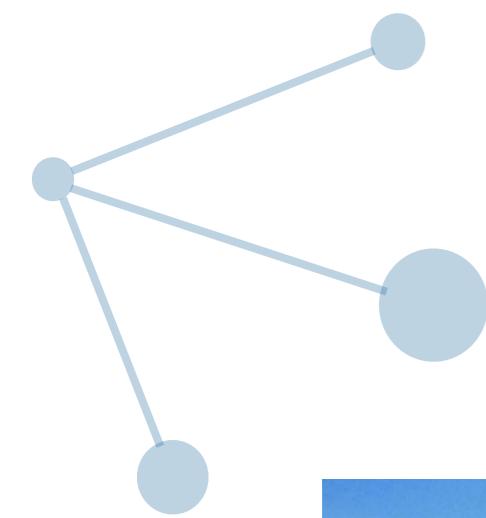


Why I Like FP

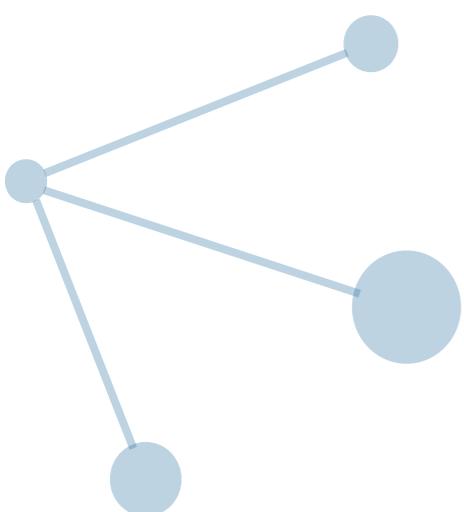
Adelbert Chang
Box, Inc.



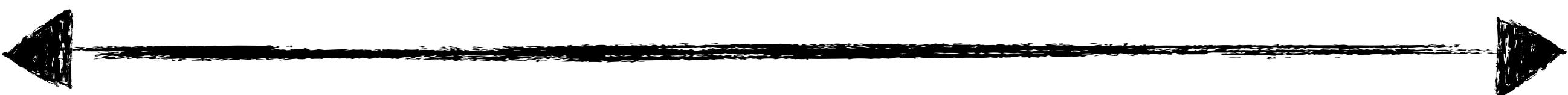
Let's go back..

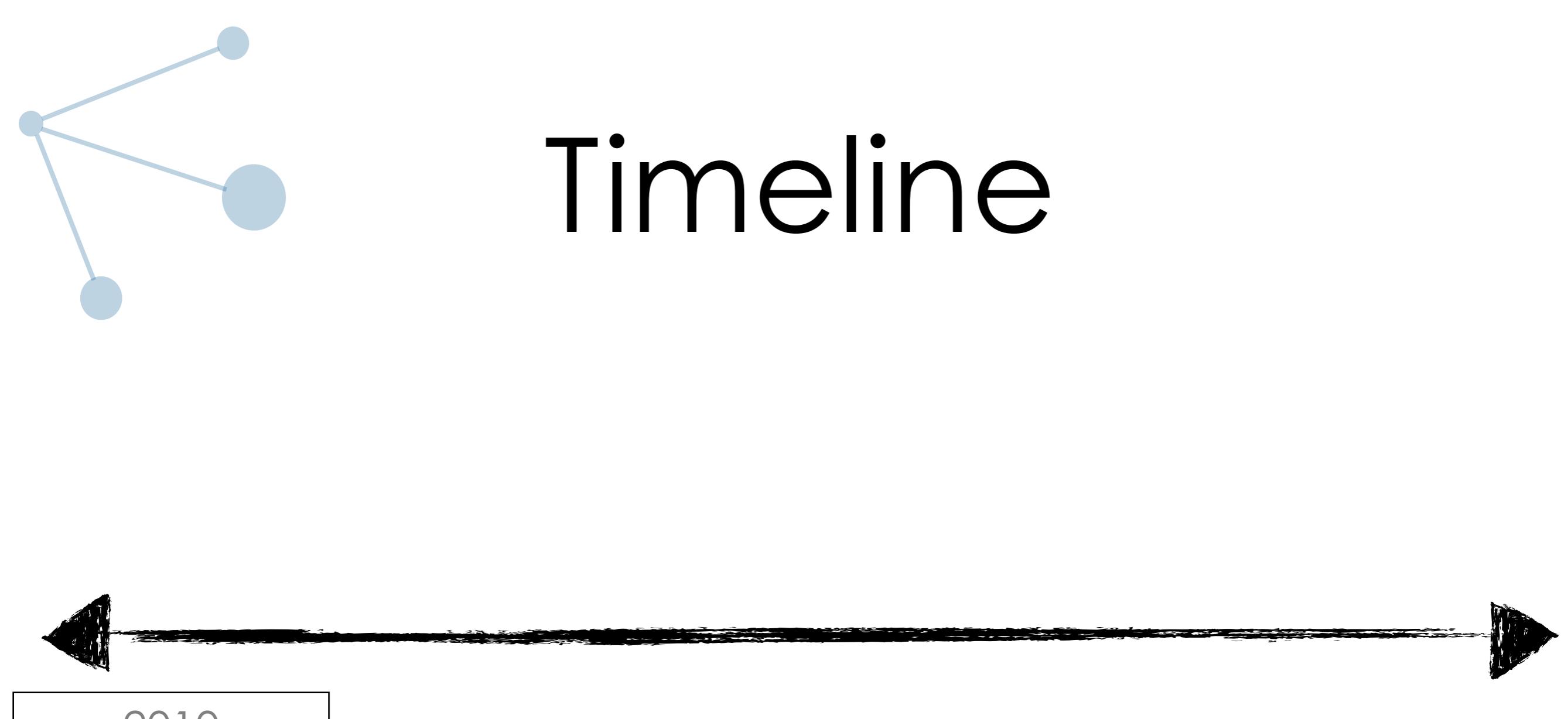


<http://gauss.cs.ucsb.edu/home/images/UCSB-from-air.jpg>



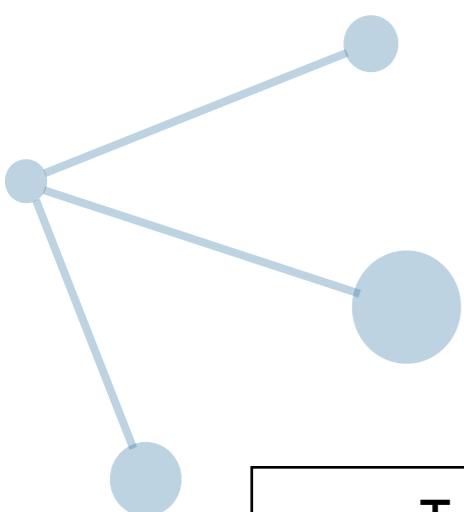
Timeline





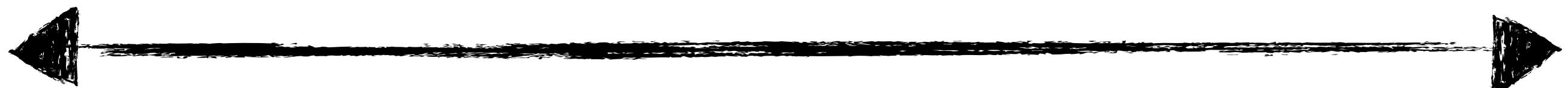
Timeline

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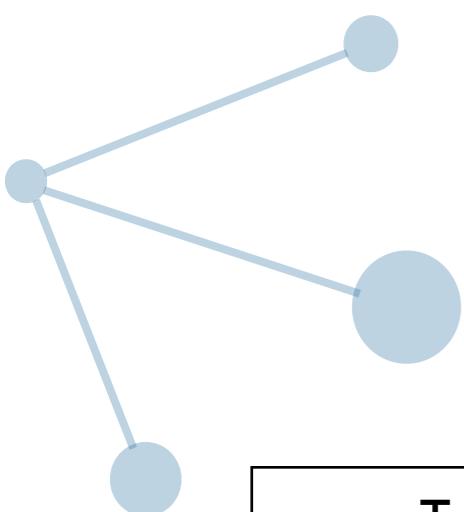


Timeline

Took first two
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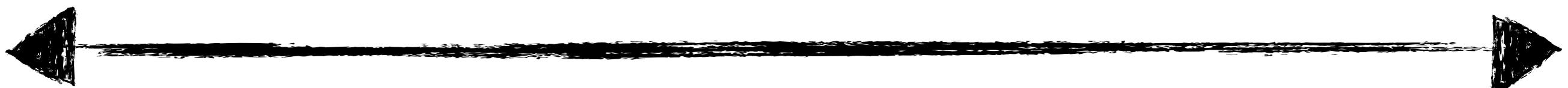


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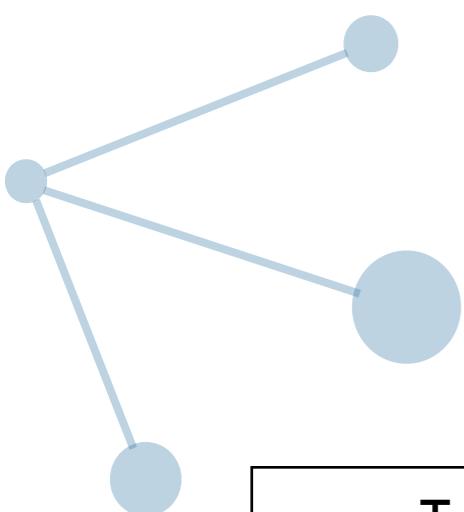
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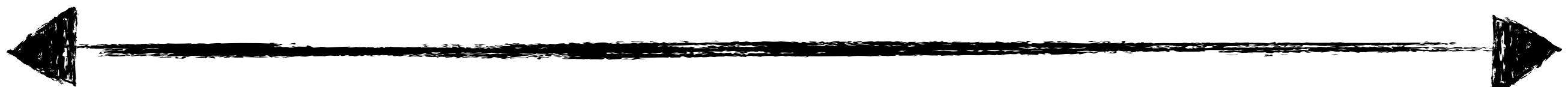
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Took upper-division math courses. Considered minoring in mathematics.
Started learning Scala.



Timeline

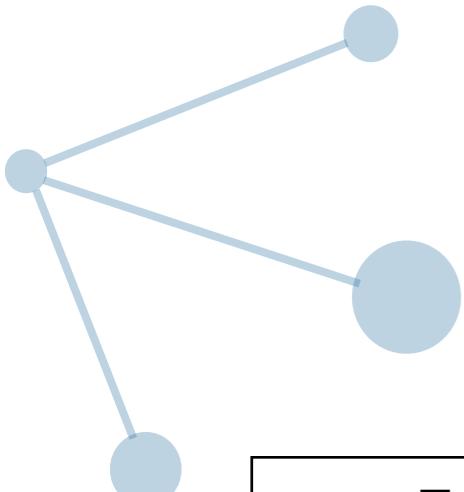
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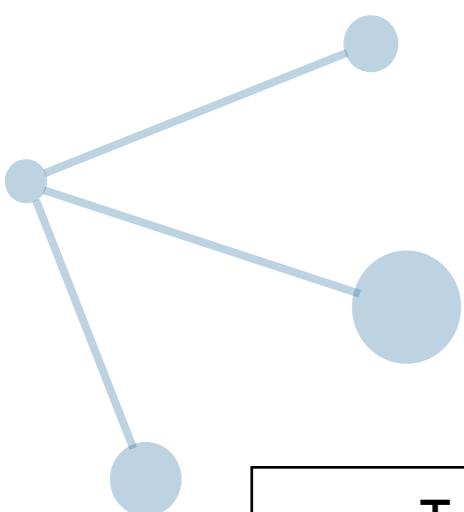
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Timeline

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2013

Presenting at an awesome FP conference.

2015



2010

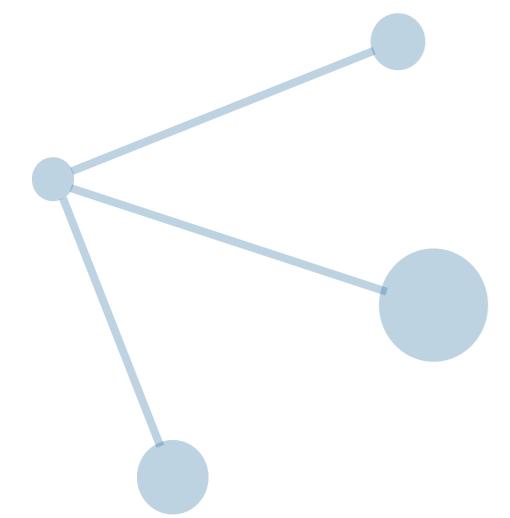
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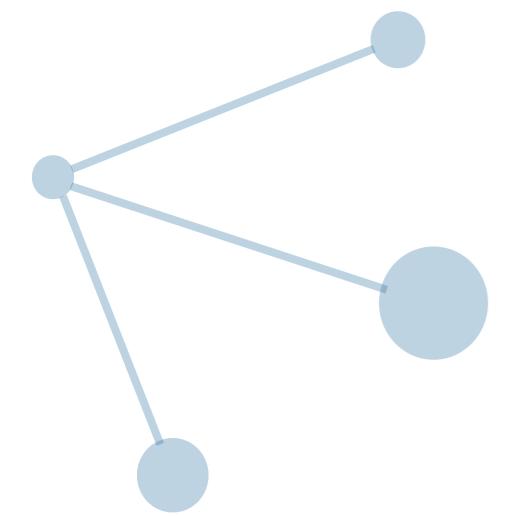
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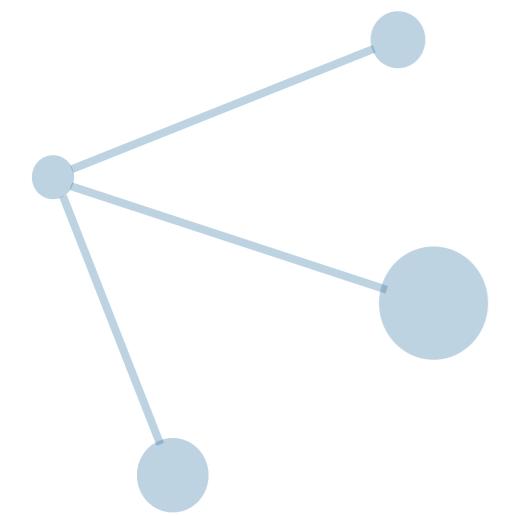


Why CS



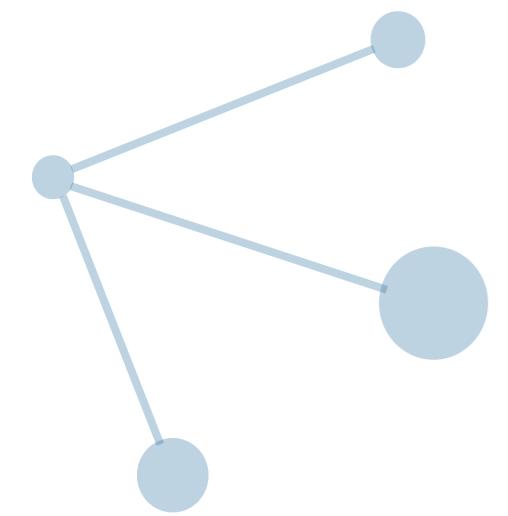
Why CS

- Computers are pretty cool



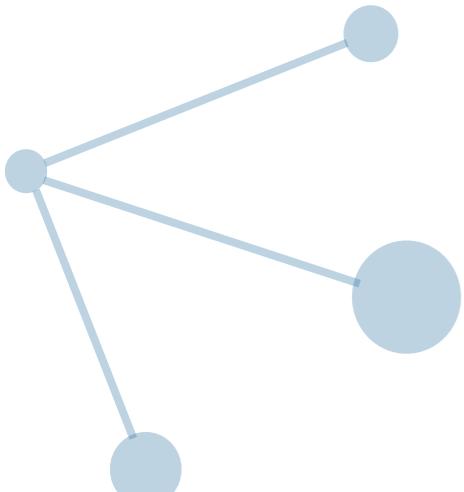
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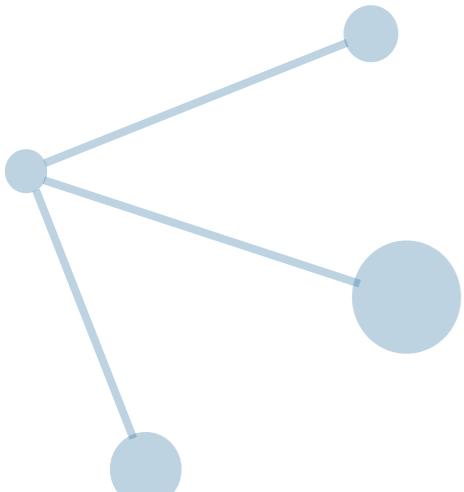
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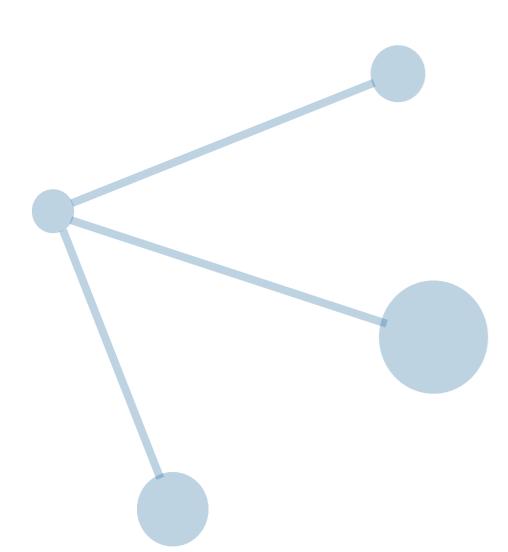
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- Rooted in logic

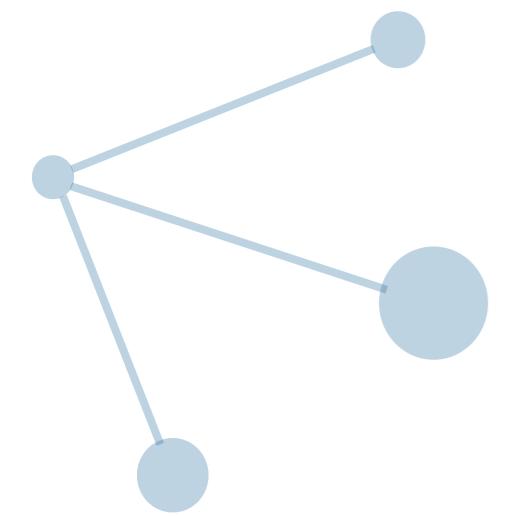


Why CS

- Computers are pretty cool
- Applied problems to solve
- Fairly interactive
- Rooted in logic
- Can be done pretty much anywhere

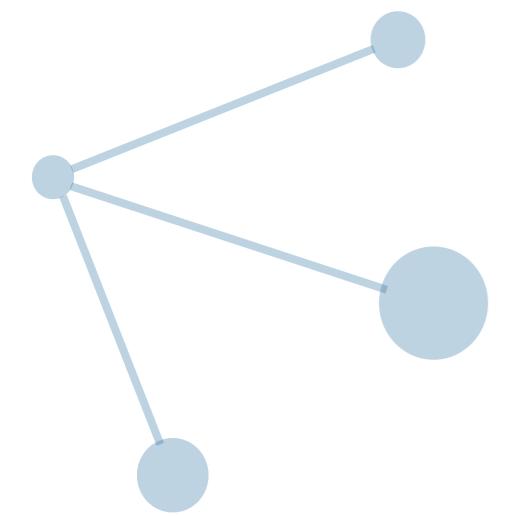


2011



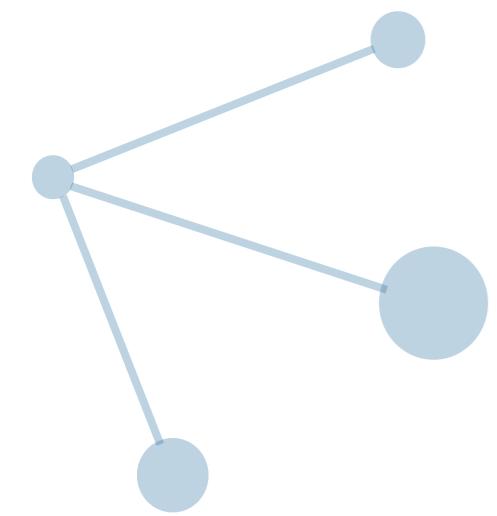
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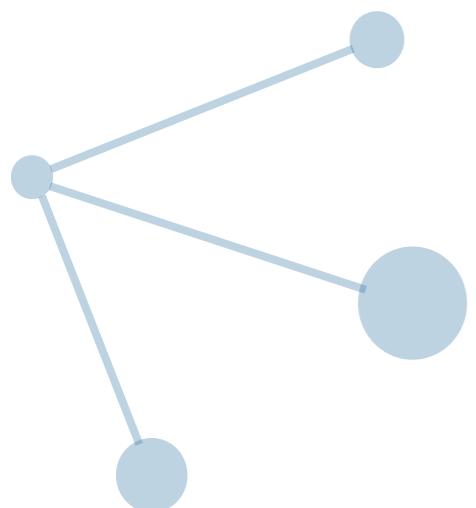
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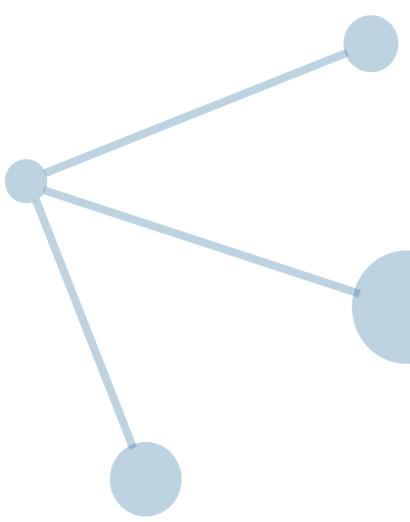
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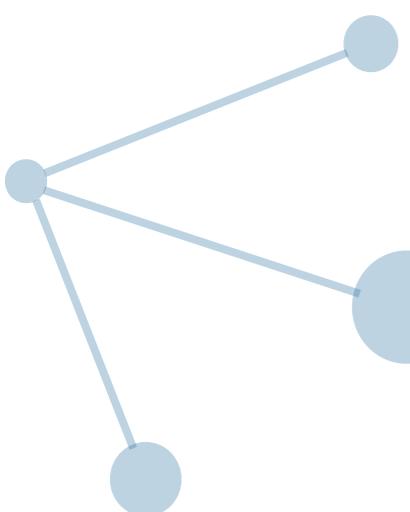
2011

- Languages learned/used: Python, C, C++
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 - Considered switching major to physics

```
swapped = True
while swapped:
    swapped = False
    for i in range(len(xs) - 1):
        if xs[i] > xs[i + 1]:
            xs[i], xs[i + 1] = xs[i + 1], xs[i]
            swapped = True
```

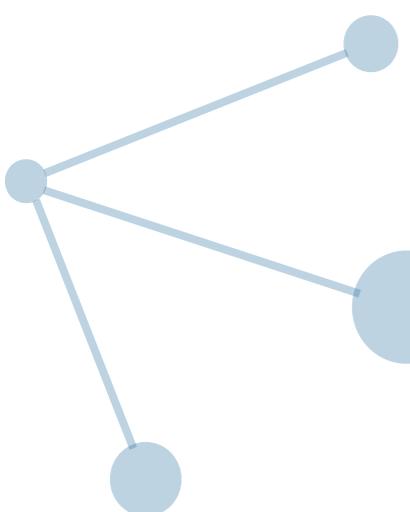


Physics and Math



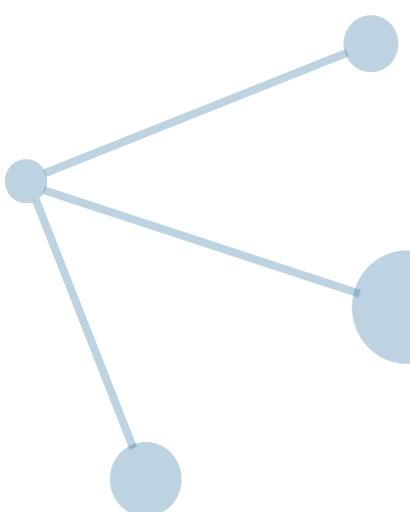
Physics and Math

- Problem solving activity



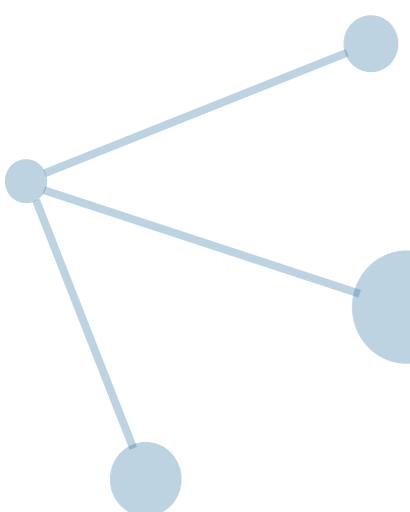
Physics and Math

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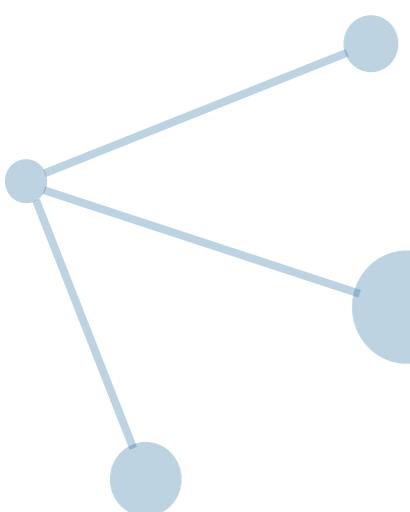
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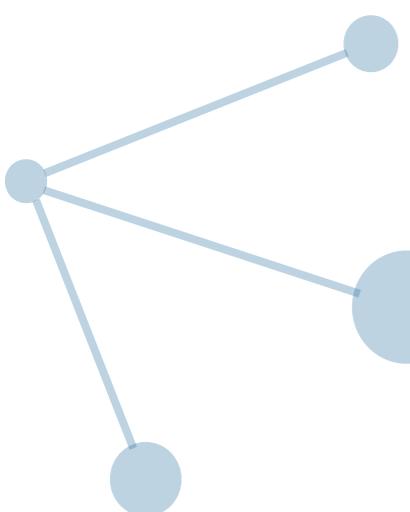
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- Example: Deriving law of motion from first principles



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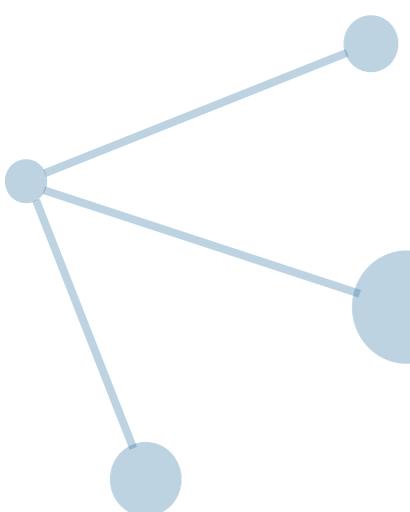
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 - $x = x_0 + v_0 t + \frac{1}{2} a t^2$



Physics and Math

Deriving $x = x_0 + v_0t + \frac{1}{2}at^2$

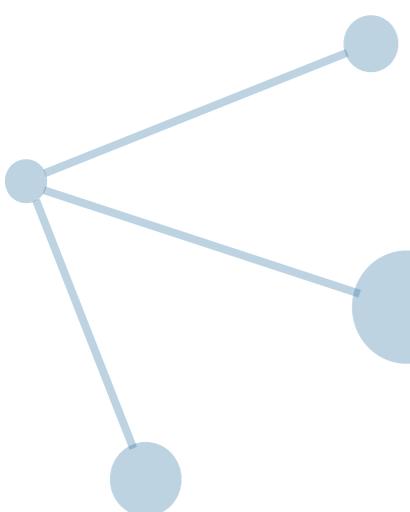




Physics and Math

Deriving $x = x_0 + v_0t + \frac{1}{2}at^2$

$$v_{\text{avg}} = \Delta x / \Delta t$$

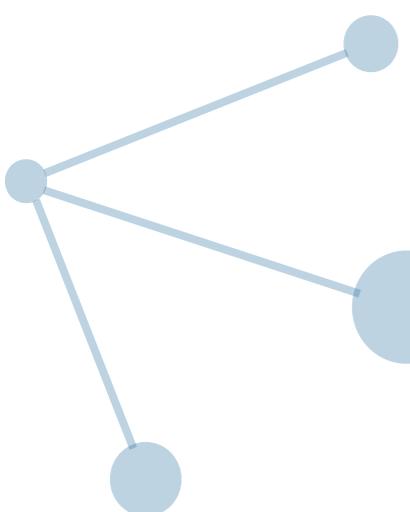


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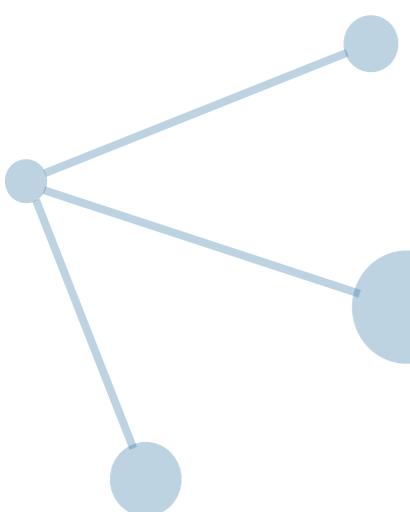
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Physics and Math

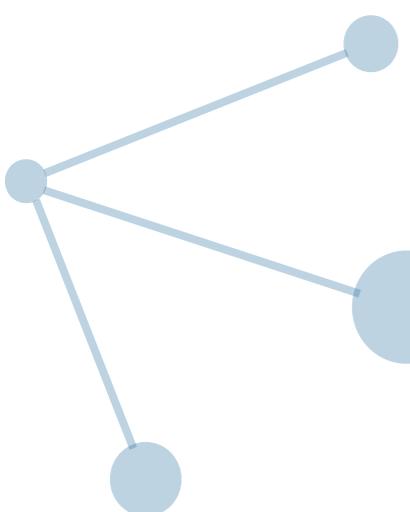
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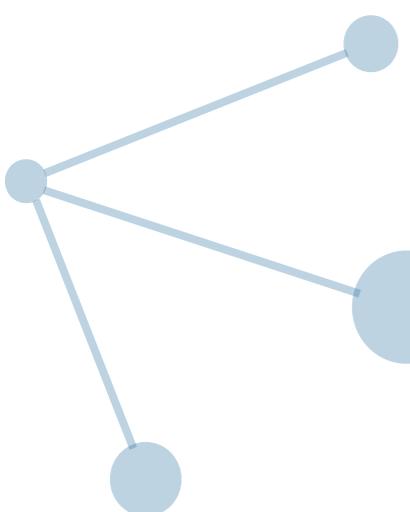
$$v_{\text{avg}} = \frac{1}{2}(v_f + v_0)$$

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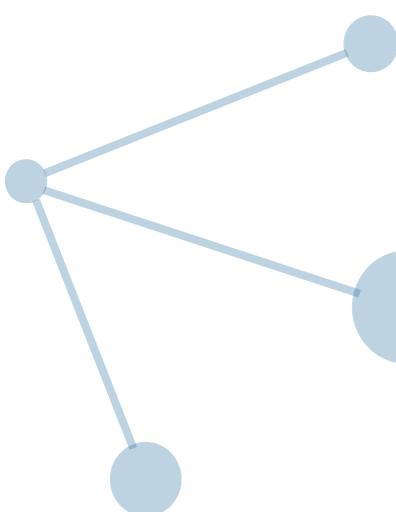
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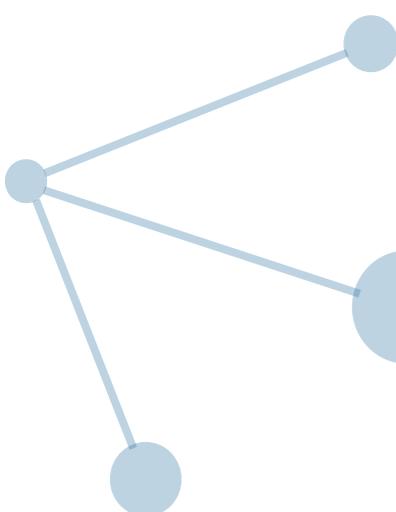
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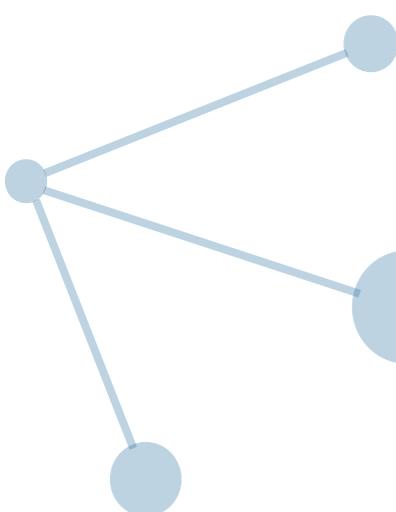
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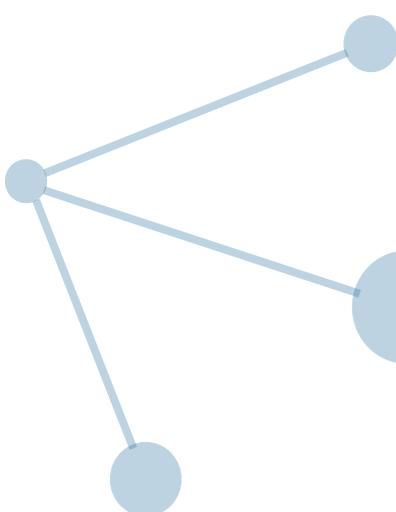
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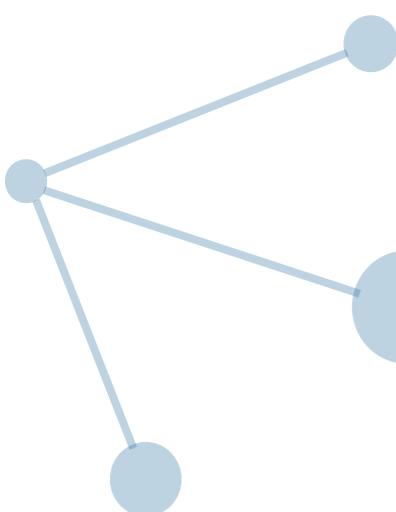
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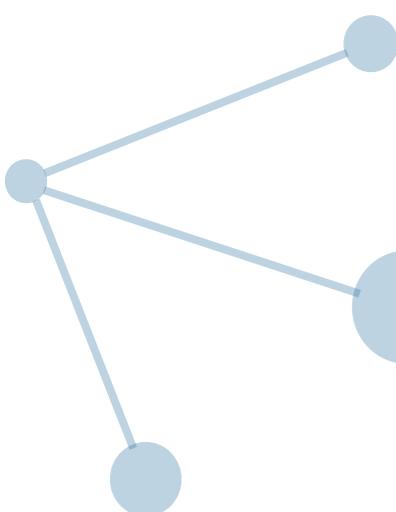
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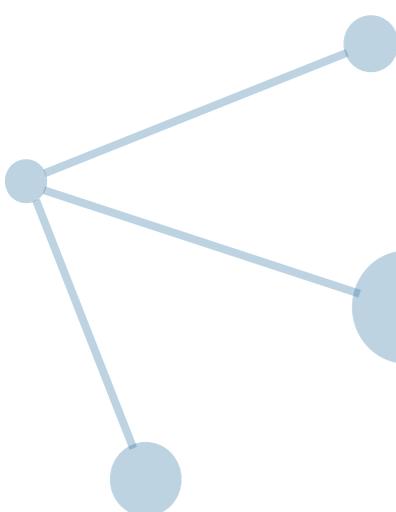
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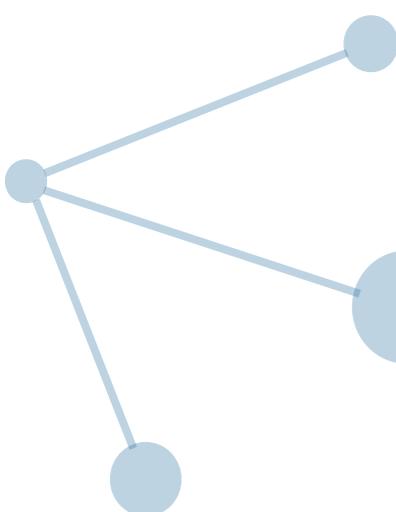
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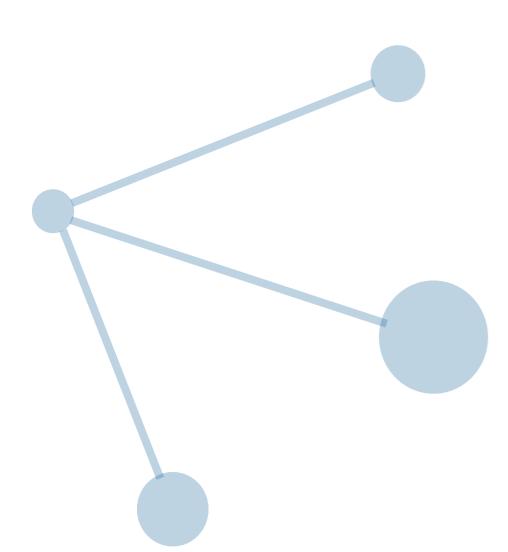
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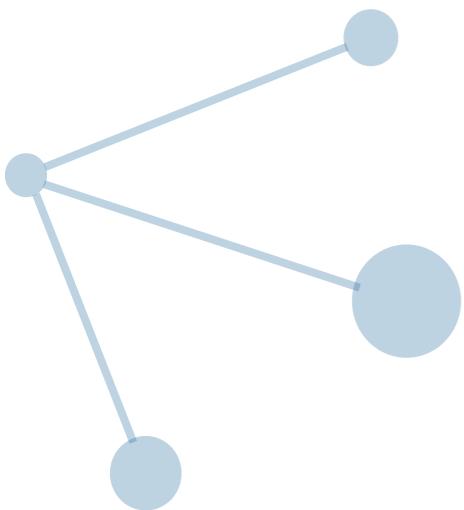
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$$x_0 + v_0 t + \frac{1}{2} a t^2$$



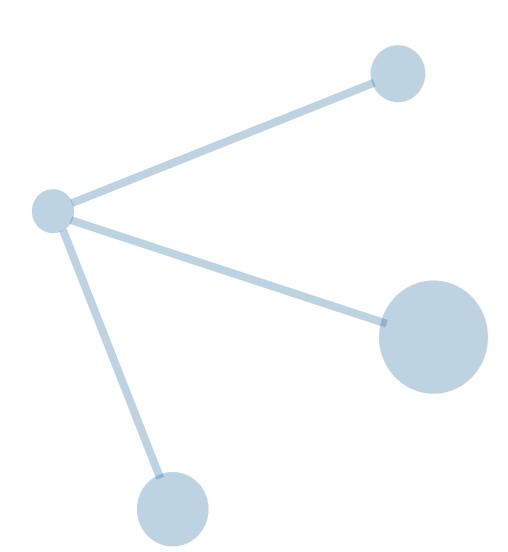
2012



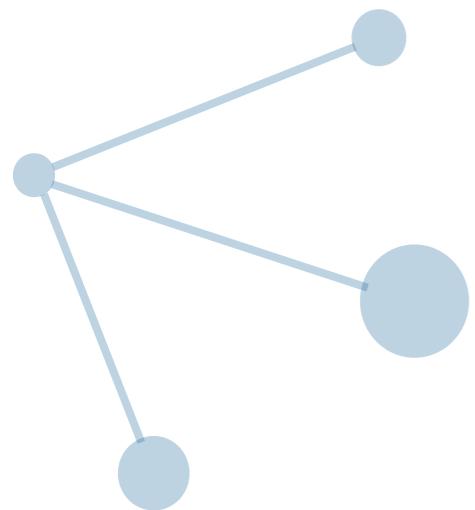
2012

```
void insert(Node* &tree, int value) {
    if (!tree) return;

    Node *trav = tree, *parent;
    while (trav) {
        parent = trav;
        if (value < trav->data) trav = trav->left;
        else if (value > trav->data) trav = trav->right;
        else break;
    }
    if (value < parent->data)
        parent->left = new Node(value);
    else
        parent->right = new Node(value);
}
```



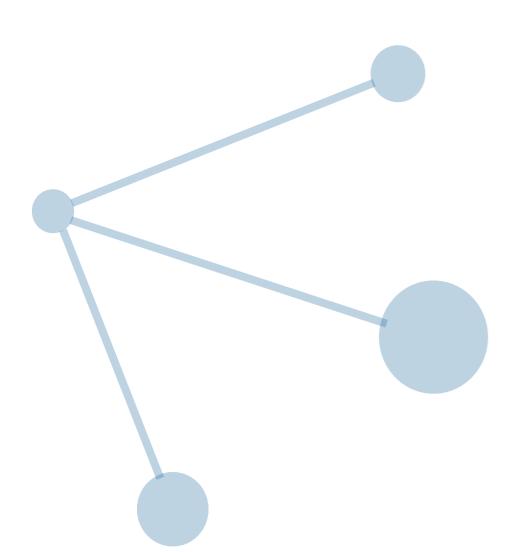
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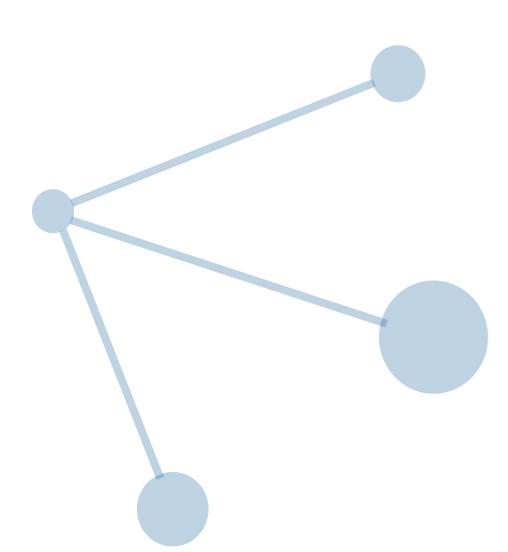
2012

```
sealed abstract class Tree
case class Branch(data: Int, left: Tree, right: Tree)
  extends Tree
case class Leaf() extends Tree

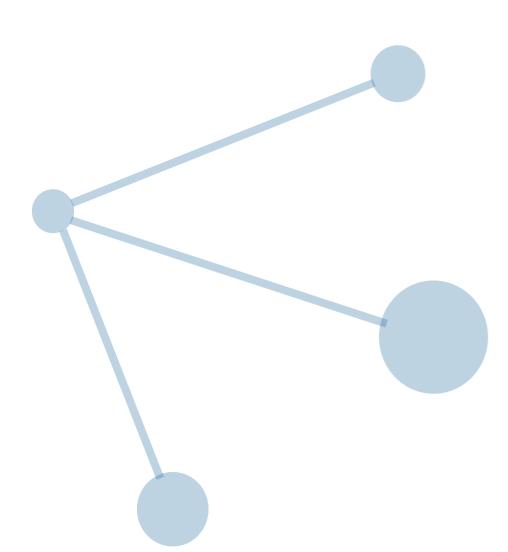
def insert(tree: Tree, value: Int): Tree =
  tree match {
    case Leaf() => Branch(value, Leaf(), Leaf())
    case b@Branch(d, l, r) =>
      if (value < d) Branch(d, insert(l, value), r)
      else if (value > d) Branch(d, l, insert(r, value))
      else b
  }
```



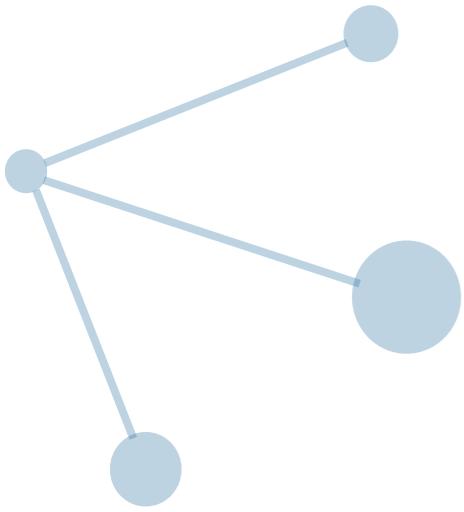
Hmm.. functional programming is pretty cool.



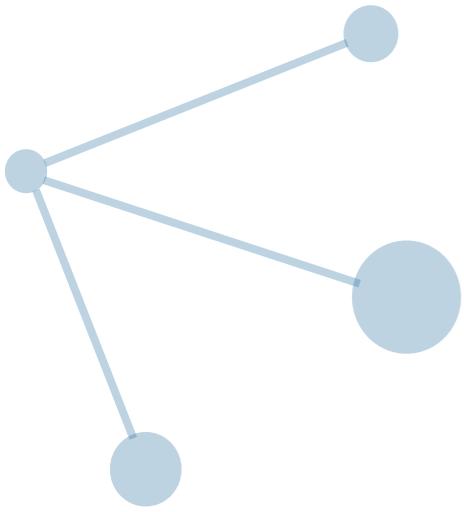
But is it as cool as physics and math?



Let's see..

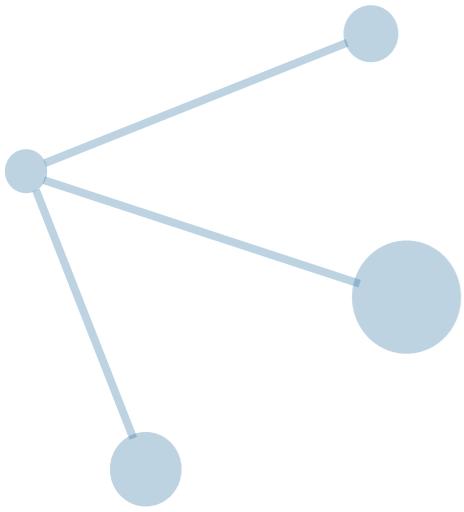


Functional Programming



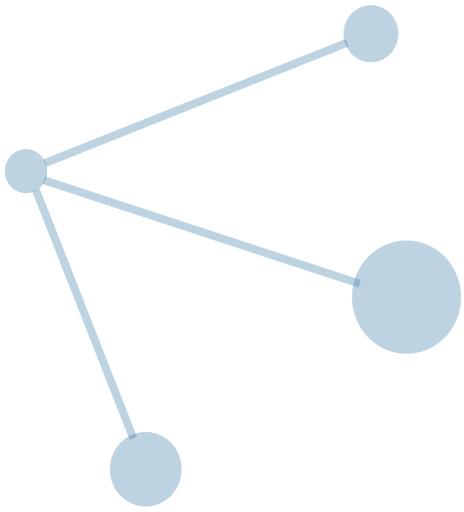
Functional Programming

programming with functions



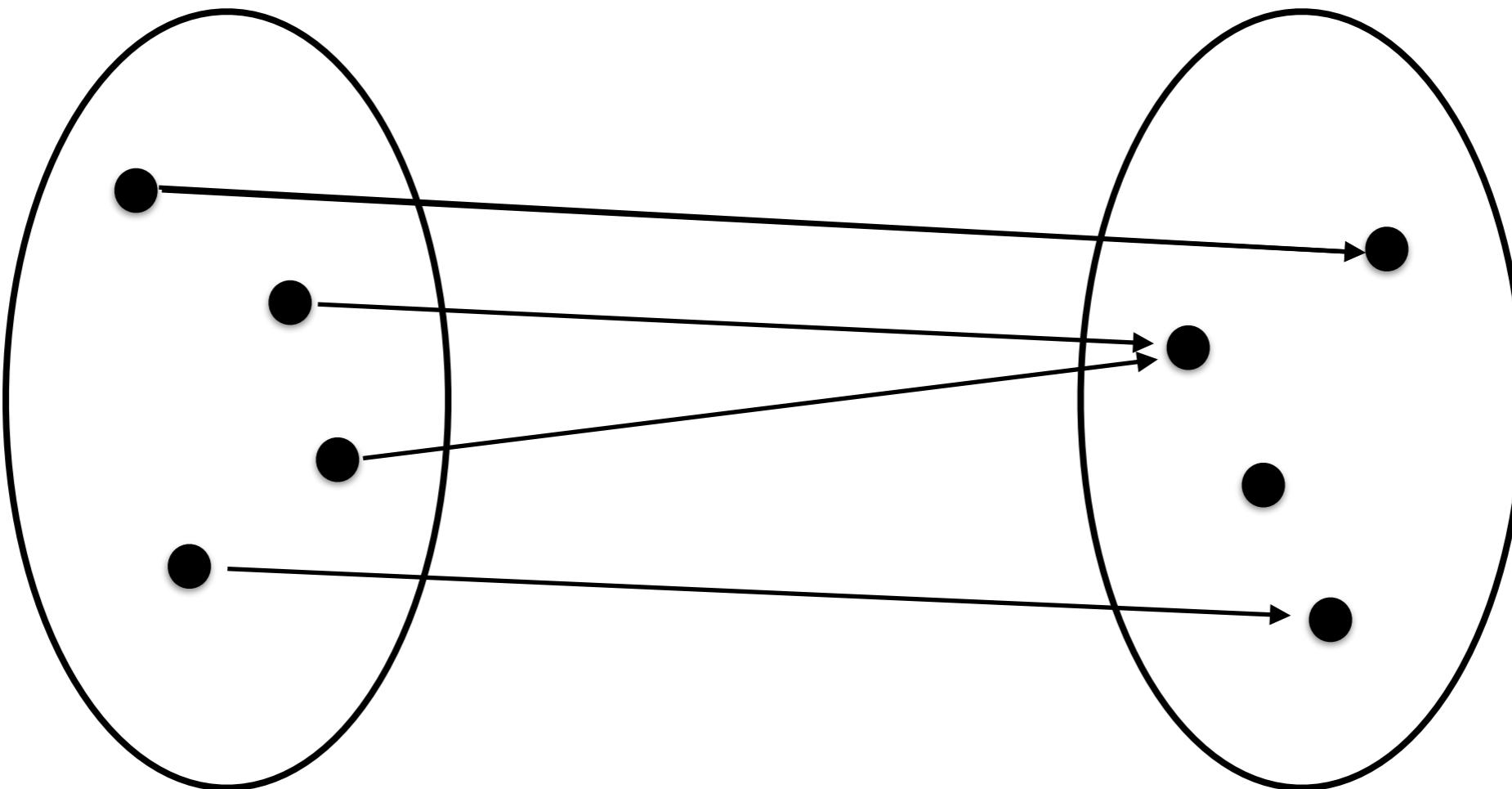
Functional Programming

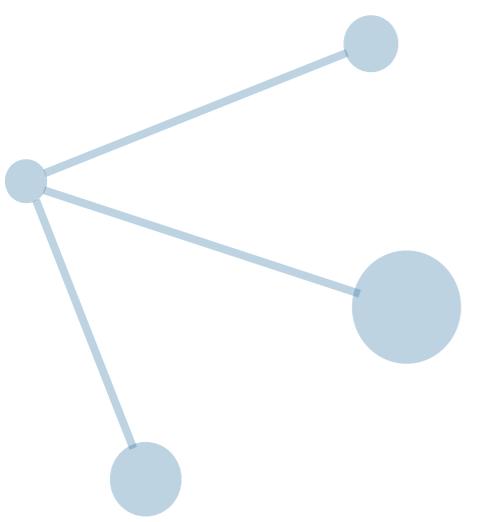
programming with **pure** functions



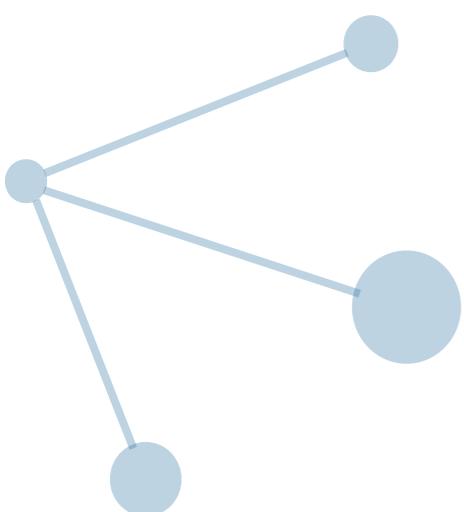
Functional Programming

programming with **pure** functions



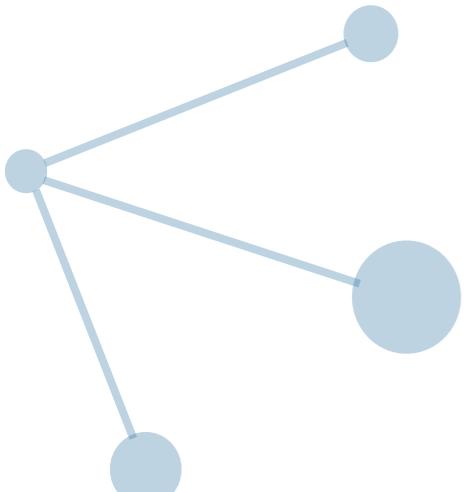


Functions



Functions

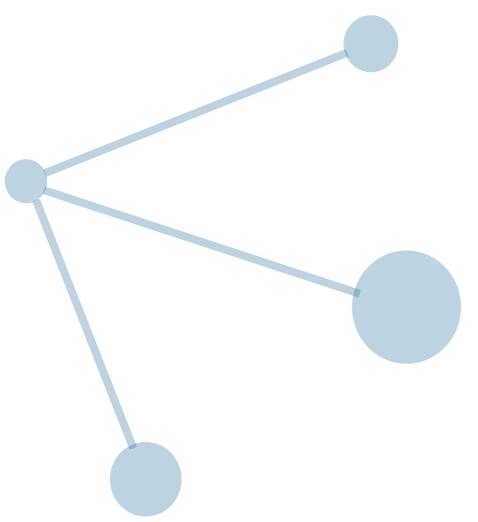
```
def parseIntPartial(s: String): Int =  
    s.toInt // can throw NumberFormatException
```



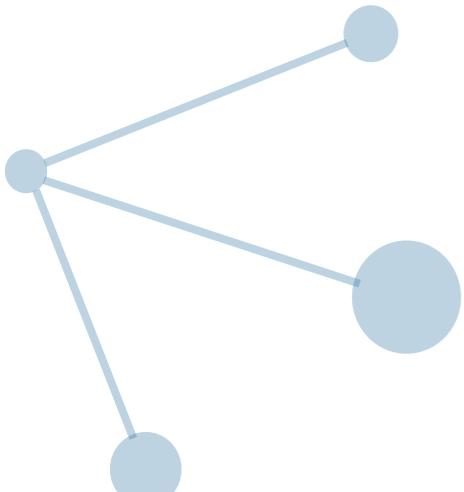
Functions

```
def parseIntPartial(s: String): Int =  
  s.toInt // can throw NumberFormatException
```

```
def parseIntTotal(s: String): Option[Int] =  
  try {  
    Some(s.toInt)  
  } catch {  
    case nfe: NumberFormatException => None  
  }
```

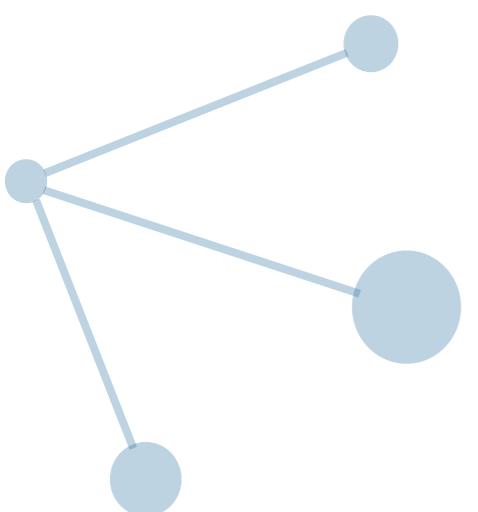


Functions



Functions

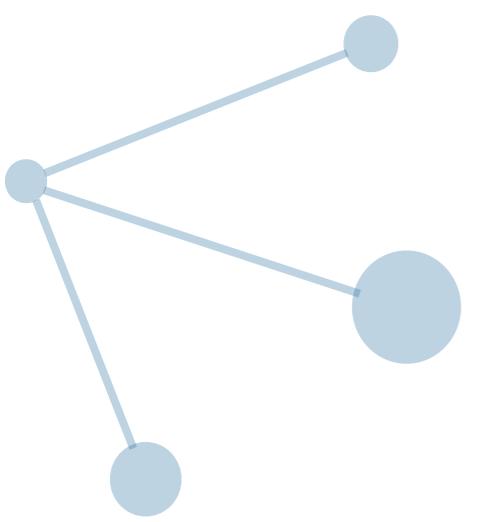
```
class Rng(var seed: Long) {  
    def nextInt(): Int = {  
        val int = getInt(seed)  
        mutate(seed)  
        int  
    }  
}
```



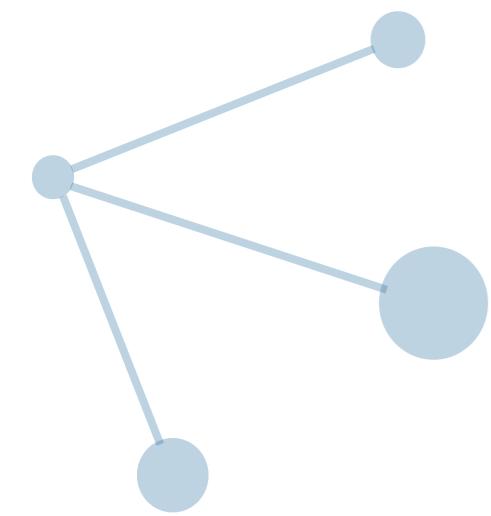
Functions

```
class Rng(var seed: Long) {  
    def nextInt(): Int = {  
        val int = getInt(seed)  
        mutate(seed)  
        int  
    }  
}
```

```
case class Rng(seed: Long) {  
    def nextInt: (Rng, Int) = {  
        val int = getInt(seed)  
        val newSeed = f(seed)  
        (Rng(newSeed), int)  
    }  
}
```

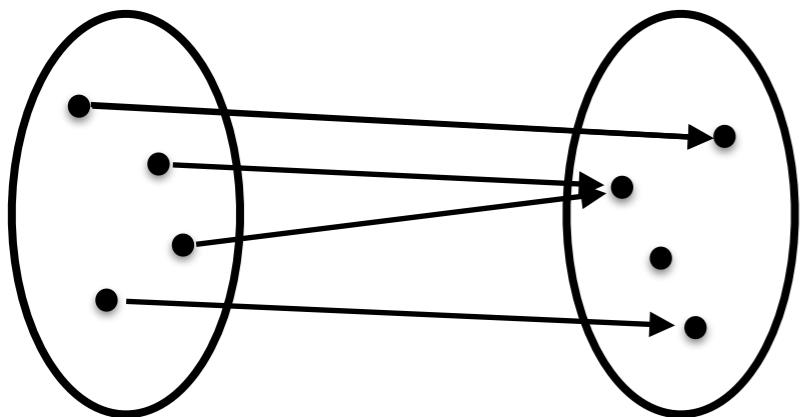


Functions

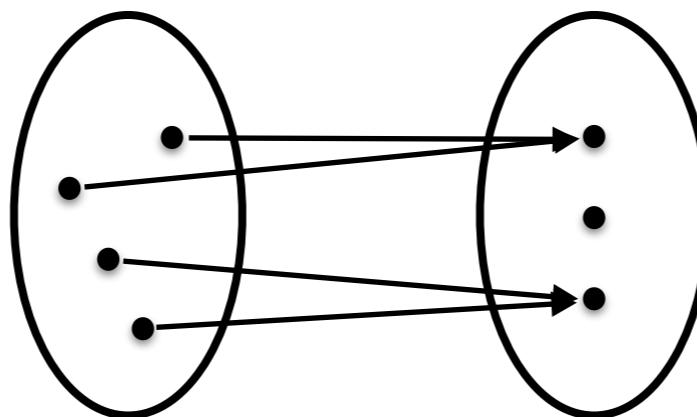


Functions

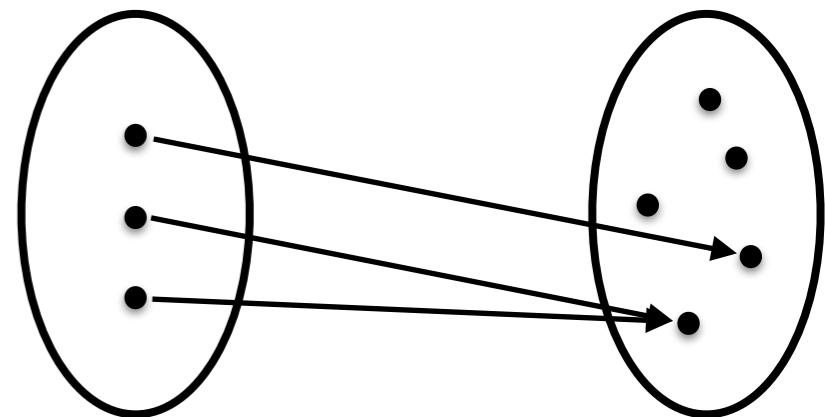
foo: A \Rightarrow B

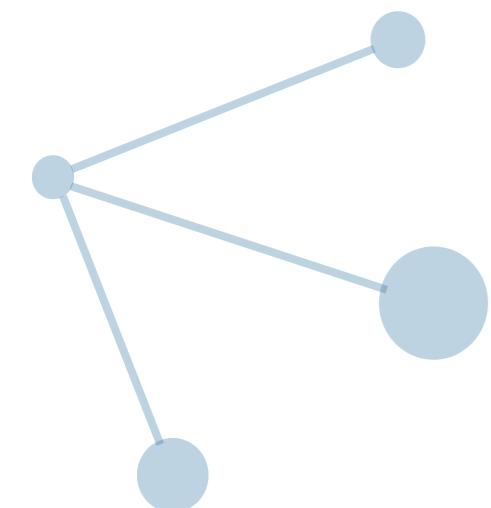


bar: B \Rightarrow C



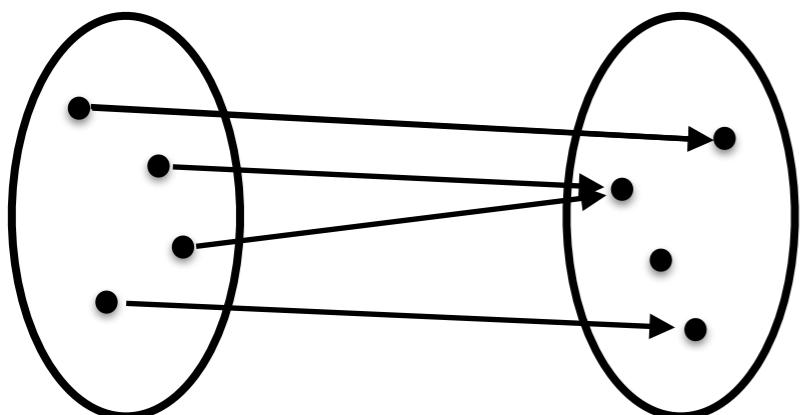
baz: C \Rightarrow D



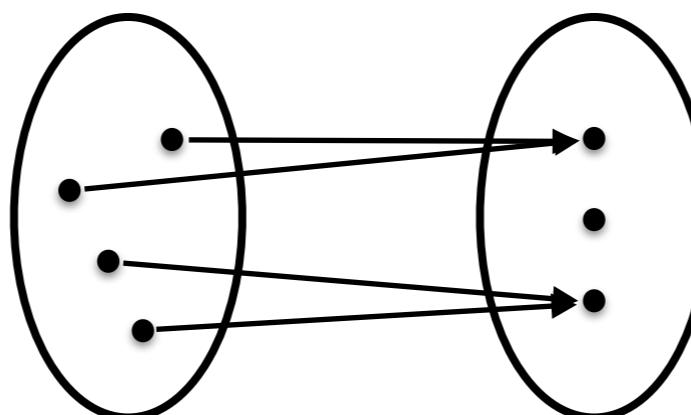


Functions

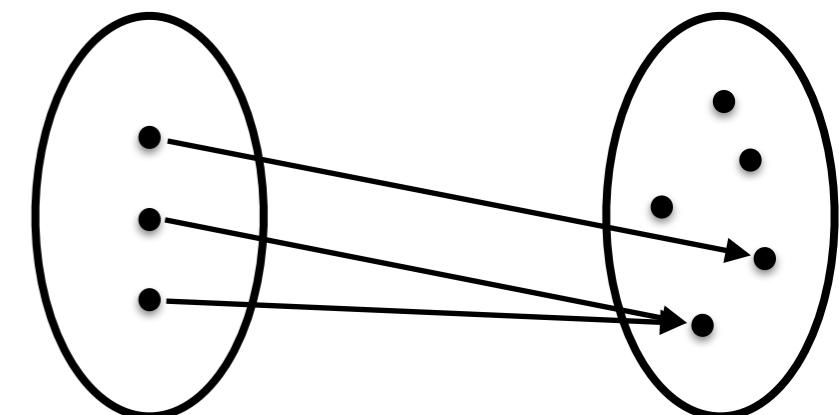
foo: A \Rightarrow B



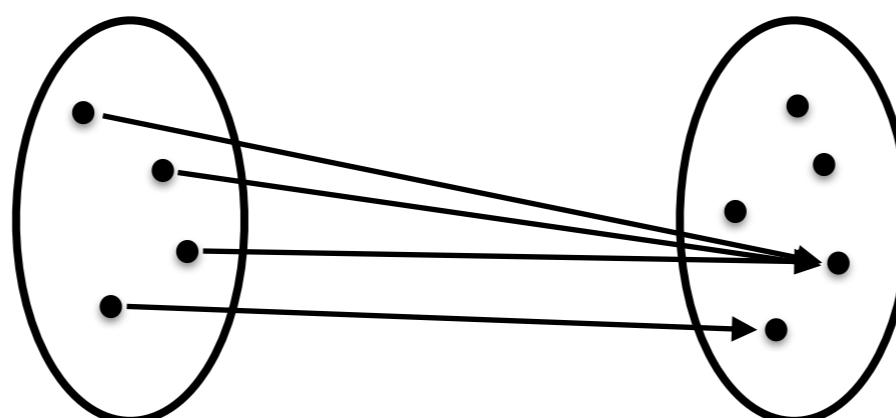
bar: B \Rightarrow C

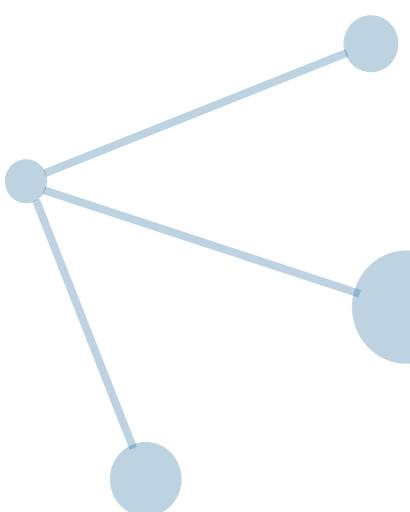


baz: C \Rightarrow D



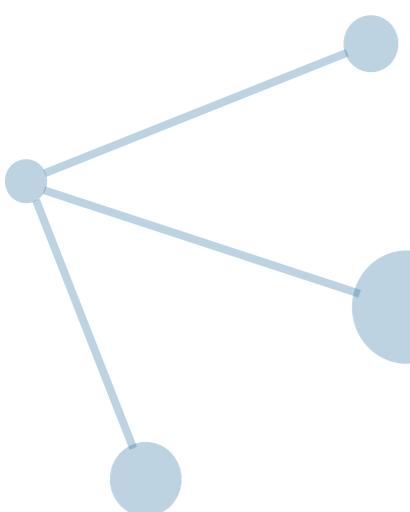
baz.compose(bar).compose(foo): A \Rightarrow D





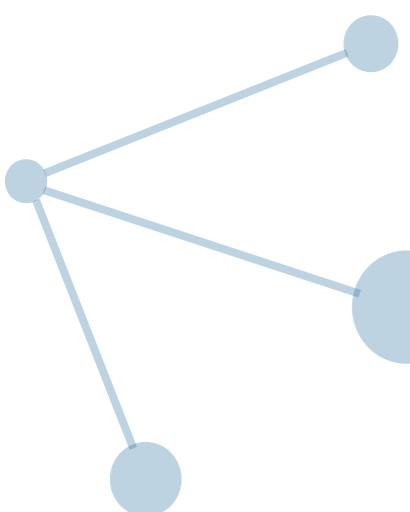
```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Reducing Values



Reducing Values

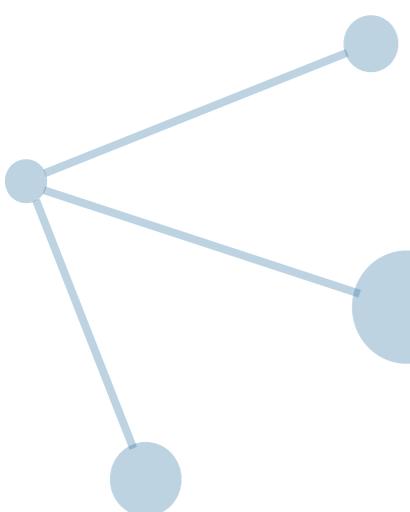
```
def sum(list: List[Int]): Int =  
    list.foldLeft(0)(_ + _)
```



Reducing Values

```
def sum(list: List[Int]): Int =  
    list.foldLeft(0)(_ + _)
```

```
def concat(list: List[String]): String =  
    list.foldLeft("")(_ ++ _)
```

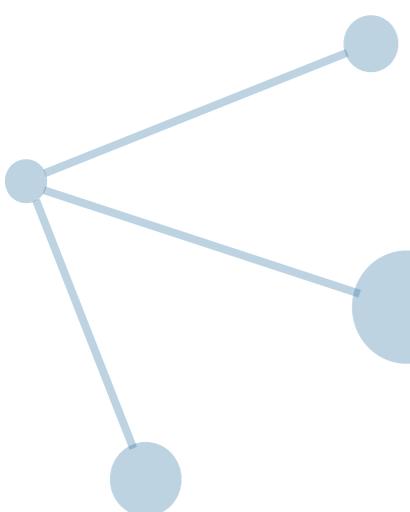


Reducing Values

```
def sum(list: List[Int]): Int =  
    list.foldLeft(0)(_ + _)
```

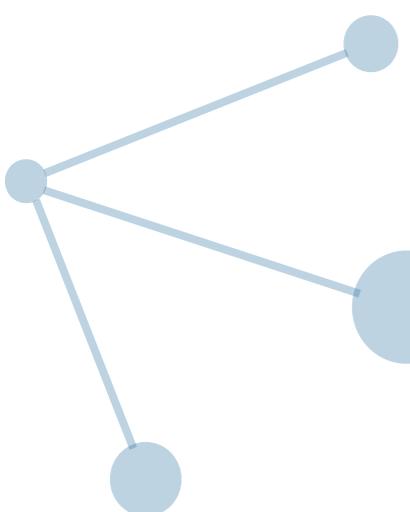
```
def concat(list: List[String]): String =  
    list.foldLeft("")(_ ++ _)
```

```
def union[A](list: List[Set[A]]): Set[A] =  
    list.foldLeft(Set.empty[A])(_ union _)
```



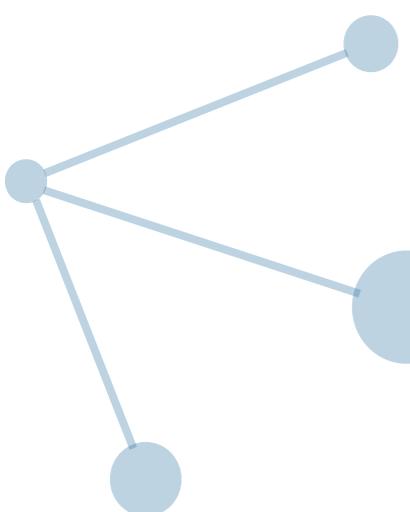
Reducing Values

```
def fold[A](list: List[A]): A =  
    list.foldLeft(IDENTITY)(BINARY_OP)
```



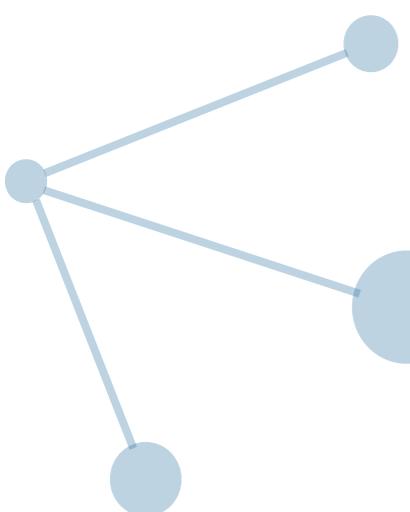
```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Reducing Values



Reducing Values

```
trait Monoid[A] {  
    def append(x: A, y: A): A  
  
    def zero: A  
}
```



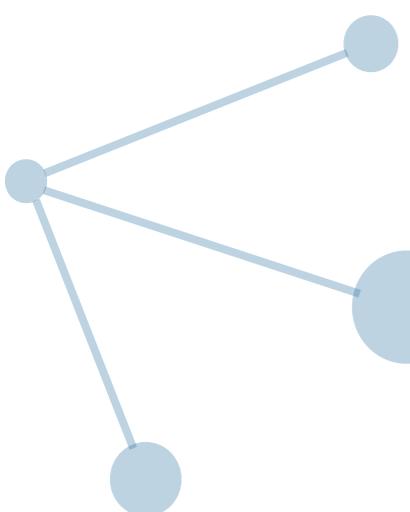
Reducing Values

```
trait Monoid[A] {
  def append(x: A, y: A): A

  def zero: A
}

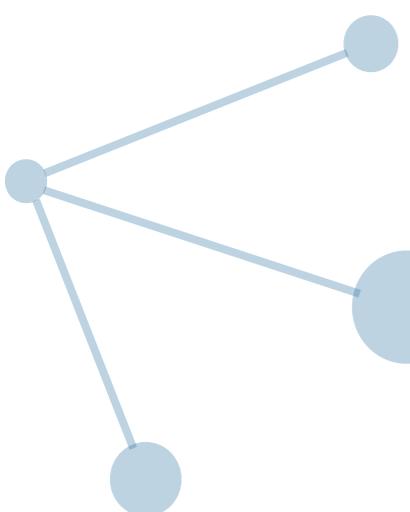
trait Foldable[F[_]] {
  def foldMap[A, B : Monoid](fa: F[A])(f: A => B): B

  def fold[A : Monoid](fa: F[A]): A =
    foldMap(fa)(identity) // identity: x => x
}
```



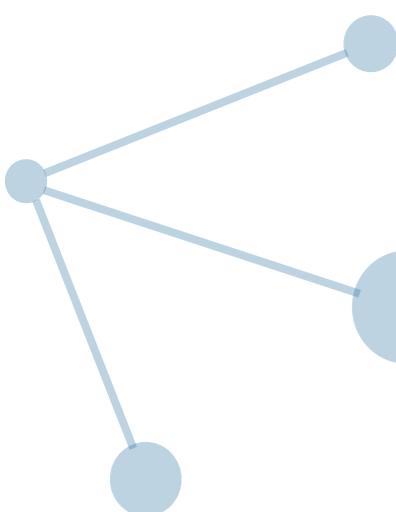
Reducing Values

| Set | Op | Identity |
|-------------|---------------|------------------|
| Numbers | + | 0 |
| Numbers | x | 1 |
| Collections | ++ | Empty collection |
| Boolean | && | TRUE |
| Boolean | | FALSE |
| A => A | ◦ | identity |
| (A, B) | position-wise | position-wise |



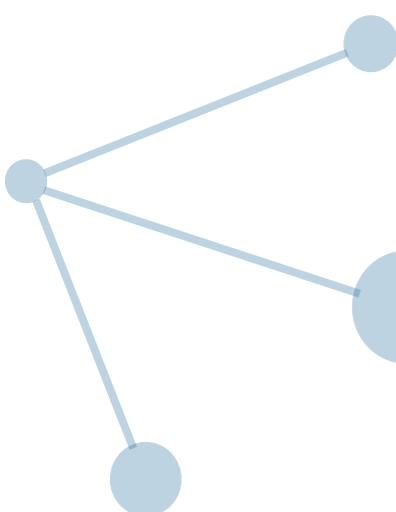
```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Reducing Values



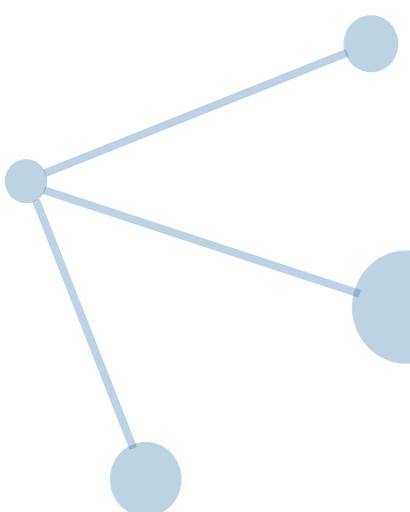
Reducing Values

```
trait Foldable[F[_]] {
    def foldMap[A, B : Monoid](fa: F[A])(f: A => B): B
    def forall[A](fa: F[A])(p: A => Boolean): Boolean =
        foldMap(fa)(a => Conjunction(p(a)))
    def exists[A](fa: F[A])(p: A => Boolean): Boolean =
        foldMap(fa)(a => Disjunction(p(a)))
    def toList[A](fa: F[A]): List[A] =
        foldMap(fa)(a => List(a))
    def length[A](fa: F[A]): Int =
        foldMap(fa)(const(1))
}
```



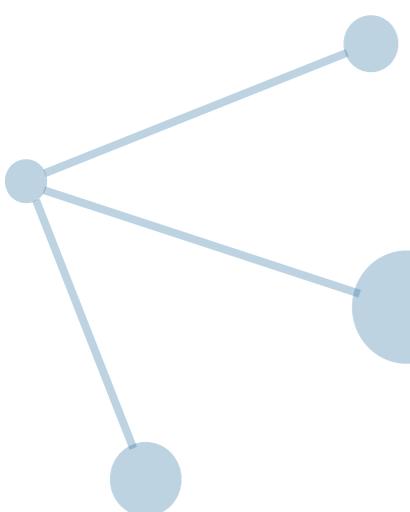
Reducing Values

```
trait Foldable[F[_]] {
    def foldMap[A, B : Monoid](fa: F[A])(f: A => B): B
    def forall[A](fa: F[A])(p: A => Boolean): Boolean =
        foldMap(fa)(a => Conjunction(p(a)))
    def exists[A](fa: F[A])(p: A => Boolean): Boolean =
        foldMap(fa)(a => Disjunction(p(a)))
    def toList[A](fa: F[A]): List[A] =
        foldMap(fa)(a => List(a))
    def length[A](fa: F[A]): Int =
        foldMap(fa)(const(1))
}
def const[A, B](a: A)(b: B): A = a
```



```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Reducing Values

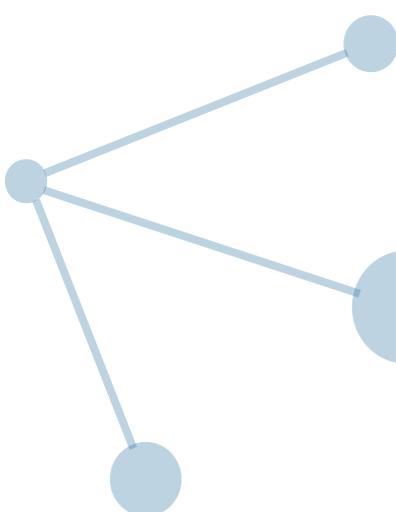


Reducing Values

```
def sum(list: List[Int]): Int = list.foldMap(identity)

def length(list: List[Int]): Int = list.foldMap(const(1))

def average(list: List[Int]): Int =
  sum(list) / length(list)
```



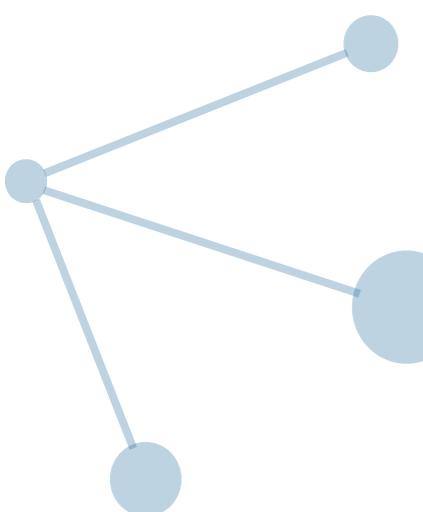
Reducing Values

```
def sum(list: List[Int]): Int = list.foldMap(identity)

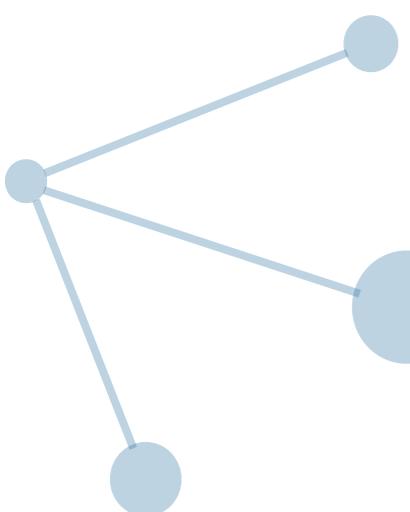
def length(list: List[Int]): Int = list.foldMap(const(1))

def average(list: List[Int]): Int =
  sum(list) / length(list)
```

```
def average(list: List[Int]): Int = {
  val (s, l) = list.foldMap(i => (i, 1))
  s / l
}
```

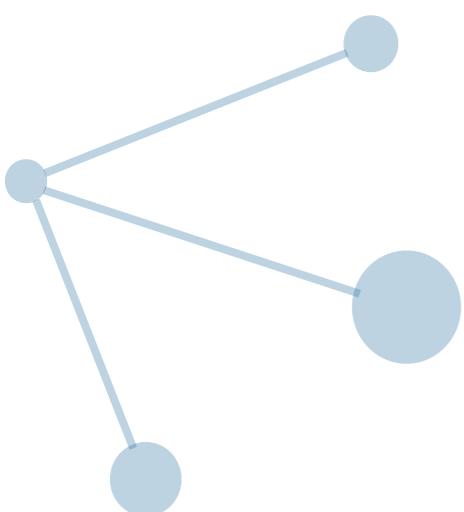


Tracking Effects



Tracking Effects

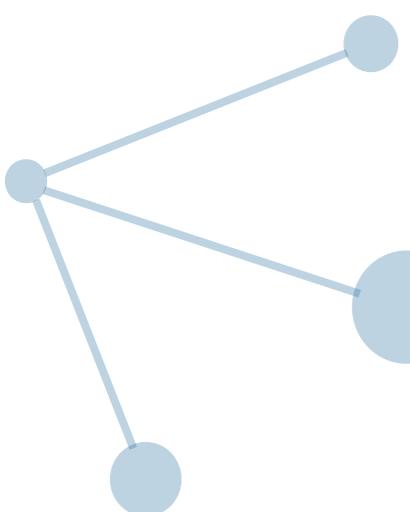
- Interesting programs will have effects
 - Optional values, error handling, asynchronous computation, input/output
 - Usual means aren't quite nice



Tracking Effects

- Interesting programs will have effects
 - Optional values, error handling, asynchronous computation, input/output
 - Usual means aren't quite nice

```
def lookup(map: Map[Foo, Bar], foo: Foo): Bar =  
  if (map.contains(foo)) map(foo) else null  
  
val bar = lookup(someMap, someFoo)  
if (bar != null) {  
  . . .  
} else {  
  . . .  
}
```

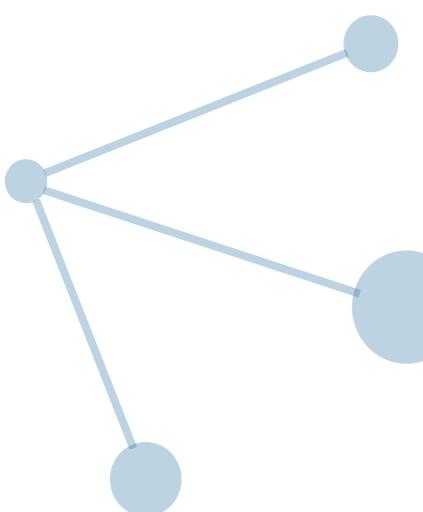


Tracking Effects

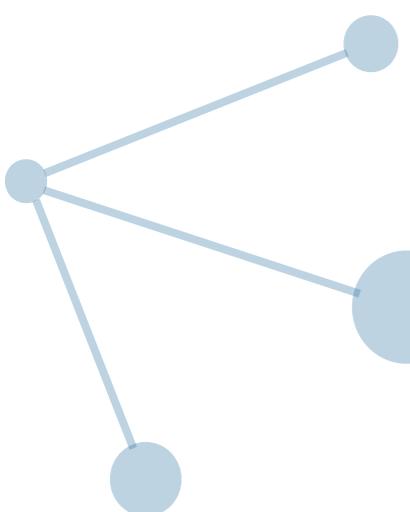
- Interesting programs will have effects
 - Optional values, error handling, asynchronous computation, input/output
 - Usual means aren't quite nice

```
def lookup(map: Map[Foo, Bar], foo: Foo): Bar =  
  map(foo)
```

```
try {  
  val bar = lookup(map, someFoo)  
} catch {  
  case e: NoSuchElementException => . . .  
}
```

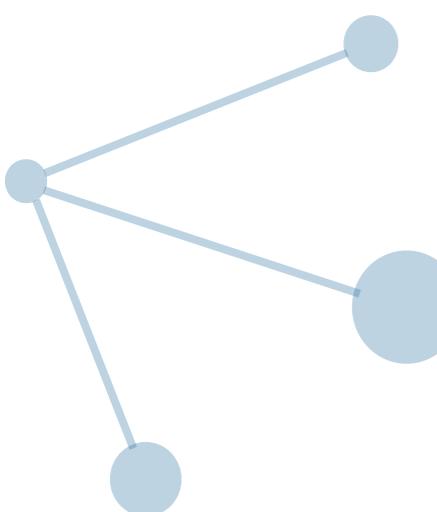


Tracking Effects



Tracking Effects

- Reify effects as values



Tracking Effects

- Reify effects as values

// A value that might exist is either..

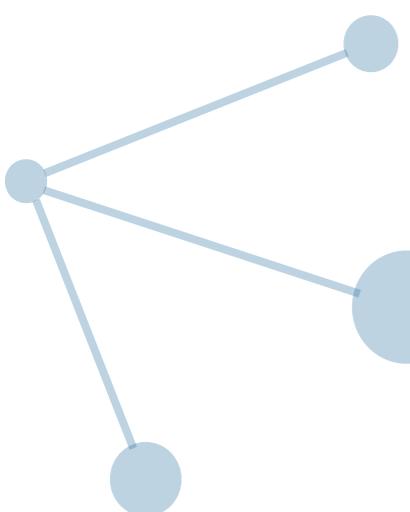
```
sealed abstract class Option[A]
```

// ..there

```
final case class Some[A](a: A) extends Option[A]
```

// .. or not there

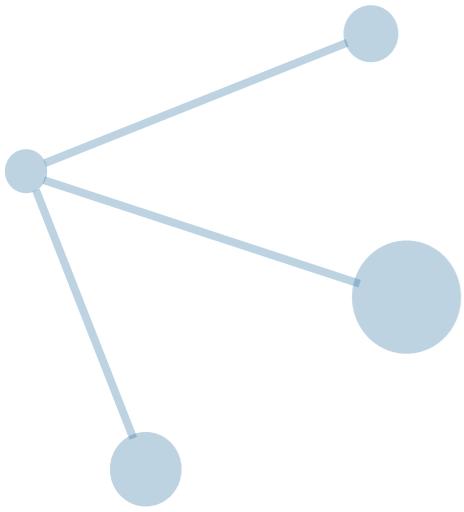
```
final case class None[A]() extends Option[A]
```



Tracking Effects

- Reify effects as values

```
def lookup(map: Map[Foo, Bar], foo: Foo): Option[Bar] =  
  if (map.contains(foo)) Some(map(foo))  
  else None
```

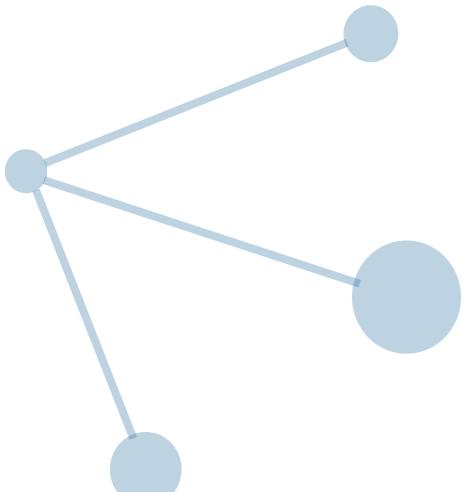


Missing values

```
sealed abstract class Option[A]
```

```
final case class Some[A](a: A) extends Option[A]
```

```
final case class None[A]() extends Option[A]
```

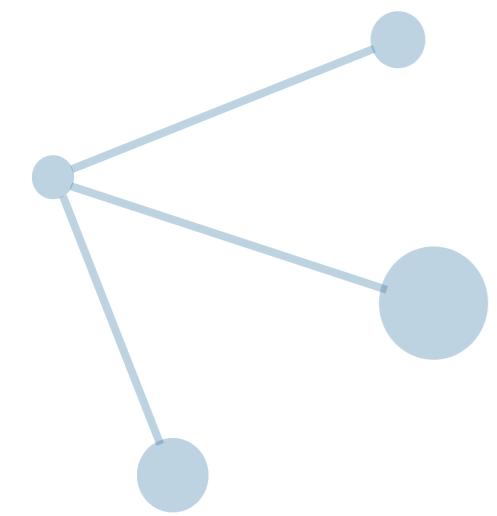


Errors

```
sealed abstract class Either[+E, +A]
```

```
final case class Success[+A](a: A) extends Either[Nothing, A]
```

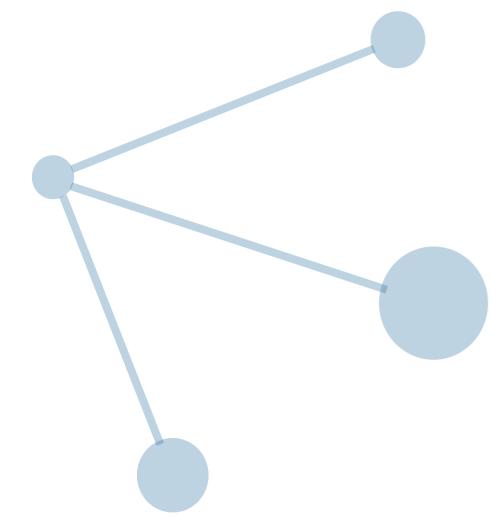
```
final case class Failure[+E](e: E) extends Either[E, Nothing]
```



Ex. Errors

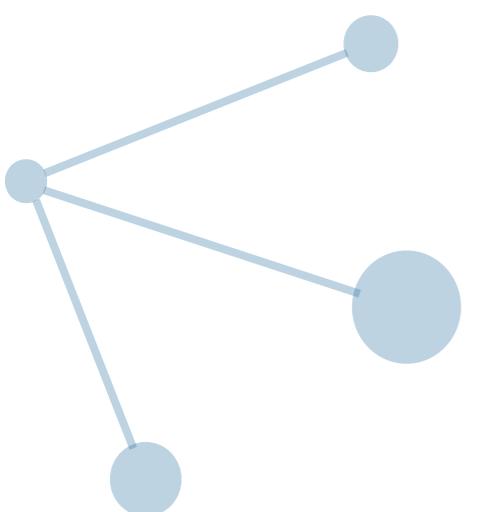
```
sealed abstract class Error
final case object UserDoesNotExist extends Error
final case object InvalidToken      extends Error

def userInfo(uid: UserId,
            tk: Token): Either[Error, UserInfo] = . . .
```



Input/Output

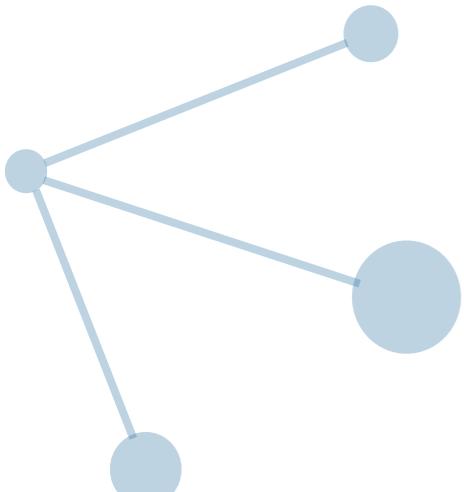
```
trait IO[A] {  
    def unsafePerformIO(): A  
}
```



Input/Output

```
trait IO[A] {  
    def unsafePerformIO(): A  
}
```

- Input/output now becomes a value that can be manipulated and more importantly, tracked



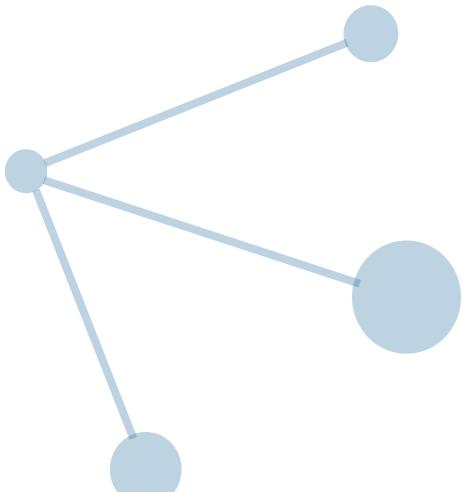
Input/Output

```
trait IO[A] {  
    def unsafePerformIO(): A  
}
```

- Input/output now becomes a value that can be manipulated and more importantly, tracked

```
val action: IO[Data] = ...  
val retryPolicy = ...
```

```
retryPolicy.retry(action) {  
    case HttpError(429) => action  
}
```

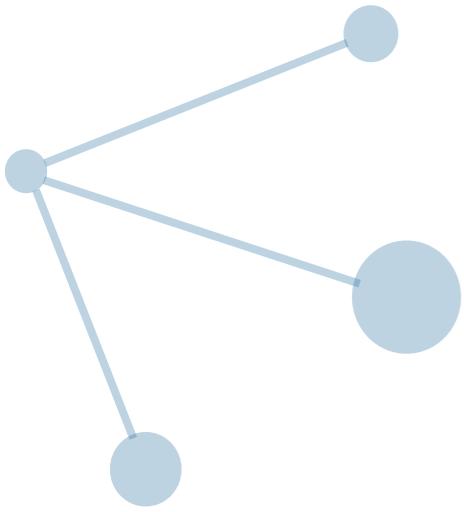


Input/Output

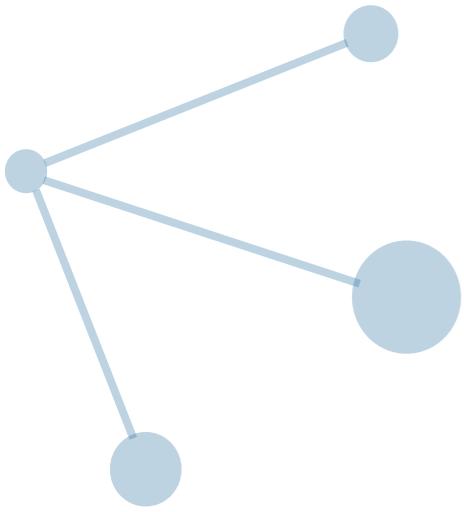
```
trait IO[A] {  
    def unsafePerformIO(): A  
}
```

- Input/output now becomes a value that can be manipulated and more importantly, tracked

```
trait SafeApp {  
    def run(args: List[String]): IO[Unit]  
  
    final def main(args: Array[String]): Unit =  
        run(args.toList).unsafePerformIO()  
}
```

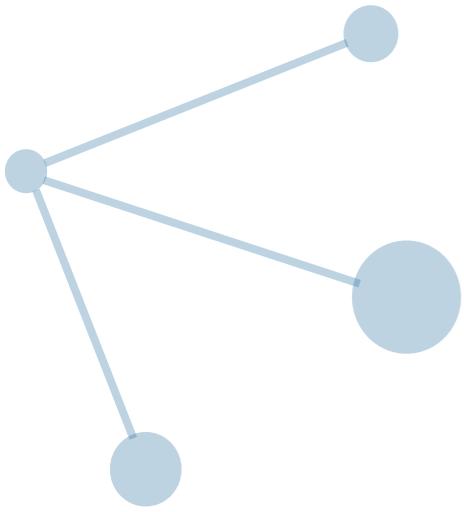


Referential Transparency



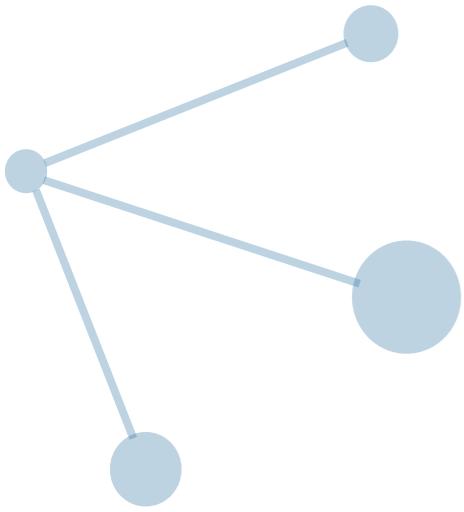
Referential Transparency

An expression **e** is referentially transparent if



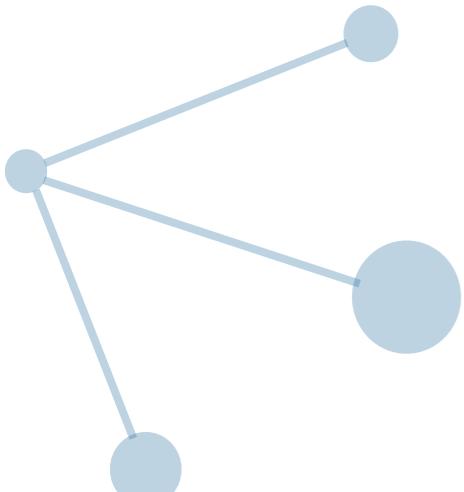
Referential Transparency

An expression **e** is referentially transparent if
for all programs **p**



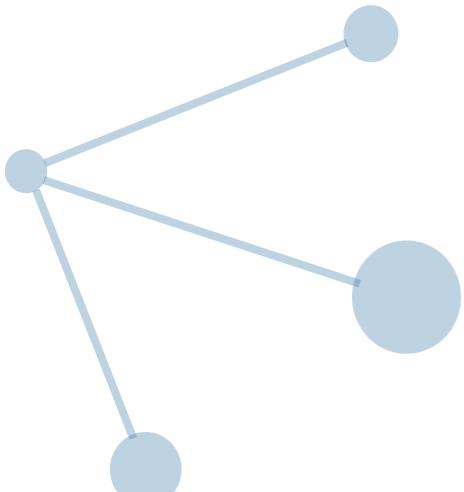
Referential Transparency

An expression **e** is referentially transparent if
for all programs **p**
every occurrence of **e** in **p**



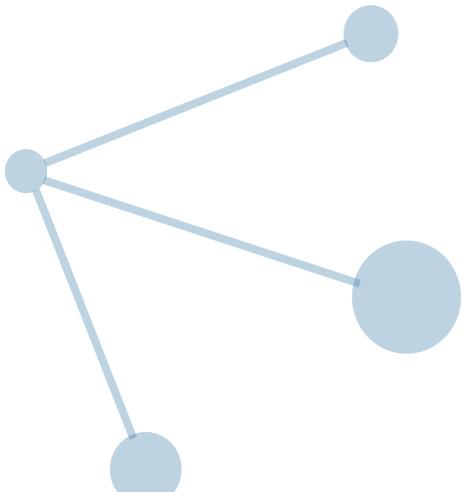
Referential Transparency

An expression **e** is referentially transparent if
for all programs **p**
every occurrence of **e** in **p**
can be replaced with the result of evaluating **e**

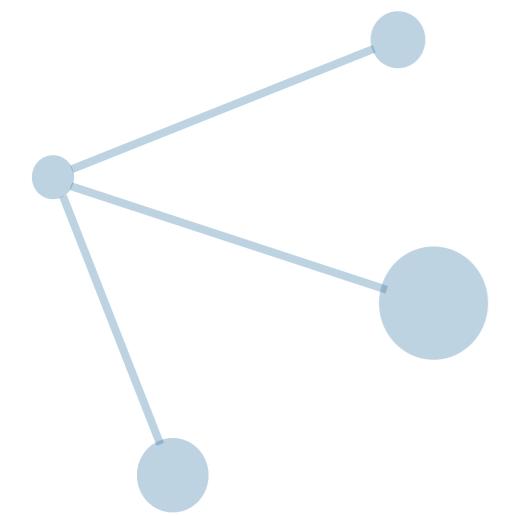


Referential Transparency

An expression **e** is referentially transparent if
for all programs **p**
every occurrence of **e** in **p**
can be replaced with the result of evaluating **e**
without changing the result of evaluating **p**.

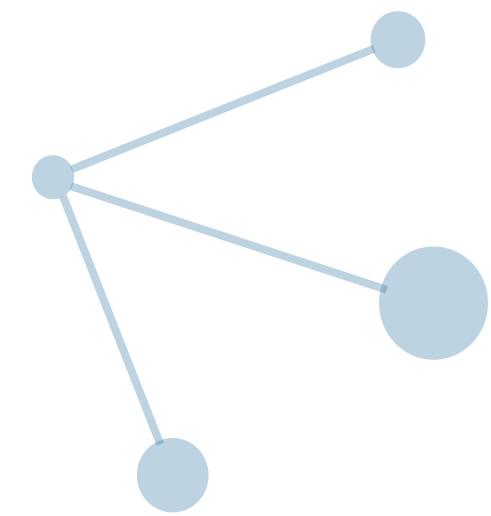


Referential Transparency



Referential Transparency

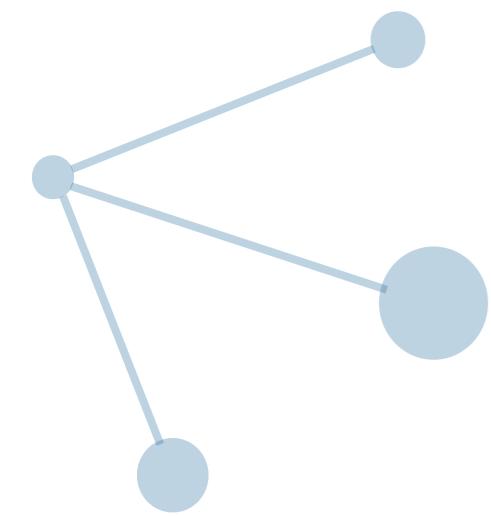
$$x^4 - 12x^2 + 36 = 0$$



Referential Transparency

$$x^4 - 12x^2 + 36 = 0$$

$$y = x^2$$

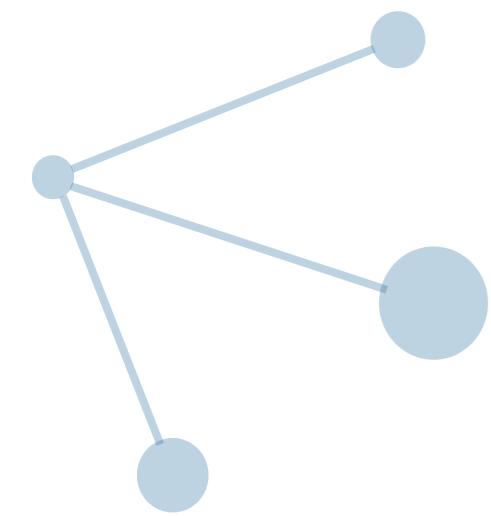


Referential Transparency

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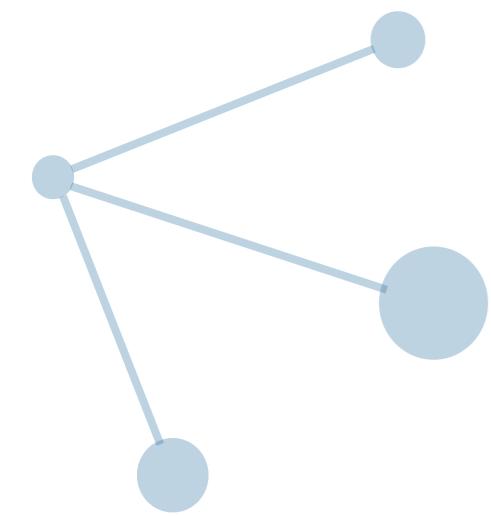
Referential Transparency

$$x^4 - 12x^2 + 36 = 0$$

$$ax^2 + bx + c = 0$$

$$y = x^2$$

$$y^2 - 12y + 36 = 0$$



Referential Transparency

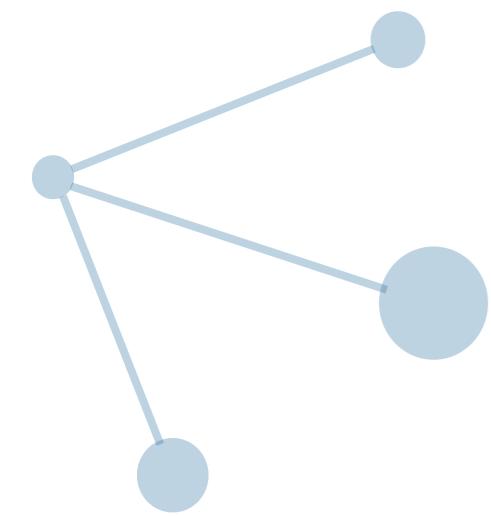
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$$y^2 - 12y + 36 = 0$$

$$ax^2 + bx + c = 0$$

$$(-b \pm \sqrt{b^2 - 4ac}) / 2a$$



Referential Transparency

$$x^4 - 12x^2 + 36 = 0$$

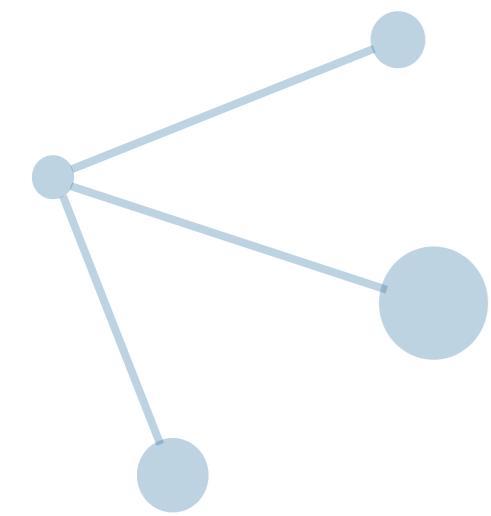
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Referential Transparency

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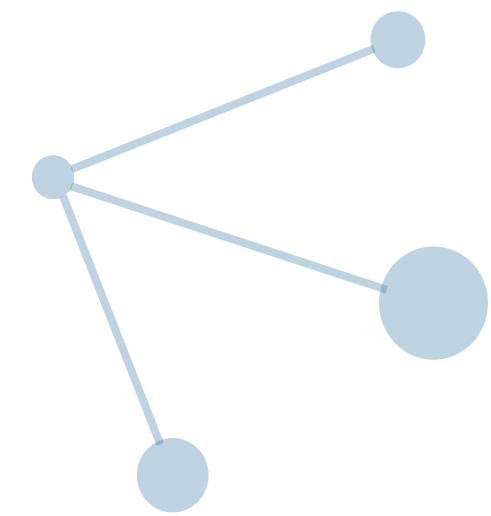
$$y^2 - 12y + 36 = 0$$

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$$a = 1$$

$$b = -12$$



Referential Transparency

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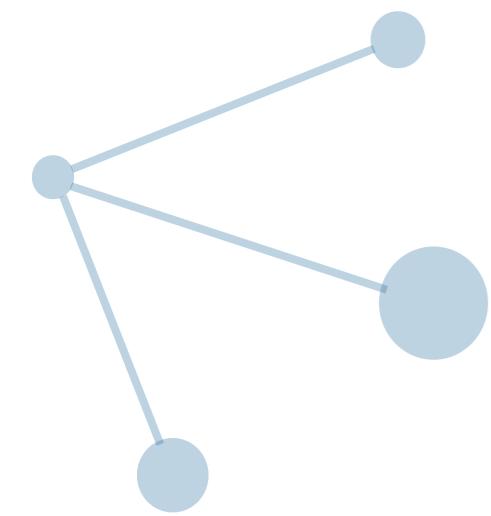
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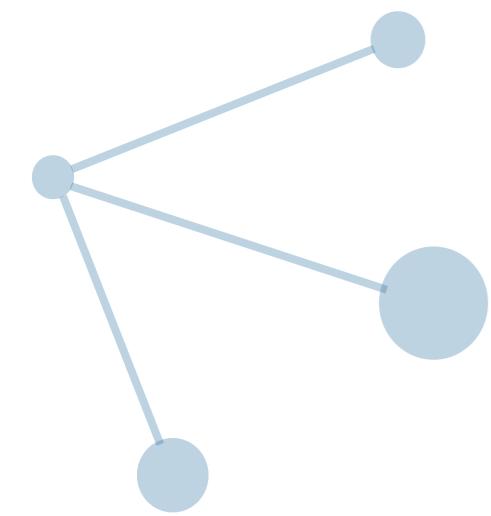
$$(-b \pm \sqrt{b^2 - 4ac}) / 2a$$

$$a = 1$$

$$b = -12$$

$$c = 36$$

$$(-(-12) \pm \sqrt{(-12)^2 - 4(1)(36)}) / 2(1)$$



Referential Transparency

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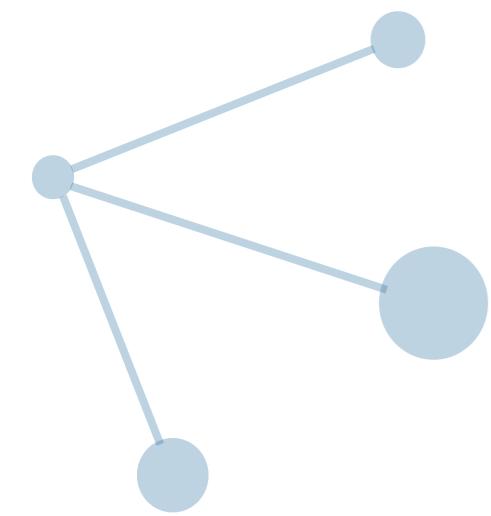
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$$12 / 2$$



Referential Transparency

$$x^4 - 12x^2 + 36 = 0$$

$$y = x^2$$

$$y^2 - 12y + 36 = 0$$

$$y = 6$$

$$ax^2 + bx + c = 0$$

$$(-b \pm \sqrt{b^2 - 4ac}) / 2a$$

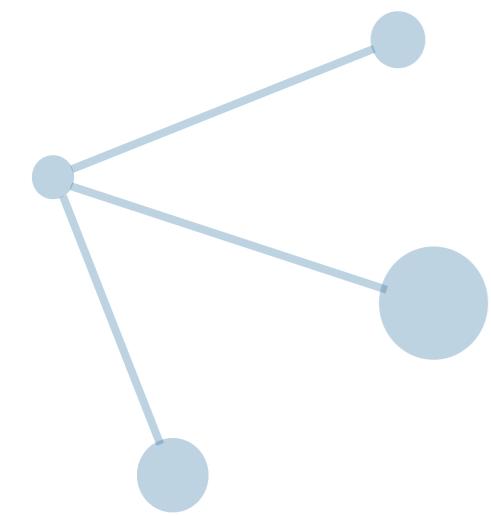
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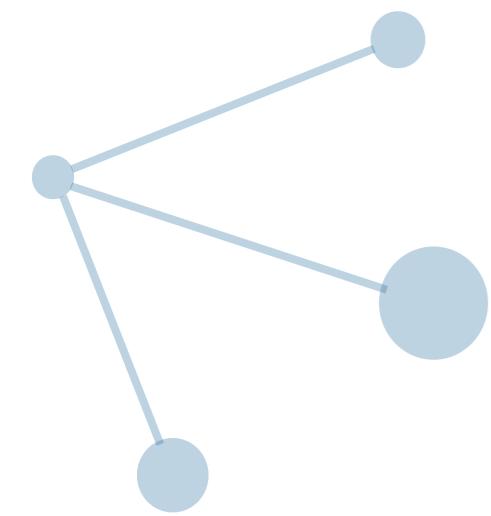
$$a = 1$$

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Referential Transparency

$$x^4 - 12x^2 + 36 = 0$$

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$$y = 6$$

$$x^2 = 6$$

$$x = \pm \sqrt{6}$$

$$ax^2 + bx + c = 0$$

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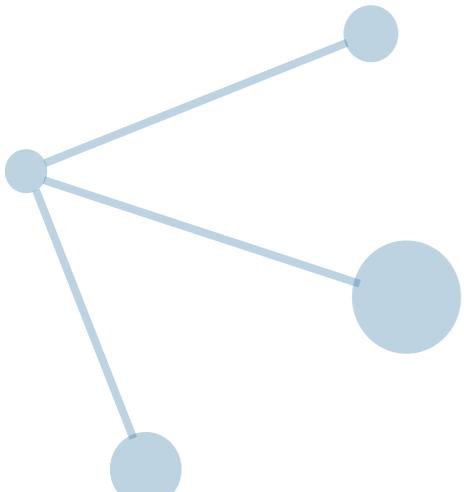
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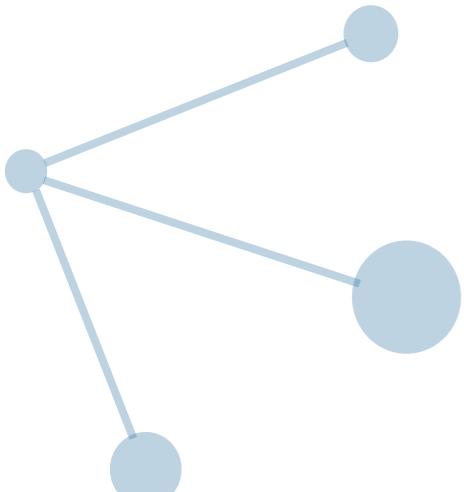
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Referential Transparency

An expression **e** is referentially transparent if
for all programs **p**
every occurrence of **e** in **p**
can be replaced with the result of evaluating **e**
without changing the result of evaluating **p**.



Referential Transparency

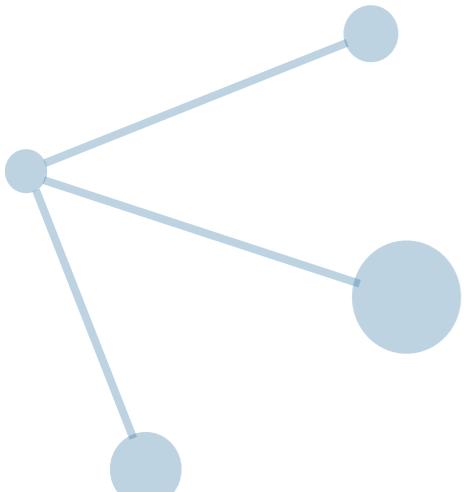
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```
val sb = new StringBuilder("Lambda ")
```

```
val lc = sb.append("Conf")
```

```
val s1 = lc.toString // Lambda Conf
```

```
val s2 = lc.toString // Lambda Conf
```



Referential Transparency

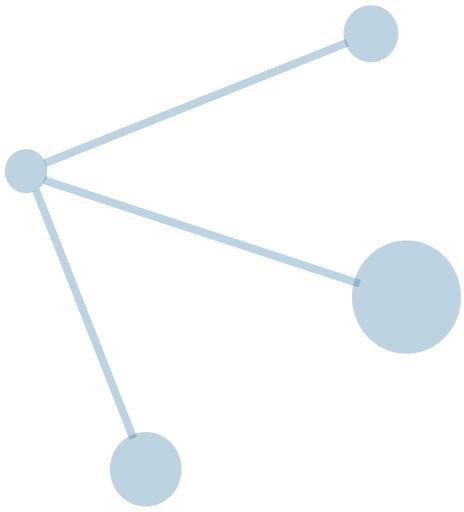
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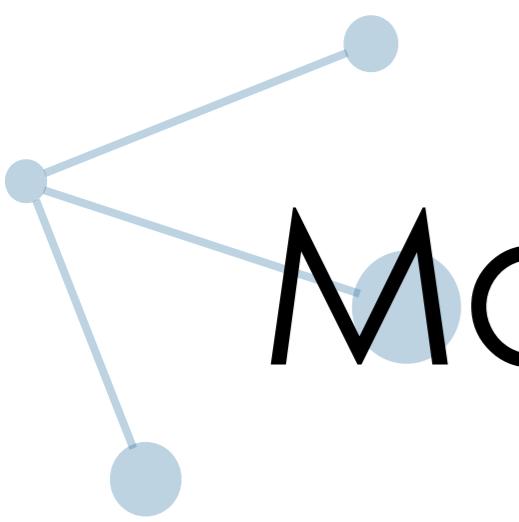
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val sb = new StringBuilder("Lambda ")
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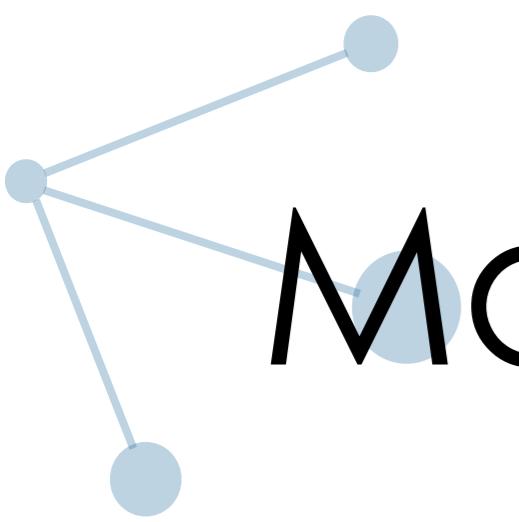
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val s1 = sb.append("Conf").toString // Lambda Conf
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```
val s2 = sb.append("Conf").toString // Lambda ConfConf
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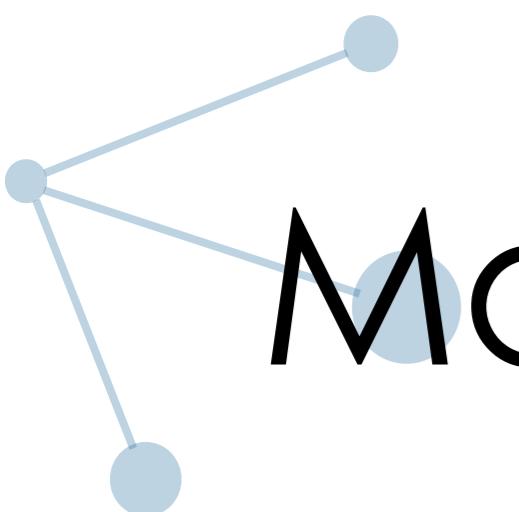
```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Manipulating Effects



Manipulating Effects

- The type signature of our functions reflect the effects involved



Manipulating Effects

- The type signature of our functions reflect the effects involved

```
def tokenFor(uid: UserId): IO[EncryptedToken]
```

```
def decrypt(token: EncryptedToken): Token
```

```
/** Client side */
val encrypted: IO[EncryptedToken] =
  tokenFor(. . .)
```

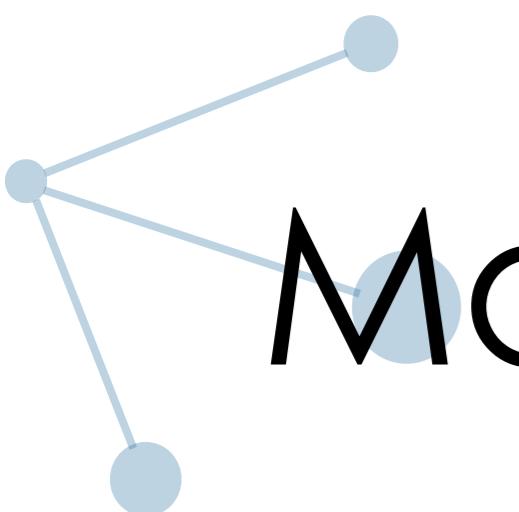
```
// Want EncryptedToken, have IO[EncryptedToken]
val decrypted = decrypt(???)
```



Manipulating Effects

```
/* Apply a pure function to an effectful value */
trait Functor[F[_]] {
    def map[A, B](fa: F[A])(f: A => B): F[B]
}

new Functor[IO] {
    def map[A, B](fa: IO[A])(f: A => B): IO[B] =
        new IO[B] {
            def unsafePerformIO(): B =
                f(fa.unsafePerformIO())
        }
}
```



```
graph LR; A(( )) --- B(( )); A --- C(( )); A --- D(( ));
```

Manipulating Effects



Manipulating Effects

```
def tokenFor(uid: UserId): IO[EncryptedToken]
```

```
def decrypt(token: EncryptedToken): Token
```

```
/** Client side */
val encrypted: IO[EncryptedToken] =
  tokenFor(...)
```

```
// Want EncryptedToken, have IO[EncryptedToken]
val decrypted: IO[Token] =
  encrypted.map(et => decrypt(et))
```



Manipulating Effects

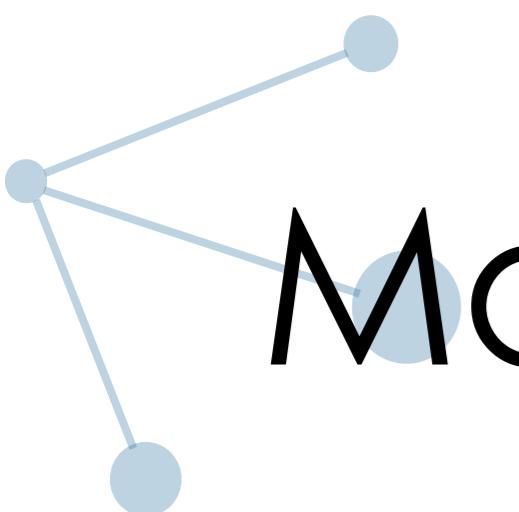
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def decrypt(token: EncryptedToken): Token
```

```
/** Client side */
val encrypted: IO[EncryptedToken] =
  tokenFor(...)
```

```
// Want EncryptedToken, have IO[EncryptedToken]
val decrypted: IO[Token] =
  encrypted.map(et => decrypt(et))
```

- Similar mechanisms for manipulating multiple effects



Manipulating Effects

```
/* Apply a pure binary function to 2 effectful values */
trait Applicative[F[_]] extends Functor[F] {
  def map2[A, B, C](fa: F[A], fb: F[B])(f: (A, B) => C): F[C]

  def pure[A](a: A): F[A]

  def map[A, B](fa: F[A])(f: A => B): F[B] =
    map2(fa, point(())) { case (a, _) => f(a) }
}
```



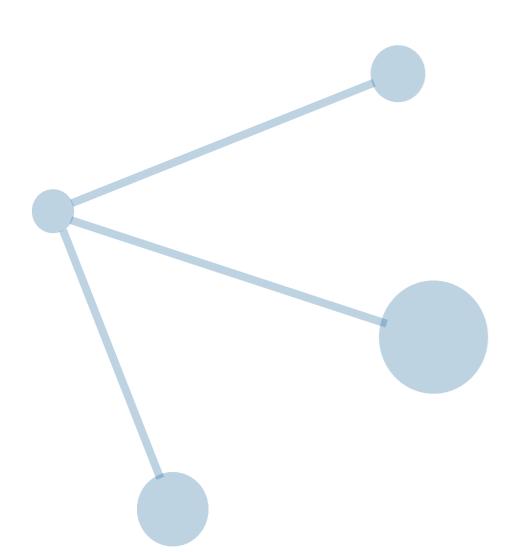
Manipulating Effects

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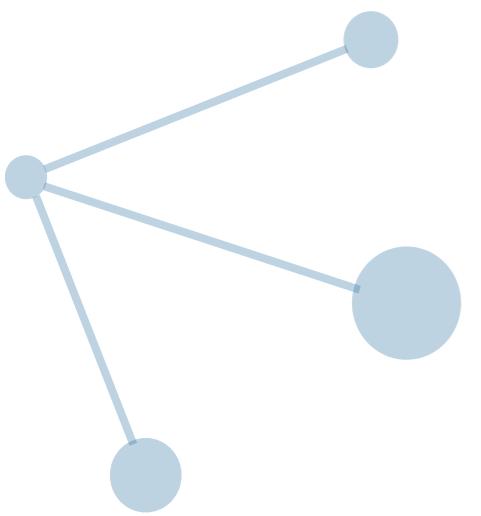
  def pure[A](a: A): F[A]
}

new Applicative[IO] {
  def map2[A, B, C](fa: IO[A], fb: IO[B])
    (f: (A, B) => C): IO[C] =
    new IO[C] { def unsafePerformIO(): C = f(fa.un..., fb.un...) }

  def pure[A](a: A): IO[A] =
    new IO[A] { def unsafePerformIO(): A = a }
}
```

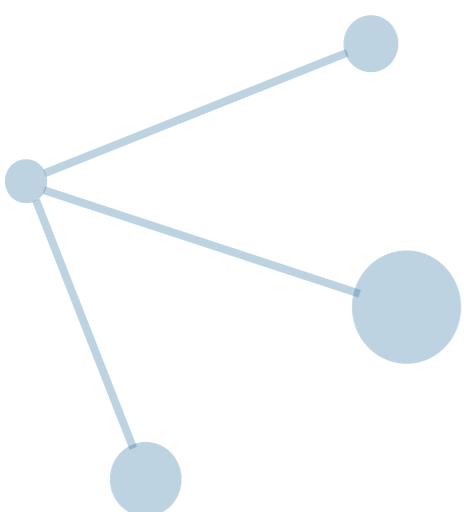


Lens



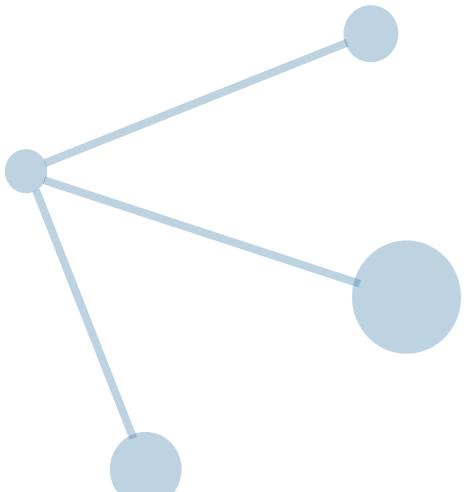
Lens

- Often want getters/setters when working with data



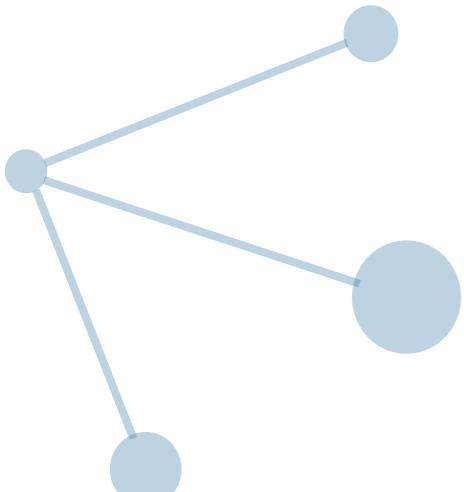
Lens

- Often want getters/setters when working with data
- If getters/setters are per-object, cannot compose



Lens

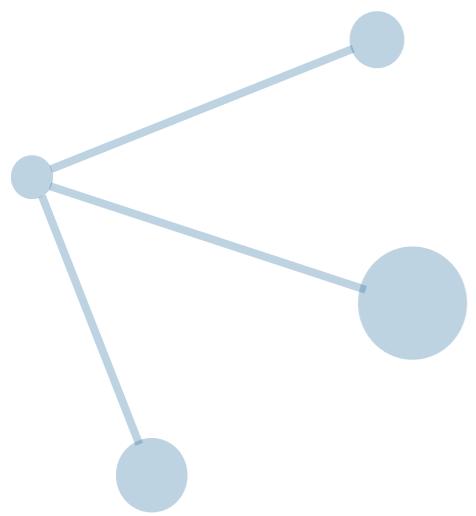
- Often want getters/setters when working with data
- If getters/setters are per-object, cannot compose
- As with effects, reify as data
 - Getter: get an **A** field in an object **S**
 - Setter: change an **A** field in an object **S**



Lens

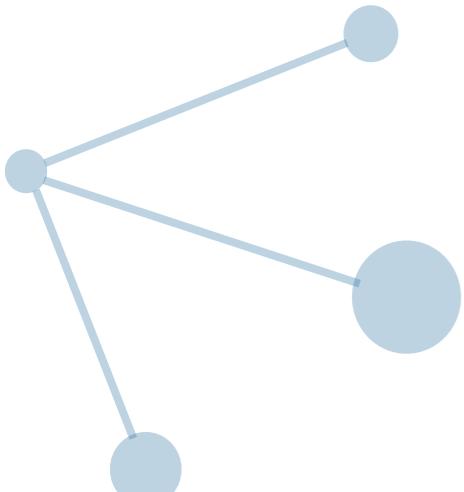
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- As with effects, reify as data
 - Getter: get an **A** field in an object **S**
 - Setter: change an **A** field in an object **S**

```
trait Lens[S, A] {  
    def get(s: S): A  
  
    def set(s: S, a: A): S  
}
```



Lens

```
trait Lens[S, A] { outer =>
    def get(s: S): A
    def set(s: S, a: A): S
    def modify(s: S, f: A => A): S =
        set(s, f(get(s)))
    def compose[B](other: Lens[A, B]): Lens[S, B] =
        new Lens[S, B] {
            def get(s: S): B = other.get(get(s))
            def set(s: S, b: B): S =
                set(s, other.set(outer.get(s), b))
        }
}
```



Lens

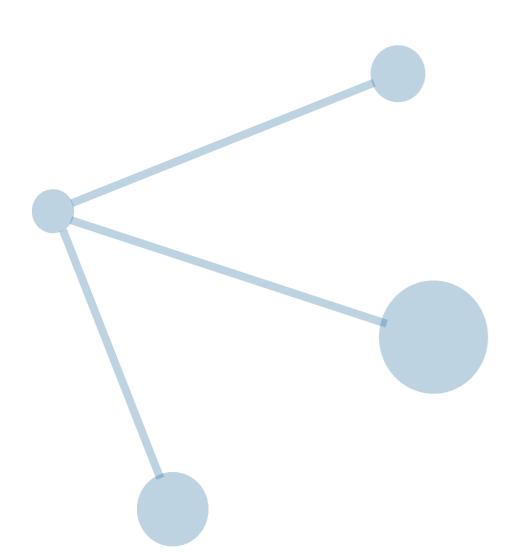
```
case class Employee(position: Position)
```

```
object Employee {  
    val position: Lens[Employee, Position] = ...  
}
```

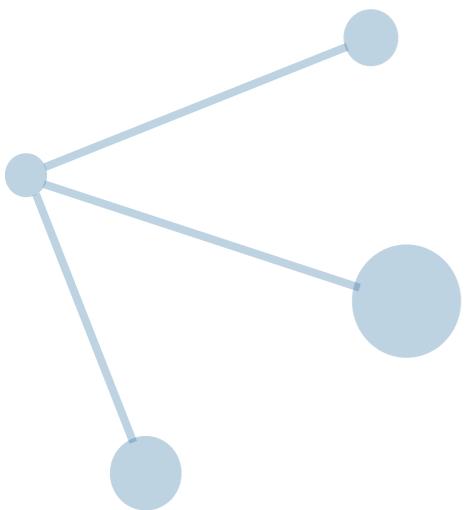
```
case class Team(manager: Employee, ...)
```

```
object Team {  
    val manager: Lens[Team, Employee] = ...  
}
```

```
/** Client side */  
val l: Lens[Team, Position] =  
    Team.manager.compose(Employee.position)  
l.set(someTeam, somePosition)
```

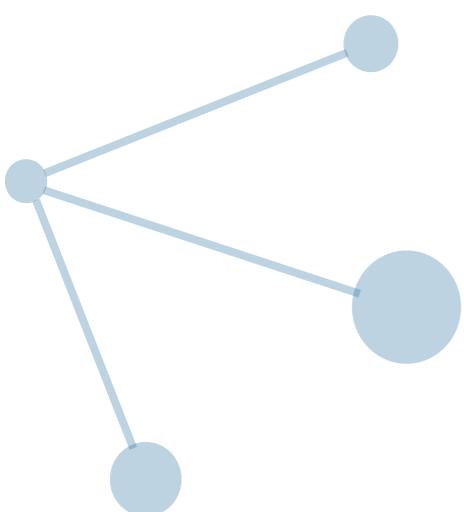


Lens



Lens

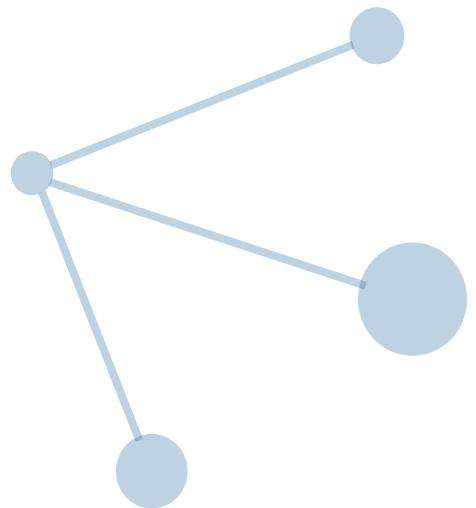
- Effectful modifications can be useful
 - Possibly failing: $A \Rightarrow \text{Option}[A]$
 - Many possible values: $A \Rightarrow \text{List}[A]$
 - IO: $A \Rightarrow \text{IO}[A]$



Lens

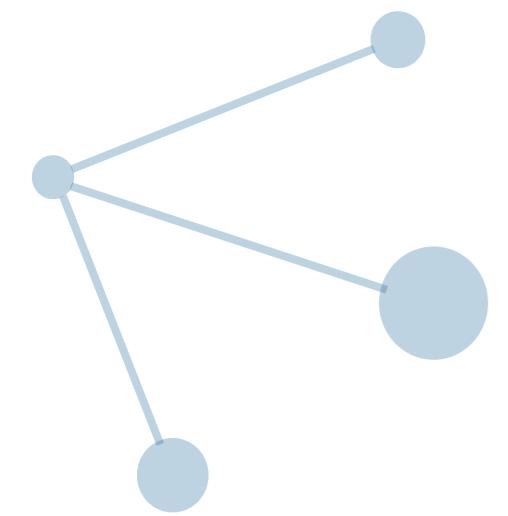
- Effectful modifications can be useful
 - Possibly failing: $A \Rightarrow \text{Option}[A]$
 - Many possible values: $A \Rightarrow \text{List}[A]$
 - IO: $A \Rightarrow \text{IO}[A]$

```
trait Lens[S, A] {  
    def modifyOption(s: S, f: A => Option[A]): Option[S]  
  
    def modifyList(s: S, f: A => List[A]): List[S]  
  
    def modifyIO(s: S, f: A => IO[A]): IO[S]  
}
```



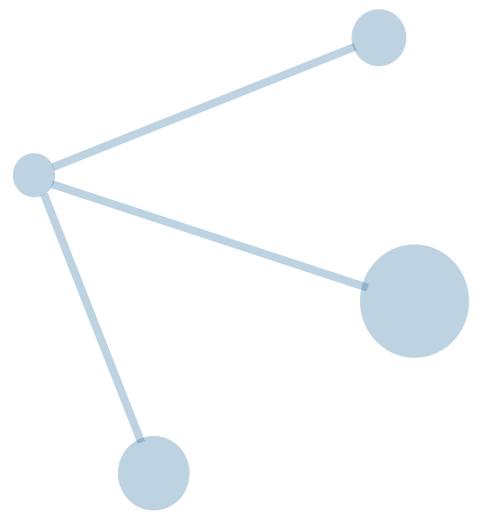
Lens

```
trait Lens[S, A] {  
    def modifyOption(s: S, f: A => Option[A]): Option[S] =  
        f(get(s)).map(a => set(s, a))  
  
    def modifyList(s: S, f: A => List[A]): List[S] =  
        f(get(s)).map(a => set(s, a))  
  
    def modifyIO(s: S, f: A => IO[A]): IO[S] =  
        f(get(s)).map(a => set(s, a))  
}
```



Lens

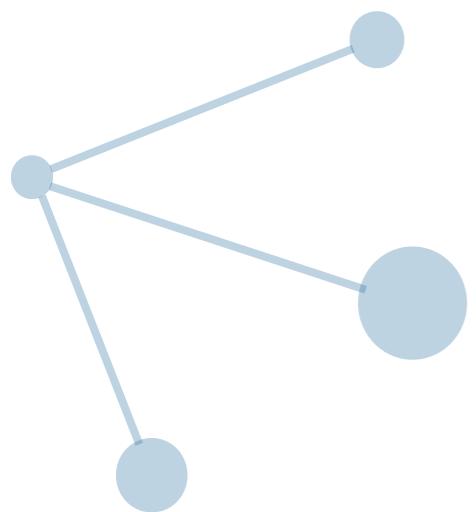
```
trait Lens[S, A] {  
    def modifyF[F[_] : Functor](s: S, f: A => F[A]): F[S] =  
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}
```



Lens

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trait Lens[S, A] {  
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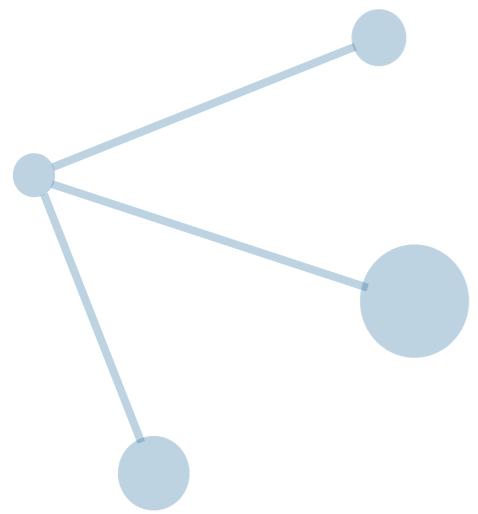
- Example functors:
 - Option, Either, Future, List, IO



Lens

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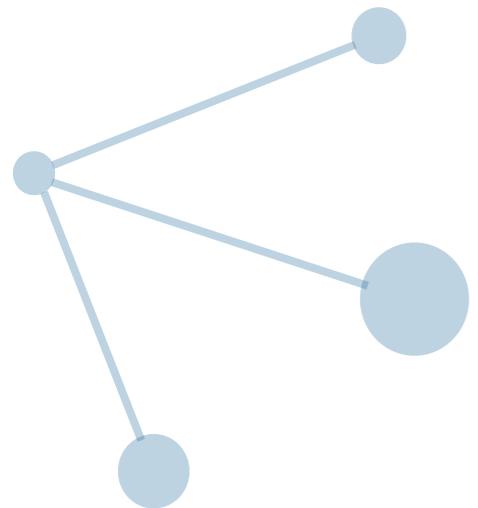
- Example functors:
 - Option, Either, Future, List, IO
- What if we want to just modify without any effect?



Id

```
type Id[A] = A
```

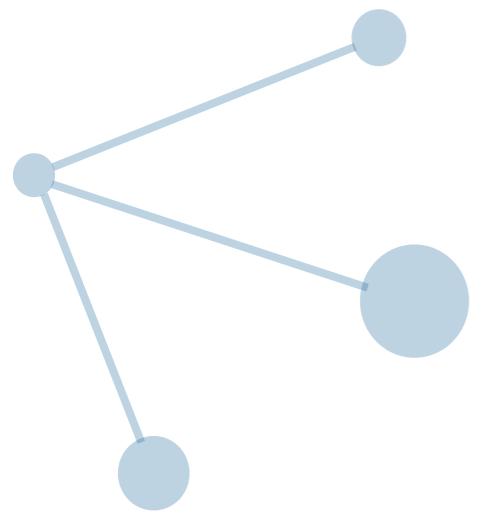
```
new Functor[Id] {  
    def map[A, B](fa: Id[A])(f: A => B): Id[B] =  
}
```



Id

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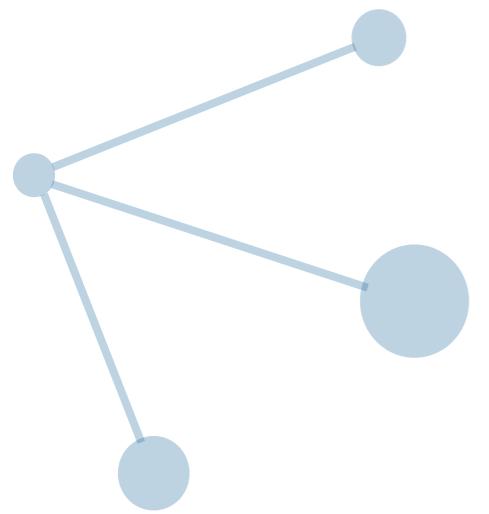
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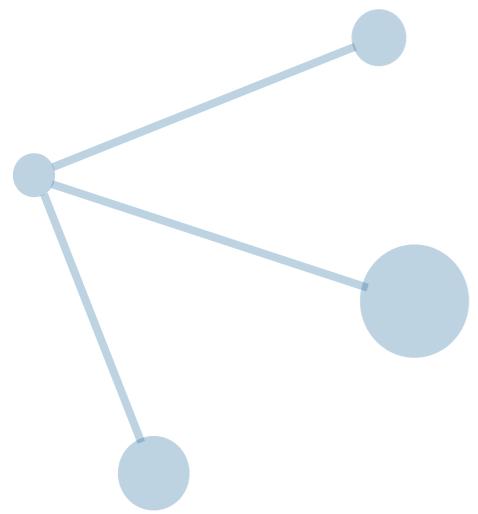
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new Functor[Id] {  
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Id

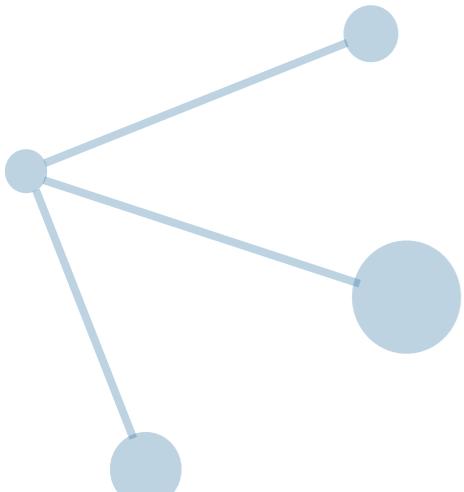
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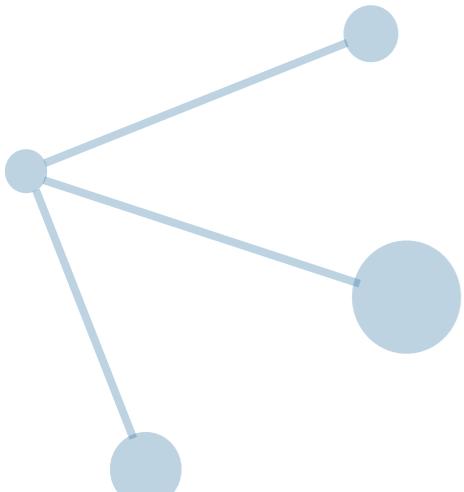
Id

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trait Lens[S, A] {  
    def modifyF[F[_] : Functor](s: S, f: A => F[A]): F[S] =  
        f(get(s)).map(a => set(s, a))  
  
    def modify(s: S, f: A => A): S = modifyF[Id](s, f)  
}
```



Setting

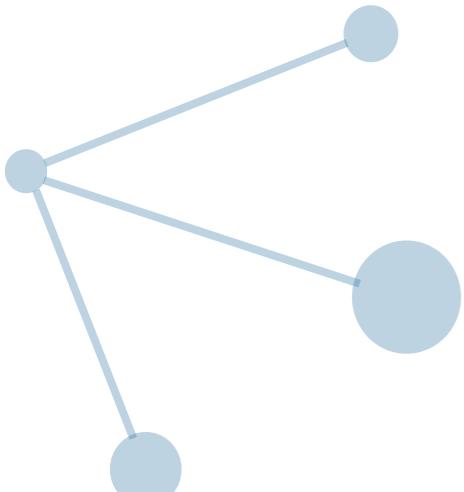
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        f(get(s)).map(a => set(s, a))  
  
    def modify(s: S, f: A => A): S = modifyF[Id](s, f)  
  
    def set(s: S, a: A): S = modify(s, const(a))  
}
```



Setting

```
trait Lens[S, A] {  
    def modifyF[F[_] : Functor](s: S, f: A => F[A]): F[S] =  
        f(get(s)).map(a => set(s, a))  
  
    def modify(s: S, f: A => A): S = modifyF[Id](s, f)  
  
    def set(s: S, a: A): S = modify(s, const(a))  
}
```

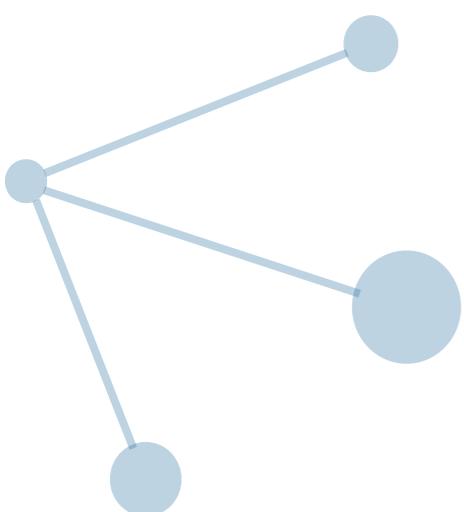
```
def const[A, B](a: A)(b: B): A = a
```



Setting

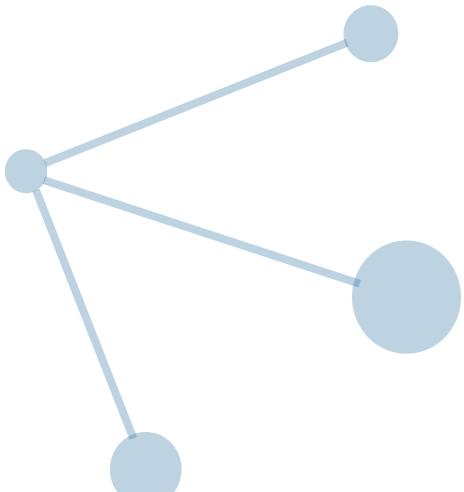
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trait Lens[S, A] {  
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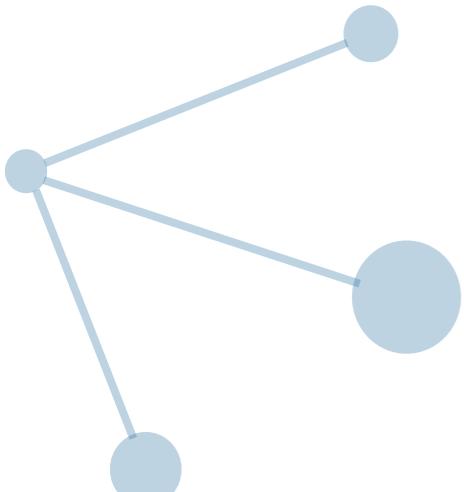
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Getting

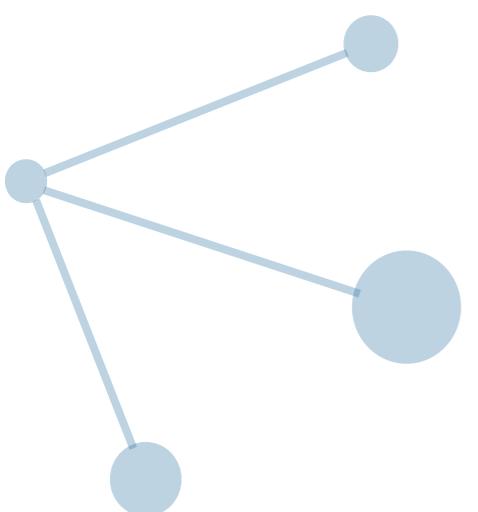
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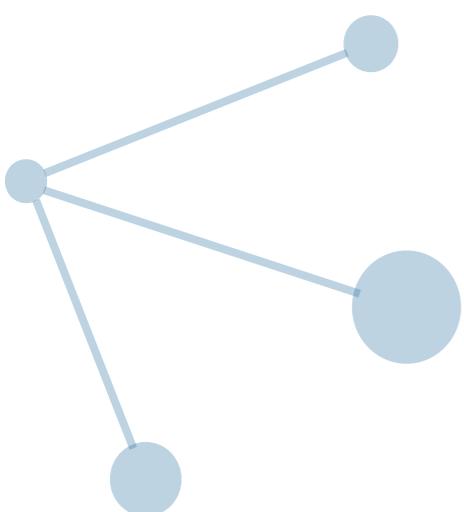
- Can we define `get` in terms of `modify`?



Getting

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trait Lens[S, A] {  
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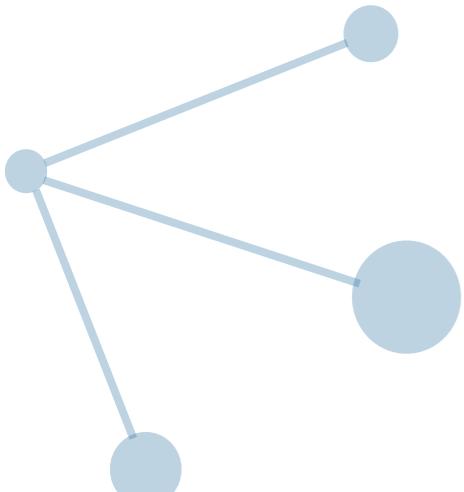
- Can we define `get` in terms of `modify`?
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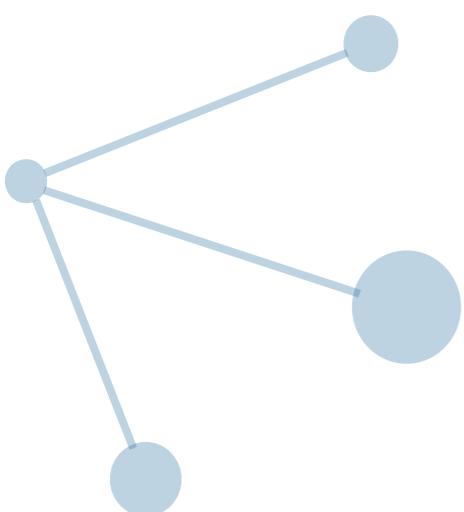
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 - Reify constant function to type-level

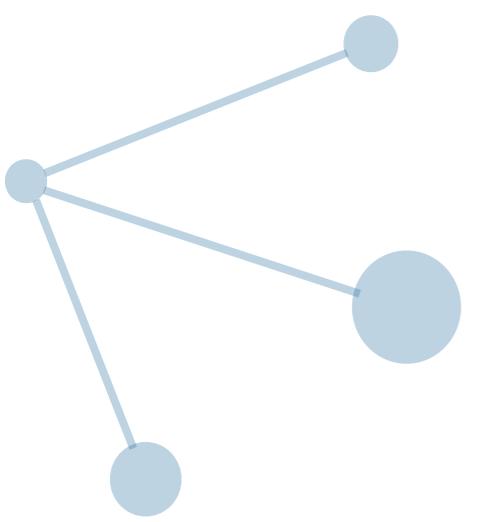


Getting

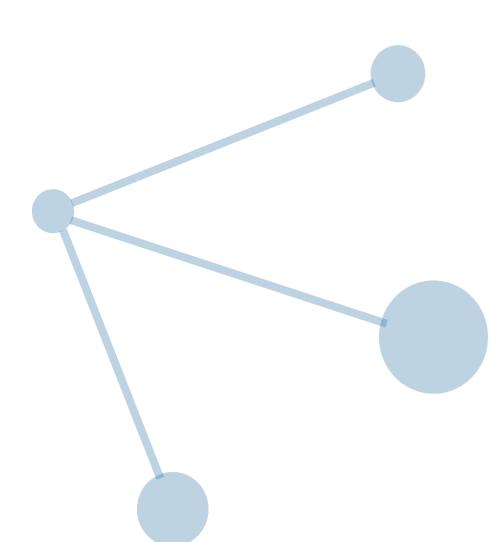
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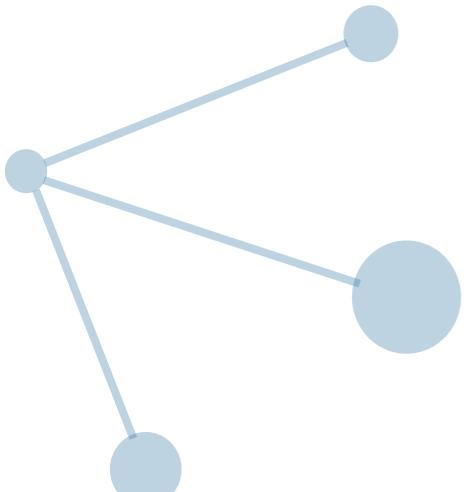


Getting



Getting

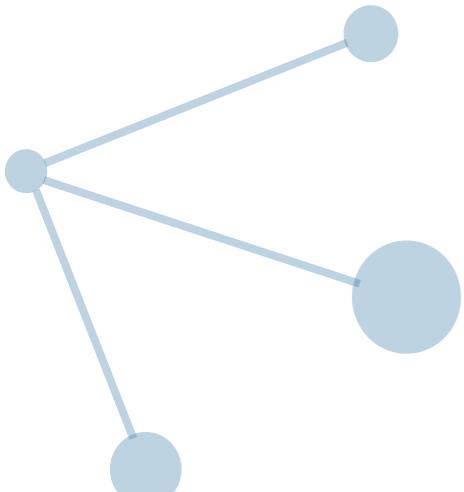
```
case class Const[z, A](get: z)
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Getting

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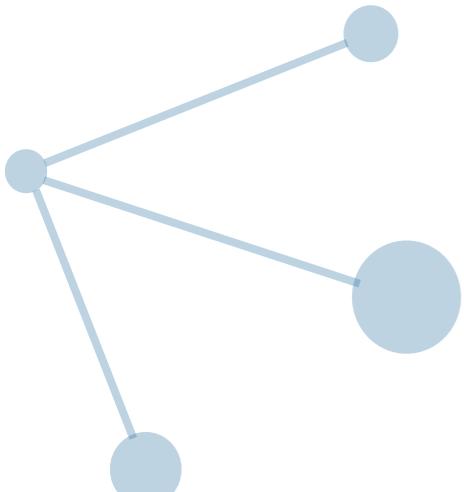
```
new Functor[Const[z, ?]] {
  def map[A, B](fa: Const[z, A])(f: A => B): Const[z, B]
}
```



Getting

```
case class Const[z, A](get: z)
```

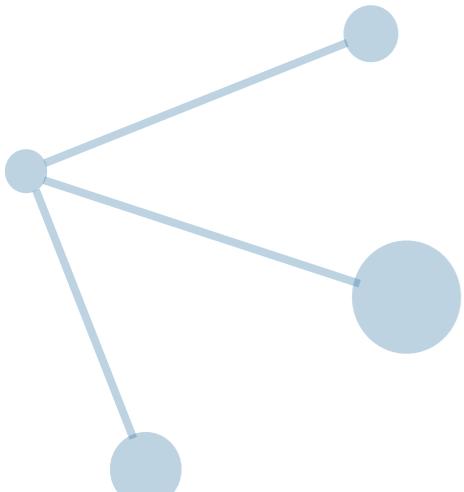
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}
```



Getting

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case class Const[z, A](get: z)
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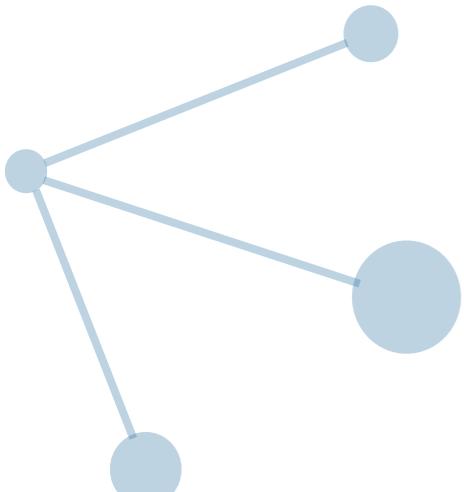
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new Functor[Const[z, ?]] {
  def map[A, B](fa: z)(f: A => B): z =
}
```



Getting

```
case class Const[z, A](get: z)
```

```
new Functor[Const[z, ?]] {
  def map[A, B](fa: z)(f: A => B): z = Const(fa.get)
}
```



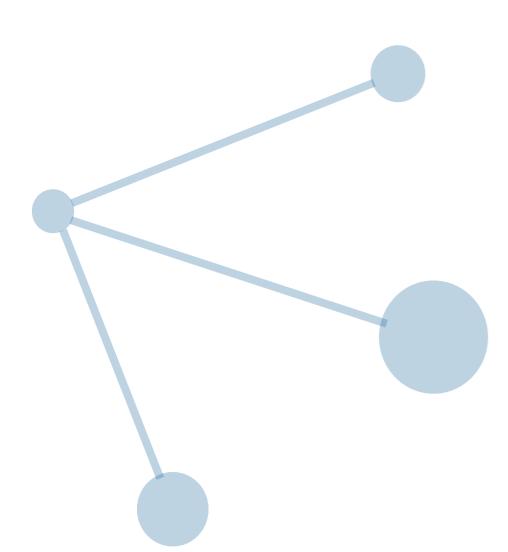
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case class Const[z, A](get: z)
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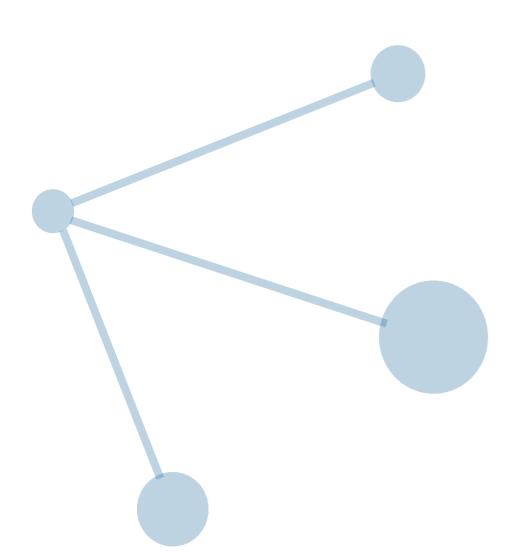
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  def map[A, B](fa: z)(f: A => B): z = Const(fa.get)
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```
trait Lens[S, z] {
  def modifyF[F[_] : Functor](s: S, f: z => F[z]): F[S]

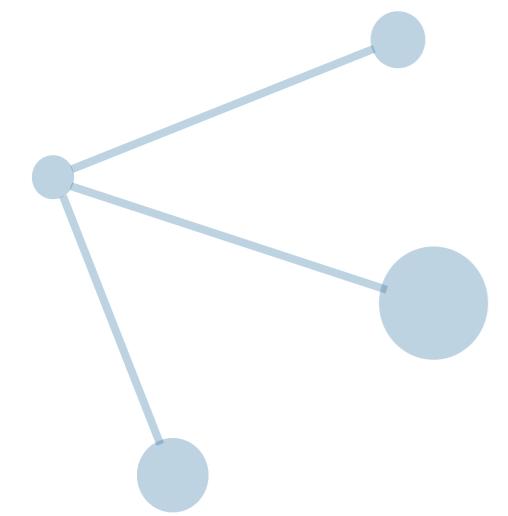
  def get(s: S): z = {
    val const: Const[z, S] =
      modifyF[Const[z, ?]](s, z => Const(z))
    const.get
  }
}
```



Id and Const are party tricks!

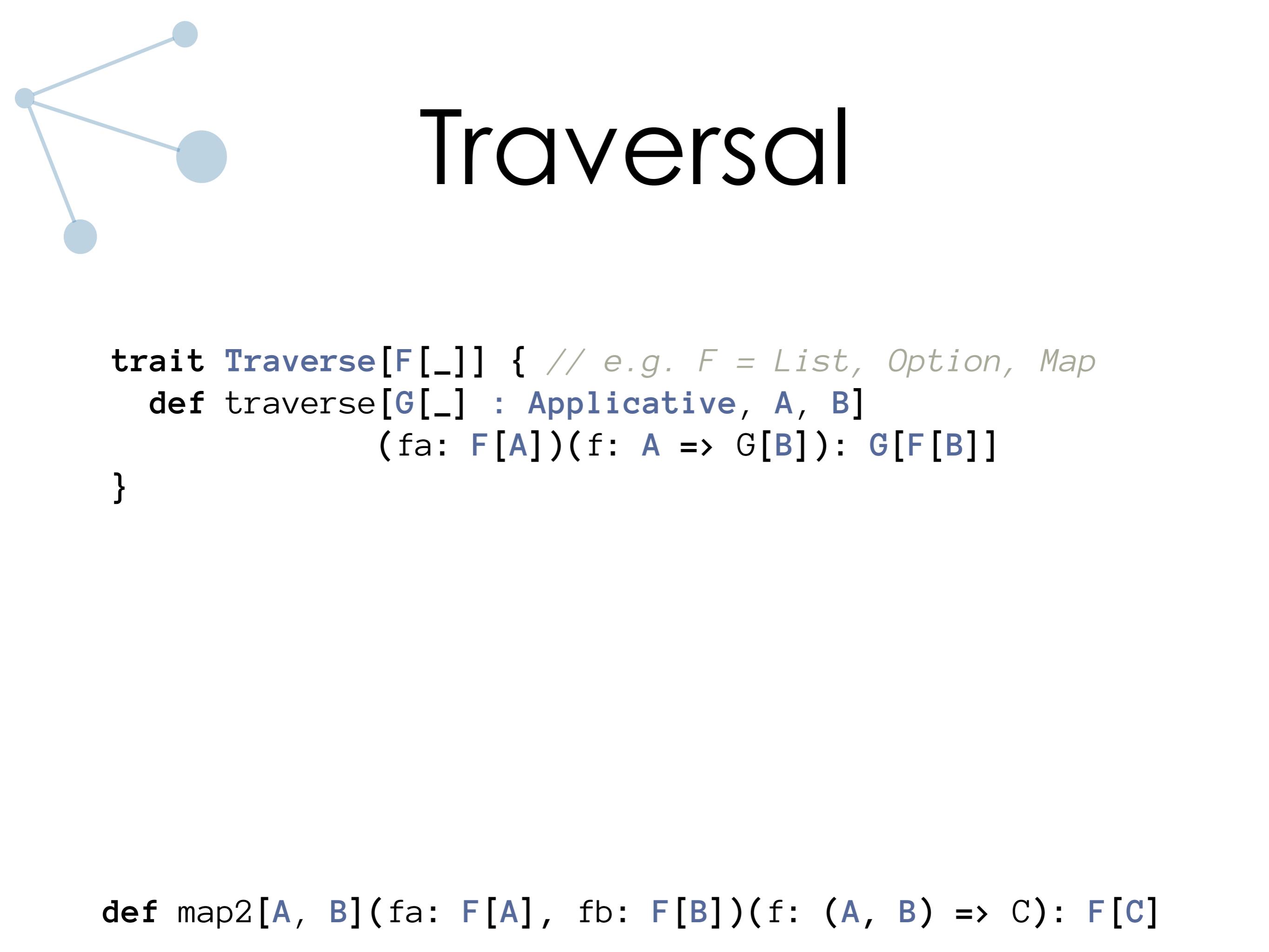


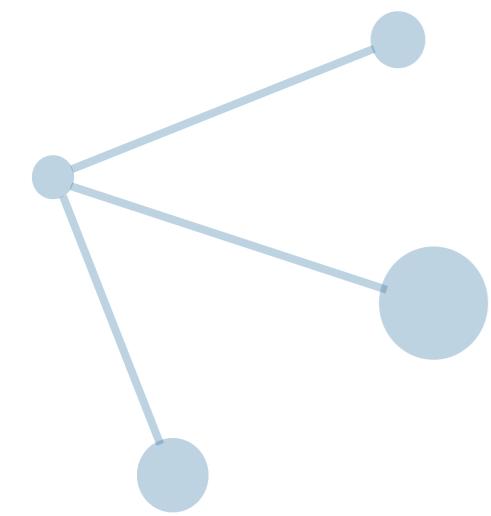
Traversal



Traversal

```
trait Traverse[F[_]] { // e.g. F = List, Option, Map
  def traverse[G[_] : Applicative, A, B]
    (fa: F[A])(f: A => G[B]): G[F[B]]
}
```



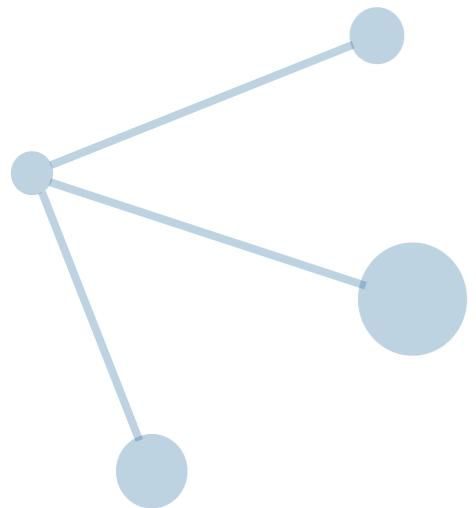


Traversal

```
trait Traverse[F[_]] { // e.g. F = List, Option, Map
    def traverse[G[_] : Applicative, A, B]
        (fa: F[A])(f: A => G[B]): G[F[B]]
}
```

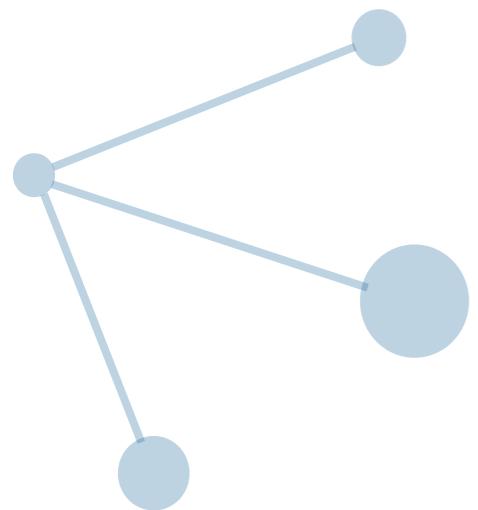
```
def scatterGather[A, B]
    (list: List[A])(f: A => Future[B]): Future[List[B]]
```

```
def map2[A, B](fa: F[A], fb: F[B])(f: (A, B) => C): F[C]
```



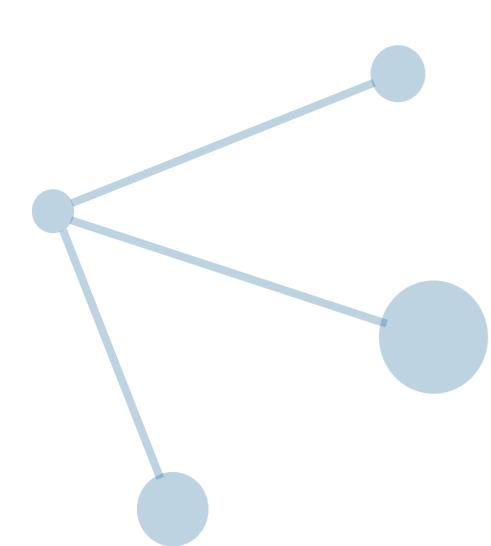
Traversal

```
new Applicative[Id] {  
    def map2[A, B, C](fa: Id[A], fb: Id[B])(f: (A, B) => C): Id[C] =  
        def pure[A](a: => A): Id[A] =  
    }
```



Traversal

```
new Applicative[Id] {  
    def map2[A, B, C](fa: Id[A], fb: Id[B])(f: (A, B) => C): Id[C] =  
        def pure[A](a: => A): Id[A] =  
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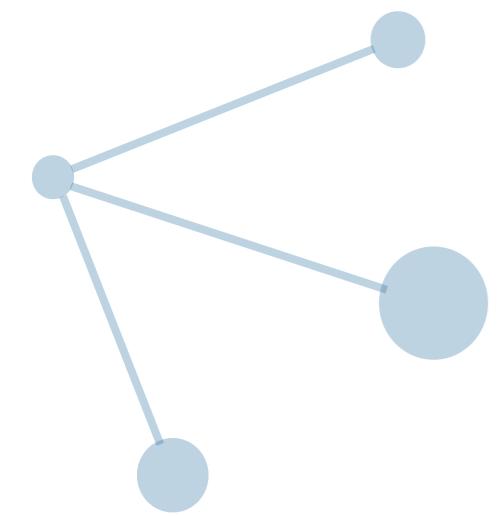
Traversal

```
new Applicative[Id] {  
    def map2[A, B, C](fa: A, fb: B)(f: (A, B) => C): C =  
  
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Traversal

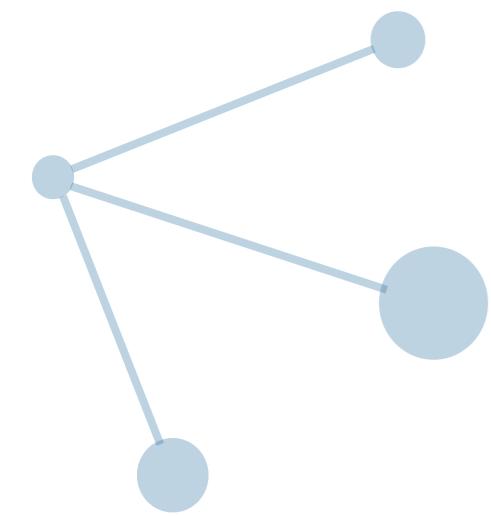
```
new Applicative[Id] {  
    def map2[A, B, C](fa: A, fb: B)(f: (A, B) => C): C = f(fa, fb)  
  
    def pure[A](a: => A): A = a  
}
```



Traversal

```
new Applicative[Const[z, ?]] {
    def map2[A, B, C]
        (fa: Const[z, A], fb: Const[z, B])
        (f: (A, B) => C): Const[z, C] =
            ...

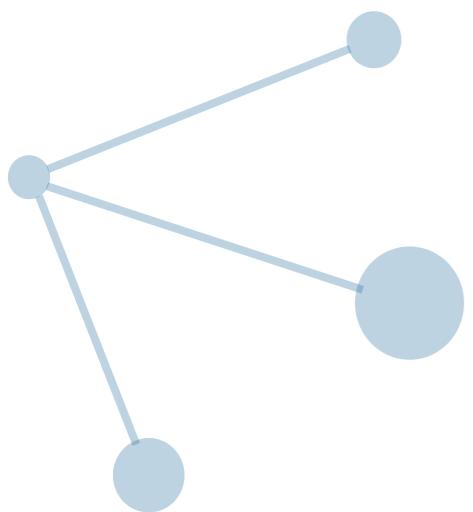
    def pure[A](a: => A): Const[z, A] =
}
```



Traversal

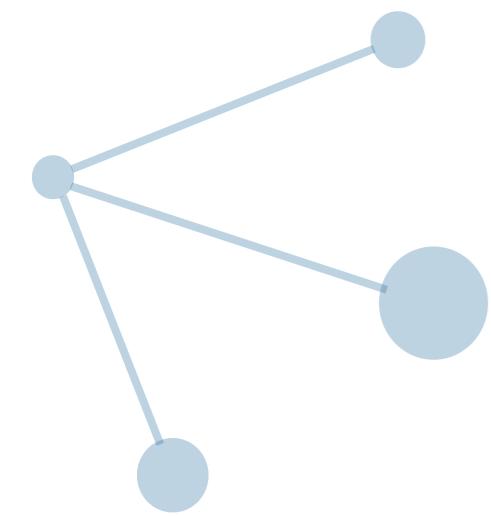
```
new Applicative[Const[z, ?]] {
  def map2[A, B, C]
    (fa: Const[z, A], fb: Const[z, B])
    (f: (A, B) => C): Const[z, C] =
  def pure[A](a: => A): Const[z, A] =
}
```

The code snippet shows the implementation of an `Applicative` trait for `Const`. It includes a `map2` method that takes two `Const` values and a function from their product type to a result type, and returns a `Const` value of the result type. The `pure` method takes a function and returns a `Const` value. Three specific terms in the `map2` definition are highlighted with red boxes: `Const[z, A]`, `Const[z, B]`, and `Const[z, C]`.



Traversal

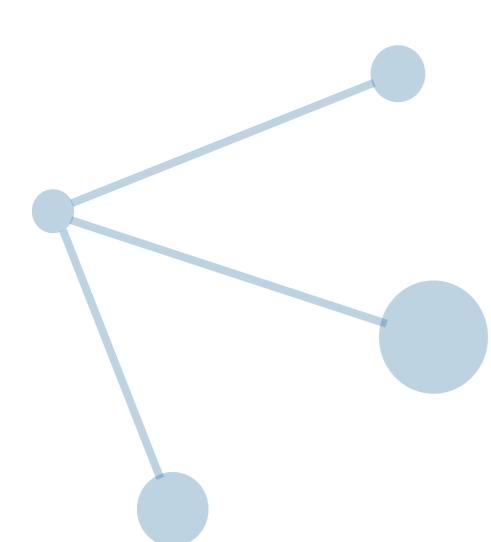
```
new Applicative[Const[z, ?]] {
  def map2[A, B, C]
    (fa: z, fb: z)
    (f: (A, B) => C): z =
  def pure[A](a: => A): z =
}
```



Traversal

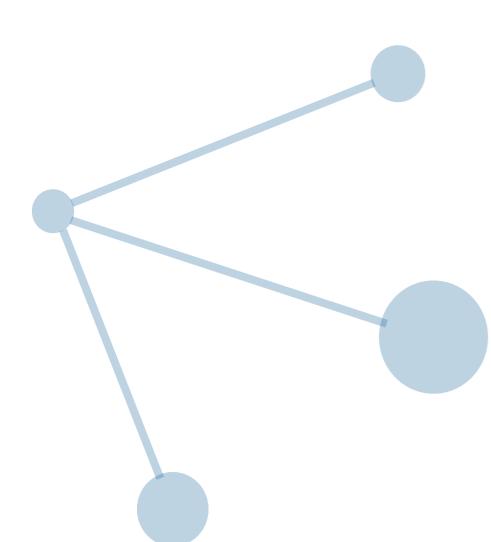
```
new Applicative[Const[z, ?]] { // If Z can form a Monoid. .
  def map2[A, B, C]
    (fa: z, fb: z)
    (f: (A, B) => C): z = fa.append(fb)

  def pure[A](a: => A): z = Monoid[z].zero
}
```



Traversal

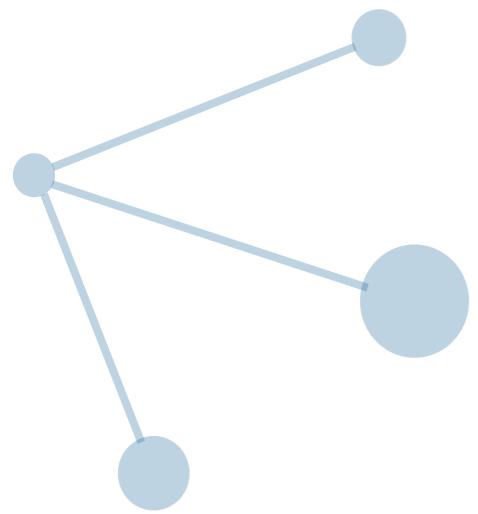
```
trait Traverse[F[_]] { // e.g. F = List, Option, Map
  def traverse[G[_] : Applicative, A, B]
    (fa: F[A])(f: A => G[B]): G[F[B]]
}
```



Traversal

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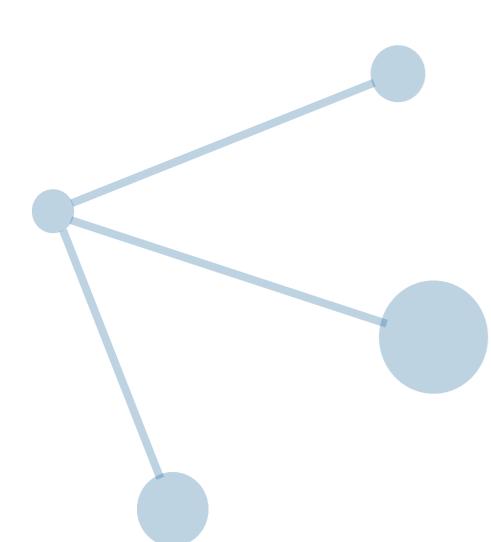
```
traverse[Id, A, B]
```



Traversal

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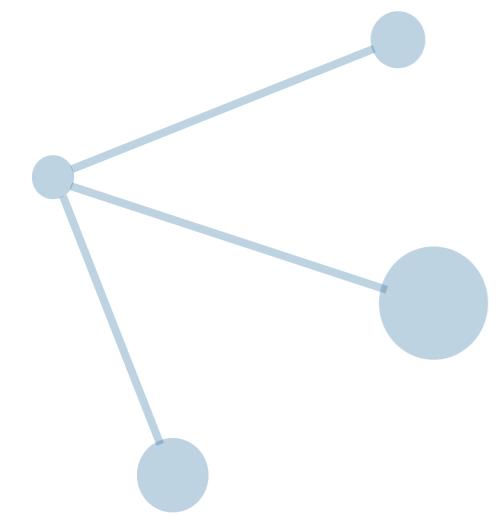
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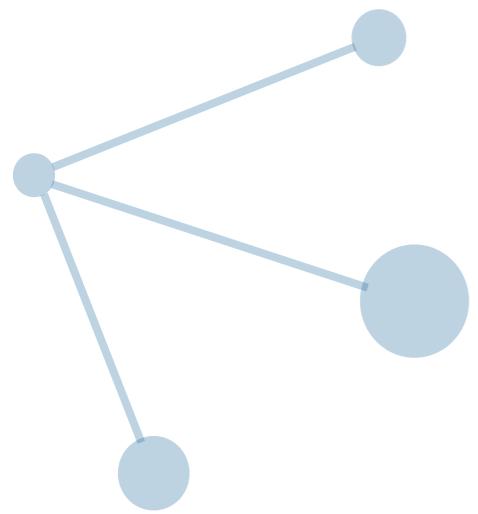
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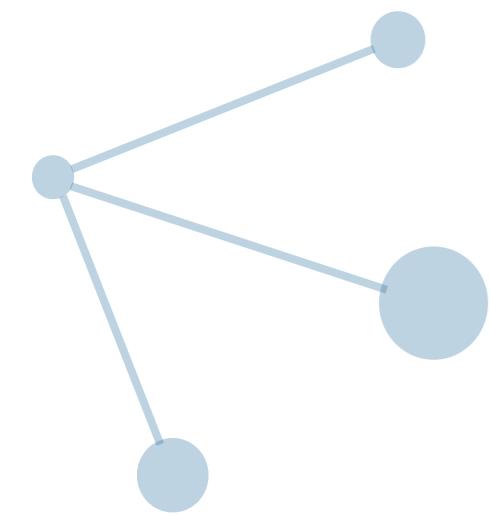


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traverse[Const[Z, ?], A, B] // If Z can form a Monoid
```

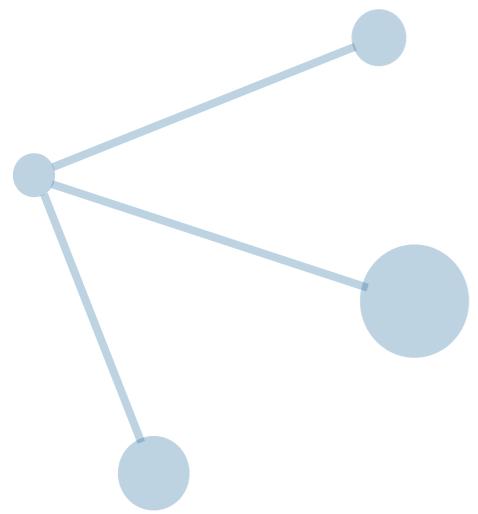


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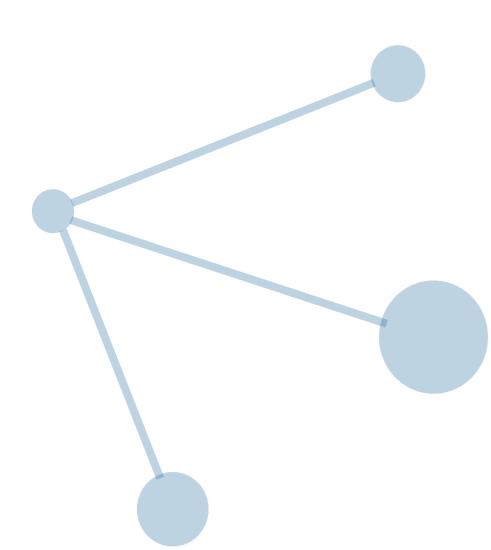


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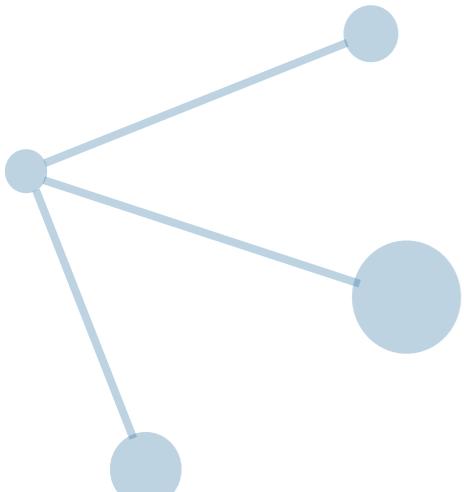
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```

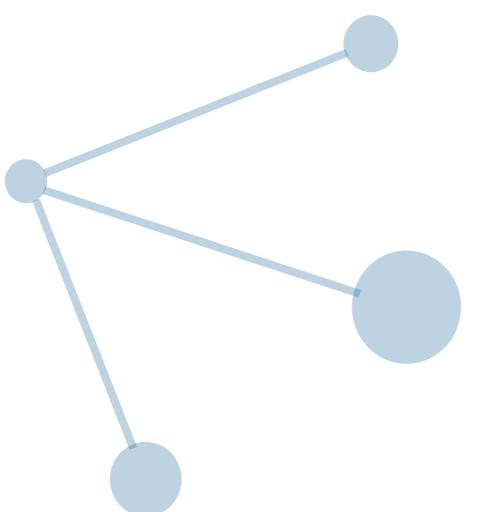


Interested?



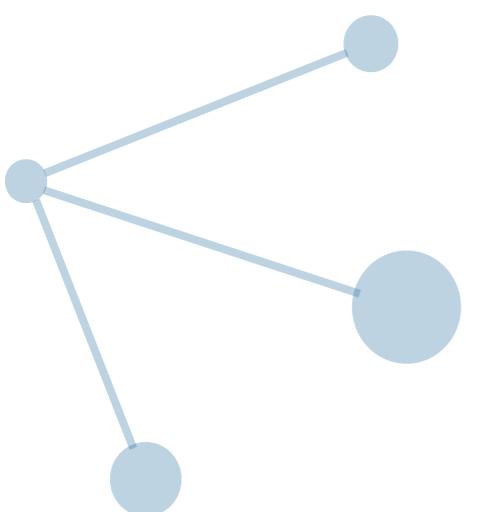
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 - Cats, Scalaz
 - Argonaut, Atto, Doobie, HTTP4S, Monocle, Remotely, Scalaz-stream, Shapeless, Typelevel.org



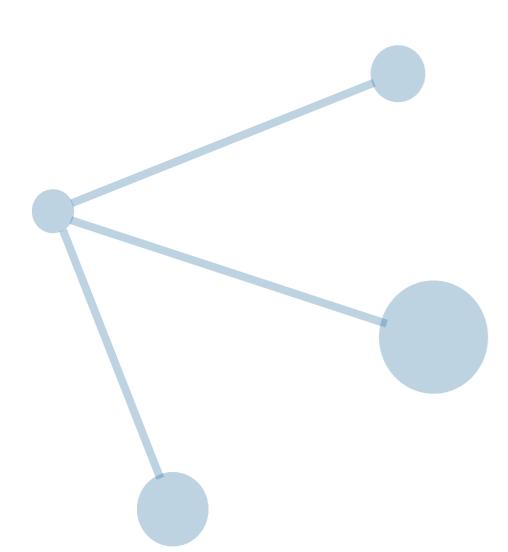
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- Algebra
 - Algebroid, Algebra, Breeze, Spire
- Big Data
 - Scalding + Algebroid, Scoobi, Spark



EOF