CS4286 Assignment 01

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Q1: Security Service and Mechanisms (Link)

- In Dec 2019, researchers found that an Elasticsearch database was exposed on the web, which includes 2.7 billion email addresses with more than 1 billion email addresses paired with plain-text password.
- Most of the emails were from Chinese domains including `qq.com`, `126.com`, while a few addresses are from Russian domains. Except email addresses and passwords, information includes phone numbers, `MD5`, `SHA1`, `SHA256` hashes of email addresses (which can be used to ease searches of relational databases) may also be exposed. After finding the exposed data, researchers immediately took steps to notified the US ISP that hosted the IP address of leaked database to take it down. However, when the database was disabled, the data was still exposed for more than a week, which might be taken advantage by adversaries.
- According to the verification by the researchers, all the emails with passwords originated from a dark web vendor who sold the records that included passwords stolen from Chinese internet giants in Jan 2017 (the event is so called “Big Asian Leak”). As mentioned by HackRead, more than 60 copies were sold for a $615 dollar price, most of the records from domains `Netease`, `Tencent`, `Sina` and `Sohu`.
- In this incident, **the type of attacker is passive adversary**, because the attacker only attempt to get unauthorized access of the data, there is no evidence to prove that there are some active adversaries starting to take advantage of the information. The attacker could be both insiders and outsiders, because no information is given about how did the dark net vendor get the records. **The main security services were compromised is data confidentiality**, since the records of users are leaked. Besides, many people prefer to use same email password combination, hackers may also use the exposed information to perform **credential stuffing**. If the hackers can gain access to an account, they may change the password and do some other **types of attack (spam, phishing, fraud). Therefore, integrity, availability and access control may also be compromised.**
- To avoid this event, an obvious mechanism is **encipherment** which provides **confidentiality**. Because in the leaked records, 1 billion passwords were stored in plain-text, it will be better to use **hash with salt** to store the password. Besides, internet company should also enhance the mechanism **access control model** which provides better **access control** of the database.

Q2: Substitution Cipher

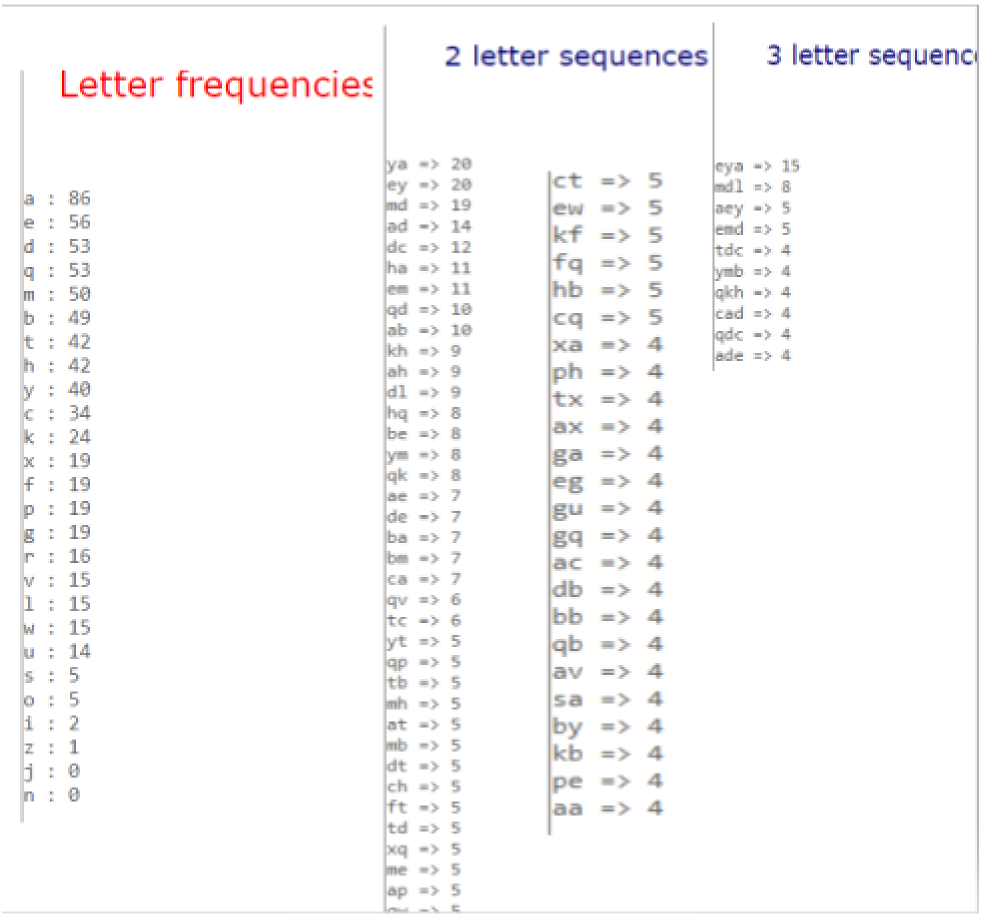
- Key:

```
plain alphabetic:
abcdefghijklmnopqrstuvwxyz
key:
TRXCAPLYMZOGVDQFJHBEKSWIUN
(or)
TRXCAPLYMZOGVDQFNHBEKSWIUJ
```

- Result:

```
in the third week of november, in the year 1895, a dense
yellow fog settled down upon london. from the monday
to the thursday i doubt whether it was ever possible
from our windows in baker street to see the loom of
the opposite houses. the first day holmes had spent in
cross-indexing his huge book of references. the second
and third had been patiently occupied upon a subject
which he had recently made his hobby—the music of the
middle ages. but when, for the fourth time, after pushing
back our chairs from breakfast we saw the greasy, heavy
brown swirl still drifting past us and condensing in
oily drops upon the window-panes, my comrade's impatient
and active nature could endure this drab existence no
longer. he paced restlessly about our sitting-room in
a fever of suppressed energy, biting his nails, tapping
the furniture, and chafing against inaction.
```

- Steps:
  - I used the [online frequency analysis tool]() to analyze the character frequency and compared it with the frequency [bigram]() and [trigram]().



- According to the letter frequencies, bigram, trigram, and two letter words, three letter words. I can find that

  ```
  e => a, t => e, z => n, he => ya, th => ey, ing => mdl...
  ```

- I use the command `tr` in Linux system to substitute the characters

  ```
  tr 'EGHINTZ' 'alymden' < cipher.txt > test.txt
  ```

- Based on the output, continue observing the words, and try whether I can do some substitution to get a word, to analyze the result better, I wrote the code to select words with specific length in `test.txt`:

  ```
  import operator
  length = input("Length of the word: ")
  with open('test.txt', 'r') as file:
      words = file.read().split()
      dic = {}
      for word in words:
          if len(word) == length:
              if not word in dic :
                  dic[word] = 1
              else:
                  dic[word] += 1
       sorted_dic = sorted(dic.items(), key=lambda kv:kv[1])
       print(sorted_dic)
  ```

- Repeat the previous three steps until all words can be found.
- **Finally, since the frequency for `J` and `N` are zero, we can get two correct keys.**

Q3: Modes of Operation and 'shift cipher'

- Plaintext: 'IAMALICE', Key: 4

A	B	C	D	E	F	G	H
0000	0001	0010	0011	0100	0101	0110	0111
I	J	K	L	M	N	O	P
1000	1001	1010	1011	1100	1101	1110	1111

- (a) Encrypt the plaintext P using CBC mode with $IV = 0001$.

$$\begin{aligned}
C_0 &= E_k(P_0 \oplus IV) = E_k(1000 \oplus 0001) = E_k(1001) = E_k(J) = N(1101) \\
C_1 &= E_k(P_1 \oplus C_0) = E_k(0000 \oplus 1101) = E_k(1101) = E_k(N) = B(0001) \\
C_2 &= E_k(P_2 \oplus C_1) = E_k(1100 \oplus 0001) = E_k(1101) = E_k(N) = B(0001) \\
C_3 &= E_k(P_3 \oplus C_2) = E_k(0000 \oplus 0001) = E_k(0001) = E_k(B) = F(0101) \\
C_4 &= E_k(P_4 \oplus C_3) = E_k(1011 \oplus 0101) = E_k(1110) = E_k(O) = C(0010) \\
C_5 &= E_k(P_5 \oplus C_4) = E_k(1000 \oplus 0010) = E_k(1010) = E_k(K) = O(1110) \\
C_6 &= E_k(P_6 \oplus C_5) = E_k(0010 \oplus 1110) = E_k(1100) = E_k(M) = A(0000) \\
C_7 &= E_k(P_7 \oplus C_6) = E_k(0100 \oplus 0000) = E_k(0100) = E_k(E) = I(1000) \\
C &= NBBFCOAI
\end{aligned}$$

- (b) Encrypt the plaintext P using CBC mode with $IV = 0101$. How does your ciphertext compare to that in 2(a).

$$\begin{aligned}
C_0 &= E_k(P_0 \oplus IV) = E_k(1000 \oplus 0101) = E_k(1101) = E_k(N) = B(0001) \\
C_1 &= E_k(P_1 \oplus C_0) = E_k(0000 \oplus 0001) = E_k(0001) = E_k(B) = F(0101) \\
C_2 &= E_k(P_2 \oplus C_1) = E_k(1100 \oplus 0101) = E_k(1001) = E_k(J) = N(1101) \\
C_3 &= E_k(P_3 \oplus C_2) = E_k(0000 \oplus 1101) = E_k(1101) = E_k(N) = B(0001) \\
C_4 &= E_k(P_4 \oplus C_3) = E_k(1011 \oplus 0001) = E_k(1010) = E_k(K) = O(1110) \\
C_5 &= E_k(P_5 \oplus C_4) = E_k(1000 \oplus 1110) = E_k(0110) = E_k(G) = K(1010) \\
C_6 &= E_k(P_6 \oplus C_5) = E_k(0010 \oplus 1010) = E_k(1000) = E_k(I) = M(1100) \\
C_7 &= E_k(P_7 \oplus C_6) = E_k(0100 \oplus 1100) = E_k(1000) = E_k(I) = M(1100) \\
C &= BFNBOKMM
\end{aligned}$$

  - The cipher text is slightly different
- (c) Use your answer from 2(b). If the MSB bit of C_2 becomes an error($C_2 = F(0101)$), what is the recovered plaintext?

$$
\begin{aligned}
P'_0 &= D_k(C_0) \oplus IV = N \oplus IV = 1101 \oplus 0101 = I(1000) \\
P'_1 &= D_k(C_1) \oplus C_0 = B \oplus C_0 = 0001 \oplus 0001 = A(0000) \\
P'_2 &= D_k(C_2) \oplus C_1 = B \oplus C_1 = 0001 \oplus 0101 = E(0100) \\
P'_3 &= D_k(C_3) \oplus C_2 = N \oplus C_2 = 1101 \oplus 0101 = I(1000) \\
P'_4 &= D_k(C_4) \oplus C_3 = K \oplus C_3 = 1010 \oplus 0001 = L(1011) \\
P'_5 &= D_k(C_5) \oplus C_4 = G \oplus C_4 = 0110 \oplus 1110 = I(1000) \\
P'_6 &= D_k(C_6) \oplus C_5 = I \oplus C_5 = 1000 \oplus 1010 = C(0010) \\
P'_7 &= D_k(C_7) \oplus C_6 = I \oplus C_6 = 1000 \oplus 1100 = E(0100) \\
P' &= IAEILICE
\end{aligned}
$$

- (d) Use your answer from 2(b). If the block C_2 is lost (receiver does not realize it is missing), what is the recovered plaintext?
  - Know Information:

$$
\begin{aligned}
C_0 &= B(0001) \\
C_1 &= F(0101) \\
C_2 &= B(0001) \\
C_3 &= O(1110) \\
C_4 &= K(1010) \\
C_5 &= M(1100) \\
C_6 &= M(1100) \\
IV &= 0101
\end{aligned}
$$

  - Decryption

$$
\begin{aligned}
P'_0 &= D_k(C_0) \oplus IV = N \oplus IV = 1101 \oplus 0101 = I(1000) \\
P'_1 &= D_k(C_1) \oplus C_0 = B \oplus C_0 = 0001 \oplus 0001 = A(0000) \\
P'_2 &= D_k(C_2) \oplus C_1 = N \oplus C_1 = 1101 \oplus 0101 = I(1000) \\
P'_3 &= D_k(C_3) \oplus C_2 = K \oplus C_2 = 1010 \oplus 0001 = L(1011) \\
P'_4 &= D_k(C_4) \oplus C_3 = G \oplus C_3 = 0110 \oplus 1110 = I(1000) \\
P'_5 &= D_k(C_5) \oplus C_4 = I \oplus C_4 = 1000 \oplus 1010 = C(0010) \\
P'_6 &= D_k(C_6) \oplus C_5 = I \oplus C_5 = 1000 \oplus 1100 = E(0100) \\
P' &= IAILICE
\end{aligned}
$$

Q4: Number Theory

- $X = 55199998, Y = 9998$
- To factorization n, we can use the prime numbers from 2 to $\sqrt{n}$, if n is divisible by one of the prime number p, then keep doing $n = n/p$ until n is not divisible, if the final result is not 1, it must be another prime number in the factorization.
- (a) Compute $43^Y \ mod \ 4286$ using the square-and-multiply method.
  - Compute binary of Y:

$$9998_{10} = 10011100001110_2 = 2^{13} + 2^{10} + 2^9 + 2^8 + 2^3 + 2^2 + 2^1$$

  - Square and multiply:
    - Code:

```python
temp = 43
for i in range(14):
    x = (temp**2)%4286
    print("43^(2^%d) mod 4286 = %d^2 mod 4286 = %d"%(i+1, temp, x))
    temp = x
```

- Result:

```
[03/02/20]seed@VM:~/Desktop$ python calc.py
43^(2^1) mod 4286 = 43^2 mod 4286 = 1849
43^(2^2) mod 4286 = 1849^2 mod 4286 = 2859
43^(2^3) mod 4286 = 2859^2 mod 4286 = 479
43^(2^4) mod 4286 = 479^2 mod 4286 = 2283
43^(2^5) mod 4286 = 2283^2 mod 4286 = 313
43^(2^6) mod 4286 = 313^2 mod 4286 = 3677
43^(2^7) mod 4286 = 3677^2 mod 4286 = 2285
43^(2^8) mod 4286 = 2285^2 mod 4286 = 877
43^(2^9) mod 4286 = 877^2 mod 4286 = 1935
43^(2^10) mod 4286 = 1935^2 mod 4286 = 2547
43^(2^11) mod 4286 = 2547^2 mod 4286 = 2491
43^(2^12) mod 4286 = 2491^2 mod 4286 = 3239
43^(2^13) mod 4286 = 3239^2 mod 4286 = 3279
43^(2^14) mod 4286 = 3279^2 mod 4286 = 2553
```

- Final answer:

```
>>> ((1849*2859*479*877*1935*2547*3279)) % 4286
2401L
```

$$43^{9998} = (1849 * 2859 * 479 * 877 * 1935 * 2547 * 3279)\ mod\ 4286 = 2401$$

- (b) Calculate $\phi(Y)$.
  - Factorization: $9998 = 2 * 4999$
  - According to Fermat's little theorem $\phi(9998) = (2-1) * (4999-1) = 4998$
- (c) $gcd(X, 928374827)$
  - By using the following code:

```python
def gcd(a,b):
    if a > b :
        if b == 1:
            return a
        print("%d = %d*%d+%d"%(b, a, a/b,a%b))
        x = gcd(b, a%b);
        return x
    else:
        return gcd(b,a)
print(gcd(55199998, 928374827))
```

- Get the answer 9:

  ```
  928374827 = 55199998*16+45174859
  55199998 = 45174859*1+10025139
  45174859 = 10025139*4+5074303
  10025139 = 5074303*1+4950836
  5074303 = 4950836*1+123467
  4950836 = 123467*40+12156
  123467 = 12156*10+1907
  12156 = 1907*6+714
  1907 = 714*2+479
  714 = 479*1+235
  479 = 235*2+9
  235 = 9*26+1
  ```

- (d) Find integers x and z such that $x \cdot X + z \cdot 928374827 = gcd(X, 928374827)$.

$$
\begin{gathered}
1 = 235 - 9 * 26 \\
= 235 - (479 - 235 * 2) * 26 = -479 * 26 + 235 * 53 \\
= -479 * 26 + (714 - 479 * 1) * 53 = -479 * 79 + 714 * 53 \\
= -(1907 - 714 * 2) * 79 + 714 * 53 = -1907 * 79 + 714 * 211 \\
= -1907 * 79 + (12156 - 1907 * 6) * 211 = 12156 * 211 - 1907 * 1345 \\
= 12156 * 211 - (123467 - 12156 * 10) * 1345 = -123467 * 1345 + 12156 * 13661 \\
= -123467 * 1345 + (4950836 - 123467 * 40) * 13661 = 4950836 * 13661 - 123467 * 547785 \\
= 4950836 * 13661 - (5074303 - 4950836 * 1) * 547785 \\
= -5074303 * 547785 + (10025139 - 5074303 * 1) * 561446 \\
= 10025139 * 561446 - 5074303 * 1109231 \\
= 10025139 * 561446 - (45174859 - 10025139 * 4) * 1109231 \\
= -45174859 * 1109231 + 10025139 * 4998370 \\
= -45174859 * 1109231 + (55199998 - 45174859 * 1) * 4998370 \\
= 55199998 * 4998370 - 45174859 * 6107601 \\
= 55199998 * 4998370 - (928374827 - 55199998 * 16) * 6107601 \\
= -928374827 * 6107601 + 55199998 * 102719986
\end{gathered}
$$

  - Therefore, $x = 102719986$, $z = -6107601$
- (e) Choose any prime number Z that is smaller than X. Calculate $X^X \ mod \ Z$
  - Choose $Z = 2$
  - $X^X \ mod \ Z = (X \ mod \ Z)^X \ mod \ Z = (55199998 \ mod \ 2)^{55199998} \mod 2 = 0$

Q5: El-Gamal

- $p = 19$ and $g = 3$
- (a) Suppose the private key is $x = 7$. Compute the public key y.

  $y = g^x \ mod \ p = 3^7 \ mod \ 19 = ((3^3 \mod 19)^2 \mod 19) * (3 \mod 19) \mod 19 = 2$

- (b) Encrypt the message $M = 8$ using the public key above and $r = 8$.

$$A = g^r \ mod\ p = 3^8 \ mod\ 19 = 6$$
$$B = My^r \ mod\ p = 8 * 2^8 \ mod\ 19 = 2^{11} \ mod\ 19 = 15$$
$$C = (A, B) = (6, 15)$$

- (c) Verify your calculation in part (b) above by decrypting the ciphertext you obtained in part (b)

$$K = A^x \ mod\ p = 6^7 \ mod\ 19 = 9$$

  - Find $9^{-1}(mod\ 19)$

$$19 = 2 * 9 + 1$$
$$19 - 2 * 9 = 1$$
$$19 + 19 * 9 - 2 * 9 = 19 * 9 + 1$$
$$19 + 17 * 9 = 19 * 9 + 1$$
$$17 * 9 \equiv 1(mod\ 19)$$
$$K^{-1} = 17$$

  - Decrypt:

$$M = BK^{-1} \ mod\ p = (15 * 17) \ mod\ 19 = 8$$

Q6: Diffie-Hellman

- $p = 19$ and $g = 10$.
- (a) Alice picks $x = 7$, and Bob picks $y = 11$. What is the shared key K resulting from the exchange?

$$\phi(19) = 18, gcd(19, 10) = 1$$
$$K = g^{xy} \ mod\ p = 10^{7*11} \ mod\ 19$$
$$= 10^{77} \ mod\ 19 = 10^{4*18+5} \ mod\ 19 = 10^5 \ mod\ 19 = 3$$

- (b) How would you modify the exchanged messages in DH to prevent its main weakness? Clearly state all your assumptions (including any additional cryptographic algorithm or material needed) and the notation you used.
  - The weakness of DH is man-in-middle attack
  - To avoid this weakness, we can use **digital signature(implemented by RSA)** to sign the shared public key:
    - Set up:
      - $n = p'q'$, where $p'q'$ are large prime (say 512 bits long each)
      - select e such that $gcd(e, (p-1)(q-1)) = 1$
      - $ed \equiv 1 \ mod\ (p-1)(q-1)$
      - Signing (Private) Key : d
      - Verification (Public) Key :(e, n)
    - Signature Generation: $S = A^d \ mod\ n$, where A is the shared public key
    - Signature Verification: If $S^e \ mod\ n = A$, output valid; otherwise, output invalid(i.e. it is changed by man-in-middle attack)
    - Assumption, the shared public keys: $A = g^x \ mod\ p$, $B = g^y \ mod\ p$ satisfy $A < n, B < n$