

CH 4.3

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MATH 324 Ch 4.3

Standard Normal R Calculations

1A) $P(0 < Z < 2.17)$?

```
pnorm(2.17) - pnorm(0)
```

```
## [1] 0.4849966
```

1B) $P(-2.5 \leq Z \leq 0)$?

```
pnorm(0) - pnorm(-2.5)
```

```
## [1] 0.4937903
```

1C) $P(-2.5 \leq Z \leq 2.5)$?

```
pnorm(2.5) - pnorm(-2.5)
```

```
## [1] 0.9875807
```

1D) $P(Z \leq 1.37)$?

```
pnorm(1.37)
```

```
## [1] 0.9146565
```

1E) $P(Z \geq -1.75)$?

```
1 - pnorm(-1.75)
```

```
## [1] 0.9599408
```

1G) $P(-1.5 \leq Z \leq 2)$?

```
pnorm(2) - pnorm(-1.5)
```

```
## [1] 0.9104427
```

1H) $P(1.37 \leq Z \leq 2.5)$?

```
pnorm(2.5)-pnorm(1.37)
```

```
## [1] 0.07913379
```

1I) $P(Z \geq 1.5)$?

```
1-pnorm(1.5)
```

```
## [1] 0.0668072
```

1J) $P(|Z| \leq 2.50) = P(-2.50 \leq Z \leq 2.50)$

```
pnorm(2.5)-pnorm(-2.5)
```

```
## [1] 0.9875807
```

Percentiles of Z

2A) $n(91)$

$\phi(z) = 0.91 = c = 1.34$

2B) $n(9)$

$\phi(z) = 0.09 = -0.91 = -1.34$

2C) $n(75)$

$\phi(z) = 0.75 = c = .675$

2D) $n(25)$

$\phi(z) = 0.25 = -0.75 = -0.675$

2E) $n(6)$

$\phi(z) = 0.06 = c = -0.1555$

3A) $\phi(c) = 0.9838$ (98.38th percentile)

found in 2.1 row and the 0.04 column so standard table = $c = 2.14$

3B) $P(0 \leq Z \leq c) = 0.291$

$$\phi(c) = 0.7910 = c = 0.81$$

3C) $1 - \phi(c) = P(Z \geq c) = 0.121$

$$1 - P(c \leq Z) = P(Z < c) = \phi(c) = 1 - 0.121 = 0.8790 = c = 1.17$$

3D) $P(-c \leq Z \leq c) = 0.668$

$$P(-c \leq Z \leq c) = \phi(c) - \phi(-c) = \phi(c) - (1 - \phi(c)) = 2 * \phi(c) - 1 = \phi(c) = 0.9920 = c = 0.97$$

3E) $P(|Z| \geq c) = 0.016$

$$1 - 0.016 = 0.9840 = 1 - P(c \leq |Z|) = P(|Z| < c) = P(-c < Z < c) = \phi(c) - \phi(-c) = 2 * \phi(c) - 1 = \phi(c) = 0.9920 = c = 2.41$$

4A) $\alpha = 0.0055$ (99.45th percentile)

```
qnorm(0.055)
```

```
## [1] -1.598193
```

4B) $\alpha = 0.09$ (100(1-.09)th percentile)

```
qnorm(0.09)
```

```
## [1] -1.340755
```

4C) $\alpha = 0.663$ (100(1-.66)th percentile)

```
qnorm(0.663)
```

```
## [1] 0.4206646
```

Word Problems for $X \sim N(\mu, \sigma^2)$

In a road-paving process, asphalt mix is transferred by truck. Let X = truck haul time be normally distributed with mean 8.46 min and standard deviation .913 min.

5A)

What is the probability that haul time will be at least 10 minutes?

$$P(X \geq 10) = P((X - \mu)/\sigma \geq (10 - 8.46)/0.913) = P(Z \geq 1.69) = P(Z \leq -1.69)$$

```
pnorm(-1.69)
```

```
## [1] 0.04551398
```

5B)

What is the probability that haul time will exceed 10 minutes?

$$P(X > 10) = P(X \geq 10) = 0.0455$$

5C)

What is the probability that haul time will be between 8 and 10 minutes?

$$P(8 \leq X \leq 10) = P((8-8.46/0.913) \leq (X - \mu/\sigma) \leq (10-8.46/0.913)) = P(-0.5 \leq Z \leq 1.69) = P(Z \leq 1.69) - P(Z \leq -0.5)$$

```
pnorm(1.69) - pnorm(-0.5)
```

```
## [1] 0.6459485
```

5D)

What value c is such that 98% of all haul times are in the interval from $8.46 - c$ to $8.46 + c$.

$$\begin{aligned} P(8.46 - c \leq X \leq 8.46 + c) &= 0.98 \\ P(-c \leq X - 8.46 \leq c) &= 0.98 \\ P(-c \leq X - \mu \leq c) &= 0.98 \\ P((-c/0.913) \leq (X - \mu/\sigma) \leq (c/0.913)) &= 0.98 \\ P((-c/0.913) \leq Z \leq (c/0.913)) &= 0.98 \\ 2P(Z \leq c/0.913) - 1 &= 0.98 \\ P(Z \leq c/0.913) &= 1 + 0.98/2 = 0.99 \\ P(Z \leq c/0.913) &= 0.99 \\ c/0.913 &= 2.33 \\ c &= 2.33 * 0.913 \\ c &= 2.127 \end{aligned}$$

5E)

If four haul times are independently selected, what is the probability that at least one of them exceeds 10 minutes?

$$\begin{aligned} P(X > 10) &= 0.0455 \\ P(\text{at least one exceeds}) &= 1 - P(\text{none of the four exceeds}) = 1 - P(X < 10)^4 = 1 - [1 - P(X > 10)]^4 \\ &= 1 - [1 - 0.0455]^4 = 0.169951 \end{aligned}$$