Linear Distinct Fractions

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Linear distinct fractions

$$\int \frac{x^2+1}{(x-1)(x-2)(x-3)}$$

$x(x-1)^2 \longrightarrow$ reducible

$x(x^2+1) \longrightarrow$ irreducible

$$\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} = \frac{x^2+1}{(x-1)(x-2)(x-3)}$$

$$A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = x^2+1$$

$$Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + C + c$$

x^2 ppl: $A + B + C = 1$

x ppl: $-5A - 4B - 3C = 0$

unit ppl: $6A + 3B + 2C = 1$

RREF: $A = 1 \quad B = -5 \quad C = 5$

$$\int \frac{1}{x-1} - \frac{5}{x-2} + \frac{5}{x-3}\,dx = \ln|x-1| - 5\ln|x-2| + 5\ln|x-3| + c$$

$$\int \frac{1}{x-1} + \frac{1}{3x+1}\,dx = \ln|x-1| + \frac{1}{3}\ln|3x+1|$$

$2x$

$$\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3}\,dx$$

$x^2 - 2x - 3 \;\overline{)\; 2x^3 - 4x^2 - x - 3}$

$-(2x^3 + 4x^2 - 6x)$

$(5x - 3)$

$$\int 2x + \frac{5x - 3}{x^2 - 2x - 3} \to (x-3)(x+1)$$

$$\frac{A}{x-3} + \frac{B}{x+1}$$

$$A(x+1) + B(x-3) = 5x - 3$$

$$Ax + A + Bx - 3B = 5x - 3$$

x 계수: $A + B = 5$

상수항: $A - 3B = -3$

$$\begin{pmatrix} 1 & 1 & 5 \\ 1 & -3 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

$A = 3,\ B = 2$

$$\int 2x + \frac{3}{x-3} + \frac{2}{x+1}\,dx$$

$$x^2 + 3\ln(x-3) + 2\ln(x+1)$$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{}{18}$

$1 \cdot 9 + 1 \cdot (2 \cdot 3) + 1 \cdot 2$

$\int \frac{6x+7}{(x+2)^2} dx \quad \frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$

$6x + 7 = A(x+2) + B$

$Ax + 2A + B = 6x + 7$

x terms: $A = 6$

constant terms: $7 = 2A + B$

$B = -5$

$\int \frac{6}{x+2} - \frac{5}{(x+2)^2} dx$

$6 \ln|x+2|$

$u = x + 2$

$du = dx$

$-5 \int \frac{1}{(x-2)} dx = -5 \int u^{-n} du$

$= -5 u^{-1}$

$= \frac{5}{u}$