MATH 242 Notes

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1. "duhs"

the duhs are things that we just need to remember. these should be muscle memory, no questions asked, no math involved:

f	$\frac{df}{dx}$	long form
$\sin(x)$	$\cos(x)$	$\int \cos(x) dx = \sin(x) + C$
$\cos(x)$	$-\sin(x)$	$\int \sin(x) dx = -\cos(x) + C$
x	1	$\int dx = x + C$
e^x	e^x	$\int e^x dx = e^x + C$
$\tan(x)$	$\sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + C$
$\ln(x)$	$\frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\arctan(x)$	$\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \arctan(x) + C$

it isn't quite a duh but the fundamentally important rule for quick integration is the power rule:

$\frac{d}{dx} x^n = n x^{n-1}$

$\Longrightarrow \frac{d}{dx} \frac{x^{n+1}}{n+1} = x^n$

so from this we can discern:

$$\int x^n dx := \begin{cases} \frac{x^{n+1}}{n+1} + C \text{ if } n \neq -1 \\ \ln|x| + C \text{ if } n = -1 \end{cases}$$

2. u-substitution

how do we solve this?

$$\int x(x^2 + 1)^{50} dx$$

u-sub is "the chain rule in reverse"

u-sub has 6 steps (in the context of the given problem):

1. $u = x^2 + 1$: identify the u (this is our u-sub we work with)
2. $\frac{du}{dx} = 2x$: now we find the u'
3. $du = 2xdx$: now we find the du alone
4. $\frac{1}{2} \int u^{50} du$: putting it back into context
5. $\frac{1}{2} \frac{u^{51}}{51} + C$: "solved"
6. $\frac{1}{2} \frac{(x^2+1)^{51}}{51} + C$: back-substitution

done.

an example:

$$\int e^{10x} dx$$

$$u = 10x \Longrightarrow \frac{du}{dx} = 10 \Longrightarrow du = 10dx$$

3. integration by parts

this is in stewart calc ch7.1

$$\int (uv)' = \int u'v + \int uv'$$

$$uv = \int vdu + \int udv$$

$$\int udv = uv - \int vdu$$

memorize:

$$\int udv = uv - \int vdu$$

worked example:

$$\int x \ln x dx$$

$$u = \ln x \text{ and } dv = xdx$$

$$du = \frac{1}{x}dx \text{ and } v = \frac{x^2}{2}$$

as per the formula:

$$\int udv = uv - \int vdu$$

$$\begin{aligned} \int x \ln x dx &= \frac{x^2}{2} \ln x \int \frac{x^2}{2} \cdot \frac{1}{x} dx \\ &= \frac{x^2}{2} \ln x - \int \frac{1}{2} x dx \\ &= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \end{aligned}$$

3.1. integrating ln(x)

$$\int \ln x dx$$

$$u = \ln x \text{ and } v = x$$

$$du = \frac{1}{x}dx \text{ and } dv = dx$$

$$\begin{aligned} \int \ln x dx &= \ln(x)x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C \end{aligned}$$

4. partial fractions

$$\frac{21}{35} = \frac{A}{5} + \frac{B}{7} = \frac{7A + 5B}{35}$$

$$7A + 5B = 21$$

this is an underdetermined system of equations, because the answers to what A and B are are not uniquely determined

Ex:

$$\int \frac{2}{x^2 - 2x} dx$$

$$\frac{2}{x^2 - 2x} = \frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$\frac{2}{\cancel{x(x-2)}} = \frac{A(x-2) + Bx}{\cancel{x(x-2)}}$$

and now you have a simple system to solve

$$Ax - 2A + Bx = Ux + Z$$

$$\begin{aligned} A + B &= 0 \\ -2A &= 2 \end{aligned}$$

$$\Rightarrow A = -1, B = 1$$

(this may also be done with reduced row echelon form as long as a proper identity matrix can be formed on the left side of the matrix. if your name is noah “alex” yurasko, you can probably do this method)

returning to the integral:

$$= \int \left(-\frac{1}{x} + \frac{1}{x-2} \right) dx$$

$$= -\ln|x| + \ln|x-2| + C$$

there are some cases where this is useful:

1. denominator has distinct linear factors (factors do not result in a power)

(he only wrote one case)

5. appendix

5.1. rules from 05/02

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = \csc(x)\cot(x)$$

5.2. identities from 05/02

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\ln(a^r) = r\ln(a)$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\tan^2(x) + 1 = \sec^2(x)$$

5.3. integrals from 05/02

$$\int \tan^2(x)$$

$$= \int \sec^2(x) - 1dx$$

$$= \tan(x) - x + C$$

5.4. table of trigonometric substitutions

expression	substitution	identity