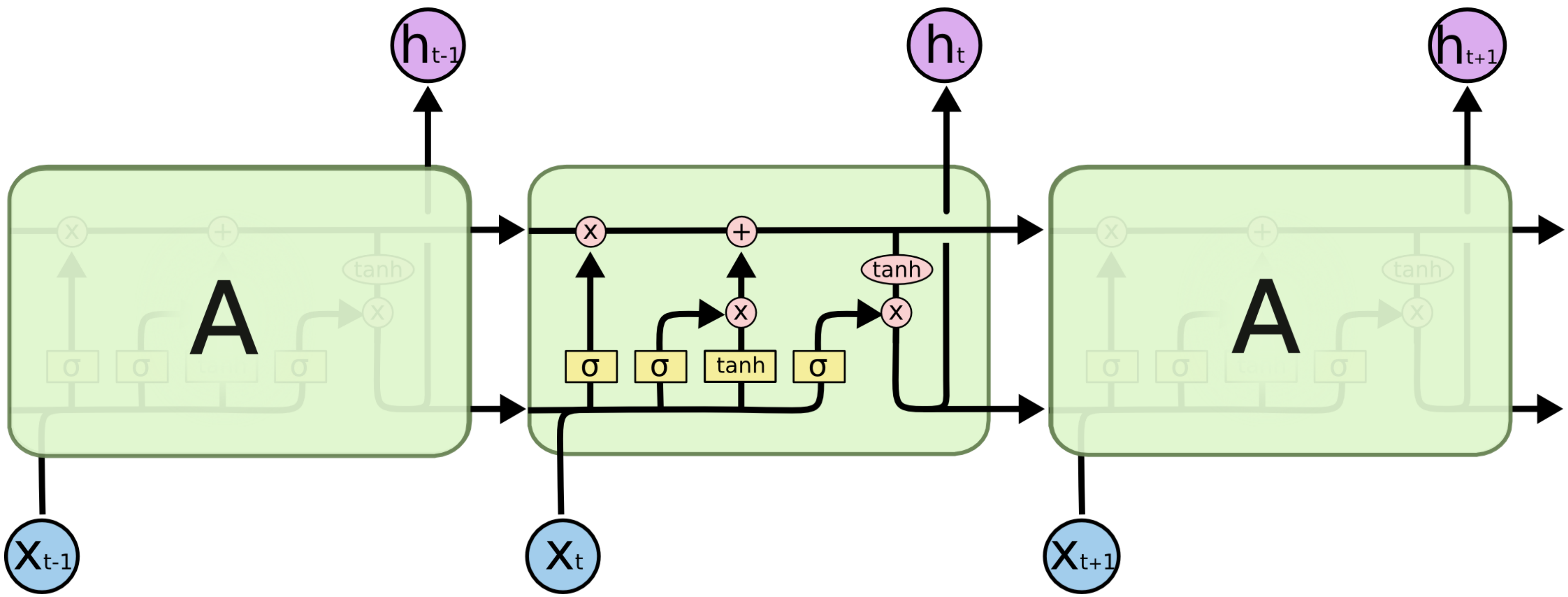
$$c_t = f \odot c_{t-1} + i \odot g$$

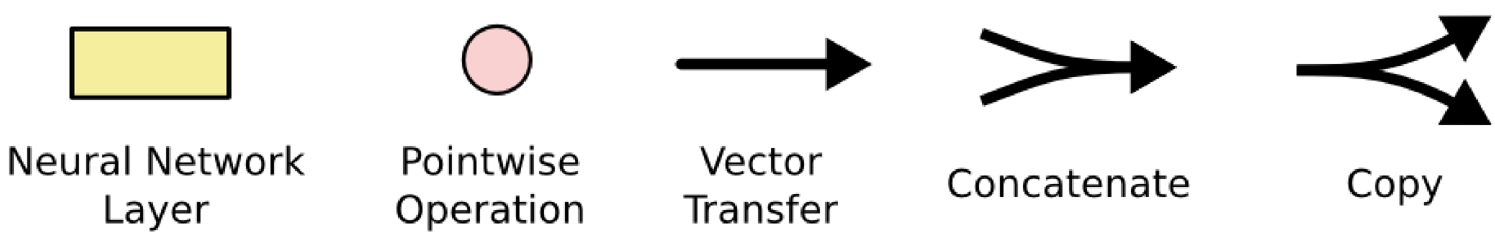
$$h_t = o \odot \tanh(c_t)$$

Cell State Example

- Want to remember person & number of a subject noun so that it can be checked to agree with the person & number of verb when it is eventually encountered.
- Forget gate will remove existing information of a prior subject when a new one is encountered.
- Input gate "adds" in the information for the new subject.

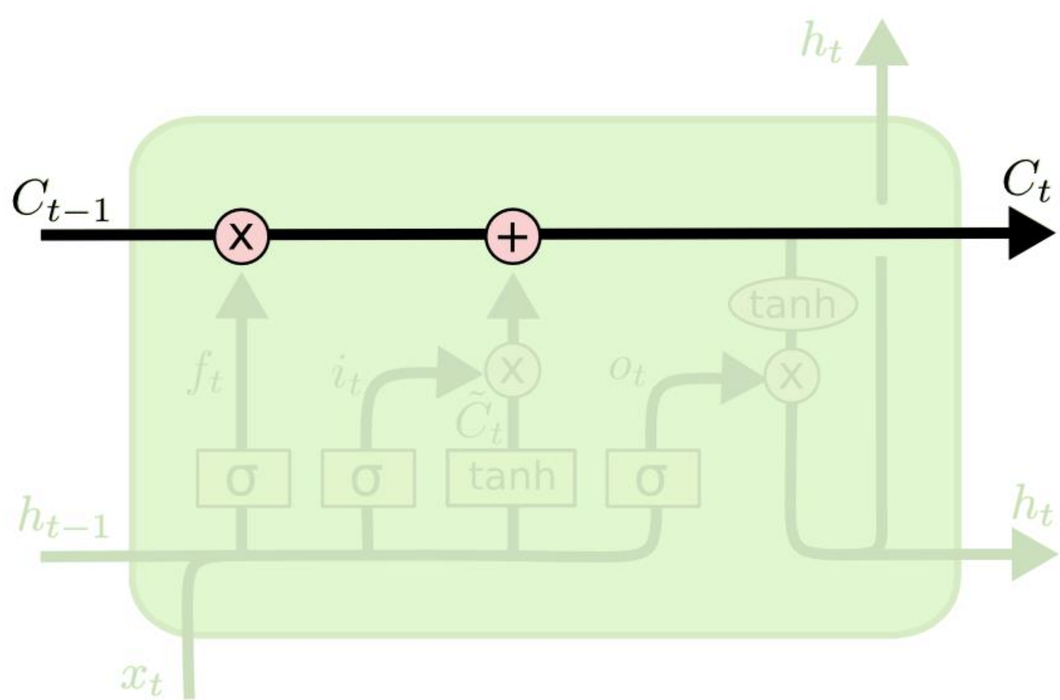
LSTM Network Architecture





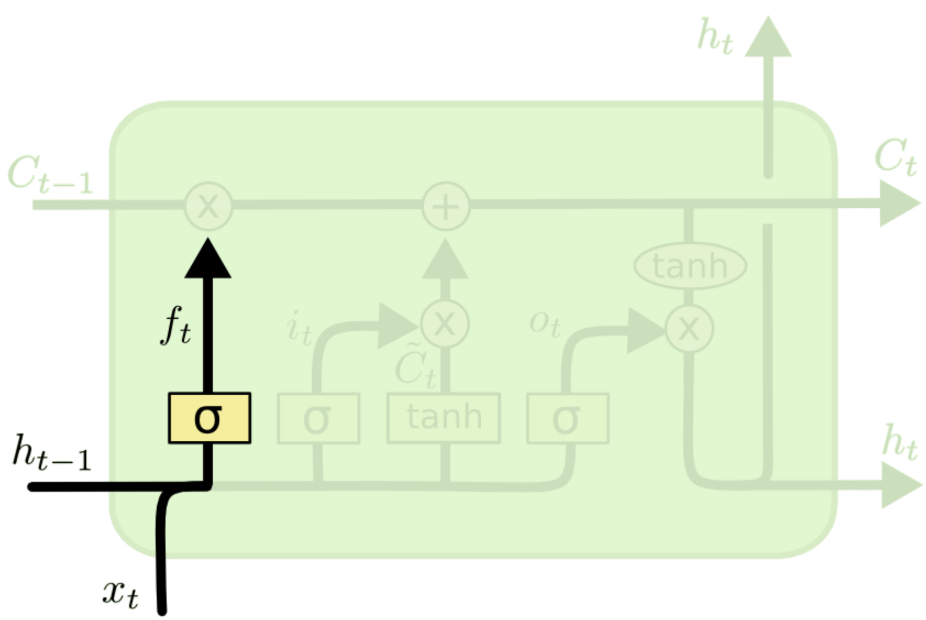
Cell State

- Maintains a vector C_t that is the same dimensionality as the hidden state, h_t
- Information can be added or deleted from this state vector via the forget and input gates.



Forget Gate

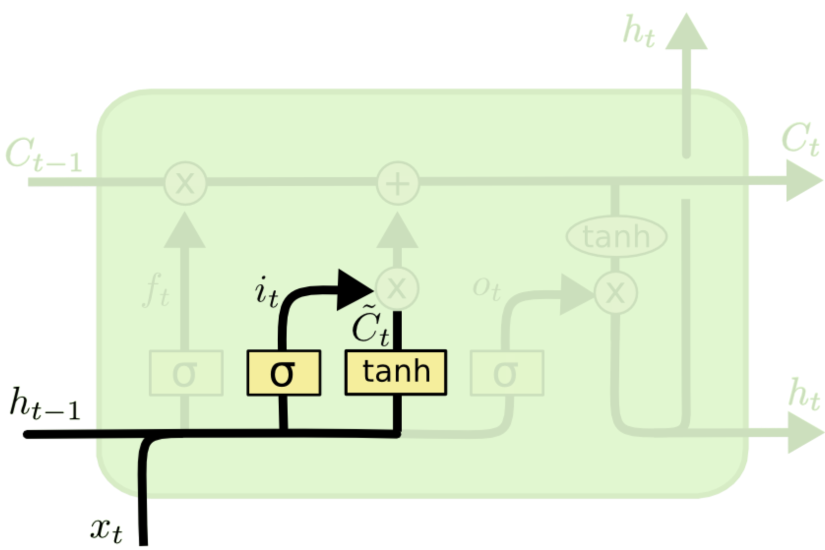
- Forget gate computes a 0-1 value using a logistic sigmoid output function from the input, x_t, and the current hidden state, h_t:
- Multiplicatively combined with cell state, "forgetting" information where the gate outputs something close to 0.



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

Input Gate

- First, determine which entries in the cell state to update by computing 0-1 sigmoid output.
- Then determine what amount to add/subtract from these entries by computing a tanh output (valued –1 to 1) function of the input and hidden state.

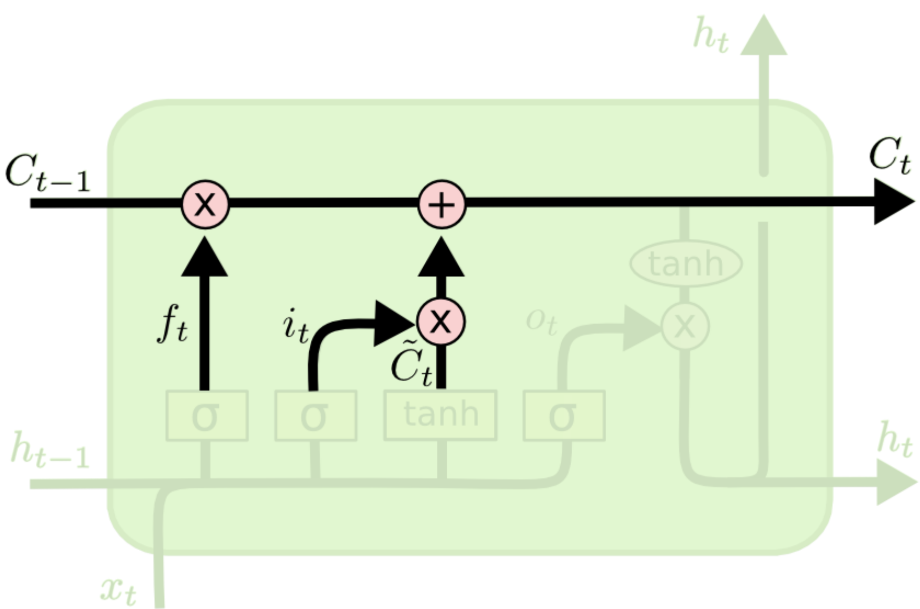


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Updating the Cell State

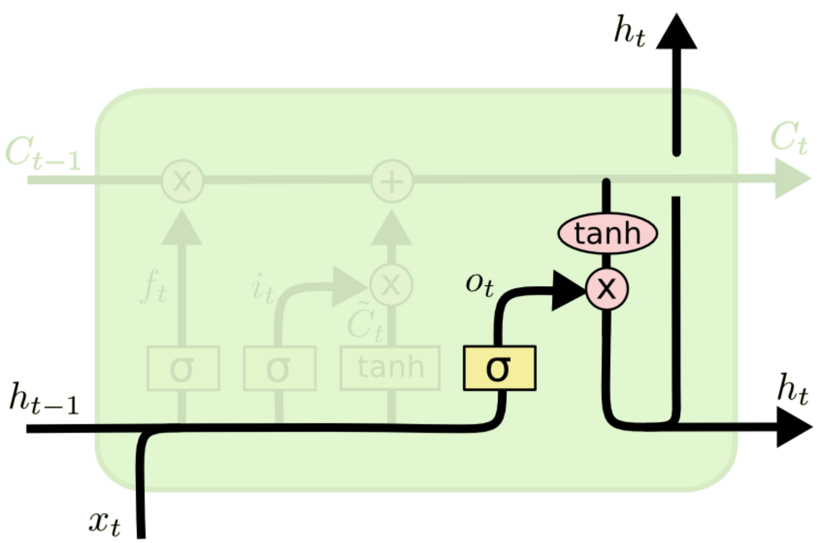
- Cell state is updated by using component-wise vector multiply to "forget" and vector addition to "input" new information.



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Output Gate

- Hidden state is updated based on a "filtered" version of the cell state, scaled to –1 to 1 using tanh.
- Output gate computes a sigmoid function of the input and current hidden state to determine which elements of the cell state to "output".

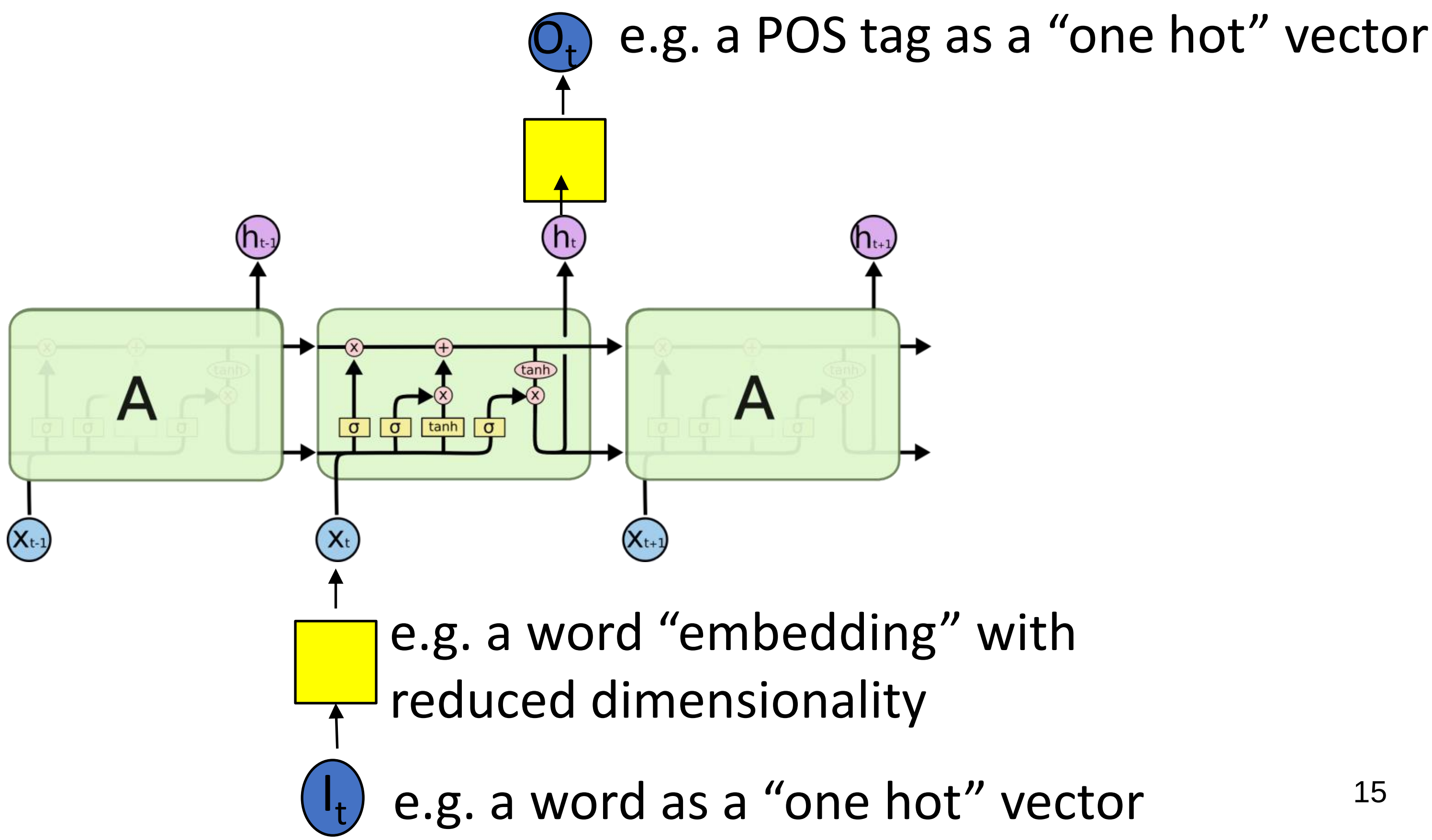


$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

Overall Network Architecture

- Single or multilayer networks can compute LSTM inputs from problem inputs and problem outputs from LSTM outputs.



LSTM Training

- Trainable with backprop derivatives such as:
  - Stochastic gradient descent (randomize order of examples in each epoch) with momentum (bias weight changes to continue in same direction as last update).
  - ADAM optimizer (Kingma & Ma, 2015)
- Each cell has many parameters (W_f, W_i, W_C, W_o)
  - Generally requires lots of training data.
  - Requires lots of compute time that exploits GPU clusters.