$f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda - 3500$

Newton Raptor's method:

$$x_{n+1} = \lambda_0 - \frac{f(\lambda)}{f'(\lambda)}$$

$f'(\lambda) = 4\lambda^3 + 39\lambda^2 - 438\lambda - 835$, $\lambda_0 = 2.675$

$$\lambda_2 = 2.675 - \frac{(2.675)^4 + 13(2.675)^3 - 219(2.675)^2 - 835(2.675) - 3580}{4(2.675)^3 + 39(2.675)^2 - 438(2.675) - 835}$$

$\lambda_2 = 11.054$

Compute $A - \lambda I$ for $\lambda = 11.054$:

$$A - 11.054 I = \begin{bmatrix} 4-11.054 & 8 & -1 & -2 \\ -2 & -9-11.054 & -2 & -4 \\ 0 & 10 & 5-11.054 & -10 \\ -1 & 13 & -14 & -13-11.054 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

using Row reduction method we get:

$v_1 = -0.0360$, $v_2 = -0.0121$, $v_3 = -0.8601$, $v_4 = 0.5086$

the eigenvector is: $v_2 = \begin{pmatrix} -0.0360 \\ -0.0121 \\ -0.8601 \\ 0.5086 \end{pmatrix}$

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$$

Newton's Raphon's ...:

$$x_{n+1} = x_0 - \frac{f(x)}{f'(x)}$$

$$f(x) = \lambda^4 + 13\lambda^3 + 219\lambda^2 - 835\lambda + 3500$$

$$f'(x) = 4\lambda^3 + 39\lambda^2 + 438\lambda - 835$$

$$\boxed{\lambda_0 = 11.054}$$

$$\lambda_3 = 11.054 - \frac{(11.054)^4 + 13(11.054)^3 - 219(11.054)^2 - 835(11.054) + 3500}{4(11.054)^3 + 39(11.054)^2 - 438(11.054) - 835}$$

$$\boxed{\lambda_3 = -5.604}$$

$$A - (-5.604)I =$$

$$\begin{pmatrix} 4-(-5.604) & 8 & -1 & -2 \\ -2 & -9-(-5.604) & -2 & -4 \\ 0 & 10 & 5-(-5.604) & -10 \\ -1 & -13 & -14 & -13-(-5.604) \end{pmatrix}$$

$$= \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$$

Solve $(A + 5.604I)v = 0$

$$\begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & 4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

Row Reduction leads to:

$V_1 \simeq 0.5634$ , $V_2 \approx -0.6161$

$V_3 \simeq 0.5495$ $V_4 \simeq -0.0334$

Thus the eigenVector is:

$$\boxed{V_3 = \begin{pmatrix} 0.5634 \\ -0.6161 \\ 0.5495 \\ -0.0334 \end{pmatrix}}$$

$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} \longrightarrow \det(A - I\lambda) = 0$$

$$\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

S

Cofactor expansion!

$$A1 = \begin{pmatrix} 4-\lambda & -1 & -2 \\ -2 & -2 & -4 \\ -1 & -14 & -13-\lambda \end{pmatrix}$$

$(4-\lambda)[(-2)(-13-\lambda) - (-4)(-14)] - (-1)[(-2)(-13-\lambda) - (-1)(-4)] + (-2)[(-2)(-14) - (-1)(-2)]$

$= (4-\lambda)(26 + 2\lambda - 56) + (1)[(26 + 2\lambda) - 4] - 2(28 - 2)$

$= (4-\lambda)(2\lambda - 30) + 2\lambda + 22 - 2(26)$

$= 8\lambda - 120 - 2\lambda^2 + 30\lambda + 2\lambda + 22 - 52$

$= \boxed{-2\lambda^2 + 40 - 150}$

$$A2 = \begin{pmatrix} 4-\lambda & 8 & -2 \\ -2 & -9-\lambda & -4 \\ -1 & -13 & -13-\lambda \end{pmatrix}$$

$(4-\lambda)[(-9-\lambda)(-13-\lambda) - (-4)(-13)] - 8[(-2)(-13-\lambda) - (-1)(-4)] + (-2)[(-2)(-13) - (-1)(-9-\lambda)]$

$= (4-\lambda)[(117 + 9\lambda + 13\lambda + \lambda^2) - 52] - 8[(26 + 2\lambda) - 4] - 2(26 - 9 - \lambda)$

$= (4-\lambda)[(\lambda^2 + 22\lambda + \underbrace{117) - 52}_{65}] - 8(2\lambda + 22) - (34 + 2\lambda)$

$= 4\lambda^2 + 88\lambda + 260 - \lambda^3 - 22\lambda^2 - 65\lambda - 16\lambda - 176 - 34 + 2\lambda$

$= \boxed{-\lambda^3 - 18\lambda^2 + 9\lambda + 50}$

$$A3 = \begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ -1 & -13 & -14 \end{pmatrix}$$

$(4-\lambda)[(-9-\lambda)(-14) - (-2)(-13)] - 8[(-2)(-14) - (-1)(-2)] + (-1)[(-2)(-13) - (-1)(-9-\lambda)]$

$= (4-\lambda)[(126 + 14\lambda) - (26)] - 8(28 - 2) + (-1)[(26) - (9 + \lambda)]$

$= (4-\lambda)(14\lambda + 100) - 8(26) - (17 - \lambda)$

$= 56\lambda + 400 - 14\lambda^2 - 100\lambda - 208 - 17 + \lambda$

$= \boxed{-14\lambda^2 - 43\lambda + 175}$

$\det(A - I\lambda) = -(10) \cdot \det A1 + (5-\lambda) \cdot \det A2 \; \cancel{+(-)(10)} \cdot \det A3$
$+ (-)(-10) \cdot \det \cdot A3$

$= (-10)(-2\lambda^2 + 40\lambda - 150) + (5-\lambda)(-\lambda^3 - 18\lambda^2 + 9\lambda + 50) + 10 \cdot (-14\lambda^2 - 43\lambda + 175)$

$= 20\lambda^2 - 400\lambda + 1500 + (5-\lambda)(\lambda^3 - 18\lambda^2 + 9\lambda + 50) + 10 + (-140\lambda^2 - 430\lambda + 175)$

Th $\quad (5-\lambda)(-\lambda^3 - 18\lambda^2 + 9\lambda + 50)$

$= -5\lambda^3 - 90\lambda^2 + 45\lambda + 250 - 140\lambda^2 - 430\lambda + 175 + 20\lambda^2 - 400\lambda + 500$
$+ \lambda^4 + 18\lambda^3 - 9\lambda^2 - 50\lambda$

$= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

Newton Raphson Method :

$$x_{n+1} = x_0 - \frac{f(x)}{f'(x)}$$

$f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

$f'(\lambda) = 4\lambda^3 + 39\lambda^2 - 438\lambda - 835 \quad , \boxed{x_0 = 2}$

$$x_1 = 2 - \frac{(2)^4 + 13(2)^3 - 219(2)^2 - 835(2) + 3500}{4(2)^3 + 39(2)^2 - 438(2) - 835}$$

$\boxed{x_1 = 2{,}675}$

$A - \lambda I \ , \ \lambda = 2{,}675$

$$A - 2{,}675 = \begin{pmatrix} 4 - 2{,}675 & 8 & -1 & -2 \\ -2 & -9 - 2.675 & -2 & -4 \\ 0 & 10 & 5 - 2{,}675 & -10 \\ -1 & -13 & -14 & -13 - 2{,}67 \end{pmatrix}$$

$$= \begin{pmatrix} 1{,}325 & 8 & -1 & -2 \\ -2 & -11{,}675 & -2 & -4 \\ 0 & 10 & 2{,}325 & -10 \\ -1 & -13 & -14 & -15{,}675 \end{pmatrix}$$

We solve $(A - 2{,}675 I)\,v = 0$ : $\begin{pmatrix} 1{,}325 & 8 & -1 & -2 \\ -2 & -11{,}675 & -2 & -4 \\ 0 & 10 & 2{,}325 & -10 \\ -1 & -13 & -14 & -15{,}675 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$

After row reduction :

$v_1 = 0{,}9583$
$v_2 = -0{,}1610$
$v_3 = 0{,}2074$
$v_4 = -0{,}1128$

$$\vec{v_1} = \begin{pmatrix} 0{,}9583 \\ -0{,}1610 \\ 0{,}2074 \\ -0{,}1128 \end{pmatrix}$$

$\boxed{\lambda_1 = 2{,}675}$

$$A=\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$\det(A-\lambda I)=0.$

$$A-\lambda I=\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

$$\det(A-\lambda I)=(4-\lambda)\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$$

$$-8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}+$$

$$(-1)\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ 1 & -13 & -13-\lambda \end{vmatrix}-(-2).$$

$$\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

From this

$$M_1=\begin{vmatrix} -9-\lambda & -2 & 4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$$

$M_1=(-9-\lambda)(5-\lambda)(-13-\lambda)-(-10)(-14)+2[10-(-13-\lambda)$
$-(-10)(-13)-4(10(-14)-(5-\lambda)(-13))$

$M_1=(-9-\lambda)(\lambda^2+8\lambda-205)+(-220+132\lambda)$

$=\lambda^3/4$
$\lambda^3-17\lambda^2+133\lambda+1845-520-20\lambda+300+52\lambda$

$=-\lambda^3-17\lambda^2+165\lambda+1625.$

$$M_3\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}.$$

$-2[10(-13-\lambda)-130].$
$+(9+\lambda)(0-10).$
$-4[0-10(-1)].$
$-2(-130-10\lambda-130)-10(9+\lambda)-40$
$520+20\lambda-90-10\lambda-4$
$=390+10\lambda$

$$M_2=\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$$

$-2[(5-\lambda)(-13-\lambda)-140]+2[0-10]-4[0-(5-\lambda)(-1)$

$-2\lambda^2-16\lambda+410-20-20+4\lambda$
$=2\lambda^2-12\lambda+370.$

$$M_4.\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

$-2[10(-14)-(5-\lambda)(-13)]+(9-\lambda)[0-(5-\lambda)$
$(-1)-2[0-10(-1)]$

$-2[-140+65-13\lambda)+(9+\lambda)(5-\lambda)-26.$

$=150+26\lambda+45-9\lambda+5\lambda-\lambda^2-20$
$=\lambda^2+22\lambda+175.$

$\det(A-\lambda I) = (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda) + 1625) - 8(-2\lambda^2 - 12\lambda + 370) - (390 + 10\lambda) +$
$2(-\lambda^2 + 22\lambda + 175)$. $\quad + 96\lambda - 2960 - 390 - 10\lambda -$

$= -4\lambda^3 - 68\lambda^2 + 660\lambda + 6500 + \lambda^4 + 17\lambda^3 - 1625\lambda + 16\lambda^2$
$2\lambda^2 + 44\lambda$.

$= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

Polynomial: $\boxed{\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0}$

$\lambda = -21.125$ : $P(-21.125) = (-21.125)^4 + 13(-21.25)^3 - 219(-21.25)^2 - 835(-21.25) + 3500 = 0$

$\lambda_4 = -21.125$

For $\lambda_4 = -21.125$

$$A - (-21.25)I = \begin{bmatrix} 4-(-21.125) & 8 & -1 & -2 \\ -2 & -9-(-21.125) & -2 & -4 \\ 0 & 10 & 5-(-21.25) & -10 \\ -1 & -13 & -14 & -13-(-21.25) \end{bmatrix}$$

$$= \begin{bmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{bmatrix}.$$

$(A - (-21.25)I)v = 0$.

$$\begin{bmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = 0.$$

Therefrore; $V_4 = \begin{bmatrix} 0.0248 \\ -0.3345 \\ -0.2223 \\ -0.9154 \end{bmatrix}$

For percentage importance.

Absolute Values;

$\lambda_1 = 2.675$.

$\lambda_2 = 11.054$

$\lambda_3 = 5.604$

$\lambda_4 = 21.125$.

% importance $= \Sigma \lambda_{n+1} \Sigma \lambda_1 \ldots$

$\Sigma \lambda_{1 \ldots 4} = 2.675 + 11.054 + 5.604 + 21.125$.

$\Sigma \lambda_1 \cdots \lambda_4 = 40.488$.

% importance of:

$\lambda_1 = \dfrac{2.675}{40.458} \times 100\%$.

% importance of $\lambda_1 = 6.612\%$.

$\lambda_2 = \dfrac{11.054}{40.458} \times 100\% = 27.322\%$.

$\lambda_3 = \dfrac{5.604}{40.458} \times 100\% = 13.85\%$.

$\lambda_4 = \dfrac{21.125}{40.458} \times 100\% = 52.215\%$.

For the Percentage Importance;

$\lambda_1 = 6.612\%$. $\qquad \lambda_3 = 13.85\%$.

$\lambda_2 = 27.322\%$. $\qquad \lambda_4 = 52.215\%$.