Bike Share Project LP Formulization

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Objective function:

$$\text{Minimize} \sum_{i}^{A} \sum_{j}^{A} S_{ij} \times x_{ij} \quad \text{where i} \neq \text{j}$$

S_{ij}: Transfer realtion indicating an allocation instance between station i and j
A: All stations ordered alphabetically
x_{ij}: Rectlinear distance between station i and j

Constraints Pre Explanation:

$\phi_i(D_i)$ can be defined as the variable vector of station "i" that consist of $\sum f_{in} - \sum f_{out}$ or vice versa (based on the demand of station i) where f_i can be defined as flow in or out from the station i

$$\phi_i(D_i) = \begin{cases} \sum_{j}^{A} S_{ji} - S_{ij} & D_i \leq 0 \\ \text{for all i in A} & \\ \sum_{j}^{A} S_{ij} - S_{ji} & D_i \geq 0 \\ \text{for all i in A} & \end{cases} \tag{1}$$

S_{ij}: Transfer realtion indicating an allocation instance between station i and j
D_i: Bike demand of station i for reaching the equilibrium

The purpose is to balance the bikes in the system by equalizing all stations' final stock to the *Equilibrium Capacity* $= \epsilon$

$$\epsilon = \frac{\sum_{i}^{A} b_i}{\sum_{i}^{A} C_i}$$

b_i: Initial bike count in the station i
C_i: Max bike capacity of station i

Constraints (Upper Bound):

α: Elasticity coefficent allowes % flexibility from ϵ

$$\frac{b_i + \phi_i(D_i)}{C_i} \leq \epsilon + \alpha \quad \text{for all i that is elements of A}$$

$$\frac{b_i + \phi_i(D_i)}{C_i} \geq \epsilon - \alpha \quad \text{for all i that is elements of A}$$

Lower bound is not used in Spicy Linprog, conversion to lower bound is needed. Since Spicy is set up so that any addition or substraction would violate the "0 coefficent" of unused decision variable instance, algebraic work is required to break up the equation and simplify for $\phi_i(D_i)$

$$\text{Upper Bound: } \frac{\phi_i(D_i)}{C_i} \leq \|\epsilon \text{ - } \frac{b_i}{C_i}\| + \alpha \quad \text{for all i that is elements of A}$$

$$\text{Lower Bound: } -\frac{\phi_i(D_i)}{C_i} \leq -\|\epsilon \text{ - } \frac{b_i}{C_i}\| + \alpha \quad \text{for all i that is elements of A}$$

Nonnegativity :

$$S_i \geq 0 \qquad \text{for all i in A}$$