Trig Sub

Tuesday, February 10, 2026 11:07 AM

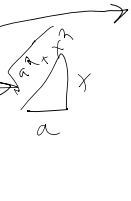
$\sqrt{a^2-x^2}$, $x = a\sin\theta$

$\sqrt{a^2+x^2}$, $x = a\tan\theta$

$\sqrt{x^2-a^2}$, $x = a\sec\theta$

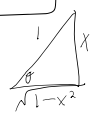
a goes on hyp

x goes on adjacent



$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$$

$x = \sin\theta$



$\sin\theta = x$

$\cos\theta = \frac{dx}{d\theta}$

$\cos\theta\,d\theta$

$$\int \frac{x^3}{\sqrt{1-x^2}} = \int \sin^3\theta\cos\theta\,d\theta$$

$$= \int \sin^3\theta\,d\theta$$

$$= \int \sin^2\theta\sin\theta\,d\theta$$

$$= \int (1-\cos^2\theta)\sin\theta\,d\theta \qquad \int u^2\,du$$

$$= \int \sin\theta\,d\theta - \int \sin\theta\cos^2\theta\,d\theta$$

$u = \cos\theta \quad du = -\sin\theta\,d\theta$

$$= -\cos\theta\,d\theta + \frac{\cos^3\theta}{3} + C$$

$$= \sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

Template

$$\int \frac{x^3}{\sqrt{1+x^2}}\,dx$$

$x = \tan\theta$

$\frac{dx}{d\theta} = \sec^2\theta$

$dx = \sec^2\theta\,d\theta$

$$\int \cos\theta\tan^3\theta\sec^2\theta\,d\theta$$

$$\int \tan^3\theta\sec\theta\,d\theta$$

$$\int \tan\theta\tan^2\theta\sec\theta\,d\theta$$

$$\int \tan\theta(\sec^2\theta - 1)\sec\theta\,d\theta$$

$\int u^2\,du = \frac{\sec^3\theta}{3}$

$= \sec\theta + C$

$$\int \tan\theta\sec\theta\sec^2\theta\,d\theta - \int \tan\theta\sec\theta$$

$u = \sec\theta$

$$I = \frac{\sec^3\theta}{3} - \sec\theta + C$$

$$= \frac{1}{3}\left(\sqrt{1+x^2}\right)^3 - \sqrt{1+x^2} + C$$

New hardest Problem

$$\int \frac{\sqrt{x^2-1}}{x}$$

$\sec\theta = x$

$\sec\theta\tan\theta\,d\theta = dx$

$$\int \frac{\sqrt{x^2-1}}{x}\,dx = \int \sin\theta\sec\theta\tan\theta\,d\theta$$

$S \cdot \frac{1}{C} \cdot \frac{S}{C}$

$$= \tan^2\theta\,d\theta = \int (\sec^2\theta - 1)\,d\theta = \tan\theta - \theta + C$$

$$= \sqrt{x^2-1} - \sec^{-1}x + C$$

this shit rough

Hw - learn these three