Implementation and Validation of Inverse Dynamics Actuation Control with Learned Kinematics for Exoskeletons - Supplementary Material

Table S 1: Body Names Exoskeleton Model

Number	Name
1	HIP_BELT
2	BACK_RAIL
3	RIGHT_SWIVEL_JOINT
4	RIGHT_LEG_RAIL
5	LEFT_SWIVEL_JOINT
6	LEFT_LEG_RAIL

Table S 2: Body Numbering Test Bench Model

Number	Name in OpenSim	Name in Paper
1	Huefte2	Trunk
2	Anbau	
3	Bein_L	Left Upper Leg
4	Gelenk_unten_L	Left Knee
5	Bein_R	Right Upper Leg
6	Gelenk_unten_R	Right Knee

Table S 3: Test Bench Parallel Kinematic Coupling Constraints

Number m	$b(m)$	$\boldsymbol{T}^{C,m}_{\mathrm{bench},b(m)}$	W_m
1	1	$\begin{matrix} 1 & 0 & 0 & 0.2 \\ 0 & \cos\left(-\frac{\pi}{4}+\theta_t\right) & -\sin\left(-\frac{\pi}{4}+\theta_t\right) & 0.52 \\ 0 & \sin\left(-\frac{\pi}{4}+\theta_t\right) & \cos\left(-\frac{\pi}{4}+\theta_t\right) & -0.48 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,4,5,6\}$
2	2	$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_t) & -\sin(\theta_t) & 0 \\ 0 & \sin(\theta_t) & \cos(\theta_t) & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{4\}$
3	3	$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_t - \theta_h) & -\sin(\theta_t - \theta_h) & 0 \\ 0 & \sin(\theta_t - \theta_h) & \cos(\theta_t - \theta_h) & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{4\}$
4	4	$\begin{matrix} 1 & 0 & 0 & 0.335 \\ 0 & \cos(13.8°) & -\sin(13.8°) & 0.0886 \\ 0 & \sin(13.8°) & \cos(13.8°) & -0.1636 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,2,3,4,6\}$
5	6	$\begin{matrix} 1 & 0 & 0 & 0.065 \\ 0 & \cos(13.8°) & -\sin(13.8°) & 0.0456 \\ 0 & \sin(13.8°) & \cos(13.8°) & 0.0112 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,2,4,6\}$

Table S 4: Exoskeleton Test Bench Coupling Coupling based on [16]

Number m	$b_{C,\text{bench}}(m)$	$b_{C,\text{exo}}(m)$	$\boldsymbol{T}_{\text{C,bench},m}^{\text{bench},b_{C,\text{bench}}(m)}$	$\boldsymbol{T}_{\text{C,exo},m}^{\text{exo},b_{C,\text{exo}}(m)}$	W_m
1	2	1	$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.0361 \\ 0 & 0 & -1 & -0.0655 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,2,3,4,5,6\}$
2	2	2	$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.093 \\ 0 & 0 & -1 & -0.188 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.26 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,3\}$
3	5	4	$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.209 \\ 0 & 0 & -1 & -0.1 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 & -0.1 \\ 0 & 1 & 0 & -0.25 \\ 0 & 0 & 1 & 0.08 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,3\}$
4	3	6	$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.209 \\ 0 & 0 & -1 & -0.1 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & -0.25 \\ 0 & 0 & 1 & 0.08 \\ 0 & 0 & 0 & 1 \end{matrix}$	$\{1,3\}$

Table S 5: Cable-based Routing Model Properties

Number j	Number n	$b_{sec}(j,n)$	${}^{\mathrm{exo},b_{sec}(j,n)}\boldsymbol{p}_{j,n}$	Cable Length
1	1	2	$[0.1378 \quad 0.07 \quad -0.0326]^{\mathrm{T}}$	0.7420
1	2	1	$[0.134 \quad 0.085 \quad 0.0889]^{\mathrm{T}}$	
1	3	1	$[0 \quad 0 \quad 0.111]^{\mathrm{T}}$	
1	4	1	$[-0.1334 \quad -0.004 \quad 0.0889]^{\mathrm{T}}$	
1	5	1	$[-0.1336 \quad -0.0925 \quad 0.091]^{\mathrm{T}}$	
1	6	6	$[0.11 \quad -0.215 \quad 0.09]^{\mathrm{T}}$	
2	1	2	$[-0.108 \quad 0.07 \quad -0.078]^{\mathrm{T}}$	0.504
2	2	1	$[0.109 \quad 0.03 \quad 0.109]^{\mathrm{T}}$	
2	3	1	$[0.109 \quad 0.03 \quad 0.109]^{\mathrm{T}}$	
2	4	1	$[0.122 \quad -0.0405 \quad 0.106]^{T}$	
2	5	1	$[0.12 \quad -0.095 \quad 0.107]^{\mathrm{T}}$	
2	6	4	$[-0.11 \quad -0.215 \quad 0.09]^{\mathrm{T}}$	
3	1	2	$[0.0335 \quad 0.07 \quad 0.0206]^{\mathrm{T}}$	0.5812
3	2	1	$[0.0312 \quad -0.0095 \quad 0.139]^{\mathrm{T}}$	
3	3	1	$[0.078 \quad -0.0118 \quad 0.1187]^{\mathrm{T}}$	
3	4	1	$[0.1042 \quad -0.013 \quad 0.1112]^{\mathrm{T}}$	
3	5	1	$[0.107 \quad -0.095 \quad 0.11159]^{\mathrm{T}}$	
3	6	4	$[0.107 \quad -0.095 \quad 0.11159]^{\mathrm{T}}$	
4..6	mirrored on y-z-planes			

Table S 6: Exoskeleton Actuator Coupling

Number j	$b_{\mathrm{act}}(j)$	$\boldsymbol{T}_{\mathrm{C,act},j}^{\mathrm{exo},b_{\mathrm{act}}(j)}$			
1	2	-0.050826	0.9987074	0	0.1378
		0.997101	0.05074491	0.056688	0.07
		0.0566147	0.00288126	-0.9983919	-0.0326
		0	0	0	1
2	2	-0.05444	0.99851	0	0.10763
		0.997190	0.0543725	0.0551838	0.07
		0.051447	0.0028052	-0.99867	-0.00778
		0	0	0	1
3	2	0.0541	0.998535	0	0.03349
		0.998361	-0.054099	0.0186576	0.07
		0.01863	-0.00100	-0.99976	0.020618
		0	0	0	1
4..6	mirrored on y-z-plane				

Table S 7: Assistive Torque Indices in Nm according to [14]

Actual Support Torque	$\mathrm{ATI}_{1000}^{\mathrm{Lower}}$	$\mathrm{ATI}_{200}^{\mathrm{Lower}}$	$\mathrm{ATI}_{1000}^{\mathrm{Hold}}$	$\mathrm{ATI}_{1000}^{\mathrm{Raise}}$	$\mathrm{ATI}_{200}^{\mathrm{Raise}}$
$\sum \tau_{\mathrm{sup,hum,actu}}$	10.74	10.94	10.28	9.10	9.84
$\sum \tau_{\mathrm{sup,hum,actu2}}$	9.23	8.70	8.09	7.41	7.64
$\sum \tau_{\mathrm{sup2,hum,actu}}$	14.09	14.15	13.19	11.91	12.79
$\sum \tau_{\mathrm{sup2,hum,actu2}}$	11.57	10.89	10.18	9.32	9.73