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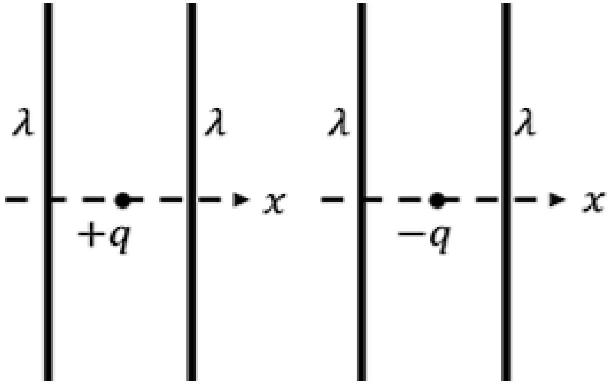
Quiz

The figure below depicts two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges $+q$ and $-q$ are kept in equilibrium between them. The point charges are confined to move in the $x-$ direction only. If they given a small displacement about their equilibrium positions, then the correct statement(s) is (are)



a) Both charges execute simple harmonic motion
b) Both charges will continue moving in the direction of their displacement
c) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of displacement
d) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of displacement

Quiz

If $\vec{F}$ derives from a gradient and its components are defined everywhere,

a) $\vec{\nabla}\vec{F} = 0$
b) $\vec{\nabla}.\vec{F} = 0$
c) $\vec{\nabla} \times \vec{F} = \vec{0}$
d) $\nabla^2\vec{F} = \vec{0}$

The curl of gradient of a vector is

a) Unity
b) Zero
c) Null vector
d) Depends on the constants of the vector

The vector function $\vec{A} = \vec{\nabla}u$ where $u(x, y)$ is a scalar function. Then $|\vec{\nabla} \times \vec{A}|$ is

a) -1
b) 0
c) 1
d) ∞

Quiz

The curl is a vector operator that describes the infinitesimal rotation of a vector field in three dimensional space

a) True
b) False

Divergence:
The divergence of a vector field A at a point is the net outward flux of A per unit volume as the volume about the point tends to zero

Gradient:
The vector that represents both the magnitude and the direction of the maximum space rate of increase of a scalar.

Curl:
The curl of a vector field is a vector whose magnitude is the maximum net circulation of A per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the net circulation maximum.

Quiz

The gradient can be replaced by which of the following?

a) Maxwell equation
b) Volume integral
c) Differential equation
d) Surface integral

Quiz

When a gradient of a scalar field is a null vector, the scalar field lies parallel to the x-axis

a) True
b) False

Quiz

The divergence of the vector $3xz\vec{i} + 2xy\vec{j} - yz^2\vec{k}$ at a point $(1,1,1)$ is

a) 7
b) 4
c) 3
d) 0

Homework

A vector $\vec{A}$ cartesian components A_x and A_y. Write the vector in terms of its radial and tangential components.

Two fundamental postulates of electrostatics in free space

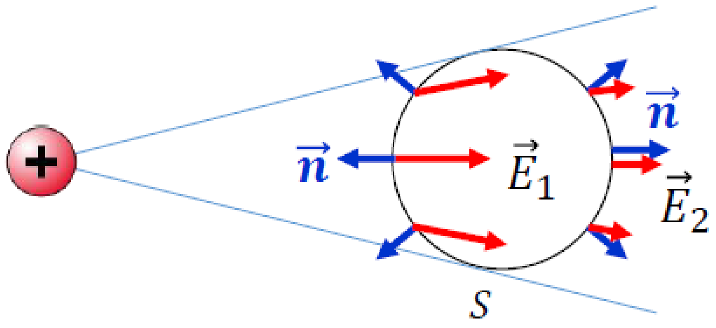
Postulates of Electrostatics in Free Space	
Differential Form	Integral Form
$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$
$\nabla \times \mathbf{E} = 0$	$\oint_C \mathbf{E} \cdot d\ell = 0$

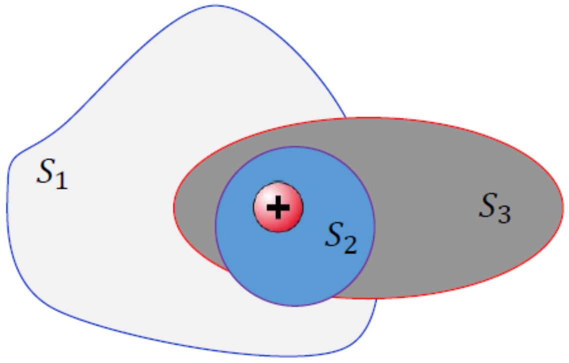
Gauss's Law

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \oint_S \mathbf{E} \cdot ds = \frac{Q}{\epsilon_0}$$

- Gauss's law does not depend on the shape of Gauss's surface

- Charge outside: $Flux = 0$ Charge Inside: $Flux = \frac{Q}{\epsilon_0}$



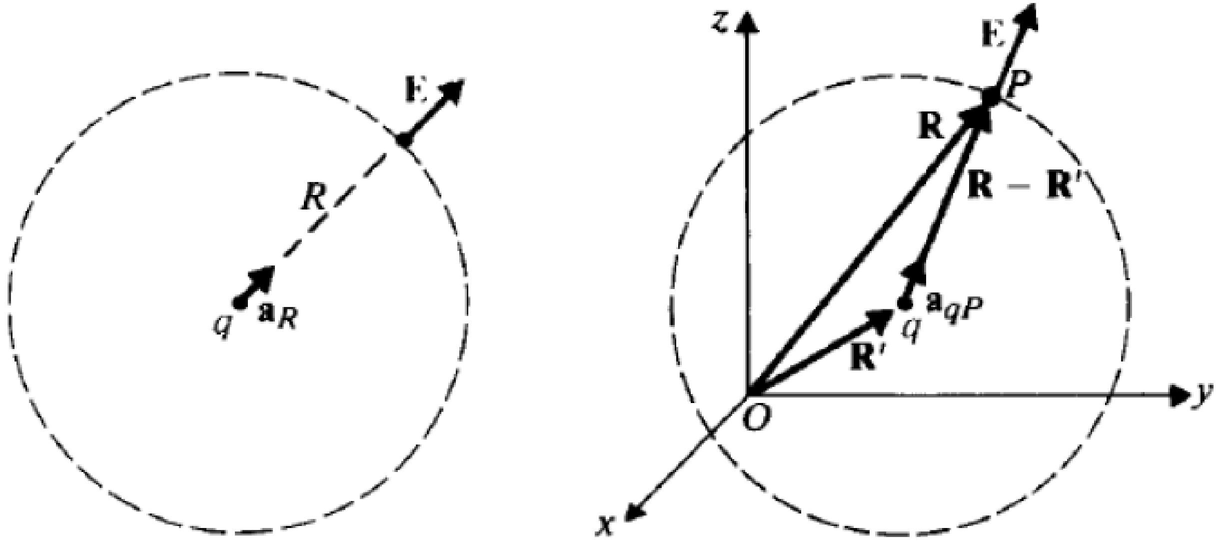


Coulomb's Law

$$\mathbf{E} = \mathbf{a}_R E_R = \mathbf{a}_R \frac{q}{4\pi\epsilon_0 R^2} \quad (\mathrm{V/m})$$

$$\mathbf{E}_p = \frac{q\,(\mathbf{R}-\mathbf{R}')}{4\pi\epsilon_0 \left|\mathbf{R}-\mathbf{R}'\right|^3} \quad (\mathrm{V/m})$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{n} \frac{q_k\,(\mathbf{R}-\mathbf{R}'_k)}{\left|\mathbf{R}-\mathbf{R}'_k\right|^3} \quad (\mathrm{V/m})$$



(a) Point charge at the origin. (b) Point charge not at the origin.

Conservative Force

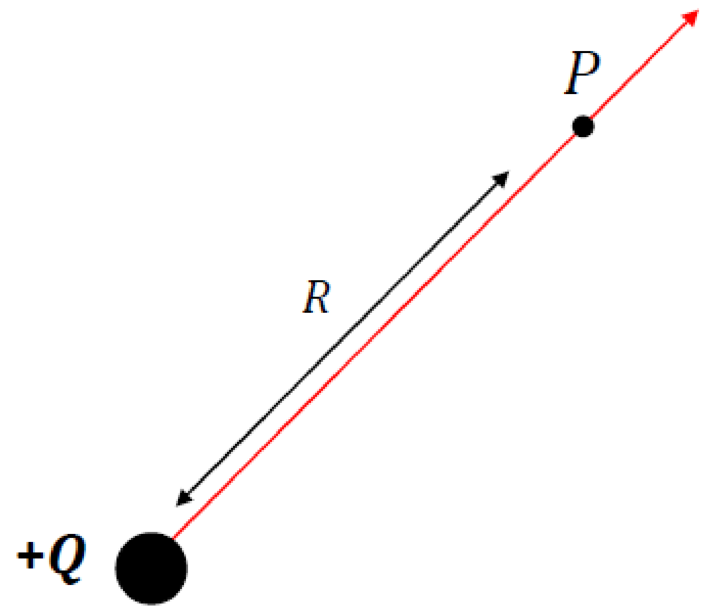
- Conservative force conserves the total energy
- Conservative force always act naturally to push the system towards lower potential energy

Electric Potential

- Electric Potential results from the work done when a test charge q is placed at P
- Work done on the object is independent of the path

$$\boxed{V(R) = \frac{W}{q_0} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{R}}$$

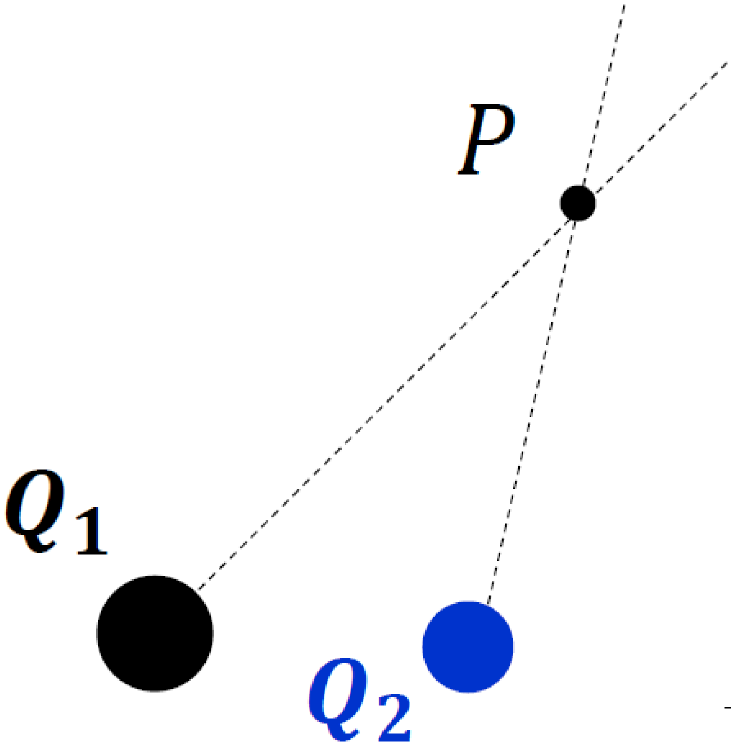
$$\boxed{V(\infty) = 0}$$



Electric Potential

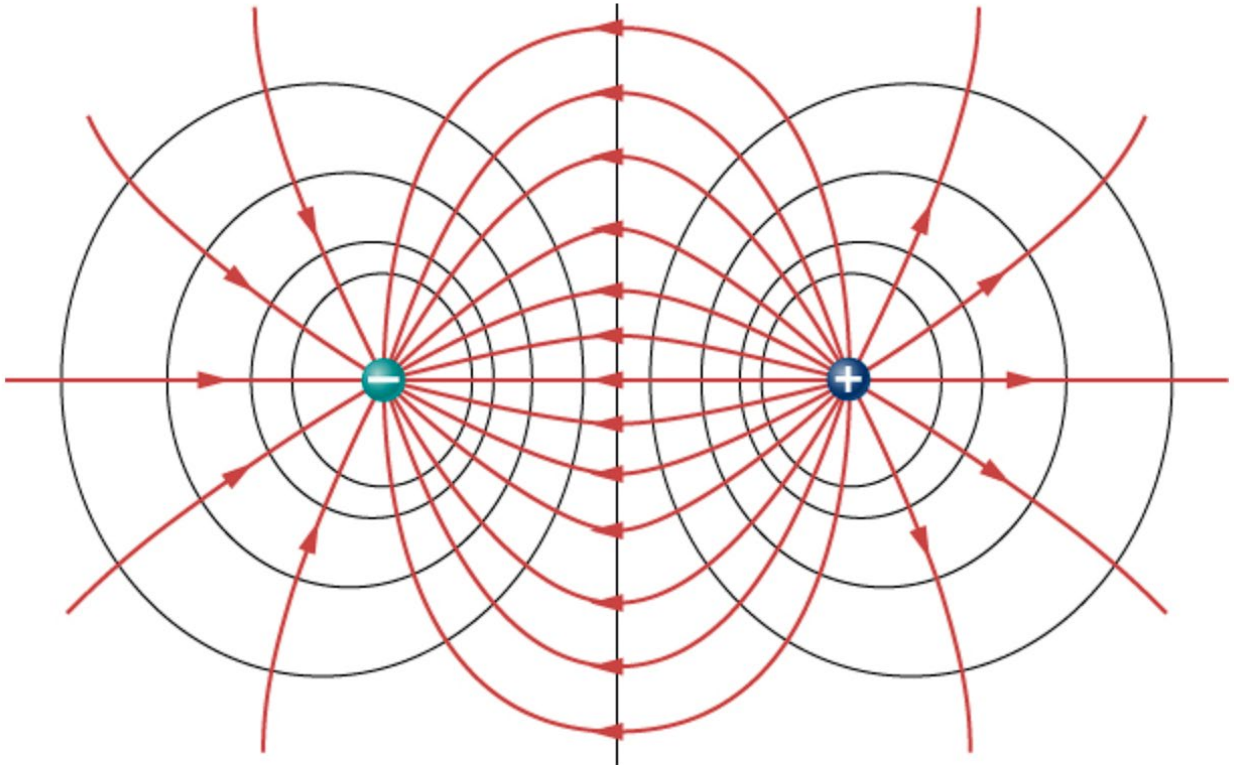
- Superposition rule

$$V(P) = V_{Q1} + V_{Q2}$$



Electric Potential

- Two different equipotentials never cross like two electric field lines

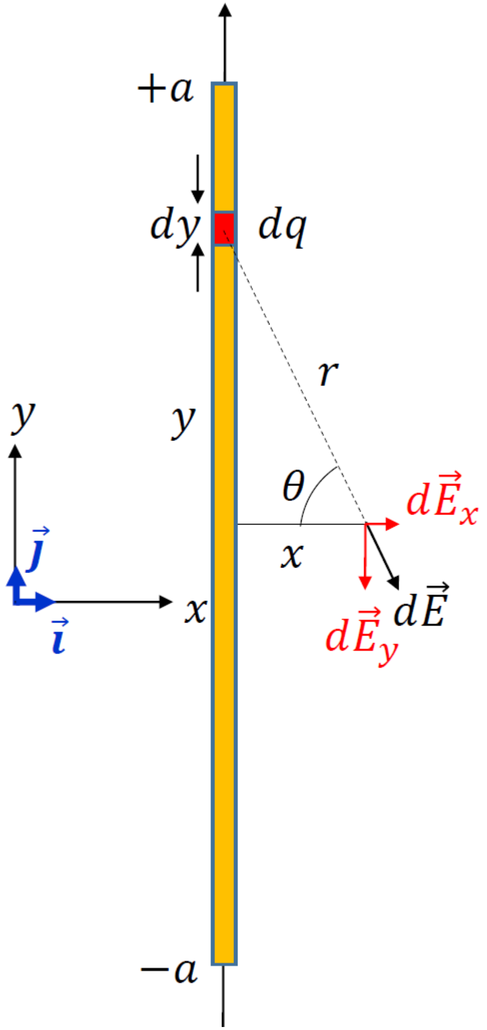


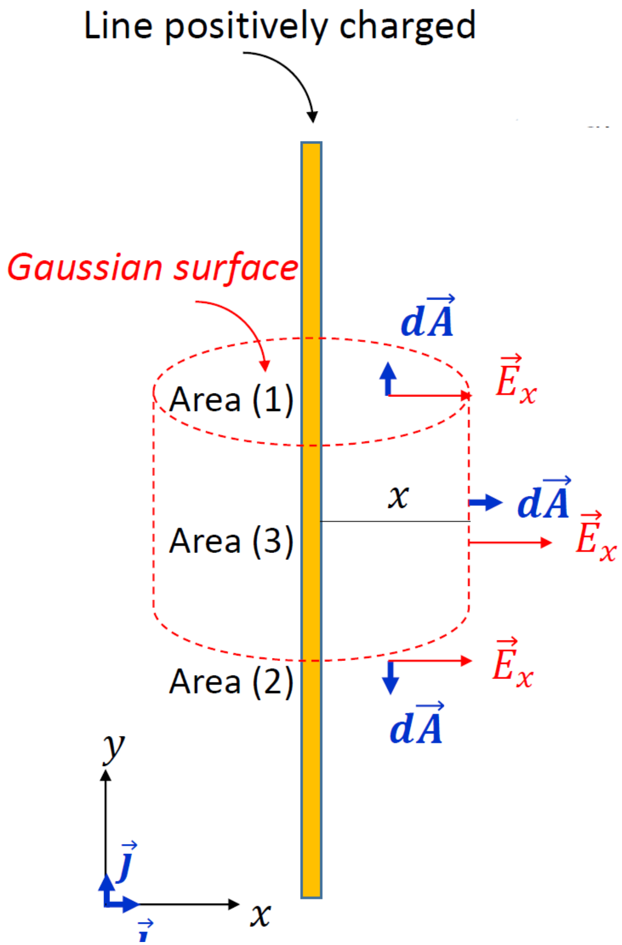
Electric Field Calculation Strategy

If symmetry conditions exist such that a Gaussian surface can be constructed over which $\mathbf{E} \cdot d\mathbf{s}$ is constant, it is always easier to determine $\mathbf{E}$ directly. If not, we can determine $\mathbf{E}$ by first finding V.

Electric Field due to Line Charge

$$\begin{cases} \text{If } a << x & \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x^2}\vec{i} \quad \text{Point charge} \\ \text{If } a = \infty & \vec{E} = \dfrac{1}{2\pi\varepsilon_0}\left(\dfrac{Q}{2a}\right)\dfrac{1}{x}\vec{u} = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{x}\vec{i} \end{cases}$$

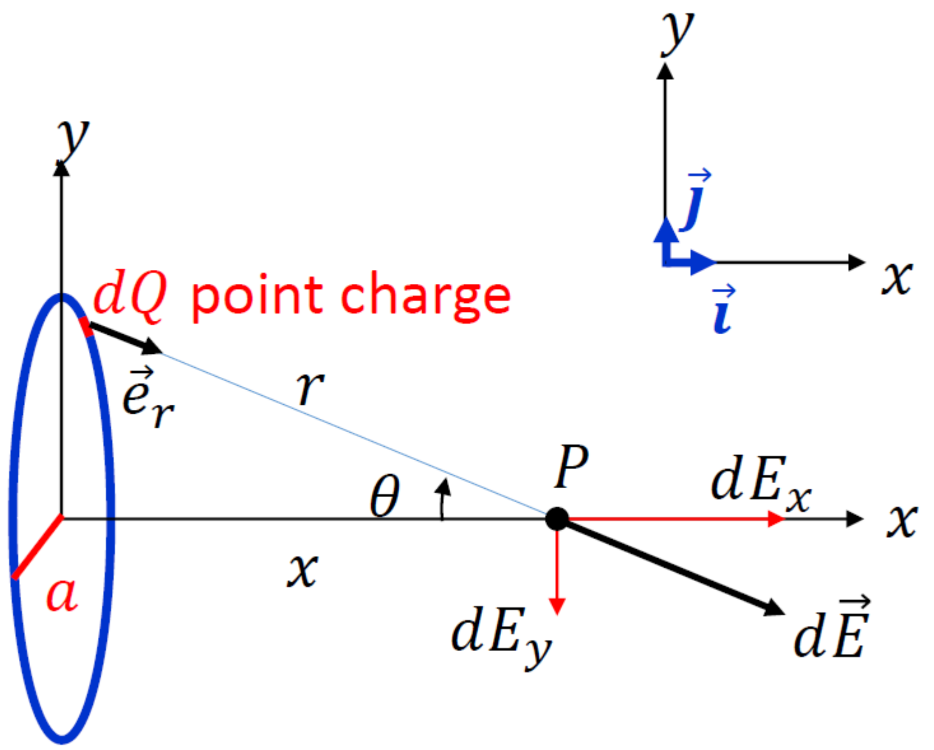




Electric Field due to Ring Charge

$$\vec{E} = E_x \vec{i} = \frac{1}{4\pi\varepsilon_0} \frac{xQ}{(x^2 + a^2)^{3/2}} \vec{i}$$

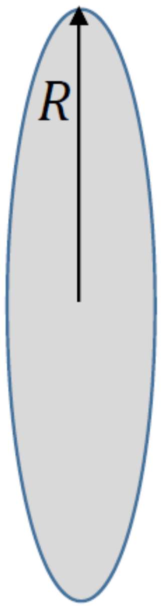
$$V = \frac{1}{4\pi\varepsilon_0} \oint_{\Gamma} \frac{dQ}{r} = \frac{1}{4\pi\varepsilon_0} \frac{1}{r} \oint_{\Gamma} dQ = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$$



Electric Field due to Disk Charge

From the ring

$$d\vec{E}_x = \frac{1}{4\pi\varepsilon_0}\frac{xdQ}{(x^2+r^2)^{3/2}}\vec{\imath}$$

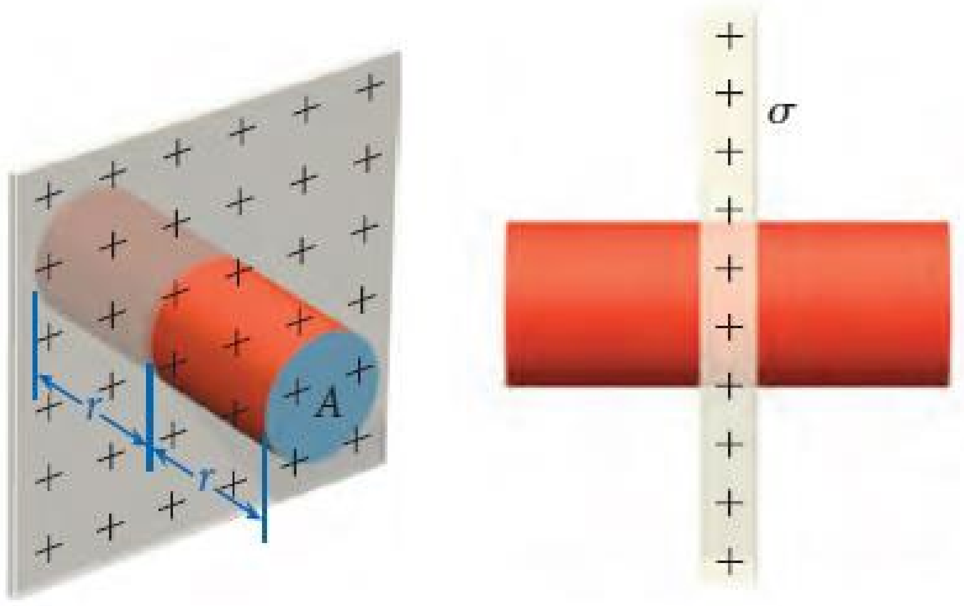


$$\vec{E}_x = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{x\sigma 2\pi r dr}{(x^2+r^2)^{3/2}}\vec{\imath} = \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{rdr}{(x^2+r^2)^{3/2}}\vec{\imath}$$

$$\boxed{\vec{E}_x = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{x\sigma 2\pi r dr}{(x^2+r^2)^{3/2}}\vec{\imath} = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{R^2/x^2+1}}\right]\vec{\imath}}$$

Electric Field due to Plane Charge

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{\imath}$$



Electric Field due to Continuous Charge

Volume Charge

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{V'} \rho \frac{\mathbf{R}}{R^3}\, dv' \quad (\mathrm{V/m}).$$

Surface Charge

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{S'} \mathbf{a}_R \frac{\rho_s}{R^2}\, ds' \quad (\mathrm{V/m})$$

Infinite Surface Charge

$$\mathbf{E} = \mathbf{a}_z E_z = \mathbf{a}_z \frac{\rho_s}{2\epsilon_0}, \qquad z > 0$$
$$\mathbf{E} = -\mathbf{a}_z E_z = -\mathbf{a}_z \frac{\rho_s}{2\epsilon_0}, \qquad z < 0$$

Line Charge

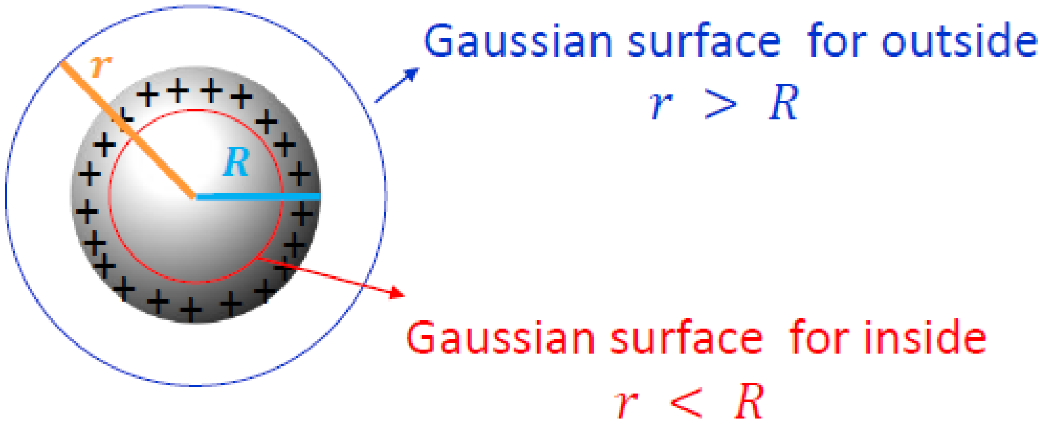
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_{L'} \mathbf{a}_R \frac{\rho_\ell}{R^2}\, d\ell' \quad (\mathrm{V/m})$$

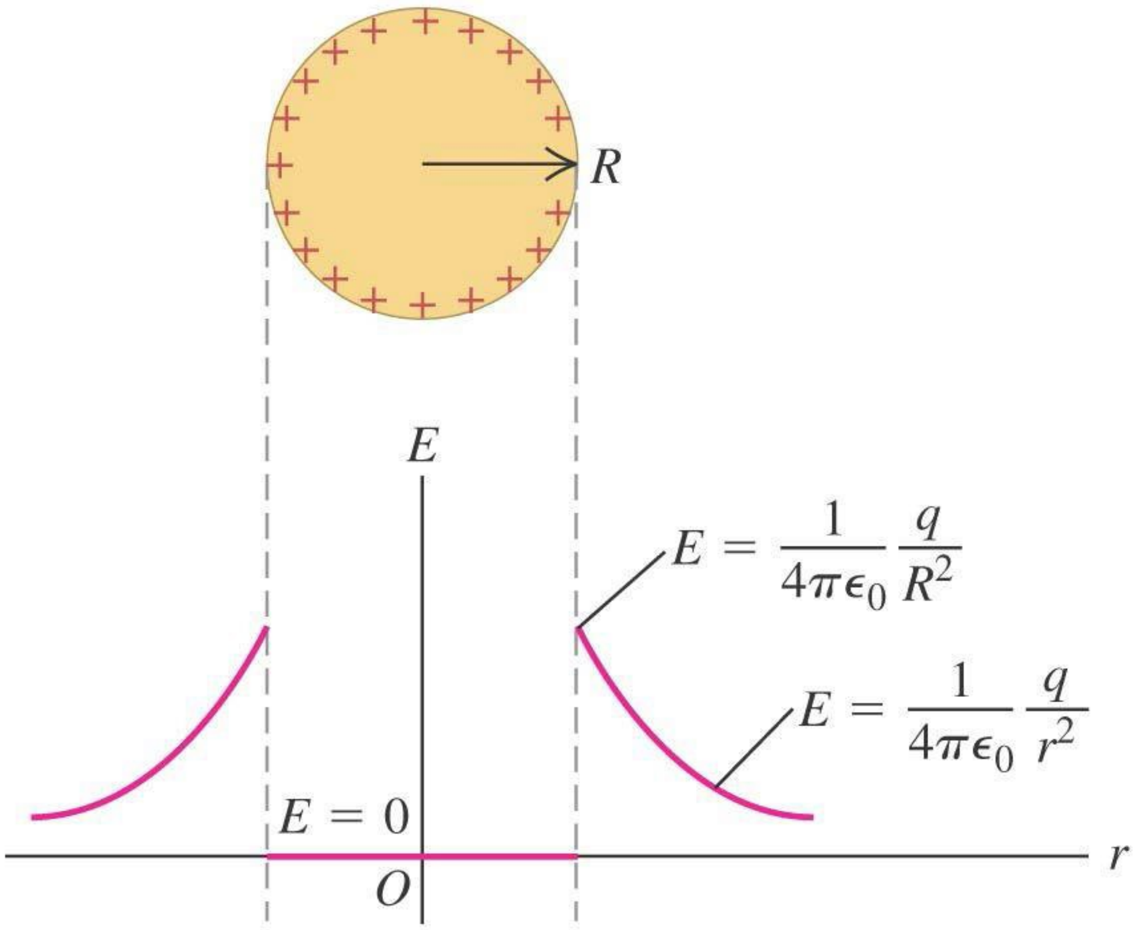
Infinite Line Charge

$$\mathbf{E} = \mathbf{a}_r \frac{\rho_\ell}{2\pi\epsilon_0 r} \quad (\mathrm{V/m})$$

Solid Conducting Sphere

$\vec{E} = \vec{0}$ everywhere inside a conducting shell





Conductors in Static Electric Field

Inside a Conductor (Under Static Conditions)
$\rho = 0$ $\mathbf{E} = 0$

Under static conditions the $\mathbf{E}$ field on a conductor surface is everywhere normal to the surface. In other words, the surface of a conductor is an equipotential surface under static conditions.

Boundary Conditions at a Conductor/Free Space Interface
$E_t = 0$ $E_n = \frac{\rho_s}{\epsilon_0}$

Exercise 1

Let's consider two following fields

1) $\vec{E} = \alpha\left[xy\vec{\imath} + 2yz\vec{\jmath} + 3xz\vec{k}\right]$

2) $\vec{E} = \alpha\left[y^2\vec{\imath} + (2xz + z^2)\vec{\jmath} + 2yz\vec{k}\right]$

(a) Which one of these fields is an electrostatic field? Justify

(b) Without doing <u>any calculation</u> can you give a hint to how to check the result?

Exercise 2

P.3–13 Determine the work done in carrying a -2 (μC) charge from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$ in the field $\mathbf{E} = \mathbf{a}_x y + \mathbf{a}_y x$

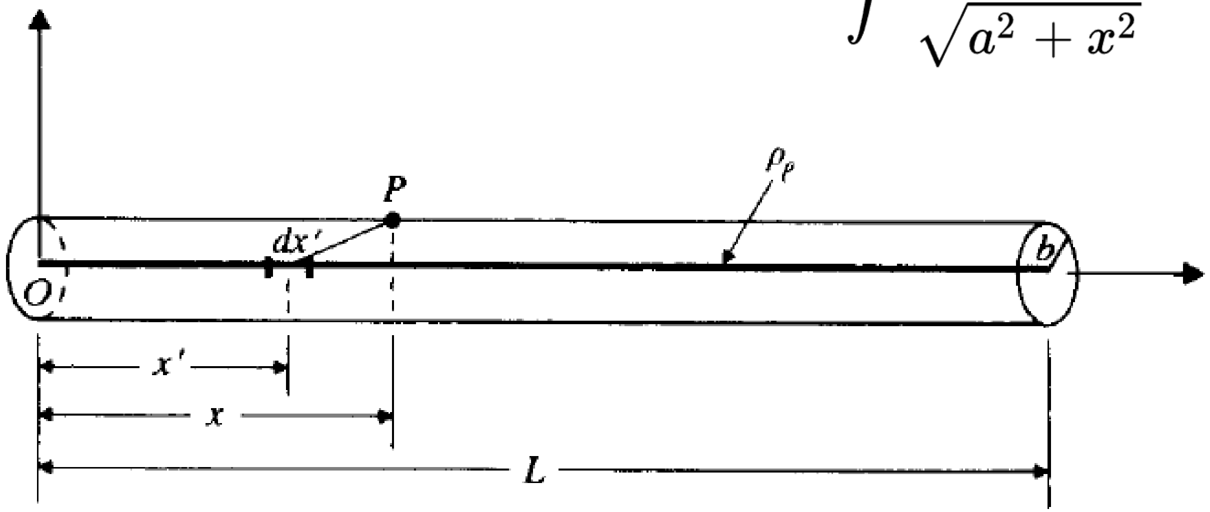
a) along the parabola $x = 2y^2$,

b) along the straight line joining P_1 and P_2.

Exercise 3

P.3–17 In Example 3–5 we obtained the electric field intensity around an infinitely long line charge of a uniform charge density in a very simple manner by applying Gauss's law. Since $|\mathbf{E}|$ is a function of r only, any coaxial cylinder around the infinite line charge is an equipotential surface. In practice, all conductors are of finite length. A finite line charge carrying a constant charge density ρ_ℓ along the axis, however, does not produce a constant potential on a concentric cylindrical surface. Given the finite line charge ρ_ℓ of length L in Fig. 3–40, find the potential on the cylindrical surface of radius b as a function of x and plot it.

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(\frac{\sqrt{x^2 + a^2} + x}{a}\right) + C$$



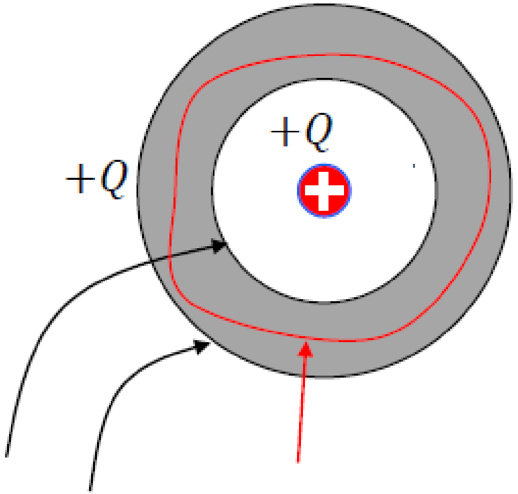
Exercise 4

A point charge $+Q$ is located at the center of a solid spherical conducting shell with inner radius R and outer radius $2R$. In addition the conducting shell has a total net charge of $+Q$.

(a) How much charges are located on the inner ($r = R$) and outer surfaces ($r = 2R$) of the conducting shell?

(b) What is the magnitude of the electric field in the regions $r < R$, $R < r < 2R$ and $r > 2R$?

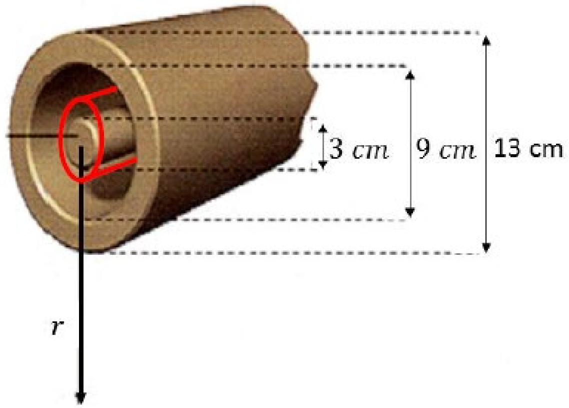
(c) Would a displacement of the charge $+Q$ off the center by $d = R/3$ vertically down cause any change? Explain



Exercise 5

The figure below shows a portion of an infinitely long, concentric cable in cross section. The inner conductor has a linear charge density of $\lambda = 6.00\ nC/m$ *and the outer conductor has no net charge.*

(a) Find the electric field for $r < 1.5\ cm, r = 1.5\ cm,\ 4.5\ cm < r < 6.5\ cm$ *and* $r > 6.5\ cm$, *where* r *is the perpendicular distance from the common axis of the cylindrical system.*
(b) What are the surface charge densities on the inside and the outside surfaces of the outer conductor?

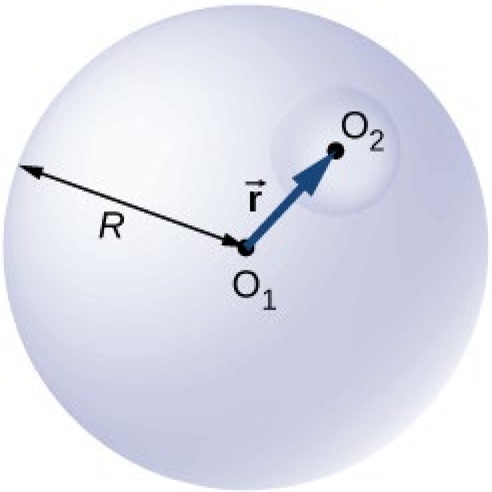


Exercise 6

Each of three charged spheres of radius a, one conducting, one having a uniform charge density within its volume, and one having a spherically symmetric charge density that varies radially as r^n (n > -3), has a total charge Q. Use Gauss's theorem to obtain the electric fields both inside and outside each sphere. Sketch the behavior of the fields as a function of radius for the first two spheres, and for the third with $n = -2$, $n = +2$.

Exercise 7

A total charge Q is distributed uniformly throughout a spherical volume that is centered at O1 and has a radius R. Without disturbing the charge remaining, charge is removed from the spherical volume that is centered at O2 (see below), where $\vec{r}$ is the displacement vector directed from O1 to O2. Find the electric field everywhere in the empty region.



Thank You

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