Maximum Likelihood

Assume that we are given a coin, and we need to find out whether the coin is biased or not (i.e. whether it's more likely to land heads than tails). Furthermore, we also want to find out how biased the coin is (i.e. how likely it will land heads the next time we flip it). In this exercise, we will use the maximum likelihood principle to determine the bias of a coin from the observed data.

Let $c \in \{0, 1\}$ be a random variable associated with a flip of a coin (0-tails,1-heads). The probability that a coin lands heads is denoted by a parameter μ, which is what we want to find. The distribution of a random variable c can then be written as: $P(c; \mu) = \mu^c(1 - \mu)^{1-c}$. Now imagine that we flip a coin Ntimes and denote the outcome of the i^{th} flip by a variable $c^{(i)}$. We assume that all flips are independent of each other. Also, assume that we observe heads Htimes.

Your task is to write a function MaxLikelihood that computes the maximum likelihood value and the maximum likelihood parameter of the observed data $D = \{c^{(1)}, c^{(2)}, ..., c^{(N-1)}, c^{(N)}\}$. Maximum likelihood parameter denotes a parameter μ_{max} that explains the observed data the best. In the context of our problem, finding the parameter that corresponds to the maximum likelihood would allow us to determine how biased the coin is.

As its inputs, your function should take a vector of possible parameters μ, a number of heads H that we observed in data D, and a total number of coin flips N. It should then do three things:

1) First, you will need to compute a likelihood value for every parameter μ in the provided input vector variable mu_list. Store all of these values into a vector variable L_list.

2) Second, select the maximum likelihood value max_L from the previously computed values in L_list.

3) Finally, select the maximum likelihood parameter max_mu by finding the parameter that is associated with the maximum likelihood value max_L.

Your Function

Save Reset MATLAB Documentation (https://www.mathworks.com/help/)

```
function [max_L,max_mu]=MaxLikelihood(mu_list,H,N)
    % compute the maximum likelihood value and its parameters
    %
    % Input:
    % - mu_list: a vector of possible parameter values, e.g. mu_list=[0:0.01:1]
    % - H: number of Heads
    % - N: number of total coin flips
    % Output:
    % - max_L: maximum likelihood value over all the parameters mu_list
    % - max_mu: a parameter value corresponding to the maximum likelihood value max_L

    % Compute a vector of likelihoods L_list
    % Every element in L_list should store a likelihood value
    % associated with its respective parameter from mu_list.
    s = H; % sum
    L_list= mu_list.^s .* (1-mu_list).^(N-s);

    % Compute the maximum likelihood value by selecting a maximum
    % value from L_list

    [m, idx] = max(L_list);

    max_L= m;

    % Find the parameter mu that corresponds to the maximum
    % likelihood value
    max_mu= mu_list(idx);
end

```

Code to call your function

Reset

```
H=10; N=25;
mu_list=[0:0.01:1];
[max_L,max_mu]=MaxLikelihood(mu_list,H,N);

```

▶ Run Function

Previous Assessment: All Tests Passed

Submit

- ✓ **Is Maximum Likelihood Value Correct?**
- ✓ **Is Maximum Likelihood Parameter Correct?**