Scalar Particles in Lattice QCD[1]

SCALAR Collaboration
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Abstract

We report a project to study scalar particles by lattice QCD simulations. After a brief introduction of the current situation of lattice study of the sigma meson, we describe our numerical simulations of scalar mesons, σ and κ. We observe a low sigma mass, $m_\pi < m_\sigma \leq m_\rho$, for which the disconnected diagram plays an important role. For the kappa meson, we obtain higher mass than the experimental value, *i.e.* $m_\kappa \sim 2m_{K^*}$.

1 Introduction

The objective of our collaboration is to understand scalar mesons in the framework of QCD. The confidence level of the sigma meson has been increasing, and an other scalar meson [1, 2], κ, has been reported by several experimental groups. In the modern hadron physics based on QCD, the chiral symmetry is an important ingredient and the sigma meson plays an essential role in it together with the pion.

The existence of the sigma meson was obscure for many years. The re-analyses of the π-π scattering phase shift have suggested a pole of the σ meson with $I = 0$ and $J^{PC} = 0^{++}$[3]. In this analysis, the chiral symmetry, analyticity, unitarity and crossing symmetry are taken into account. Contributions of the σ pole were observed in the decay processes such as D$\rightarrow \pi\pi\pi$[4] and $\Upsilon(3S) \rightarrow \Upsilon\pi\pi$ [5]. In 1996 PDG(Particle Data Group), "f_0(400-1200) or σ" appeared below 1 GeV mass region, and "f_0(600) or σ" in the 2002.

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If the sigma meson exists, it is natural to consider the κ meson as a member of the nonet scalar states of chiral SU(3)$\otimes$SU(3) symmetry. Recently, the κ with $I = 1/2$ is reported with mass $m_\kappa \sim 800$ MeV [6, 7].

We believe that it is very important now to investigate these scalar mesons by lattice QCD in order to establish scalar meson spectroscopy as a sound and important piece of hadron physics. Lattice QCD provides a first-principle approach of hadron physics, and allows us to study non-perturbative aspects of quark-gluon dynamics. It is a relativistic formulation, and quarks are described as Dirac fermions. It is *not* a model, and apart from numerical limitations, there are no approximations. It is not a bound state calculation: neither a potential model nor Bethe-Salpeter calculation.

Lattice QCD is usually formulated in the Euclidean path integral as

$$Z = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \bar{\psi}D\psi} = \int \mathcal{D}U \det D\, e^{-S_G}, \tag{1}$$

where S_G is the gluon kinetic action, whose continuum limit is $-\int d^4x \mathrm{Tr} F_{\mu\nu}^2/4$ We construct a state with a definite quantum number and measure the decay in the channel,

$$G(x,y) = \frac{1}{Z}\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi H(y)H(x)^\dagger\, e^{-S_G - \bar{\psi}D\psi} = \langle H(y)H(x)^\dagger\rangle \to e^{-m|x-y|} \tag{2}$$

where $H(x)$ is a hadron operator. For scalar mesons, $H(x) = \bar{\psi}_\nu^{af_1}\Gamma_{f_1f_2}\psi_\nu^{af_2}$. The indices a, f_i and ν stand for color, flavor and Dirac indices, respectively. $H(x)^\dagger|0\rangle$ is a state whose quantum number is specified by the operator H.

We must be very careful of the limitation of the present lattice QCD calculations:

1. Sufficient statistics: Gauge configurations are generated by the Monte Carlo simulation, and there are statistical errors like experimental data.

2. Continuum limit: Numerical simulations are performed at a finite lattice spacing, a, and we must take $a \to 0$ limit at the end.

3. Infinite volume limit: Lattice volume should be large enough to include a hadron.

4. Chiral extrapolation: u and d quark masses on the current lattice are still large and extrapolated to zero.

The last point may cause systematic bias. π mesons are not sufficiently light and our sigma meson cannot decay into 2 π, *i.e.*, its width is zero. Any other particle, *e.g.* ρ, Δ and N^*, also has zero width in lattice QCD calculations in the literature. In the case of the sigma meson, this flaw should be kept in mind, since the two-pion component may be important.

There have been several attempts at lattice study of sigma mesons. To our knowledge, the first such calculation was carried out by deTar and Kogut [8], where the so-called disconnected diagram, or the OZI forbidden type diagram is discarded. The channel was called 'valence' sigma, σ_V. They measured the screening masses and observed that σ_V is much heavier than the π meson at the zero temperature, while σ_V and π degenerate as the temperature increases over T_c. Kim and Ohta calculated the valence sigma mass with staggered fermions for the lattice spacing $a = 0.054$ fm and lattice size $48a = 2.6$ fm [9]. They obtained $m_\sigma/m_\pi = 1.4 \sim 1.6$ by varying $m_\pi/m_\rho = 0.65 \sim 0.3$.

Lee and Weingarten [10] have stressed the importance of the mixing the scalar meson and glueballs and concluded that $f_0(1710)$ is the lightest scalar glueball dominant particle, while $f_0(1390)$ is composed of mainly the u and d quarkonium. Alford and Jaffe analyzed the possibility that the sigma particle is an exotic state, i.e., $qq\bar{q}\bar{q}$ by a lattice QCD calculation [11].

All these calculations are in the quench approximation, *i.e.*, the fermion determinant in Eq.(1) is dropped, which corresponds to ignoring quark pair creation and annihilation diagrams. McNeile and Michael observed that the σ meson masses in the quench approximation and in the full QCD simulation are very different [12]. They also considered the mixing with the glueballs. They obtained a very small sigma meson mass, even smaller than the π mass, in the full QCD case.

There are two ongoing projects of σ meson spectroscopy : Riken-Brook haven-Columbia (RBC) collaboration [13] and Scalar collaboration [14, 15]. The two approaches are complementary. The RBC collaboration employs the domain wall fermions, which respect the chiral symmetry, but include a quench approximation, while Scalar collaboration uses Wilson fermions, which break the chiral symmetry at a finite lattice spacing, but performs the full QCD simulation. RBC reported their simulation at $a^{-1} = 1.3$ GeV on a $16^3 \times 32$ lattice. They remedied the quench defect with the help of the chiral perturbation. They observed that the masses of the non-isosinglet scalar (a_0) and the singlet scalar (σ) are almost degenerate when the quark mass is heavy (above s quark mass regions), and that as the quark mass decreases, the a_0 mass remains almost constant, but the mass of the σ decreases.

2 Propagators

The quantum numbers of the σ meson are $I = 0$ and $J^{PC} = 0^{++}$; we adopt the σ meson operator of

$$\hat{\sigma}(x) \equiv \frac{\bar{u}(x)u(x) + \bar{d}(x)d(x)}{\sqrt{2}}, \tag{3}$$

where u and d indicate the corresponding quark spinors, and we suppress the color and Dirac indices. The σ meson propagator is written as

$$G_\sigma(y,x) \quad = \quad \frac{1}{Z}\int \mathcal{D}U\mathcal{D}\bar{u}\mathcal{D}u\mathcal{D}\bar{d}\mathcal{D}d \left(\hat{\sigma}(y)\hat{\sigma}(x)^\dagger\right) e^{-S_G - \bar{u}Du - \bar{d}Dd}. \tag{4}$$

By integrating over u, $\bar{u}$, d and $\bar{d}$ fields, the σ meson propagator is given by

$$\begin{aligned} G_\sigma(y,x) \quad = - \quad & \langle \mathrm{Tr}D^{-1}(x,y)D^{-1}(y,x)\rangle \\ & +2\langle(\sigma(y) - \langle\sigma(y)\rangle)(\sigma(x) - \langle\sigma(x)\rangle)\rangle \end{aligned} \tag{5}$$

where $\sigma(x) \equiv \mathrm{Tr}D^{-1}(x,x)$. "Tr" represents a summation over color and Dirac spinor indices. In Eq.(5), D^{-1}'s are u and d quark propagators. Here we assumed that the u and d quark propagators are equivalent because u and d quark masses are almost the same. From Eq.(5), we can see that the σ propagator consists of two terms. The first term corresponds to the connected diagram, *i.e.*, a $\bar{q}q$-type meson.

The second term is the "disconnected" diagram; it is the correlation of $\sigma = \mathrm{Tr}D^{-1}$ at two points x and y. The term "disconnected" is not appropriate since the corresponding matrix element is, of course, not factorized; quark lines are connected by gluon interactions. Nevertheless, we use this jargon in the following.

The quantum number of the σ meson ($I = 0$ and $J^P = 0^+$) is the same as that of the vacuum, and the vacuum expectation value of the σ operator, $\langle\sigma(x)\rangle$, does not vanish. Therefore, the contribution of $\langle\sigma(x)\rangle$ should be subtracted from the σ operator.

For κ^+, the operator is $H(x) = \sum_a \sum_\nu \bar{s}^a_\nu u^a_\nu$, and we have only the connected diagram,

$$G_\kappa(y,x) = -\langle \mathrm{Tr}D_s^{-1}(x,y)D^{-1}(y,x)\rangle, \tag{6}$$

where D_s^{-1} is the s quark propagator.

3 Numerical simulations

In this project, we intend to use standard well-established techniques for numerical calculations, and want to see the outcome. We employ Wilson fermions and the plaquette gauge action. Full QCD simulations were done by the Hybrid Monte Carlo (HMC) algorithm.

CP-PACS performed a very large-scale simulation of light meson spectroscopy in the full QCD calculation [16]. We use here the same values of the simulation parameters, *i.e.*, $\beta = 4.8$ and $\kappa = 0.1846$, 0.1874, 0.1891, except lattice size; our lattice, $8^3 \times 16$, is smaller. We employ the point source and sink; smaller lattice size leads to larger mass due to a higher state mixture. In other words, our mass values on the small size lattice should be considered as the upper limit. We have checked that the values of m_π and m_ρ are consistent with those of CP-PACS.

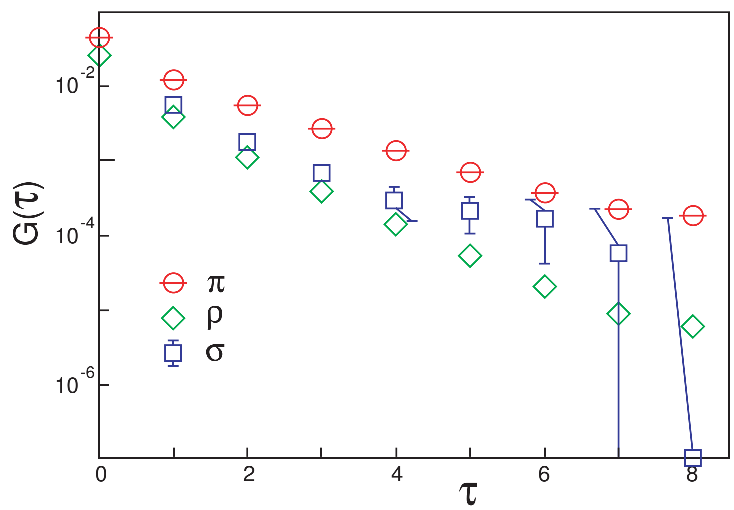

Figure 1: Propagators of π, ρ and σ for $\kappa = 0.1891$.

It is very difficult to evaluate the disconnected part of the propagator, since we must calculate $\mathrm{Tr}D^{-1}(x,x)$ for all lattice sites x. We used the Z_2 noise method to calculate the disconnected diagrams and the subtraction terms of the vacuum $\langle\sigma\rangle$. Each of these terms is of the order of ten, and $\langle(\sigma - \langle\sigma\rangle)(\sigma - \langle\sigma\rangle)\rangle$ becomes less than 10^{-4}, as shown in Fig.2. Therefore, high accuracy is required for the calculation. One thousand random Z_2 numbers are generated. Our numerical results show that the values of the first and the second terms in Eq.(5) are of the same order. Therefore, in order to obtain the signal correctly as the difference between these terms, high-precision numerical simulations and careful analyses are required. We have investigated the relationship between the amount of Z_2 noise and the achieved accuracy in Ref.[14]. Gauge configurations were created by HMC in the SX5 vector supercomputer, and most disconnected propagator calculations by the Z_2 noise method were mainly performed on the SR8000 parallel machine at KEK.

The propagators of π, ρ and σ for $\kappa = 0.1891$ are shown in Fig.1. The connected and disconnected parts of the σ propagator are shown in Fig.2. It is difficult to obtain σ propagator at large τ since the precision of our calculation is limited to $O(G(\tau)) \sim 10^{-4}$.

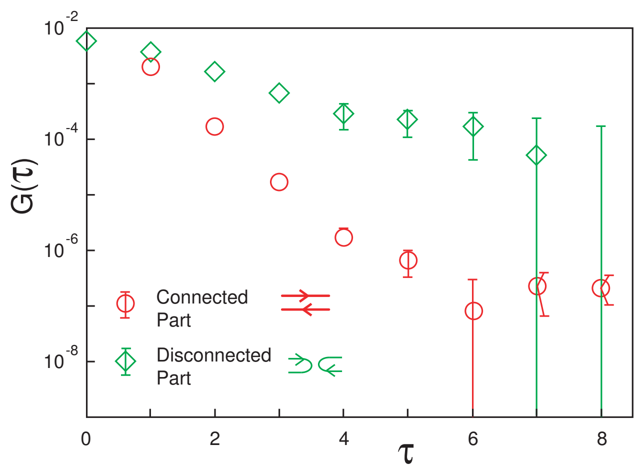

Figure 2: Propagators of the connected and disconnected diagram of σ for $\kappa = 0.1891$.

From our results, we evaluate the critical value of the hopping parameter $\kappa_c = 0.195(3)$ and lattice space $a = 0.207(9)$ fm (CP-PACS has obtained $\kappa_c = 0.19286(14)$ and $a = 0.197(2)$ fm). Figure 3 (left) shows masses of ρ, σ and π as functions of $1/\kappa$. We find that m_σ/m_ρ at the chiral limit is 0.33 ± 0.09.

Finally, we present our preliminary result for κ meson using the same common configurations. The s quark is treated as a valence, *i.e.*, is used only in the propagator (6), not as a sea quark. We adopt the same hopping parameter values, $\kappa = 0.1846$, 0.1874 and 0.1891 for u and d quarks. We calculate three values of the hopping parameter for the s quark: $\kappa_s = 0.1835$, 0.1840 and 0.1845. For each κ_s, we calculate masses of κ, K^* and K mesons, and extrapolate them to the chiral limit. Then we tune the s quark hopping parameter, κ_s, to give the best experimental values for m_{K^*} and m_K. Figure 3 (right) shows $m_\kappa a$, $m_{K^*}a$ and $m_K a$ as functions of $1/\kappa$ for $\kappa_s = 0.1840$. Our preliminary analysis shows that value of m_κ/m_{K^*} at the chiral limit is around 2.0.

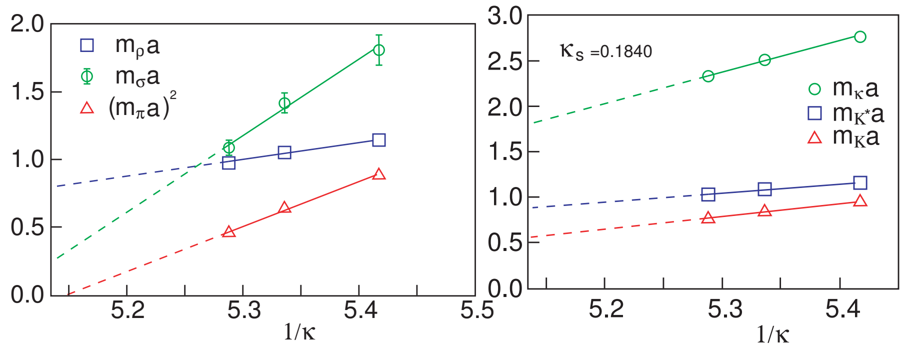

Figure 3: Left : m_ρ, m_σ and m_π^2 in the lattice unit as functions of the inverse hopping parameter. Right : m_κ, m_{K^*} and m_K in the lattice unit as functions of the inverse hopping parameter. The s quark hopping parameter is $\kappa_s = 0.1840$.

4 Concluding Remarks

We have reported our exploratory study of the scalar mesons based on the full QCD lattice simulation with dynamical fermions. Our results indicate the existence of a light σ in the region $m_\pi < m_\sigma \leq m_\rho$.

An interesting observation is that the disconnected part gives a significant contribution, and this diagram makes the σ meson light. This cannot be accessed in the framework of the non-relativistic quark model. This point should be kept in mind in future phenomenological analyses of the sigma meson, and also in lattice studies. Note that the κ meson does not have such a mechanism, and therefore these two scalar meson masses can be different. The κ and the valence σ (connected part) have the same structure in their propagators, but s quark mass is heavier than those of u and d. Therefore $m_\kappa > m_{conn}$, where m_{conn} is a mass corresponding to the connected part, σ_V.

The calculations reported here have limitations discussed in Sect.1. We expect that these defects will be gradually overcome. In lattice calculation, once someone establishes the scale of simulation required to obtain meaningful results, many improved and large-scale works succeed. Therefore in a few years, the lattice study of scalar mesons will provide important and fundamental information to deepen our understanding of hadron physics.

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