

Wilson versus Clover fermions: A case for improvement

Rajan Gupta^{a*}

^aGroup T-8, MS B-285, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, USA

We present evidence for improvement with tadpole improved clover fermions based on an analysis of the chiral behavior of B_K and the quark condensate. Also presented are a comparison of the mass splittings in the baryon octet and decuplet, a calculation of c_A using standard 2-point correlation functions, and the problem of zero modes of the Dirac operator.

1. B_K

With Wilson fermions, straightforward calculations of B_K using the 1-loop improved $\Delta S = 2$ operator fail due to the large mixing with the wrong chirality operators [1]. Since this mixing is an artifact of lattice discretization, one hopes that it can be significantly reduced by improving the action. By comparing results obtained using the Wilson and the tadpole improved clover action ($c_{SW} = 1.4785$) on the same quenched gauge lattices (170 lattices of size $32^3 \times 64$ at $\beta = 6.0$) we show that this is indeed the case.

Fig. 1 shows the Wilson and clover data as a function of $a^2 M_K^2$. For each data set, B_K is written as the sum of two parts — the contribution of the diagonal (the 1-loop tadpole improved LL) operator, and the mixing term which is proportional to α_s . The general form, ignoring chiral logarithms and terms proportional to $(m_s - m_d)^2$, for $p_i = p_f = M_K$ is [1]

$$B_K(M_K) = \frac{\langle \bar{K}^0(p_f) | \mathcal{O} | K^0(p_i) \rangle}{(8/3)f_K^2 M_K^2}$$

$$= \frac{\alpha}{M_K^2} + \beta + \gamma + (\delta_1 + \delta_2 + \delta_3)M_K^2 + \dots$$

The coefficients α, β, δ_1 are pure artifacts, therefore their value can be used to quantify improvement. Of these α is the most serious as it causes B_K to diverge in the chiral limit.

The divergence, in the limit $M_K \rightarrow 0$, of the diagonal term due to a non-zero α is evident in Fig. 1 for Wilson fermions. This arti-

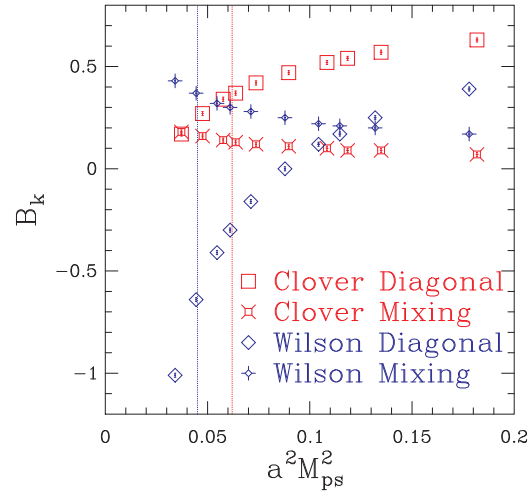


Figure 1. Data for B_K for Wilson and clover fermions. The diagonal ($\Delta S = 2$) and mixing operators contribution are shown separately. Vertical lines are at $M_K = 495$ MeV.

fact is only partially cancelled by the 1-loop mixing operator. The situation is considerably improved with clover fermions. The corresponding values at $M_K = 495$ MeV are $B_K(\text{Wilson}) = -0.29(1)$ whereas $B_K(\text{clover}) = 0.50(1)$. This improvement arises because the two dominant artifacts α and β are significantly reduced; $\alpha(W) = 0.0460(14)$ versus $\alpha(C) = -0.0125(6)$, and $\beta(W) = 0.048(46)$ versus $\beta(C) = -0.006(37)$.

As explained in [1], the contributions proportional to α, β, δ_1 can be removed completely by studying the momentum dependence of the matrix elements. Short of calculating the mixing co-

*In collaboration with T. Bhattacharya and S. Sharpe.

efficiently non-perturbatively, the way to remove the artifacts in $\gamma, \delta_2, \delta_3$ is to extrapolate to $a = 0$. We have done the calculation at $\beta = 6.0$ only, where our final results are $B_K(\text{NDR}, 2 \text{ GeV}) = 0.68(7)$ and $0.58(7)$ for Wilson and clover formulations respectively. The benchmark value, including $a \rightarrow 0$ extrapolation, is $B_K(\text{Staggered}) = 0.628(41)$, as obtained by the JLQCD collaboration [2].

2. QUARK CONDENSATE

The chiral condensate $\langle \bar{\psi}\psi \rangle$ is not simply related to the trace of the Wilson quark propagator $\langle S_F(0, 0) \rangle$. The breaking of chiral symmetry by the r term introduces contact terms that need to be subtracted non-perturbatively from $\langle S_F(0, 0) \rangle$ [3]. This has not proven practical. Instead, the methods of choice are to either evaluate the right hand side of the continuum Ward Identity

$$\begin{aligned} \langle \bar{\psi}\psi \rangle^{\text{WI}} &\equiv \langle 0 | S_F(0, 0) | 0 \rangle \\ &= \lim_{m_q \rightarrow 0} m_q \int d^4x \langle 0 | P(x) P(0) | 0 \rangle, \end{aligned} \quad (1)$$

or cast the Gell-Mann, Oakes, Renner relation

$$\langle \bar{\psi}\psi \rangle^{\text{GMOR}} = \lim_{m_q \rightarrow 0} -\frac{f_\pi^2 M_\pi^2}{4m_q}. \quad (2)$$

in terms of lattice correlation functions [4]. These estimates have errors of both $O(a)$ and $O(ma)$, and at fixed a are therefore expected to agree only in the chiral limit. A comparison of the efficacy of the two methods is shown in Fig. 2.

We find that a reliable extrapolation to the chiral limit can be made using a linear fit, and the two methods give consistent results for both Wilson and clover fermions. Also, the $O(ma)$ corrections are significantly smaller for clover fermion.

3. BARYON MASS-SPLITTINGS

In Ref. [5] we presented a detailed analysis of mass-splittings in the baryon octet and decuplet with Wilson fermions. We had found a large non-linear dependence on quark mass for the $\Sigma - \Lambda$, $\Sigma - N$, and $\Xi - N$ splittings. Extrapolation of the data to the physical masses including these non-linearities gave estimates consistent

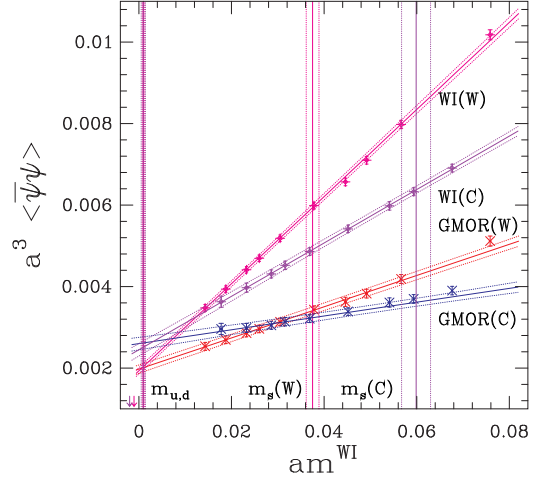


Figure 2. Data for $\langle \bar{\psi}\psi \rangle$ versus the quark mass for Wilson (W) and clover (C) fermions. The vertical lines show $m_{u,d}$ and m_s for the two actions. The $1/a$ are $2.33(4)$ and $1.99(5)$ GeV for Wilson and Clover fermions.

with observed values. On the other hand we had found a surprisingly good linear fit to the decuplet masses, and the splittings were underestimated by 25 – 30%. The data with clover fermions show the same qualitative features. As an illustration, we show a comparison of the $\Sigma - \Lambda$ splitting in Fig. 3. Details of the analysis will be published elsewhere [6].

4. DETERMINATION OF c_A

The improvement coefficient for the axial current, c_A , is calculated using the axial WI [7]. If the clover coefficient c_{SW} is tuned to its non-perturbative value 1.769 at $\beta = 6.0$ [7], the sum $(m_1 + m_2)$ of quark masses defined by

$$\frac{\sum_{\vec{x}} \langle \partial_\mu [A_\mu + ac_A \partial_\mu P]^{(12)}(\vec{x}, t) J^{(21)}(0) \rangle}{\sum_{\vec{x}} \langle P^{(12)}(\vec{x}, t) J^{(21)}(0) \rangle}$$

should be independent of t and the initial pseudoscalar state created by J , up to corrections of $O(a^2)$. We vary the composition of the initial state by using $J = P$ or A_4 and by using “wall” or “Wuppertal” smearing functions in the calculation of the quark propagators. The results in Fig. 4 show a large dependence on the initial state

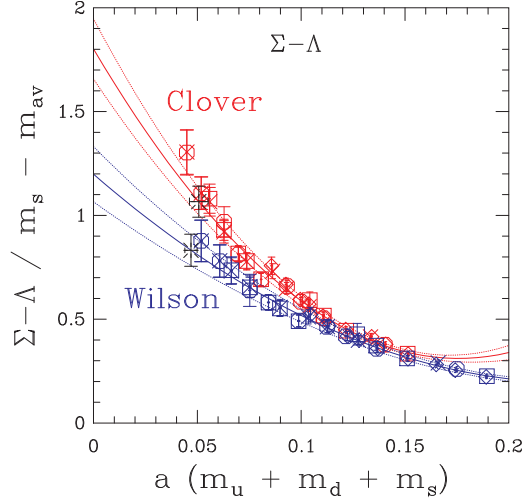


Figure 3. Behavior of the $\Sigma - \Lambda$ splitting versus the sum of the three quark masses. The leading mass dependence $m_s - (m_u + m_d)/2$ has been divided out.

for Wilson fermions and almost none already for $c_{SW} = 1.4785$! We estimate $c_A = -0.026(2)$ from this clover data, whereas the ALPHA collaboration report $c_A = -0.083(5)$ at $c_{SW} = 1.769$ [7]. We are repeating the calculation at $c_{SW} = 1.769$ to understand this difference.

5. ZERO MODES

The explicit breaking of chiral symmetry in Wilson-like fermions gives rise to the problem of “exceptional configurations” in the quenched theory. The cause is that the Wilson r term breaks the anti-hermitian property of the massless Dirac operator. As a result, zero modes of the Dirac operator extend into the physical region $\kappa < \kappa_c$. Thus, on a given configuration, as the quark mass is lowered and approaches the first of the unphysical modes, one encounters exceptionally large fluctuations in the correlation functions. Such configurations dominate the ensemble average and as discussed in [8] there is no basis for excluding them. Tuning c_{SW} reduces the $O(a)$ chiral symmetry breaking artifacts as shown above, however, it does not reduce this problem [8]. We find, by comparing fluctuations in 2-point and 3-

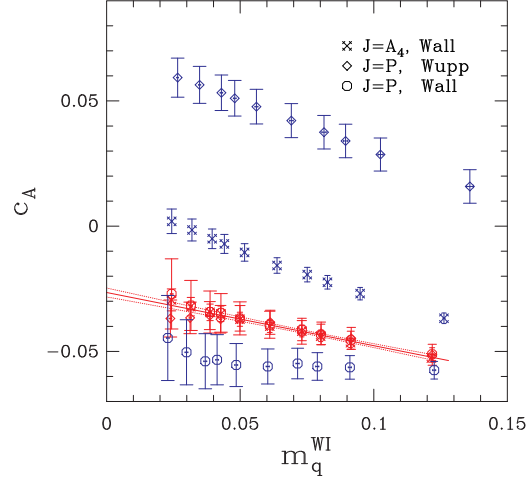


Figure 4. Wilson and clover fermion results for c_A as a function of $(m_1 + m_2)$, and for three different sources J . The clover data is seen to be independent of J and shows a linear behavior.

point correlation functions between Wilson and Clover fermions, that the problem, in fact, gets worse. A deeper understanding of the persistence of the zero mode problem even though the chiral behavior is improved is missing.

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