

# Expected Utility

PS 171B - Week 2

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# Utility Functions

Take an **outcome** as an input, assigns it a **numerical value**.

Can only do this when preference orderings of outcomes are **transitive**.

## Knowledge Check

I strictly prefer Guinness to Heineken, strictly prefer Heineken to Bud Light, and strictly prefer Guinness to Bud Light. Can a utility function be assigned to my preferences? If so, give an example.

# Values and Beliefs

**Values** involve how much we (dis)like particular outcomes. However, we are often not able to pick outcomes with certainty. **Beliefs** help measure this uncertainty... given our choice of a certain action or policy, with what probability does an outcome occur?

# Expected Utility

Basically, how good do we expect the outcome of our action/policy to be?

Involves multiplying the conditional probability of all possible outcomes given the action taken by the utility of the outcome.

## Knowledge Check

Its Friday night, sections are done, time to go out. I've got two choices: Flaming Saddles or Cabo Cantina. My values and beliefs are as follows:

Values:  $U(G) = 10$ ,  $U(H) = 8$ ,  $U(B) = 1$

Beliefs:  $Pr(G|FS) = 0.5$ ,  $Pr(H|FS) = 1$ ,  $Pr(B|FS) = 1$ ,  
 $Pr(G|CC) = 0.25$ ,  $Pr(H|CC) = 0.75$ ,  $Pr(B|CC) = 1$

What are the expected utilities of choices? What set of beliefs would change this preference ordering?

# Practice Problem: Shepsle 2.6

You are presented with two pairs of lotteries, each with three outcomes:  $x = \$2.5$  million,  $y = \$0.5$  million, and  $z = \$0$ .

$P_1$  and  $P_2$  are the first pair, with probabilities  $P_1 = (p_1(x), p_1(y), p_1(z)) = (0, 1, 0)$  and  $P_2 = (0.1, 0.89, 0.01)$ .

$P_3$  and  $P_4$  are the second pair, with probabilities  $P_3 = (0, 0.11, 0.89)$  and  $P_4 = (0.1, 0, 0.9)$ .

Empirically, most people express strict preferences for  $P_1$  to  $P_2$  and  $P_4$  to  $P_3$ . Is this behavior consistent with the theory of expected utility?

**HINT:** You don't actually need to know anything about the utility function to solve this problem. Set up the empirical preferences as inequalities of expected utilities, and try to coax out a contradiction. Yes, this will involve algebra.

# Practice Problem: Group Preferences

Two more people have joined me on my Friday adventure: Alex and Bo. We've also added one more option: stay in. Our expected utilities are as follows:

EU	Derek	Alex	Bo
FS	14.0	8.5	10.0
CC	9.5	12.0	9.0
SI	8.0	10.0	12.5

Does the above profile of preferences give a transitive social ordering over policy alternatives?

# Practice Problem: More Groups

Let's say the previous expected utilities amongst group members were all due to differences in beliefs, but we value each outcome in the same fashion such that:

$$U(G) = 10, U(H) = 8, U(B) = 1$$

For now, let's remove Bud Light from our consideration... its basically water anyway. Suppose the **absence** of any beverage has a utility of 0 and that utilities are independent. Is there a transitive social ordering over outcomes?