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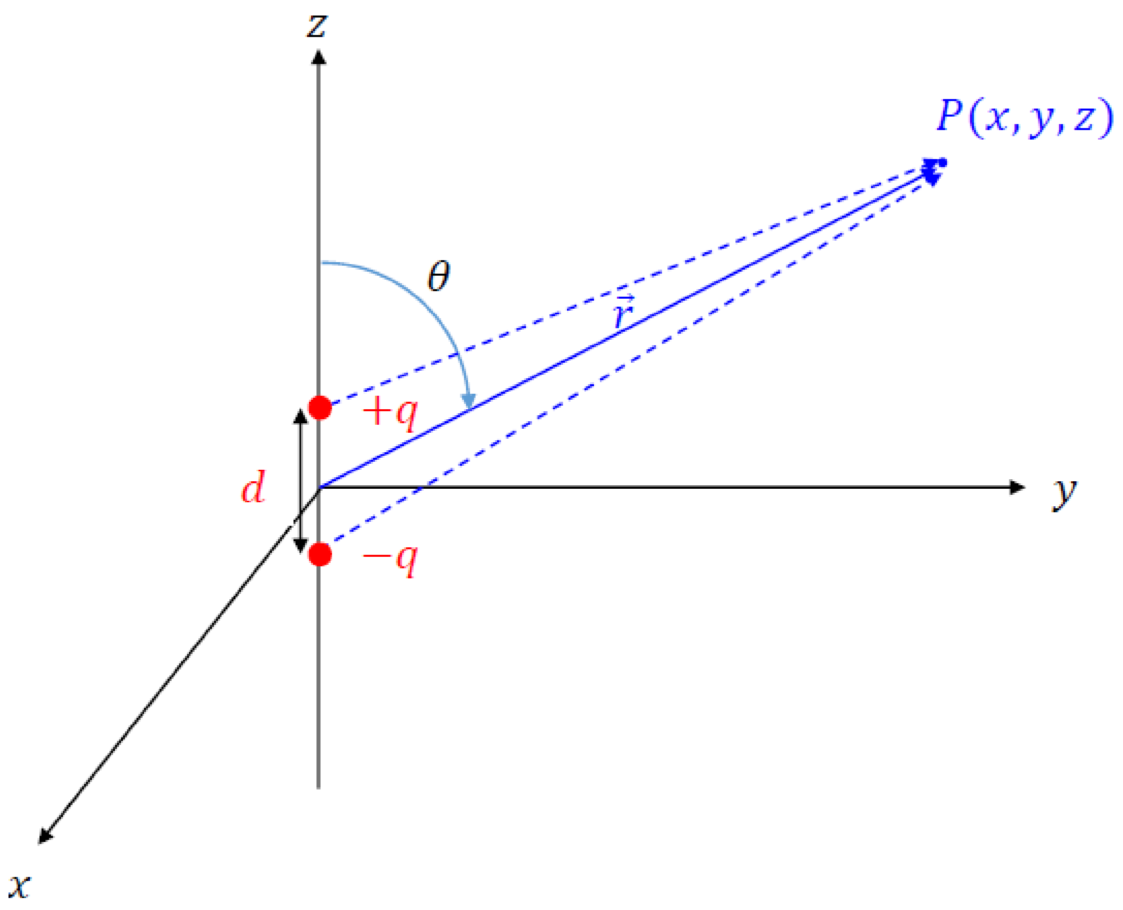
Electric Dipoles



$$V(P) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{\left(z-\frac{d}{2}\right)^2 + x^2 + y^2}} - \frac{q}{\sqrt{\left(z+\frac{d}{2}\right)^2 + x^2 + y^2}}\right]$$

$$\boxed{V(P) = \frac{q}{4\pi\varepsilon_0}\frac{zd}{r^3}} \qquad \boxed{r >> d}$$

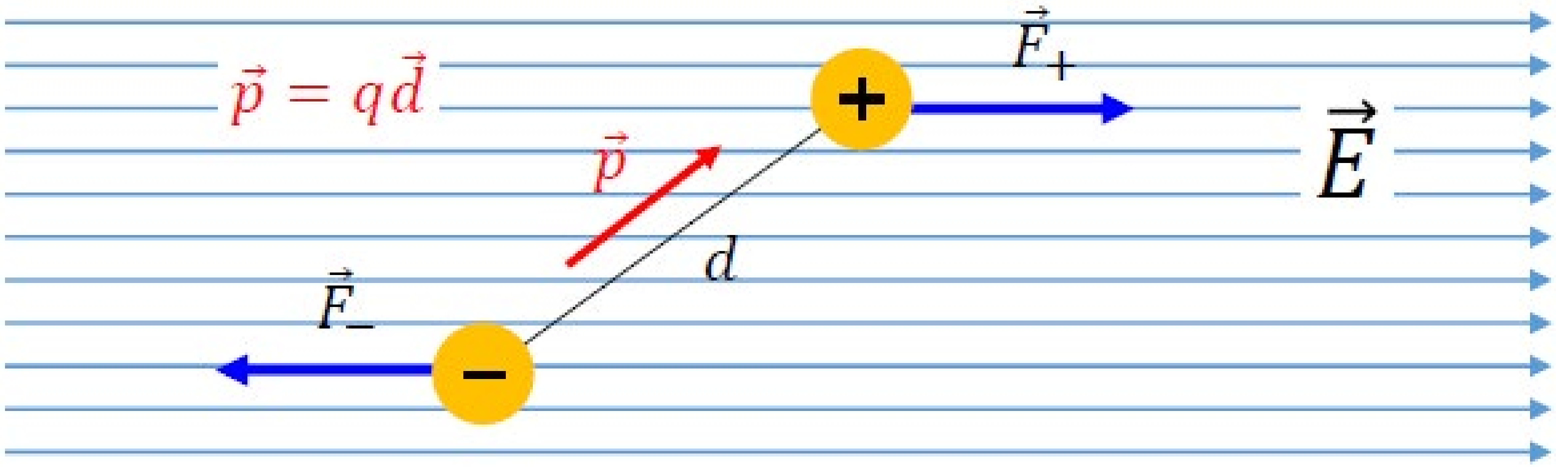
$$\vec{\boldsymbol{p}} = \boldsymbol{q}\vec{\boldsymbol{d}}$$

$\vec{\boldsymbol{p}} = \boldsymbol{\alpha_e}\vec{\boldsymbol{E}}_{\boldsymbol{loc}}$ $\qquad [p] = \mathrm{Cm} \quad [\alpha_e] = \mathrm{F}m^2$

$\boldsymbol{\alpha_e}$ = atomic (electronic) polarizability

$$V(\vec{r}) = \frac{\vec{p}.}{4\pi\varepsilon_0}\boxed{\frac{\vec{e}_r}{r^2}} = \frac{1}{4\pi\varepsilon_0}\vec{p}.\boxed{\vec{\nabla}\left(-\frac{1}{r}\right)}$$

Electric Dipoles



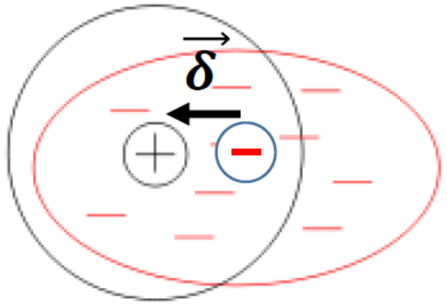
$$U = -pE\cos\theta = -\vec{p}.\vec{E}$$

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}[3(\vec{p}_1.\vec{e}_r)\vec{e}_r - \vec{p}_1]$$

Dipole $\vec{p}_2$in field $\vec{E}_1$due to dipole $\vec{p}_1$

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\vec{p}_2.[3(\vec{p}_1.\vec{e}_r)\vec{e}_r - \vec{p}_1]$$

Crude model: electronic polarization



Electronic cloud = charged sphere $E(r) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$

Force exerted on the nucleus (charge Q)
inside the cloud at distance δ from the center

$$F_{cloud} = QE(\delta) = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R^3}\delta$$

Equilibrium position is reached when $\vec{F}_{cloud} + \vec{F}_{ext} = \vec{0}$ $\Rightarrow$ $-\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R^3}\delta + QE_{ext} = 0$

Equilibrium distance $\delta = 4\pi\varepsilon_0 R^3 \frac{E_{ext}}{Q}$

Induced dipole $\vec{\boldsymbol{p}} = \boldsymbol{Q}\vec{\boldsymbol{\delta}}$ $p = Q\delta = 4\pi\varepsilon_0 R^3 E_{ext}$

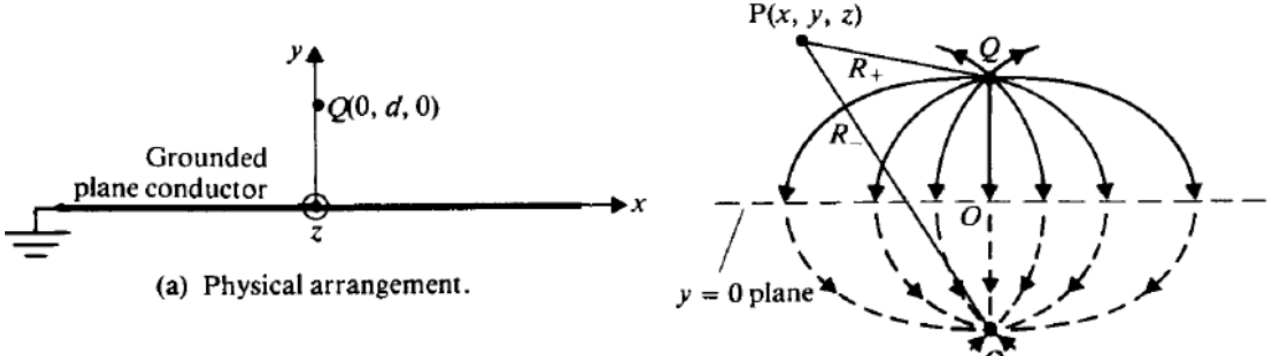
$\vec{p} = \alpha_e \vec{E}_{loc}$ $\vec{E}_{loc} = \vec{E}_{ext}$

Electronic polarizability of atoms

$$\boldsymbol{\alpha_e} = \frac{p}{E_{ext}} = 4\pi\varepsilon_0 R^3$$

Image Method

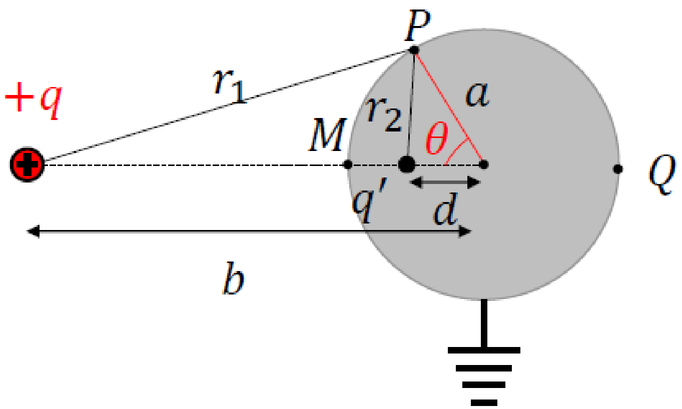
- Infinite Conducting Plane



(a) Physical arrangement.

(b) Image charge and field lines.

- Conducting Sphere



$$V(M) = V(Q) = 0 \Rightarrow \begin{cases} d = \dfrac{a^2}{b} \\ q' = -\dfrac{a}{b} q \end{cases}$$

New Concepts

- Polarization
- Bound charges (linear, surface, volume)
- Permittivity of materials $\varepsilon = \varepsilon_0 \varepsilon_r$ ($\varepsilon_r = 1$ in vacuum)
- Electric susceptibility χ ⇨ $\varepsilon = \varepsilon_0(1+\chi)$ ($\chi = 0$)
- Vector displacement $\vec{D}$

Dielectric

- A dielectric has the ability to get **polarized** by an **external** applied field
  ⇔ The applied external field <u>induces electric dipoles</u> inside the dielectric

- Polarization occurs in both **polar** and **nonpolar** materials

- Although any kind of substance is **polarizable** to some extent, the effect of <u>polarization is important only in dielectric materials</u>

- The dielectric is an insulator **BUT** an insulator is not necessarily a dielectric

Dielectric

- Polarized charge density on the surface

$$\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n$$

- Polarized charge density inside the dielectric

$$\rho_p = -\nabla \cdot \mathbf{P}.$$

- Electric displacement

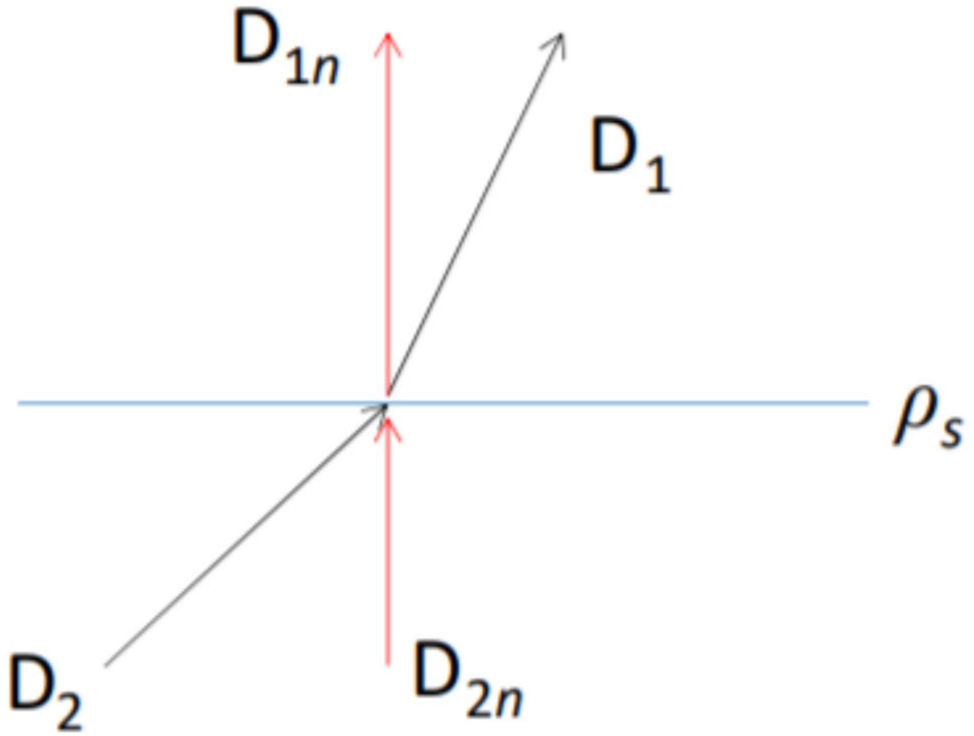
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad (\mathrm{C/m^2}).$$

$$\nabla \cdot \mathbf{D} = \rho \qquad (\mathrm{C/m^3}),$$

$$\begin{aligned}\mathbf{D} &= \epsilon_0 (1 + \chi_e) \mathbf{E} \\ &= \epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E} \qquad (\mathrm{C/m^2}),\end{aligned}$$

Boundary Conditions for Electrostatic Fields

Tangential components,	$E_{1t} = E_{2t}$;
Normal components,	$\mathbf{a}_{n2} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$.



$$D_{1n} - D_{2n} = \rho_s \quad (\mathrm{C/m^2})$$

Capacitor

- Capacitance: $$Q = CV$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Exercise 1

A layer of porcelain is 80 mm long, 20 mm wide and 0.7 μm thick. Calculate its capacitance with ε_r = 6

Solution:

Given data:

Permitivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{F m}^{-1}$

Thickness $d = 0.7\ \mu\text{m (or) } 0.7 \times 10^{-6}\ \text{m}$

$$\text{Area} = l \times b \times h$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 6 \times 80 \times 20 \times 10^{-6}}{0.7 \times 10^{-6}}$$

$$\boxed{C = 1.21 \times 10^{-7}\ \text{F}}$$

Exercise 2

The dielectric constant of a helium gas at NTP is 1.0000684. Calculate the electron polarizability of helium atoms if the gas contains 2.7×10^{26} atoms/m^3 and hence calculate the radius of helium atom ($\varepsilon_0 \; = \; 8.854 \; \times \; 10 – 12 \; Fm^{-1}$)

Exercise2 Sol

Solution:

Given data:

$$\text{Relative permitivity} \quad \varepsilon_r = 1.0000684$$

$$\text{No. of atoms in the gas } N = 2.7 \times 10^{26} \text{ atoms/m}^3$$

$$\text{Permitivity of free space } \varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

i) We know polarization

$$P = \varepsilon_0 (\varepsilon_r - 1) E$$

$$\text{and } P = N\alpha_e E$$

From the above two equations, we can write

$$N\alpha_e = \varepsilon_0 (\varepsilon_r - 1)$$

$$\alpha_e = \frac{\varepsilon_0 (\varepsilon_r - 1)}{N}$$

$$= \frac{8.854 \times 10^{-12} (1.0000684 - 1)}{2.7 \times 10^{26}}$$

$$\boxed{\alpha_e = 2.242 \times 10^{-42} \text{ Fm}^2}$$

ii) Electronic polarizability

$$\alpha_e = 4\pi\varepsilon_0 R^3$$

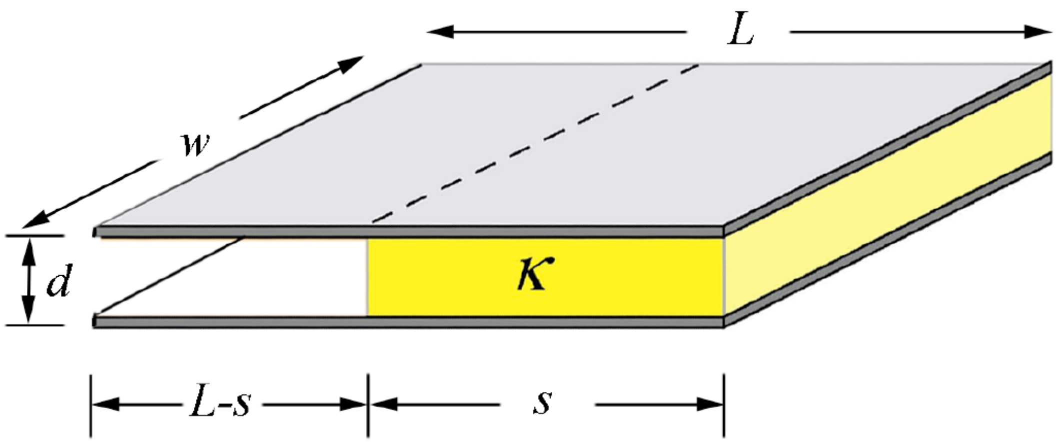
$$R = \frac{\alpha_e}{4\pi\varepsilon_0}$$

$$= \frac{2.242 \times 10^{-42}}{4 \times 3.14 \times 8.854 \times 10^{-12}}$$

Radius of helium atom $\boxed{R = 0.0201 \times 10^{-30} \text{ m}}$

Exercise 3

A dielectric rectangular slab has length s, width w, thickness d, and dielectric constant κ. The slab is inserted on the right hand side of a parallel-plate capacitor consisting of two conducting plates of width w, length L, and thickness d. The left hand side of the capacitor of length $L-s$ is empty. The capacitor is charged up such that the left hand side has surface charge densities $\pm\sigma_L$ on the facing surfaces of the top and bottom plates respectively and the right hand side has surface charge densities $\pm\sigma_R$ on the facing surfaces of the top and bottom plates respectively. The total charge on the entire top and bottom plates is $+Q$ and $-Q$ respectively. The charging battery is then removed from the circuit. ***Neglect all edge effects.***



Exercise 3

a) Find an expression for the magnitude of the electric field E_L on the left hand side in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

b) Find an expression for the magnitude of the electric field E_R on the right hand side in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

c) Find an expression that relates the surface charge densities σ_L and σ_R in terms of κ, s, w, L, ε_0, and d as needed.

d) What is the total charge $+Q$ on the entire top plate? Express your answer in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

e) What is the capacitance of this system? Express your answer in terms of κ, s, w, L, ε_0, and d as needed.

f) Suppose the dielectric is removed. What is the change in the stored potential energy of the capacitor? Express your answer in terms of Q, κ, s, w, L, ε_0, and d as needed.

Exercise3 Sol

a) Find an expression for the magnitude of the electric field E_L on the left hand side in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

Using Gauss's Law $E_L = \dfrac{\sigma_L}{\varepsilon_o}$

b) Find an expression for the magnitude of the electric field E_R on the right hand side in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

Using Gauss's Law for dielectrics $E_R = \dfrac{\sigma_R}{\kappa\varepsilon_o}$

c) Find an expression that relates the surface charge densities σ_L and σ_R in terms of κ, s, w, L, ε_0, and d as needed.

The potential difference on the left side is $E_L d = \dfrac{\sigma_L d}{\varepsilon_o}$. On the right hand side it is

$E_R d = \dfrac{\sigma_R d}{\kappa\varepsilon_o}$. Since these must be equal we must have $\dfrac{\sigma_R}{\kappa} = \sigma_L$

d) What is the total charge $+Q$ on the entire top plate? Express your answer in terms of σ_L, σ_R, κ, s, w, L, ε_0, and d as needed.

$$Q = \sigma_L (L - s) w + \sigma_R s w$$

Exercise3 Sol

e) What is the capacitance of this system? Express your answer in terms of κ, s, w, L, ε_0, and d as needed.

The potential difference is $E_L d = \dfrac{\sigma_L d}{\varepsilon_o}$, so the capacitance is

$$C = \frac{Q}{|\nabla V|} = \frac{\sigma_L (L-s) w + \sigma_R s w}{\dfrac{\sigma_L d}{\varepsilon_o}} = \frac{\varepsilon_o w}{d}\left[(L-s) + \frac{\sigma_R}{\sigma_L} s\right] = \frac{\varepsilon_o w}{d}\left[(L-s) + \kappa s\right] = C$$

f) Suppose the dielectric is removed. What is the change in the stored potential energy of the capacitor? Express your answer in terms of Q, κ, s, w, L, ε_0, and d as needed.

Since the battery has been removed, the charge on the capacitor does not change when we do this. So the change in the energy stored is

Exercise3 Sol

$$\text{change in energy} = \frac{1}{2}\frac{Q^2}{C_o} - \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2}\left(\frac{1}{C_o} - \frac{1}{C}\right)$$

$$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon_o L w} - \frac{d}{\varepsilon_o w\left[(L-s)+\kappa s\right]}\right) = \frac{Q^2 d}{2\varepsilon_o w}\left[\frac{1}{L} - \frac{1}{(L-s)+\kappa s}\right]$$

$$\text{change in energy} = \frac{Q^2 d}{2\varepsilon_o w}\left[\frac{(L-s)+\kappa s - L}{\left[L\left[(L-s)+\kappa s\right]\right]}\right] = \frac{Q^2 d}{2\varepsilon_o w}\left[\frac{(\kappa-1)s}{\left[L\left[(L-s)+\kappa s\right]\right]}\right].$$

Thank You

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