Advanced Quantum Mechanics II

final Exam

Spring Semester 2023

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1. Consider a particle of mass m in the following spherical potential well,

$$V(r) = \begin{cases} -V_0 < 0 & \text{for} \quad r \leq a \\ 0 & \text{for} \quad a < r \end{cases}$$

Find the eigenfunctions $\psi_{nlm}(r, \theta, \phi)$ and the associated eigenenergies. Consider both bound and continuum states. Explain clearly the radial and angular parts of the wave functions, and all of the quantum numbers that appear. Moreover, state clearly the unknown parameters and the conditions needed to specify them simultaneously. (**25 Points**)

2. Consider the proton to be a spherical shell of charge of radius R. Using first order perturbation theory calculate the change in the binding energy of hydrogen due to the non-point-like nature of the proton. Does the sign of your answer make sense physically? Explain. Note: You may use the approximation $R << a_0$ throughout this problem, where a_0 is the Bohr radius. (**25 Points**)

3. Choose appropriate sections of sinusoidal functions (half or one full cycle) to be used as the trial (variational) wave functions for the ground state and the first excited state of the potential $V(x) = A|x|$. Let the spatial extent of trial wave functions be denoted by 'a_0' and 'a_1', respectively. Draw a diagram clearly indicating the potential and the trial wave functions. Find an upper bound for the energy of the first excited state using the variational principle. Note: we can also use the variational method for the first excited state provided our trial wavefunction is orthogonal to the exact wavefunction of the ground state. Comment on the validity of your method for the first excited state.(**25 Points**)

4. Consider a particle of mass m in a one-dimensional simple harmonic potential with natural frequency ω_0. At $t = 0$ the oscillator is known to be in its ground state. The oscillator is also acted on by the following perturbation potential,

$$V(x) = Axe^{-t/\tau}, \quad 0 \leq t < \infty.$$

Using first order perturbation theory, calculate the probability for the oscillator to make transition to any other state at $t > 0$. (**25 Points**)

You might find the following relations useful:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega_0}}(\mp ip_x + m\omega_0 x), \quad a_+\psi_n = \sqrt{n+1}\psi_{n+1}, \quad a_-\psi_n = \sqrt{n}\psi_{n-1}, \quad \psi_{1,0,0} = \sqrt{\frac{1}{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$