Magnetostatic Simulation of an Accelerator Magnet

make your own finite-element solver

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Introduction

what we will do ...

In this exercise, we start from the Maxwell equations and end with the magnetic field in an accelerator magnet. Except for the construction of the geometry and the definition of the materials, we will implement every step ourselves.

make sure that you bring with you ...

1. your laptop,
2. a recent version of FEMM (http://femm.foster-miller.net),
3. Matlab (www.mathworks.com) or Octave (http://www.gnu.org/software/octave/)
4. the slides of the previous lectures.

work together ...

1. in small groups (2 or 3 persons).
2. divide the work,
3. but keep informed about the “what and how” the other is doing.
4. adapt a careful attitude, double check everything, in numerical simulation is a small error sufficient to blow up the universe.

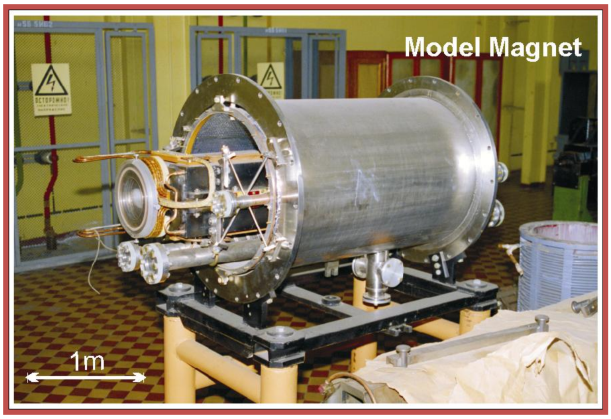
the goal of an exercise is to learn, therefore ...

1. ask everything you do not know or you are not sure of.
2. repeat an exercise (maybe with slightly different parameters) until you understand its solution thoroughly.

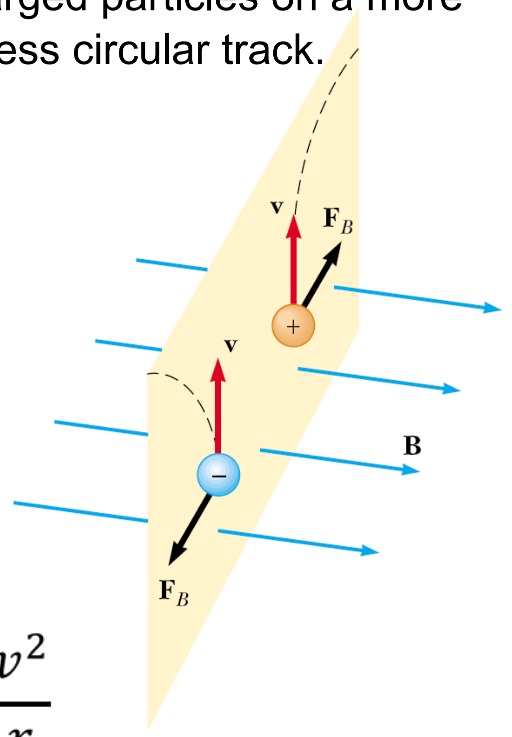
enjoy ...

Accelerator Magnet

about the problem to be simulated ...



This is an accelerator magnet. Many of them are installed in synchrotrons in order to keep accelerated charged particles on a more or less circular track.

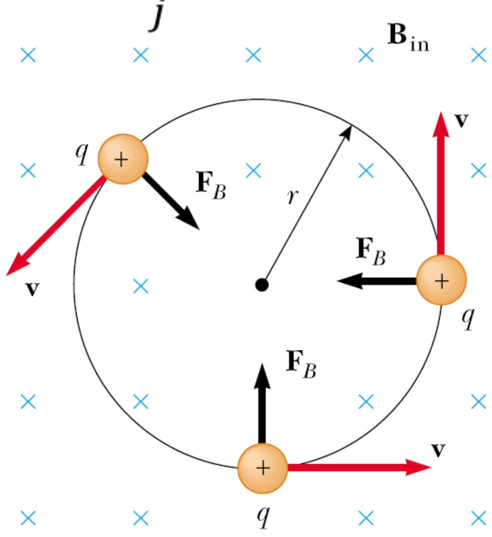


The force on an ion with mass *m* and charge *q*, moving at a velocity *v* is

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

When no other forces are present, this force is the centripetal force of a circular motion

$$\sum_j \vec{F}_j = m\vec{a}_c \quad \Rightarrow \quad qvB = m\frac{v^2}{r}$$

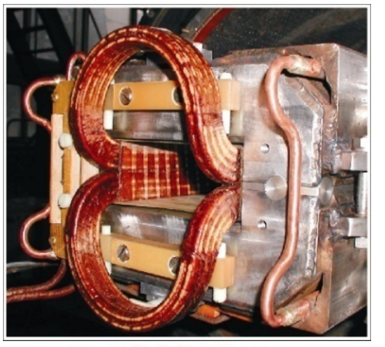


The radius of the ion trajectory is $r = \dfrac{mv}{qB}$

(a) Building smaller synchrotrons (small *r*) or (b) achieving higher velocities (large *v*) necessitates designing for higher magnetic fields (large *B*).

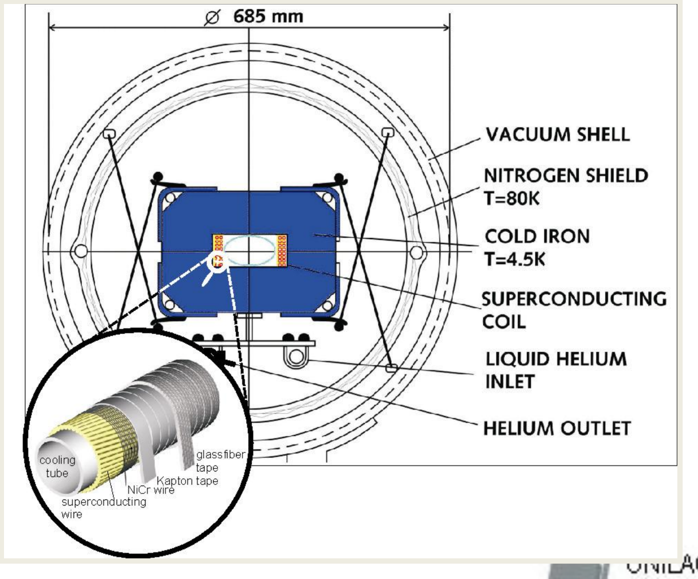
[1] (figures and basic information) R.A. Serway, J.W. Jewett, Physics for Scientists and Engineers, 6th ed, Thomson, 2004.
[2] (further reading) M. Wilson, Superconducting Magnets, Clarendon, Oxford, 2002.

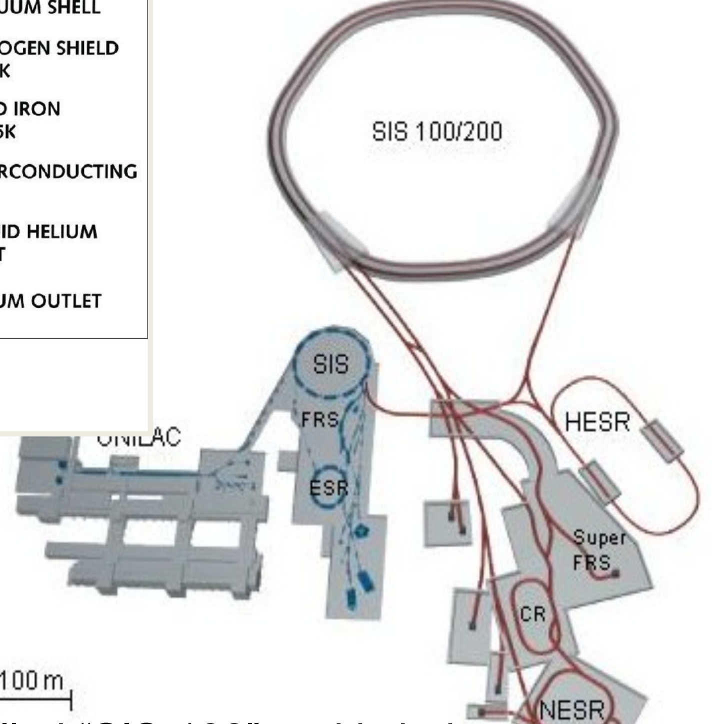
Background



where the model comes from ...

The magnet is known as the Nuclotron magnet (Dubna, Russia).





Currently, the design is called “SIS-100” and is being modified for use in the Facility for Antiproton and Ion Research (FAIR) which is currently planned at the Gesellschaft für Schwerionenforschung (GSI, Facility for Heavy Ion Research, www.gsi.de) in Darmstadt, Germany.

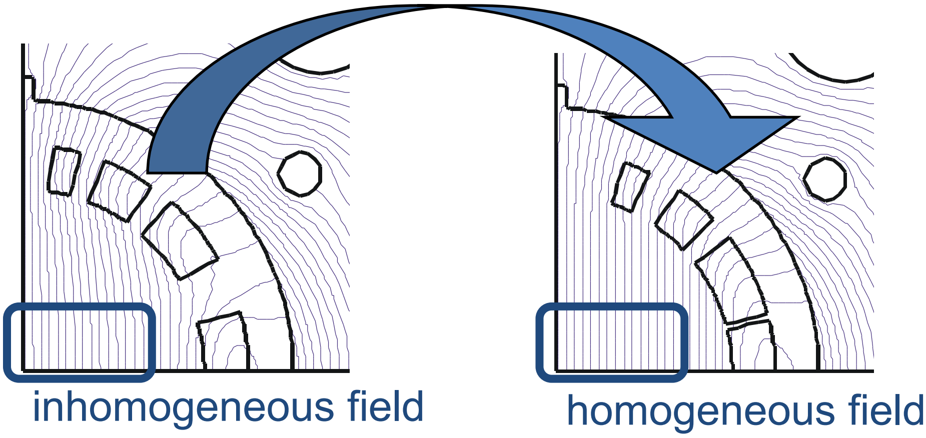
The current technological challenges are

1. superconductive coils,
2. cooling (fluid Helium),
3. reducing losses (eddy-currents, hysteresis, ...) and
4. guaranteeing a homogeneous dipole magnetic field for all operation modes.

Our Goal

what we want to achieve ...

1. validating the magnitude and the homogeneity of the aperture field,
2. checking the saturation of the iron parts and
3. possibly improving the design on these points.



how to achieve this goal ...

by 2D magnetostatic field simulations based on

1. the geometry of the magnet cross-section,
2. the currents through the magnet windings and
3. the given material characteristics.

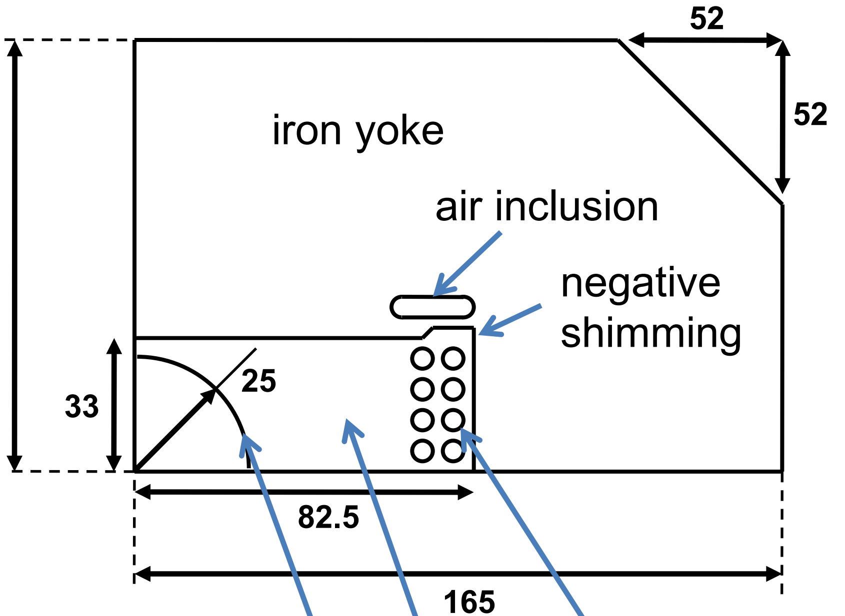
with which means ...

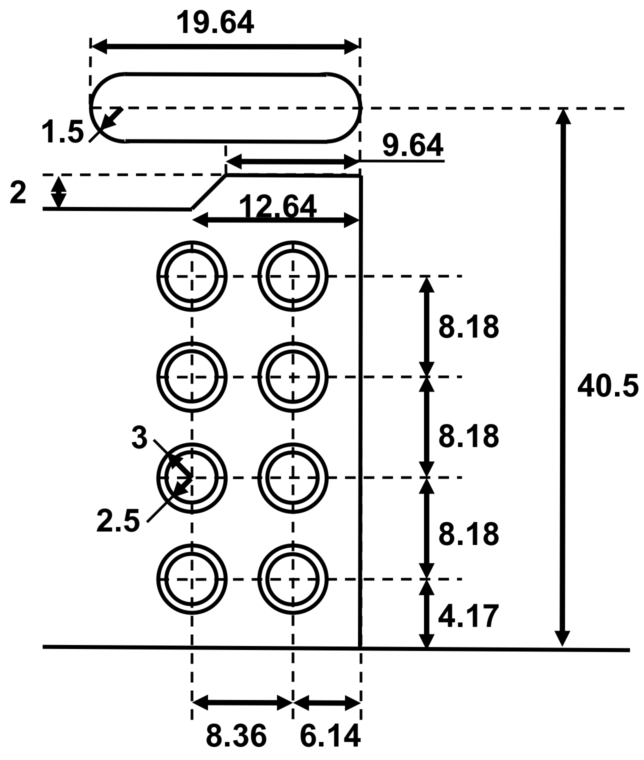
1. a freeware package for 2D FE modelling (FEMM) and
2. an own implementation of the nonlinear, magnetostatic formulation and the 2D FE discretisation (Matlab or Octave).

Geometry

how it looks like ...

all dimensions are in millimeter





- enter this geometry in FEMM;
- work bottom-up (first points, then lines/arc, then surfaces);
- a quick guide of FEMM is at the next page.

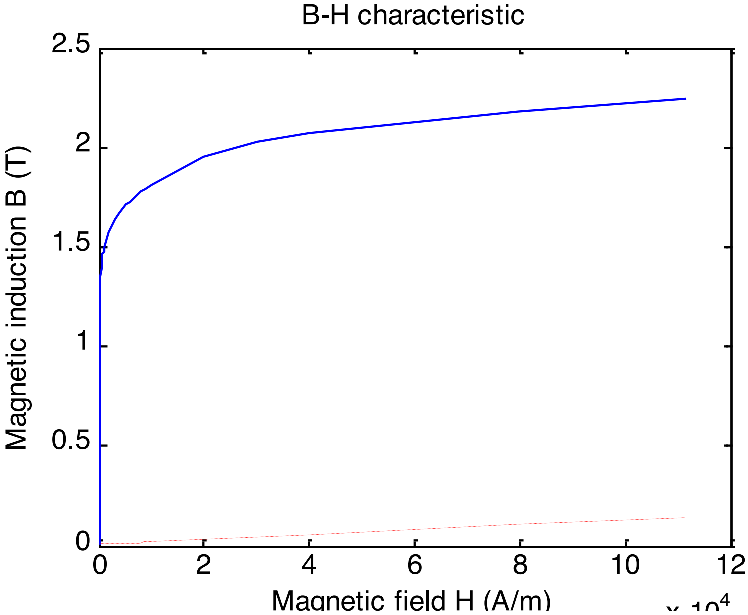
FEMM

quick guide

- use FEMM by applying the buttons directly under the menu, one-by-one, from left to right up to and including  .
- adding model items is done by the left mouse button or the tab-key.
- editing model items is done by the right mouse button followed by the space-bar.
- *geometry*: FEMM is a bottom-up modeller:
  - first define points,
  - then connect them to line or arc segments,
  - and finally, indicate block labels.
  - the responsability for avoiding crossing lines is at you.
- define materials and boundaries (menu-item “Properties”) and then assign them to the edges and the blocks, respectively.
- save your model and have a look in the .fem file with the editor of your choice, you will recognise your inputs.
- for further information, use the manual (menu-item Help, Help Topics).

Materials

linear or nonlinear ...



B (T)	H (A/m)
0.010	9.9997048e+000
0.020	1.9999410e+001
0.035	2.9999114e+001
0.050	3.9998819e+001
0.070	4.9998524e+001
0.107	5.9998229e+001
0.140	6.9997933e+001
0.200	7.9997638e+001
0.330	9.9997048e+001
1.020	1.9999410e+002
1.250	2.9999114e+002
1.350	3.9998819e+002
1.400	4.9998524e+002
1.430	5.9998229e+002
1.450	6.9997933e+002
1.470	7.9997638e+002
1.480	8.9997343e+002
1.495	9.9997048e+002
1.580	1.9999410e+003
1.640	2.9999114e+003
1.675	3.9998819e+003
1.710	4.9998524e+003
1.730	5.9998229e+003
1.760	6.9997933e+003
1.780	7.9997638e+003
1.795	8.9997343e+003
1.810	9.9997048e+003
1.950	1.9999410e+004
2.030	2.9999114e+004
2.070	3.9788735e+004
2.180	7.9577469e+004
2.250	1.1140846e+005

2

- define material properties and excitations (each of the windings carries 6045.76 A).
- for the first simulation, use a linear material for the iron parts (relative permeability: 1000).

3

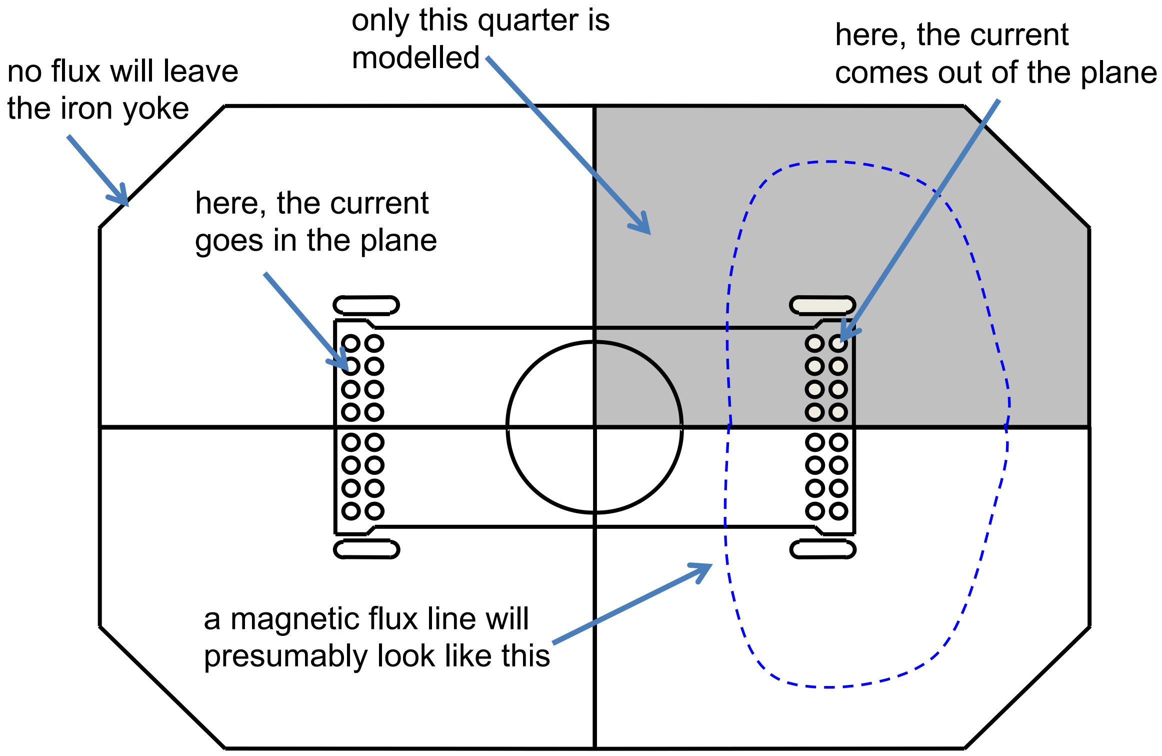
- define boundary conditions (see next page).
- define problem settings (the magnet is 3 meter long).

4

- assign materials to the blocks of the geometry.
- assign boundary conditions to some of the line and arc segments of the geometry.

Boundary Conditions

restricting the computational domain ...



The presence of the other parts outside the computational domain is *faked* by *boundary conditions (BCs).*

Magnetic flux lines stay parallel to boundaries with *electric BCs*.

Magnetic flux lines cross boundaries with *magnetic BCs* perpendicularly.

Magnetic flux lines that leave the model under an angle different from 0 or 90 degrees, indicate a *Robin's* (or *combined) BCs*.

For a magnetic-vector-potential formulation, electric BCs correspond to Dirichlet BCs (constraint on the magnetic vector potential), whereas magnetic BCs correspond to Neumann BCs (constraint on the magnetic field strength).

are you uncertain …

try the model with different BCs and have a careful look to the magnetic-flux-line plot

Solve

let FEMM do it for you ...

5

- create a mesh 

Behind the FEMM packages (by David Meeker), there is the Triangle package (by Jonathan Shewchuk) for meshing (in this case, triangularisation). Meshing is a tedious task which has to be carried out in exact arithmetic in order to avoid geometric failures due to truncation and overflow errors.

6

- solve the problem 

This is the most time consuming part of a simulation. However, for a small 2D model like this, the computation time is fully acceptable. For large 3D models, however, this may become prohibitive.

7

- look to the solution 
  Besides a magnetic-flux line plot, you can ask for a plot of the magnetic flux density (magnitude or arrows).
- compare the results to what you would expect.

can you “read” the magnetic-flux-line plot ?

- where is the highest magnetic flux density?
- where is the lowest magnetic flux density?
- which fact is giving evidence of the homogeneity of the aperture field?
- what is the function of the negative shimming and the air inclusion?
- what would happen to the magnetic flux lines when the current changes sign?
- where do you expect ferromagnetic saturation?
- what happens when you interchange Dirichlet and Neumann boundary conditions?

Own Implementation

doing it yourself ...

road map

- formulation
- discretisation
- implementation in Matlab
    1. 2D, linear, magnetostatic solver
        - getting information out of FEMM data files
        - check this information (e.g. using figures)
        - constructing the system of equations
        - applying boundary conditions
        - solving the system
        - post-processing
        - write the solution to a FEMM data file
    2. 2D, nonlinear, magnetostatic solver
        - reading a nonlinear material characteristic
        - finding elements belonging to the iron parts
        - setting up a nonlinear iteration by successive substitution and/or Newton

8

- take a look to the FEMM data files (extensions .fem and .ans). Figure out which information is needed for setting up an own finite-element solver.

9

- some of the auxiliary routines are provided.
- write the main routines yourself!

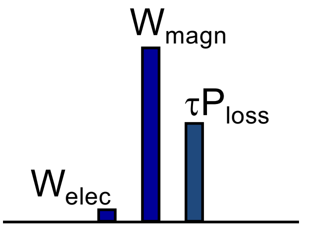
Routines

curl.m	**computing B from A**
curlcurl_ll.m	**assembling a curl-curl stiffness matrix for linear reluctivities**
curlcurl_ll_nonlinear.m	**assembling a curl-curl stiffness matrix and magnetisation vector for nonlinear reluctivities**
current_Pstr.m	**assembling righthandside vectors for coils**
driver.m	**main file**
edgemass_ll.m	**assembling a mass matrix for linear conductivities**
findlab.m	find the indices in a array of strings where a certain label is found
mesh_linear_shape_functions.m	compute all information for linear finite-element shape functions
nlin_evaluate.m	evaluate a nonlinear material characteristic using a data structure for nonlinear materials
nlin_initialise.m	initialise a data structure for nonlinear materials
plot_point.m	plot geometry points
ppder.m	derive a spline-polynomial
pyth.m	compute the magnitude of a vector field
read_femm.m	read a problem from a FEMM .ans file
save_femm.m	save a problem to a FEMM .ans file
savedivide.m	divide to vectors and deal with zero denominators
sis100_geometry.m	drivers from creating the SIS100 geometry
viewequi.m	draw an equipotential plot
write_femm_geometry.m	write geometric information to a FEMM .fem file

- more detailed information can be obtained by typing “help filename” in Matlab.
- files in which you should add some implementation are indicated in boldface. You have to insert some implementation at places indicated by “IMPLEMENTATION POINT”.

Formulation

where we turn the Maxwell equations into the magnetostatic formulation in function of the magnetic vector potential ...



Maxwell's equations

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

10

neglect electrical energy with respect to magnetic energy and losses

1

11

integrate in space and define magnetic vector potential

2

12

integrate in space and define electric scalar potential

3

material laws (simplest form)

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H} = \frac{1}{\upsilon} \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

4

13

combine 1, 2, 3 *and* 4

Discretisation (1/2)

where we go over from the continuous to the discrete level ...

(strong) formulation

$$\nabla\times\left(\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}=\vec{J}_s$$

14

apply the weighted residual method, i.e., (a) multiply with (vectorial) test functions $\vec{w}_i$ *and (b) integrate over the computational domain* Ω

15

apply the vector-calculation formula

$$(\nabla\times\vec{v})\cdot\vec{w}=\nabla\cdot(\vec{v}\times\vec{w})+\vec{v}\cdot(\nabla\times\vec{w})$$

16

apply Gauss's theorem $\int_V \nabla\cdot\vec{v}\,\mathrm{d}V=\int_{\partial V}\vec{v}\cdot\mathrm{d}\vec{A}$

17

assume that only (a) Dirichlet boundary conditions and (b) homogeneous Neumann boundary conditions are applied

Discretisation (2/2)

and end up with a system of equations ...

weak formulation

$$\int_V \left(\nu \nabla \times \vec{A} \cdot \nabla \times \vec{w}_i + \sigma \frac{\partial \vec{A}}{\partial t} \cdot \vec{w}_i \right) \mathrm{d}V = \int_V \vec{J}_\mathrm{s} \cdot \vec{w}_i \, \mathrm{d}V$$

18

insert the discretisation for the magnetic vector computational domain.

$$\vec{A}(x,y,z) = \sum_j u_j \vec{w}_j(x,y,z)$$

19

- *denote the coefficients.*
- *write the resulting algebraic system of equations.*

Reduction to 2D

simplify ...

- when the geometry/excitation/boundary conditions remain the same along the axis of the device,
- when the current is perpendicular to the cross-section,
- then the flux is aligned with the cross-section,
- then $\vec{A} = (\_\_\_, \_\_\_, \_\_\_)$ 20
- and $\vec{B} = (\_\_\_, \_\_\_, \_\_\_)$ 21

apply the edge functions $\vec{w}_j = \dfrac{N_j(x,y)}{\ell_z}\,\vec{e}_z$ (see remark (*1) next page below)

both as test and trial functions,

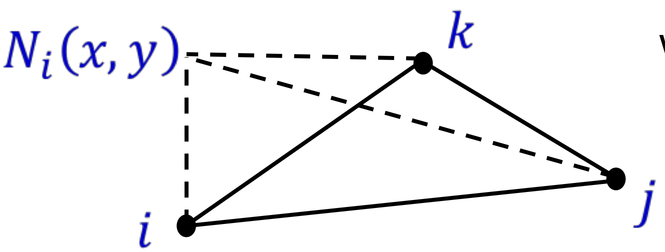
- then $K^{\mathrm{FE}}_{\upsilon,ij} =$ 22
- and $M^{\mathrm{FE}}_{\sigma,ij} =$ 23
- and $f^{\mathrm{FE}}_{i} =$ 24

Compute Coefficients (1/2)

down to the implementation ...

We use linear finite-element “hat” functions for $N_i(x,y)$.
These functions are 1 at (x_i, y_i) and are 0 at all other nodes.
in element A_{ijk} (ijk ordered counterclockwise):

$$N_i(x,y) = \frac{a_i + b_i x + c_i y}{2S_{ijk}}$$



where $\begin{cases} a_i = x_j y_k - x_k y_j \\ b_i = y_j - y_k \\ c_i = x_k - x_j \\ S_{ijk} = \text{element area} \end{cases}$

a helpful rule is $\displaystyle\int_{V_e} N_i^\alpha N_j^\beta N_k^\gamma \, \mathrm{d}V = 2\ell_z S_{ijk} \frac{\alpha!\,\beta!\,\gamma!}{(\alpha+\beta+\gamma+2)!}$

The integration of the coefficients is organised element-by-element, i.e., by summing up the *local* matrices of the individual elements (V_e denotes the prism with triangular cross-section S_{ijk} and length ℓ_z).

Remark: Notice the fact that (a) true edge functions with unit 1/m are defined and (b) the coefficients are computed by *volume* integration. This is rather uncommon but has many advantages, e.g., for circuit coupling [1]. More common is, however, using $\vec{w}_j = N_j(x,y)\vec{e}_z$ and integrating over the cross-sections [2,3,4].

[1] J. Gyselinck, Twee-dimensionale dynamische eindige-elementenmodellering van statische en roterende elektromagnetische energieomzetters, PhD, Universiteit Gent, Belgium 2000.

[2] A. Kost, Numerische Methoden in der Berechnung elektromagnetischer Felder, Springer, Berlin, 1996.

[2] P.P. Silvester , R.L. Ferrari, Finite Elements for Electrical Engineers, 2nd ed, Cambridge UP, 1996.

[3] J.P. Bastos, N. Sadowski, Electromagnetic Modeling by Finite Element Methods, Marcel Dekker Ltd, 2003.

Compute Coefficients (2/2)

further down to the implementation ...

Compute the local coefficient matrices and vectors:

25

$$K_{\nu,ij}^{(e)} =$$

26

$$M_{\sigma,ij}^{(e)} =$$

27

$$f_i^{(e)} =$$

- In the case of triangular elements, two 3-by-3 matrices and one 3-by-1 vector are found for each element.
- The assembling process is the procedure bringing such local contributions together in an overall algebraic system of equations.

- Think how assembling can be organised in an efficient way when using Matlab.
- Implement these formulae in curlcurl.m, edgemass.m and current_load.m.

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Magnetic Flux Density

- The magnetic flux density is

$$\begin{cases} B_x = \dfrac{\partial A_z}{\partial y} \\[2ex] B_y = -\dfrac{\partial A_z}{\partial x} \end{cases}$$

- The magnetic flux density in element *e* is expressed in function of the degrees of freedom **u** and the shape functions by

$$\boxed{B_x = \qquad\qquad\qquad\qquad}$$

$$\boxed{B_y = \qquad\qquad\qquad\qquad}$$

Remark: Notice the inversion of coordinates and the minus sign in the above formulae. This is typical for the curl-curl case. In case of an electrostatic formulation in terms of the electric scalar potentials, the formulae for E_x and E_y would look completely different.

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- Derive the coefficients for B_x and B_y.
- Implement these in `curl.m`.
- Compute the element-wise magnitude of the magnetic flux density (use the function `pyth.m`).
- Search for the element with the highest magnetic flux density.
- Compute the magnetic energy relying upon these values for the magnetic flux density.

Magnetic Energy

The magnetic energy equals (only in the linear case)

$$W_{\mathrm{magn}} = \frac{1}{2} u^{\mathrm{T}} K_{\nu} u$$

where **u** is the vector of degrees of freedom
and K_{ν} is the curl-curl reluctance matrix.

30

- Post-process for the magnetic energy using FEMM

$$\boxed{W_{\mathrm{magn}} = \qquad\qquad}$$

31

- Compute the magnetic energy based on the FEMM solution and the reluctance matrix computed by your own.
  (the FEMM solution is the third column in prb.node)
  (the values in prb.node are line-integrated magnetic vector potentials (in Wb), despite of the fact that FEMM uses and stores magnetic vector potentials (in Tm) in the .ans file.)

The magnetic energy also equals (only in the linear case)

$$W_{\mathrm{magn}} = \frac{1}{2} L I^2$$

where I is the applied current
and L is the inductance.

- Compute the inductance of the magnet

32

$$\boxed{L = \qquad\qquad}$$

Boundary Conditions

- The only boundary conditions present are homogeneous Dirichlet boundary conditions.
- Unconstrained nodes (subscript b, index set `idxdof`) are distinguished from constrained nodes (subscript c, index set `idxdir`).
- The unconstrained system would be

$$\begin{bmatrix} K_{\mathrm{bb}} & K_{\mathrm{bc}} \\ K_{\mathrm{cb}} & K_{\mathrm{cc}} \end{bmatrix} \begin{bmatrix} u_{\mathrm{b}} \\ u_{c} \end{bmatrix} = \begin{bmatrix} f_{\mathrm{b}} \\ f_{\mathrm{c}} \end{bmatrix}$$

- Adding constraints leads to ($\mathbf{I}_{cq}\mathbf{y}_{b}$ denotes the boundary-integral term)

$$\begin{bmatrix} K_{\mathrm{bb}} & K_{\mathrm{bc}} & 0 \\ K_{\mathrm{cb}} & K_{\mathrm{cc}} & I_{cq} \\ 0 & I_{\mathrm{qc}} & 0 \end{bmatrix} \begin{bmatrix} u_{\mathrm{b}} \\ u_{\mathrm{c}} \\ y_{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} f_{\mathrm{b}} \\ f_{\mathrm{c}} \\ u_{\mathrm{c}} \end{bmatrix}$$

- The values for $\mathbf{u}_{c}$ are known. Eliminating the Lagrange multipliers $\mathbf{u}_{q}$ leads to the constrained system

$$K_{\mathrm{bb}} u_{\mathrm{b}} = f_{\mathrm{b}} - K_{\mathrm{bc}} u_{\mathrm{c}}$$

33

- "Shrink" the unconstrained system up to the constrained system.
- Solve the system of equations.
- "Inflate" the solution vector to a full solution vector including the constrained nodes.
- Compute the magnetic energy and compare to previously obtained values.
- Write the solution to a FEMM .ans file.
  (be aware of the fact that we work with line-integrated magnetic vector potentials (Wb), whereas FEMM uses magnetic vector potentials (Tm), the save_femm routine make the necessary conversion, have a look inside.)
- Plot your own solution using FEMM.

Aperture Field (1/2)

- The magnetic vector potential inside a circle with reference radius r_{ref} in the aperture can be expressed by

$$A_z(r,\varphi) = \sum_{p \in P} \left(a_{\mathrm{p}} \cos(p\varphi) + b_{\mathrm{p}} \sin(p\varphi) \right) \left(\frac{r}{r_{\mathrm{ref}}} \right)^p$$

- Then, the magnetic flux density is

$$\begin{cases} B_x(r,\varphi) = \displaystyle\sum_{p \in P_0} \frac{p}{r} \left(-a_{\mathrm{p}} \sin((p-1)\varphi) + b_{\mathrm{p}} \cos((p-1)\varphi) \right) \left(\frac{r}{r_{\mathrm{ref}}} \right)^p \\ B_y(r,\varphi) = \displaystyle\sum_{p \in P_0} \frac{p}{r} \left(-a_{\mathrm{p}} \cos((p-1)\varphi) - b_{\mathrm{p}} \sin((p-1)\varphi) \right) \left(\frac{r}{r_{\mathrm{ref}}} \right)^p \end{cases}$$

- The Fourier coefficients of the magnetic flux density evaluated at the reference circle are

$$\begin{cases} \mathcal{F}(B_x) = \left(\dfrac{p}{r_{\mathrm{ref}}} b_{\mathrm{p}}, -\dfrac{p}{r_{\mathrm{ref}}} a_{\mathrm{p}} \right) \\ \mathcal{F}(B_y) = \left(\boxed{-\dfrac{p}{r_{\mathrm{ref}}} a_{\mathrm{p}}}, \boxed{-\dfrac{p}{r_{\mathrm{ref}}} b_{\mathrm{p}}} \right) \end{cases}$$

- These coefficients are called
  harmonic components
  and *skew harmonic components*
  (here under the assumption of a vertical main dipole field).

Aperture Field (2/2)

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- Find the nodes in the mesh which lie on the reference circle (use a geometric tolerance!) and order them along the circle.
- Extract the magnetic vector potential at the quarter of the reference circle which is inside the computational domain (notice: our solution consists of line-integrated magnetic vector potentials!).

35

- Combine this signal with itself until you have a periodic signal.
- Make a Fourier transformation to obtain the coefficients for A_z .
- Compute the harmonic and skew harmonic components.

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- Make bar plots of both, discard the main dipole component and plot again.
- Fill the most important components in the table.

	harmonic component	skew harmonic component
dipole		
quadrupole		
sextupole		

The quality of the aperture field of an accelerator magnet is determined by the ratio of the magnitude of the higher harmonic components with respect to the main component (in this case the vertical dipole component). In typical magnets, this ratio is below 10^{-4}. What about this magnet?

Nonlinear Material Properties

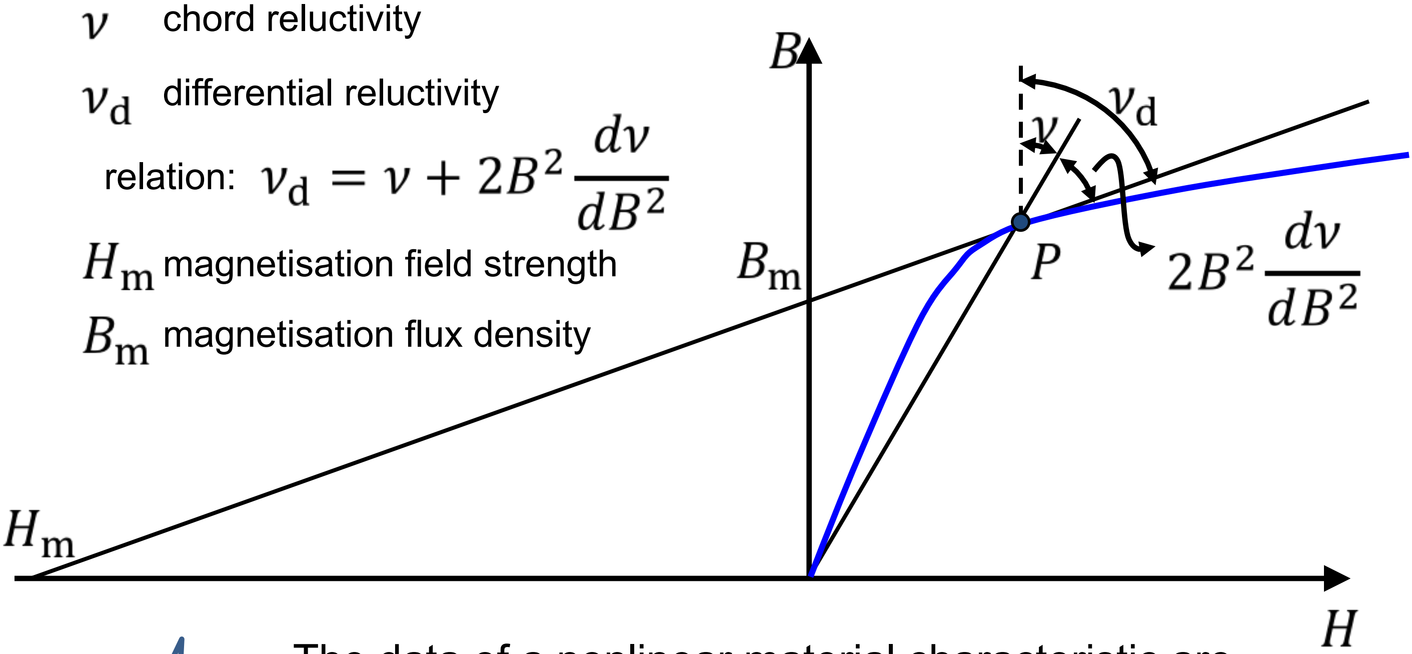
ν chord reluctivity

ν_d differential reluctivity

relation: $\nu_\mathrm{d} = \nu + 2B^2 \dfrac{d\nu}{dB^2}$

H_m magnetisation field strength

B_m magnetisation flux density



- The data of a nonlinear material characteristic are kept in a data structure (`nlin` in `driver.m`).
- The data can be evaluated given a set of magnetic flux densities.
- Check the results for the magnetic field strength, chord reluctivity, differential reluctivity, derivative of the reluctivity and coercitive field strength.

Remark: The coercitive field strength and the remanence here have another meaning than for the case of a permanent magnet. Here, H_c and B_r are defined by the crossing points of the line tangent to the BH-characteristic in the actual working point. Hence, H_c and B_r will change when the working point changes. The points (H_c,0) and (0,B_r) are no valid working points of the nonlinear material.

linearisations in the working point P :

1. successive substitution: $H = \nu B$
2. Newton: $H = H_\mathrm{m} + \nu_\mathrm{d} B$

Successive Substitution

algorithm: $\nabla\times\left(\nu\left(\vec{A}_n\right)\nabla\times\vec{A}^*_{n+1}\right)=\vec{J}_s$

The convergence of a successive-substitution approach is poor. For that reason, commonly, relaxation is applied. We will use relaxation with a fixed relaxation factor.

relaxation: $\vec{A}_{n+1}=\alpha\vec{A}^*_{n+1}+(1-\alpha)\vec{A}_n$

(relaxation factor α)

The convergence of the nonlinear iteration is checked on the basis of a convergence criterion. Monitoring the convergence of the magnetic energy is the most appropriate. Here, we apply a simpler criterion, based on the relative difference between two successively obtained solutions.

convergence criterion: $\varepsilon_{\text{nlin}}=\dfrac{\left\|\vec{A}_{n+1}-\vec{A}_n\right\|}{\left\|\vec{A}_{n+1}\right\|}$

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- Implement the successive-substitution approach with relaxation and a convergence check.
- Solve the nonlinear problem and compare to results obtained with FEMM.
- What are the main differences between the linear and the nonlinear solution.

Newton (1/2)

algorithm (1):

$$\nabla\times\left(\bar{\bar{\nu}}_{\mathrm{d}}\left(\vec{A}_n\right)\nabla\times\delta\vec{A}_{n+1}\right)=\vec{J}_{\mathrm{s}}-\nabla\times\left(\nu\left(\vec{A}_n\right)\nabla\times\vec{A}_n\right)$$

The above form is commonly used. However, it may be more convenient to think about the Newton method as being similar to the successive-substitution approach, only differing concerning the linearisation of the working point. By introducing $\vec{H}=\vec{H}_{\mathrm{m}}\left(\vec{A}_n\right)+\bar{\bar{\nu}}_{\mathrm{d}}\left(\vec{A}_n\right)\vec{B}$ in the magnetostatic equation, we arrive at

algorithm (2):

$$\nabla\times\left(\bar{\bar{\nu}}_{\mathrm{d}}\left(\vec{A}_n\right)\nabla\times\vec{A}_{n+1}\right)=\vec{J}_{\mathrm{s}}-\nabla\times\vec{H}_{\mathrm{m}}\left(\vec{A}_n\right)$$

Here, no increments are needed and only a matrix assembly is necessary for the left side of the equation. Notice that the differential reluctivity is a tensor (2-by-2 in the 2D case, 3-by-3 in the 3D case)!

The main challenge is the computation of the differential reluctivity tensor. In an element *e* where the previous solution for the magnetic flux density is given by

$$B^{(e)}=\begin{bmatrix}B_x^{(e)}\\ B_y^{(e)}\end{bmatrix}$$

the differential reluctivity tensor is

$$\bar{\bar{\nu}}_{\mathrm{d}}^{(e)}=\nu^{(e)}\bar{\bar{1}}+2B^{(e)}\nu_{\mathrm{d}}^{(e)}B^{(e)\mathrm{T}}$$

where $\nu^{(e)}$ and $\bar{\bar{\nu}}_{\mathrm{d}}^{(e)}$ are the chord and differential reluctivities obtained by evaluating the material characteristic with input $B^{(e)}$.

Newton (2/2)

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- Implement the nonlinear system matrix and additional righthandside term in `curlcurl_nonlinear.m`.
- Set up an Newton iteration and check for convergence.
- Write the solution to a FEMM .ans file and compare with results obtained by FEMM.

Remark: The convergence of a Newton approach should be good enough to be convergent without relaxation.

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- Compare the convergence of the successive-substitution approach with the convergence of the Newton approach. Try to find an optimal relaxation factor for the successive-substitution approach.

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- Compute the harmonic components and the skew harmonic components based on the nonlinear solution.
- Compare to the values obtained for the linear solution.

	harmonic component	skew harmonic component
dipole		
quadrupole		
sextupole		