Advanced Quantum Mechanics II

Midterm Exam

Spring Semester 2023

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1. Compute the eigenvalues and eigenvectors of the rotation operator for spin-$\frac{1}{2}$, that is $\mathcal{D}^{1/2}[R(\hat{n},\phi)]$, where $\hat{n} = (1,\theta,\varphi)$. Relate your results to the time evolution of a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B} = B_0\hat{n}$.

2. Consider a particle of mass m in the following spherical shell potential,

$$V(r) = \begin{cases} 0 & \text{for} \quad 0 \leq r < a \\ -V_0 < 0 & \text{for} \quad a \leq r \leq b \\ 0 & \text{for} \quad b < r \end{cases}$$

Find the eigenfunctions $\psi_{nlm}(r,\theta,\phi)$ and the associated eigenenergies. Consider both bound and continuum states. Explain clearly the radial and angular parts of the wave functions, and all of the quantum numbers that appear. Moreover, state clearly the unknown parameters and the conditions needed to specify them simultaneously. **(20 Points)**