

Quantum Field Theory I

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Problem Set 2

Please answer the following questions and provide us by the end of the TA class on Esfand 22.

1. Show vector Klein-Gordon Lagrangian has global $SO(3)$ ($\vec{\chi} \rightarrow \vec{\chi}' = e^{\frac{-i}{\hbar} \vec{\theta} \cdot \vec{J}}$) symmetry and then find Noether current of this symmetry.

$$\mathcal{L} = \frac{1}{2} \partial^\mu \vec{\chi} \partial_\mu \vec{\chi} - \frac{m^2}{2} \vec{\chi} \cdot \vec{\chi}, \quad (1)$$

where $\vec{\chi} = (\chi_1 \ \chi_2 \ \chi_3)^T$.

2. Please answer to problem 3.1 of [Sch14]
3. Please answer to problem 3.5 of [Sch14]
4. Please answer to problem 3.6 of [Sch14]
5. Find canonical energy-momentum tensor for electromagnetic field using equation 3.35 of [Sch14].
6. (Bonus) Find Noether current of Lorentz $SO(1, 3)$ symmetry in general Lagrangian.
7. (Bonus) Find symmetric energy-momentum tensor for electromagnetic field using variation by metric.

References

- [Sch14] Matthew Dean Schwartz. *Quantum field theory and the standard model*. Cambridge University Press, New York, 2014.