Worksheet 1: Basic epidemiological models

Due in noon Wednesday Week 3

This example sheet is for credit. Example sheets 1 and 2 can help you. Please make sure code is well commented and shows your working for these answers. Code must be uploaded alongside your pdf solutions.

20 marks will be awarded for presentation (including LaTeXed solutions and clarity of answers). Your write up (including figures) should be no more than 6 pages using size 12 font and 2.5cm margins. You do not have to copy out the questions asked, but the solutions must make sense as a stand alone document. 15 marks will be given for appropriate use of code with comments.

In 2009 there was a pandemic of "Swine Flu" (Influenza A H1N1) which impacted the UK. Here we will use a simple SIR model without demography to explore the infection dynamics in the UK:

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= -\beta SI/N \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \beta SI/N - \gamma I \\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma I\end{aligned}$$

and with contact rate, β, of 0.62 days^{-1}, a mean infectious duration of 2.6 days. We will assume the population size of the UK is $N = 62,000,000$.

1. Using Matlab, simulate the ODE dynamics of the outbreak for 250 days starting from a single infection on day 0 (i.e. $(S, I, R) = (N-1, 1, 0)$ is the initial condition) and assuming no interventions are put in place.
   (a) Plot the dynamics of all model classes over time assuming that no controls are in place, ensuring to label your graph appropriately. [6 marks]
   (b) Compute (i) the final size of the epidemic, R_∞ (noting the method you used), (ii) R_0 (by hand), and (iii) the expected outbreak duration (defined as the time during which there is at least 1 infection remaining). Give (i) and (iii) to the nearest whole number. [4 marks]
   (c) Plot how the effective reproduction number, $R_e(t)$, changes over time. [2 marks]
   (d) Plot how the growth rate, $r(t)$, changes over time. [3 marks]
2. The total reported cases in the UK for the 2009 pandemic was actually around 910,000. Since this is very different from our toy SIR model we will introduce to other key factors which could have played a role in transmission:
   - Underreporting of infection
   - Prior immunity

We will therefore extend our SIR model to have two different infection classes, with I_r representing infected individuals with symptoms severe enough to be *reported*, and with I_n representing those without symptoms severe enough to be reported:

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= -\beta S(I_r + I_n)/N \\ \frac{\mathrm{d}I_r}{\mathrm{d}t} &= p\beta S(I_r + I_n)/N - \gamma I_r \\ \frac{\mathrm{d}I_n}{\mathrm{d}t} &= (1-p)\beta S(I_r + I_n)/N - \gamma I_n \\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \gamma(I_r + I_n).\end{aligned}$$

Throughout the rest of this assignment we will assume that, due to reporting delays and the short infection period, cases are reported on the day the infected person recovers.

(a) Explain what the new parameter p represents. You may assume that $p = 0.04$. Write down the new initial condition if 13% of the population already have prior immunity when the first infection is introduced to the UK (you may assume this first infection does not cause symptoms severe enough to be reported). [2 marks]

(b) Run your model again, plotting both current infection prevalence and daily reported infection incidence across the 250-day period. Explain how you compute daily reported incidence. [8 marks]

(c) What is the cumulative number of reported infections at the end of your simulation now? If a cross-section serological survey was conducted to estimate the true number of immune people at the end of the outbreak what would be the expected result (you may assume the test used for the survey is 100% sensitive and specific at detecting immunity). [4 marks]

3. Although no lockdowns were imposed during the swine flu pandemic, summer school holidays in the UK did have an impact on contacts between people.

(a) Import the data "SwineFluCaseData.csv" into Matlab as a table. Plot the weekly reported case data as bar plot and explain how the model outputs you generated in question 2 differ from this data set. You do not need to include this plot in the write up. [5 marks]

(b) Now re-run your model again, assuming that (i) infection was introduced at the beginning of epidemiological week 17, (ii) that summer school holidays were from the beginning of week 30 to the end of week 35, and (iii) that during the holidays there was a contact rate reduction by a factor 0.6. Choose a suitable week for the simulation time end. Plot the infection prevalence dynamics using different colours to highlight the pre-school holiday, school holiday, and post-school holiday phases. [10 marks]

(c) How does this restriction impact $R_e(t)$? Include a figure [3 marks]

(d) What about the final size of the outbreak, and the duration of the outbreak? [2 marks]

(e) Plot the weekly reported case incidence on top of your data bar plot and include this figure in your write up. Discuss the match between your simulated outbreak and the case data [you may wish to mention: peak heights, timings, potential issues with case reporting in the data, omissions in the model] [6 marks]

4. Read the New York Times article on Moodle (which is also available here `https://www.nytimes.com/2020/05/01/opinion/sunday/coronavirus-herd-immunity.html`)

   (a) Explain how the concept of overshoot is relevant to your example here with reference to the herd immunity threshold and your computed outbreak sizes from questions 1–3. Discuss how immunity and school holidays impact overshoot in this example. You may wish to illustrate your explanation with a graph. [12 marks]