Nonlinear model predictive control with guaranteed stability based on pseudolinear neural networks*

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Abstract: A nonlinear model predictive control problem based on pseudo-linear neural network (PNN) is discussed, in which the second order on-line optimization method is adopted. The recursive computation of Jacobian matrix is investigated. The stability of the closed loop model predictive control system is analyzed based on Lyapunov theory to obtain the sufficient condition for the asymptotical stability of the neural predictive control system. A simulation was carried out for an exothermic first-order reaction in a continuous stirred tank reactor.It is demonstrated that the proposed control strategy is applicable to some of nonlinear systems.

Keywords: pseudolinear neural networks (PNN); nonlinear model predictive control; continuous stirred tank reactor (CSTR); asymptotic stability

1. Introduction

Model predictive control (MPC) is widely used in the process industry [1-5]. Because of the universal approximation ability of neural networks, the NN based MPC plays an important role in the nonlinear model predictive control [6]. The problems of neural network based model predictive control (NNMPC) can be solved by either direct optimization [7] to train an NNMPC controller with optimized data [8], or solving a universal dynamic matrix control (UDMC) problem [9,10].

In direct optimization of NNMPC, the on-line calculation of Jacobian matrix is a heavy load. In order to reduce the burden, Noriega and Wang [11] proposed a recursive approach to calculate Jacobian matrix. A similar recursive method is proposed in this paper.

The stability of the closed loop for NNMPC is an important issue in the NN based control. Zheng [12] and Santos [13] discussed the robust stability problems of MPC. In this paper, a Lyapunov approach based analysis for the asymptotic stability of the closed loop NNMPC is carried out, and the sufficient condition of local asymptotic stability is derived.

A simulating study is to be performed to illustrate the applicability of the presented approach.

2. Pseudolinear neural networks and training

The pseudolinear neural network (PNN), proposed by author in a previous work [14], is adopted for modeling the nonlinear process. Its architecture is shown in Fig. 1.

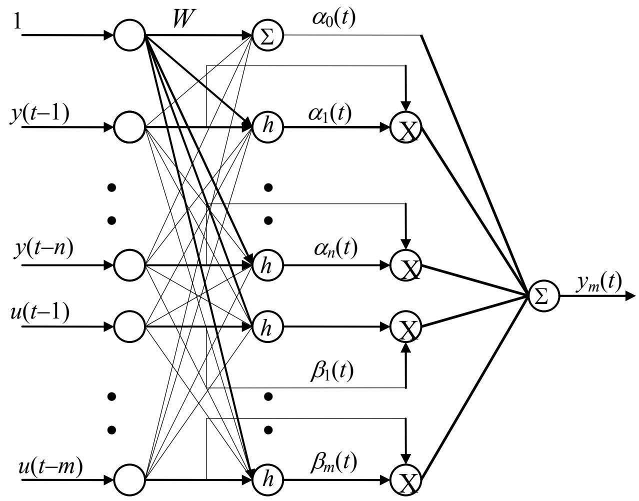

Fig. 1 The architecture of pseudolinear neural networks

In Fig.1, the X circles symbolize multiplication neurons; the Σ ones addition, and the h ones nonlinear neurons. At any discrete time t, the inputs of PNN are $\boldsymbol{I}(t)=[y(t-1),\ldots,y(t-n),u(t-1),\ldots,u(t-m)]^{\mathrm{T}}$; $\alpha_j(t)$ and $\beta_j(t)$ are the output of the jth of the first hidden neuron, and $y_m(t)$ is the output of the network, W represents the weightings from the input layer to the first hidden layer, θ_i is the threshold. The input-output relationships of the PNN are

$$y(t)=\alpha_1(t)y(t-1)+\ldots+\alpha_n(t)y(t-n)+\ldots+ \beta_1(t)u(t-1)+\ldots+\beta_m(t)u(t-m)\,. \tag{1}$$

$$\alpha_i(t)=h(\sum_{j=1}^{n+m}W_{ij}I_j(t)+\theta_i)\,,\ i=1,2,\ldots,n\,; \tag{2a}$$

$$\beta_i(t)=h(\sum_{j=1}^{n+m}W_{n+i,j}I_j(t)+\theta_{n+i})\,,\ i=1,2,\ldots,m\,. \tag{2b}$$

$$h(x)=\frac{1-\ \exp(x)}{1+\ \exp(x)}\,. \tag{3}$$

It is obviously from (1) that PNN model is a quasilinear one as to the input and output of system, on which the controller design and system analysis will be convenient under some conditions.

The recursive prediction error method (RPE) is adopted in training the PNN [15]. The RPE algorithm is of second convergent order, which brings out the faster convergence than BP algorithm. If the learning rate η is correctly chosen, the convergence of training process can be guaranteed [15].

3. PNN based model predictive control

The MPC problem is formulated as follows, which in every sample period, should be solved.

$$\min J = \frac{1}{2}\boldsymbol{E}_{t,P}^{\mathrm{T}}\boldsymbol{E}_{t,P} + \frac{1}{2}\lambda\Delta\boldsymbol{U}_{t,M}^{\mathrm{T}}\Delta\boldsymbol{U}_{t,M}, \tag{4a}$$

s.t.

$$\boldsymbol{U}_{\min} \le \boldsymbol{U}(t) \le \boldsymbol{U}_{\max}, \tag{4b}$$

where $\boldsymbol{E}_{t,P}$ is the error vector, $\Delta\boldsymbol{U}_{t,M}$ is the increment of control, $\Delta\boldsymbol{U}_{t,M} = [\Delta u(t), \Delta u(t+1), \ldots, \Delta u(t+M-1)]^{\mathrm{T}}$, P is the length of prediction, M is the length of the control.

$$\boldsymbol{E}_{t,P} = \boldsymbol{R}_{t,P} - \widehat{\boldsymbol{Y}}_{t,P}, \tag{5}$$

where $\boldsymbol{R}_{t,P}$ and $\widehat{\boldsymbol{Y}}_{t,P}$ are setpoint vector and predictive vector, respectively.

$$\begin{aligned}\boldsymbol{R}_{t,P} &= [r(t+1), r(t+2), \ldots, r(t+P)]^{\mathrm{T}} \\ \widehat{\boldsymbol{Y}}_{t,P} &= [\widehat{y}(t+1), \widehat{y}(t+2), \ldots, \widehat{y}(t+P)]^{\mathrm{T}}, \\ \boldsymbol{E}_{t,P} &= [e(t+1), e(t+2), \ldots, e(t+P)]^{\mathrm{T}}\end{aligned} \tag{6}$$

and the control vector is

$$\boldsymbol{U}_{t,M} = [u(t+1), u(t+2), \ldots, u(t+M)]^{\mathrm{T}}. \tag{7}$$

To solve the MPC problem (4), a gradient-based method is applied firstly to the unconstrained problem (4a) whose solution can be obtained, then check whether the constrained condition (4b) is satisfied or not. If the constraints are violated, then the $\boldsymbol{U}(t)$ takes the bounded $U_{\min}$ or $U_{\max}$. In the case that control $\boldsymbol{U}$ is within the constraints, an iteration of calculating $\boldsymbol{U}_{t,M}$ is necessary.

$$\boldsymbol{U}_{t,M}^{\mathrm{T}}(k+1) = \boldsymbol{U}_{t,M}^{T}(k) + \frac{\eta}{1+\lambda\eta}\left(\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}(k)}\right)^{\mathrm{T}}\boldsymbol{E}_{t,P}, \tag{8}$$

where the initial $\boldsymbol{U}_{t,M}^{\mathrm{T}}(0) = \boldsymbol{U}_{t-1,M}^{\mathrm{T}}$; and $\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}(k)}$ is the Jacobian matrix as follows.

$$\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}(k)} = \begin{bmatrix} \frac{\partial\widehat{\boldsymbol{y}}(t+1)}{\partial\boldsymbol{u}(t)} & 0 & 0 & \cdots & 0 \\ \frac{\partial\widehat{\boldsymbol{y}}(t+2)}{\partial\boldsymbol{u}(t)} & \frac{\partial\widehat{\boldsymbol{y}}(t+2)}{\partial\boldsymbol{u}(t+1)} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \frac{\partial\widehat{\boldsymbol{y}}(t+M)}{\partial\boldsymbol{u}(t+M-1)} \\ \vdots & & & & \vdots \\ \frac{\partial\widehat{\boldsymbol{y}}(t+P)}{\partial\boldsymbol{u}(t)} & \cdots & \cdots & \cdots & \frac{\partial\widehat{\boldsymbol{y}}(t+P)}{\partial\boldsymbol{u}(t+M-1)} \end{bmatrix} \tag{9}$$

With Jacobian matrix $\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}(k)}$ is obtained, the predictive output $\widehat{\boldsymbol{Y}}_{t,P}$ and error $\boldsymbol{E}_{t,P}$ can be calculated based on the $\boldsymbol{U}_{t,M}(k+1)$, therefore the control vector $\boldsymbol{U}_{t,M}$ can be gotten by Equation (8).

In order to simplify the problem, Consider only the unconstrained MPC problem (4a). When the second order based optimization is applied, it can be obtained that

$$\Delta\boldsymbol{U}_{t,M}^{\mathrm{T}} = -\eta\boldsymbol{H}_{t,M}^{-1}\left(\frac{\partial\boldsymbol{J}}{\partial\boldsymbol{U}_{t,M}}\right)^{\mathrm{T}}. \tag{10}$$

where $\boldsymbol{H}_{t,M}$ is a Hessian matrix consisting of the second order partial derivatives. Then calculate the control output $\boldsymbol{U}(t)$. It is time consuming to compute $\boldsymbol{H}_{t,M}$, whose approximation is[16]

$$\boldsymbol{H}_{t,M} = \left(\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}}\right)^{\mathrm{T}}\left(\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}}\right) + \lambda\boldsymbol{I}_M = \boldsymbol{J}^{\mathrm{T}}\boldsymbol{J} + \lambda\boldsymbol{I}_M, \tag{11}$$

$$\frac{\partial\boldsymbol{J}}{\partial\boldsymbol{U}_{t,M}} = -\left(\frac{\partial\widehat{\boldsymbol{Y}}_{t,P}}{\partial\boldsymbol{U}_{t,M}}\right)^{\mathrm{T}}\boldsymbol{E}_{t,P} + \lambda\Delta\boldsymbol{U}_{t,M}^{\mathrm{T}}. \tag{12}$$

So, it can be obtained that

$$\Delta\boldsymbol{U}_{t,M}^{\mathrm{T}} = \eta(\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J} + \beta\boldsymbol{I}_M)^{-1}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{E}_{t,P}, \tag{13}$$

where $\beta = \lambda(1+\eta)$.When control length M is set to 1, the predictive control is of a simple form as

$$\Delta\boldsymbol{U}(t) = \frac{\eta\boldsymbol{J}^{\mathrm{T}}\boldsymbol{E}_{t,P}}{\beta + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{J}}. \tag{14}$$

To calculate Jacobian matrix, let the neural network be

$$\widehat{y}(t+1) = f[y(t), \ldots, y(t-n+1), u(t), \ldots, u(t-m+1)], \tag{15}$$

$$\boldsymbol{I}(t) = [y(t), \ldots, y(t-n+1), u(t), \ldots, u(t-m+1)]^{\mathrm{T}}. \tag{16}$$

Applying the chain rule to Eq. (15), the element of Jacobian matrix (9) can be calculated recursively,

$$\frac{\partial\widehat{\boldsymbol{y}}(t+p)}{\partial\boldsymbol{u}(t+l)} = \frac{\partial\boldsymbol{f}(\boldsymbol{I}(t+p-1))}{\partial\boldsymbol{u}(t+l)} + \sum_{i=1}^{p-1}\frac{\partial\widehat{\boldsymbol{y}}(t+p)}{\partial\widehat{\boldsymbol{y}}(t+l)}\frac{\partial\widehat{\boldsymbol{y}}(t+l)}{\partial\boldsymbol{u}(t+l)}. \tag{17}$$

Denote the non-zero element of (17) be

$$a_l(\boldsymbol{I}_j) = \frac{\partial\widehat{\boldsymbol{y}}(t+p)}{\partial\widehat{\boldsymbol{y}}(t+l)}, 1 \le l \le n, \tag{18}$$

$$b_l(\boldsymbol{I}_j) = \frac{\partial\widehat{\boldsymbol{y}}(t+l)}{\partial u(t+l)}, 1 \le l \le m, \tag{19}$$

$$c_l(\boldsymbol{I}_j) = \frac{\partial\widehat{\boldsymbol{y}}(t+p)}{\partial\widehat{\boldsymbol{y}}(t+l)}. \tag{20}$$

Eq. (20) can be rewritten as

$$c_{p,l} = b_{p-l+1}(I_p) + \sum_{i=1}^{p-l}a_i(I_p)c_{p-i,l}, \quad 1 \le p \le P, 0 \le l \le M-1. \tag{21}$$

When the neural network takes the PNN forms (1) to

(3), there are

$$a_l(\boldsymbol{I}_j)=\frac{\partial\hat{y}(t+p)}{\partial\hat{y}(t+l)}=\alpha_l(\boldsymbol{I}_j)+\sum_{i=1}^{n+m+1}h'(\bullet,\boldsymbol{I}_j)w_{i,l}\boldsymbol{I}_j\,,\quad 1\le l\le m\,; \tag{22}$$

$$b_l(\boldsymbol{I}_j)=\frac{\partial\hat{y}(t+p)}{\partial\boldsymbol{u}(t+l)}=\beta_l(\boldsymbol{I}_j)+\sum_{i=1}^{n+m+1}h'(\bullet,\boldsymbol{I}_j)w_{i,l}\boldsymbol{I}_j\quad 1\le l\le n\,. \tag{23}$$

Because the vector $\boldsymbol{I}_j$ may include the future control $\boldsymbol{u}(t+l)$, $0< l< M$, the iteration values of Eq.(8) should be used.

4. The asymptotical stability of PNN based MPC

A closed loop control system should be stable. In this section, Lyapunov theory is to be employed to analyze the asymptotical stability of PNN based unconstrained MPC, so as to derive the sufficient condition of the asymptotical stability for the neural predictive control system.

Suppose the Lyapunov function is defined as follows.

$$V=\frac{1}{2}\boldsymbol{E}_P^{\mathrm{T}}\boldsymbol{E}_P+\frac{1}{2}\eta\Delta\boldsymbol{E}_P^{\mathrm{T}}\Delta\boldsymbol{E}_P\,. \tag{24}$$

Take the partial derivative of V with respect to t, then ,

$$\frac{\partial V}{\partial t}=\boldsymbol{E}_P^{\mathrm{T}}\frac{\partial\boldsymbol{E}_P}{\partial\boldsymbol{U}_M}\frac{\partial\boldsymbol{U}_M}{\partial t}+\eta\Delta\boldsymbol{E}_P^{\mathrm{T}}\frac{\partial\boldsymbol{E}_P}{\partial\boldsymbol{U}_M}\frac{\partial\boldsymbol{U}_M}{\partial t}\,, \tag{25}$$

and approximately,

$$\Delta V=\boldsymbol{E}_P^{\mathrm{T}}\frac{\partial\boldsymbol{E}_P}{\partial\boldsymbol{U}_M}\Delta\boldsymbol{U}_M+\eta\Delta\boldsymbol{E}_P^{\mathrm{T}}\frac{\partial\boldsymbol{E}_P}{\partial\boldsymbol{U}_M}\Delta\boldsymbol{U}_M\,. \tag{26}$$

Since

$$\Delta\boldsymbol{E}_P=\frac{\partial\boldsymbol{E}_P}{\partial\boldsymbol{U}_M}\Delta\boldsymbol{U}_M\,, \tag{27}$$

$$\begin{aligned}\Delta V&=-\eta\boldsymbol{E}_P^{\mathrm{T}}\boldsymbol{J}(\beta\boldsymbol{I}_M+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{E}_p+\\&\quad\eta^2\boldsymbol{E}_P^{\mathrm{T}}\boldsymbol{J}(\beta\boldsymbol{I}_M+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J}(\beta\boldsymbol{I}_M+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{E}_P\\&=-\eta\boldsymbol{E}_P^{\mathrm{T}}\boldsymbol{Q}(\boldsymbol{I}_P-\eta\boldsymbol{Q})\boldsymbol{E}_p\end{aligned} \tag{28}$$

where $\boldsymbol{Q}=\boldsymbol{J}(\beta\boldsymbol{I}_M+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}$ is a symmetric matrix. If $\boldsymbol{I}_P-\eta\boldsymbol{Q}$ is a semi-positive definite, then $\Delta V\le 0$.

For a real symmetric matrix $\boldsymbol{Q}$, there exists an orthogonal transformation matrix $\boldsymbol{S}$ [17] such that the $\boldsymbol{Q}$ can be transformed to

$$\boldsymbol{Q}=\boldsymbol{S}\,\mathrm{diag}\{\lambda_1,\lambda_2,\ldots,\lambda_M,\ldots\}\,\boldsymbol{S}^{\mathrm{T}}\,. \tag{29}$$

Then, $\boldsymbol{I}_P-\eta\boldsymbol{Q}$ is of the form

$$\boldsymbol{I}_P-\eta\boldsymbol{Q}=\boldsymbol{S}\,\mathrm{diag}\{1-\eta\lambda_1,1-\eta\lambda_2,\ldots,1-\eta\lambda_M,\ldots\}\,\boldsymbol{S}^{\mathrm{T}}\,. \tag{30}$$

If λ is so selected that

$$1-\eta\max(\lambda_i)>0\,, \tag{31}$$

i.e.

$$1-\eta\rho(\boldsymbol{Q})>0\,, \tag{32}$$

where $\rho(\boldsymbol{Q})$ is the spectrum radius of $\boldsymbol{Q}$. The semi-positive definite can be guaranteed if η is correctly chosen. Since $\boldsymbol{Q}=\boldsymbol{J}(\beta\boldsymbol{I}_M+\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}$, the norm of $\boldsymbol{Q}$, $\|\boldsymbol{Q}\|<1$. Choose

$$1-\eta\|\boldsymbol{Q}\|>0\,. \tag{33}$$

Therefore,

$$0<\eta<1/\|\boldsymbol{Q}\|<1/\rho(\boldsymbol{Q})<1\,. \tag{34}$$

From the above result, the asymptotical stability of the closed-loop PNN based MPC control system based on the second order optimization can be concluded as follows.

Theorem 1. If the Lyapunov function is defined as equation (24), the sufficient condition for the stability of the predictive control of open loop stable system based on second order optimization is

$$0<\eta<1\,. \tag{35}$$

Remark 1. The sufficient condition for the stability of the predictive control is derived under unconstrained condition when the control $\boldsymbol{U}$ does not violate the constraint, which is equivalent to the unconstrained situation. If the $\boldsymbol{U}$ achieves the constraints, the bounded control input will results in a bounded output under the assumption that the open loop system is stable for a large number of industrial processes, then the proposed control algorithm will bring the system to a stable equilibrium. Furthermore, a constrained control problem can be converted into an unconstrained one by using Lagrangian multiplier.

5. Simulated example

Consider an exothermic first-order reaction in a continuous stirred tank reactor (CSTR). The output of the system is the extent of reaction, while the input to the system is the coolant flow rate to the cooling jacket. The dynamic nonlinear system [18] can be described by

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}t}&=-x_1+\alpha(1-x_1)\exp\left(\frac{x_2}{1+x_2/\gamma}\right)\\\frac{\mathrm{d}x_2}{\mathrm{d}t}&=-x_2+\alpha(1-x_1)\exp\left(\frac{x_2}{1+x_2/\gamma}\right)+\beta(u-x_2)\,,\\y&=x_1\end{aligned} \tag{36}$$

where x_1 and x_2 are respectively the extent of reaction and the dimensionless temperature of the reactor contents; u is the input, which is the dimensionless flow rate of heat-transfer fluid through the cooling jacket. The simulation took in the parameter values as $\alpha=0.0072$, $\beta=0.3$, $\mu=8.0$ and $\gamma=20.0$. In modeling phase, the system is simulated by a multisine signal. The amplitude of the input signal is bounded within [−0.6, 0.6], and the output of the system physically within [0, 1]. The architecture of PNN for identification is 7-7-1, where $n = m = 3$, and the RPE algorithm is used to update the weighting of PNN. The whole training process uses 800 iterations. In order to overcome the inaccuracy of PNN model, the controller structure is a composite one as

$$u(t) = u_{fb}(t) + u_{ff}(t)\,, \tag{37}$$

where $u_{fb}(t)$ is the output of feedback controller, $u_{ff}(t)$ is the output of predictive controller described by Eq. (14), with $\eta = 0.20$, $\lambda = 0.40$, and $K_{max} = 5$. In simulated closed loop control, $u_{fb}(t)$ is a proportional controller, $u_{fb}(t) = k_P e(t)$ and $k_P = 5.0$. The set-point of the system is

$$y_d(t) = \begin{cases} 0.15, & \text{if } 0 < t \le 40, \text{ and } t > 120 \\ 0.24, & \text{if } \quad 40 < t \le 120 \end{cases}. \tag{38}$$

The closed loop response is shown Fig.2 where the response of linear model based one-step-ahead controller plus P-controller is provided for comparison.

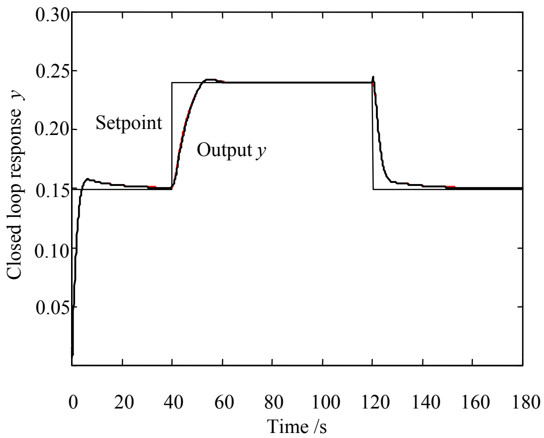

Fig 2. Closed loop response of CSTR

6. Conclusions

The proposed pseudo-linear neural network based model predictive approach employing the second order on-line optimization is demonstrated applicable to nonlinear process control systems. However, the closed-loop asymptotical stability of the neural control system is only local because of the approximation of partial derivative.

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