Nuclear and Elementary Particle Physics

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Problem Set 4

This is an optional problem set for a maximum of two points as extra credit, to be added to your final grade. You can solve and send them to us by 11:59 PM of 2nd Bahman.

1. Suppose we have a collection of protons and neutrons at temperature T. Explain how and which heavier nuclei can be produced, as the temperature drops. Such a mechanism, called the Big Bang Nucleosynthesis, is believed to have occurred in the early universe, and after the recombination, these nuclei have become neutral atoms.

2. Using atomic mass tables in Atomic Mass Data Center (nds.iaea.org/amdc), compute the binding nergy of the "last neutron" that is, the energy required to remove a neutron from the following nuclei.

   (a) H^2

   (b) C^{13}

   (c) U^{235}

3. Complete the following reactions and determine their Q-values.

   (a) $Al^{27}(d,p)$

   (b) $Li^6(p,\alpha)$

   (c) $U^{235}(n,2n)$

   (d) $U^{236}(\alpha,n)$

4. In nuclear reactors a newly-formed radioactive isotope A may be transformed into another isotope B by neutron absorption befor it has had an opportunity to decay. Neutron absorption occurs at a rate proportional to the amount of isotope A present in the system. If the proportionality constant is denoted by c, and the rate of production (atoms of A/sec) is denoted by $R(t)$, show that the number of atoms of isotope A present in the reactor at time t is given by

$$n(t) = n_0 e^{-(\lambda+c)t} + e^{-(\lambda+c)t} \int_0^t e^{(\lambda+c)t'} R(t') dt', \tag{1}$$

   where n_0 is the number of atoms of A present at $t=0$.

5. Estimate quantitatively the neutrino flight path required for neutrino oscillations.

   (a) Consider first the oscillation, mainly between ν_μ and ν_τ, mediated by θ_{23}. Assume a pure ν_μ source. Using the parameters of this oscillation given in the NuFIT (nu-fit.org), compute the position of the first maximum for ν_τ appearance and the position of the succeeding zero, for ν_μ energies of 1 GeV and 20 GeV (for neutrinos from an accelerator source).

(b) Now consider the oscillation between ν_e and other species that gives rise to the oscillation of solar neutrinos. Compute the position of the first maximum for ν_μ appearance (or maximal ν_e disappearance) and the position of the succeeding zero, for ν_e energies of 1 MeV (for reactor or solar neutrinos) and 1 GeV and 20 GeV.

6. This problem will give you a chance to dip into the tables of elementary particle properties produced by the Particle Data Group (pdg.lbl.gov) and to use this information to understand better the systematics of ψ family particle decays.
To work this problem, you should recall that a decay rate in quantum mechanics is given by a partial width $\Gamma(\mathrm{A}\to\mathrm{f})$, with units of energy. A partial width gives the rate of a basic quantum mechanical process. The total width of a resonance is

$$\Gamma_A = \mathrm{X\ f}\ \Gamma_A(\mathrm{A}\to\mathrm{f})$$

That is, it is the sum of the rates for all possible decay processes. The lifetime of the resonance is $\tau = \hbar/\Gamma_A$. The branching ratio to the decay channel f, the probability that a particular decay of A gives the final states f, is

$$\mathrm{BR}(\mathrm{A}\to\mathrm{f}) = \Gamma(\mathrm{A}\to\mathrm{f})/\Gamma_A$$

Usually, it is easiest to meaure branching ratios, but the real physics is in the actual rates. To ob tain these, we must extract the partial widths from the information that we are given.

(a) The J/ψ can decay in four different ways. (1) decay by $c\bar{c}$ annihilation directly to hadrons, (2) decay by $c\bar{c}$ annihilation to a virtual photon (a short-lived state of electromagnetic fields), which then materializes into an e^+e^- or μ^+ μ^- pair. The J/ψ is produced in e^+e^- annihilation by e^+e^- annihilation into a virtual photon which then materializes as a J/ψ. This decay is the reverse of that process, (3) decay by $c\bar{c}$ annihilation to a virtual photon, which then materializes into hadrons, (4) decay to 1 photon plus hadrons. There is also a decay to 3 photons with a very small branching ratio (about 10^5).
Look up the listing for the J/ψ at the Particle Data Group (pdg.lbl.gov). The heading "pdgLive" gives the most recently updated information. Look under $c\bar{c}$ to find the information for the J/ψ. The entry J/$\psi\to$ggg gives the branching ratio for direct decays to hadrons, mode (1) above. Similarly, the entry J/$\psi\to\gamma$gg gives the branching ratio for mode (4) above. Write the branching ratio for each of the decay modes (1)–(4). (These should add up to 100%, within the measurement errors.) Using the tabulated total width, find the partial width for each channel.

(b) The ψ(2S) can decay by the 4 modes above and also by 3 additional modes: (5) decay to the heavy lepton τ^+ τ^-, (6) decay to J/ψ plus hadrons (π π, π θ , or η), (7) radiative decay to the 1P states χ_c. Using the information in the entry for the ψ(2S), write the branching ratio for each of the decay modes (1)–(7). (Again, these should add up to 100%, within the measurement errors.) Using the tabulated total width, find the partial width for each channel.

(c) Compute the ratios of the partial widths be tween the J/ψ and the ψ(2S) for each of the processes (1)–(4). How do these ratios compare? Why would this result be expected?