Integration By Parts

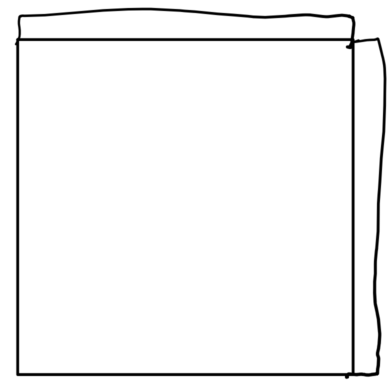
Monday, February 2, 2026 11:23 AM

$(uv)' = u'v + uv'$

$\int (uv)' = u'v + uv'$

$uv = \int v\,du + \int u\,dv$

$\int u\,dv = vv - \int v\,dv$



$$\boxed{\text{Memorize: } \int u\,dv = uv - \int v\,du}$$

$\int \frac{\ln|x|}{x}\,dx$ $\int \frac{1}{x\ln|x|}$

u-sub u-sub

$\int x\ln(x)\,dx$

ub

$\int x\ln(x)\,dx$

$u = x$ $v = \frac{1}{x} + c$

$du = 1$ $dv = \ln|x|\,dx$

$u = \ln|x|$ $v = \frac{1}{2}x^2 + c$

$du = \frac{1}{x}\,dx$ $dv = x\,dx$

L - Logs

I - Inverse Trig

A -

T - trig

E - exponential

$\int u\,dv = uv - \int v\,du$

$\ln(x) \cdot \frac{1}{2}x^2 - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$

$\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x\,dx$

$\frac{1}{2}x^2 \ln(x) - \frac{x^2}{4} + C$

$\int xe^x\,dx \qquad \int u\,dv = uv - \int v\,du$

$u = x \qquad v = e^x$

$du = 1\,dx \qquad dv = e^x\,dx$

$xe^x - \int e^x \cdot 1\,dx$

$xe^x - e^x + C$

$\int xe^{-x} \qquad \int u\,dv = uv - \int v\,du$

$u = x \qquad v = -e^{-x}$

$du = 1\,dx \qquad dv = e^{-x}$

$x \cdot -e^{-x} + \int -e^{-x}\,dx$

$-xe^{-x} - e^{-x} + C$

$\int \ln(x)\,dX \qquad \int u\,dv = uv - \int v\,du$

$u = \ln(x) \qquad v = X$

$du = \frac{1}{X}dX \qquad dv = 1\,dX$

$\ln(x)X - \int \cancel{X}\frac{1}{\cancel{X}}\,dX$

$\ln(x)X - \int 1\,dX$

$\ln(x)X - X + C$

$X(\ln(x) - 1) + C$

$\int X\cos(x)\,dX =$

$u = X \qquad V = \sin(x)$

$du = 1\,dX \qquad dv = \cos\,dX$

$X\sin(x) - \int \sin(x)\,dX$

$X\sin(x) - -\cos(x) + C$

$X\sin(x) + \cos(x) + C$

always a product in your answer

Practice

$\int X\sin X\,dX \qquad \int X\cos X\,dX \qquad \int X\ln X\,dX$

More difficult

More difficult

$$\int x^2 e^x \, dx = x^2 e^x - \int x^2 e^x \, dx$$

$v = x^2 \quad u = e^x$

$$= x^2 e^x - 2 \int 2x e^x \, dx$$

$\tilde{u} = 2x \quad \tilde{v} = e^x$

$v = 2x\,dx \quad du = e^x \, dx$

$\tilde{du} = 2 \quad d\tilde{v} = e^x \, dx$

$$= x^2 e^x - 2 \left\{ x e^x - \int e^x \, dx \right\}$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

U-Sub-Practice

$$\int e^{-x} \, dx = -\frac{1}{1} \int e^u \, du$$

$u = -x$

$$= -1 e^u$$

$du/dx = -1$

$$= -e^{-x}$$

$du = -1\,\cancel{dx}$

Back to difficult parts

Back to difficult parts

$$\underbrace{\int e^x \sin x\,dx}_{I} = e^x\sin(x) - \int e^x\cos(x)\,dx$$

$u = \sin(x)$ $\quad v = e^x$ $\qquad \tilde{u} = \cos(x)$ $\quad \tilde{v} = e^x$

$du = \cos(x)\,dx$ $\quad dv = e^x\,dx$ $\qquad d\tilde{w} = -\sin(x)$ $\quad d\tilde{u} = e^x\,dx$

$$= e^x\sin(x) - \{e^x\cos(x) - \int e^x\sin(x)\,dx$$

$$I = e^x\sin x - e^x\cos x - I$$

$$2I = e^x\sin x - e^x\cos x$$

$$I = \frac{1}{2}\left(e^x\sin x - e^x\cos(x)\right) + C$$

$$\int \tan^{-1} dx = x\tan^{-1}(x) - \int x \cdot \frac{1}{1+x^2}$$

$u = \tan^{-1}$ $\quad v = x$

$du = \frac{1}{1+x^2}$ $\quad dv = dx$

$\tilde{u} = 1+x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|$

$= \frac{1}{2}\ln|1+x^2)$

$\int x$

$\int \frac{x}{1+x^2}$

$x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C$