General Physics II

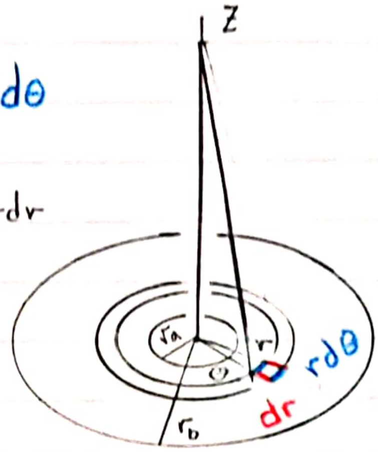
Solutions to the Midterm Exam — Spring 2023

1 a) $V(z) = \int_{r_a}^{r_b}\int_{0}^{2\pi} \frac{k\,dq}{(r^2+z^2)^{1/2}}$, $dq = \sigma\, dA = \sigma\, dr\, r\,d\theta$

$= 2\pi k\sigma \int_{r_a}^{r_b} \frac{r\,dr}{(r^2+z^2)^{1/2}}$, let $u = r^2+z^2 \Rightarrow du = 2r\,dr$

$= \pi k\sigma \int \frac{du}{\sqrt{u}} = \pi k \frac{u^{1/2}}{1/2} = 2\pi k\sqrt{r^2+z^2}\Big|_{r_a}^{r_b}$

$= 2\pi k\sigma\left(\sqrt{r_b^2+z^2} - \sqrt{r_a^2+z^2}\right)$



$\sigma = \frac{q}{\pi(r_b^2-r_a^2)}$

b) $\vec{E}(z) = -\vec{\nabla}V(z) = -\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)V(z)$

$= -\hat{z}\frac{d}{dz}V(z) = +\hat{z}\,\frac{2\pi k q}{\pi(r_b^2-r_a^2)}\left(\frac{z}{\sqrt{r_a^2+z^2}} - \frac{z}{\sqrt{r_b^2+z^2}}\right)$

c) $V(0) = \frac{2\pi k q}{r_b^2-r_a^2}(r_b-r_a) = \frac{2\pi k q}{r_b+r_a}$, $\vec{E}(0) = 0 \Leftrightarrow$ Symmetry

$V(z) = \frac{2\pi k q}{\pi(r_b^2-r_a^2)}\left[z\sqrt{1+\left(\frac{r_b}{z}\right)^2} - z\sqrt{1+\left(\frac{r_a}{z}\right)^2}\right] \xrightarrow[z\gg r_a]{z\gg r_b} \frac{2kq}{r_b^2-r_a^2}\, z\left[\cancel{1}+\frac{r_b^2}{2z^2} - \cancel{1} - \frac{r_a^2}{2z^2}\right]$

$= \frac{2kq}{r_b^2-r_a^2}\,\frac{r_b^2-r_a^2}{2z} = \frac{kq}{z}$ looks like a point charge.

$E_z(z) = \frac{2kq}{r_b^2-r_a^2}\left[\left(1+\frac{r_a^2}{z^2}\right)^{-\frac{1}{2}} - \left(1+\frac{r_b^2}{z^2}\right)^{-1/2}\right] \xrightarrow[z\gg r_a]{z\gg r_b} \frac{2kq}{r_b^2-r_a^2}\left[\left(\cancel{1}-\frac{r_a^2}{2z^2}\right) - \left(\cancel{1}-\frac{r_b^2}{2z^2}\right)\right]$

$= \frac{kq}{r_b^2-r_a^2}\,\frac{r_b^2-r_a^2}{z^2} = \frac{kq}{z^2}$ looks like a point charge.

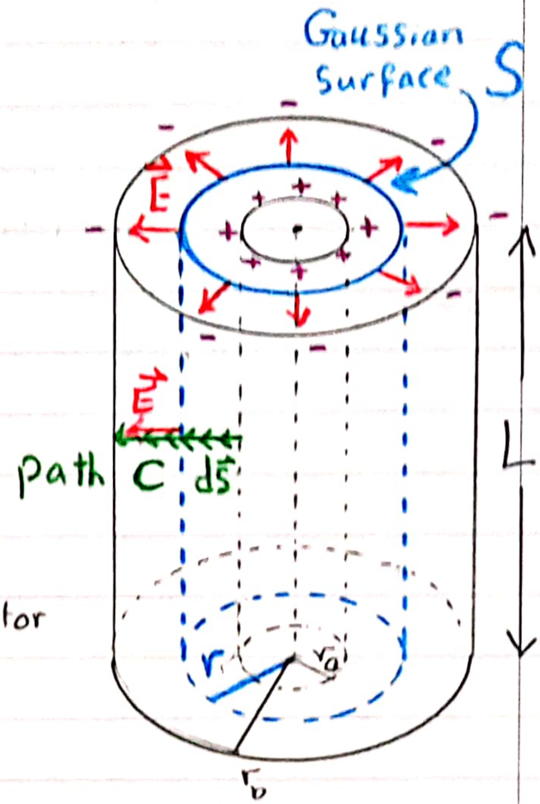
2. a) $\oint_S \vec{E}\cdot d\vec{A} = EA = E\,2\pi r L = \dfrac{q_{in}}{\epsilon_0} = \dfrac{\lambda L}{\epsilon_0}$

$$\Rightarrow E = \frac{\lambda \cancel{L}}{2\pi\epsilon_0 \cancel{L}}\frac{1}{r} \quad \text{for} \quad r_a < r < r_b$$

b) $\Delta V = \int_{C^+}^{-} \vec{E}\cdot d\vec{s} = \int_{r_a}^{r_b} \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{dr}{r} = \dfrac{\lambda}{2\pi\epsilon_0}\ln\left(\dfrac{r_b}{r_a}\right)$

$$\equiv \frac{q}{C_c} = \frac{\lambda L}{C_c}$$

$$\Rightarrow C_c = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)} = \text{Capacitance of a Cylindrical capacitor}$$

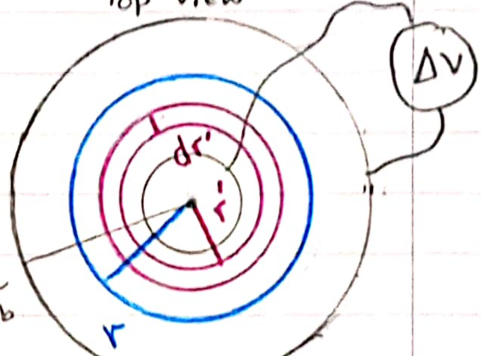


c) $U(r) = \int u(r')\,dV \quad \int_0^r \frac{1}{2}\epsilon_0 \vec{E}(r')^2\,dV$

since $E=0 \Rightarrow$ for $r<r_a$
$$= \int_{r_a}^{r}\left[\frac{1}{2}\epsilon_0\left(\frac{\lambda}{2\pi\epsilon_0 r'}\right)^2\right] L\,2\pi r'dr'$$

$$= \frac{\lambda^2 L}{4\pi\epsilon_0}\int_{r_a}^{r}\frac{dr'}{r'} = \frac{\lambda L}{2}\,\frac{\lambda}{2\pi\epsilon_0}\ln\frac{r}{r_a}$$

Top View



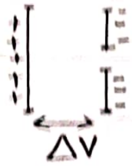
For $r \to r_b \Rightarrow U(r) = \dfrac{1}{2}\lambda L\,\dfrac{\lambda L}{\dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}} = \dfrac{1}{2}q\,\Delta V$

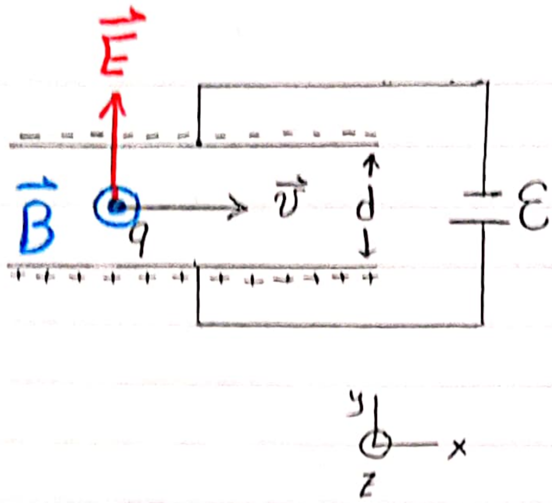
$$= \frac{1}{2}q\,\frac{q}{C_c} = \frac{1}{2}q\,\Delta V$$

✓

3 Electric gun:

$$q\Delta V = \frac{1}{2}m(v^2 - \cancel{v_0^2}^{\,0})$$





No deflection $\Rightarrow$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0 \Rightarrow \vec{B} = \frac{E}{v}\hat{z}$$

$$\Rightarrow B = \frac{E}{\sqrt{2q\Delta V/m}} = \frac{\mathcal{E}/d}{\sqrt{2q\Delta V/m}}$$