Lattice study of "$f_0(600)$ or σ"

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We investigate the propagator of "$f_0(600)$ or the σ" by the full-QCD simulation with Wilson fermions. We calculate the mesonic correlator in the $I = 0$, $J^P = 0^+$ channel on the $8^3 \times 16$ lattice. Plaquet action and Wilson fermion action are adopted. A coupling constant β is set to 4.8 and three kinds of hopping parameter, κ=0.1846, 0.1874 and 0.1891 are assayed. The disconnected diagram in the propagator is evaluated through taking average over 500 or 1000 Z2 noise. Simulations with the larger hopping parameter provide us with less noisy results. Though the statistics is not yet enough, our results indicate the existence of a pole with a mass in almost the same order as that of the ρ.

1. Introduction

In many text books, we can find the intuitive explanation of the spontaneous breaking of the the chiral symmetry based on the titled "Mexican Hat" potential, of which the angular and radial modes correspond to the π and σ mesons, respectively. Also in the the Nambu Jona-Lasinio model, the σ-degree of freedom plays an important role for the chiral symmetry breaking of QCD in the low-energy region. Despite its importance, the experimental identification of the light scalar meson is a long standing puzzle and the σ had disappeared from the list of Particle Data Group (PDG) for over 20 years. But, the scenes around the σ are changing recently; re-analyses of π-π scattering phase shift strongly suggest the existence of the $I = 0$ and $J^{PC} = 0^{++}$ meson and significant contributions of the σ in the decay channels of heavy particles such as D$\rightarrow \pi\pi\pi$ are reported [1,2]. The σ may reveal itself also in $\Upsilon(3S) \rightarrow \Upsilon\pi\pi$ channel[3]. In 1996 PDG, "f_0(400-1200) or σ" appeared bellow 1 GeV mass region. In the 2002 edition, it appears as "$f_0(600)$ or σ" and as the 105th branch of the D^+ decay, "D $\rightarrow \sigma + \pi$ " is cited [4]. Now it is an important task of the lattice calculation to confirm the properties of the σ meson based on QCD.

In quenched QCD, DeTar and Kogut first measured the σ meson screening mass in a lattice simulation [5], Alford and Jaffe discussed the possible light scalar mesons as $\bar{q}^2q^2$ states [6], and masses and mixing of $q\bar{q}$ states and a glueball have been investigated by Lee and Weingarten [7]. However, as we have already reported in the previous proceedings [8], the connected and disconnected diagrams give almost the same amount of the contributions, indicating that the dynamical quark effect seems to be essential in the calculation.

McNeile and Michael computed the mixed isosinglet scalar masses of $q\bar{q}$ and glueball states in two kinds of situation, i.e, with and without the dynamical quark effects [9]. We focus our atten-

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tion to explore whether the σ meson exists below 1 GeV in this work. Based on the quite similar motivation, Prelovsek presented in the present conference the calculation of the quenched scalar meson correlator with the domain wall fermions, where the disconnected diagram is taken into account effectively through an approximate formula based on the chiral perturbation theory [10].

2. Formulation of the σ propagator

As for the σ meson, we adopt the operator as

$$\sigma(x) \equiv \sum_{c=1}^{3}\sum_{\alpha=1}^{4} \frac{\bar{u}_\alpha^c(x)u_\alpha^c(x)+\bar{d}_\alpha^c(x)d_\alpha^c(x)}{\sqrt{2}} \ , \qquad (1)$$

where u and d are the u-quark and d-quark Dirac spinors, respectively. The indices c and α denote color and Dirac spinor indices, respectively. Quantum numbers of the operator are $I=0$ and $J^P=0^+$. Because our simulation is full QCD, any kind of the state which has the same quantum number can contribute to the correlation as a virtual state. The σ meson propagator is given by

$$\begin{aligned} & G(y,x) = \langle \sigma(y)\sigma(x)^\dagger \rangle \qquad (y_0 > x_0) \\ = \ & \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \\ & \sum_{a,b=1}^{3}\sum_{\alpha,\beta=1}^{4} \frac{\bar{u}_\beta^b(y)u_\beta^b(y)+\bar{d}_\beta^b(y)d_\beta^b(y)}{\sqrt{2}} \\ \times \ & \left(\frac{\bar{u}_\alpha^a(x)u_\alpha^a(x)+\bar{d}_\alpha^a(x)d_\alpha^a(x)}{\sqrt{2}}\right)^\dagger e^{-S_g-\bar{u}Wu-\bar{d}Wd} \qquad (2) \\ = \ & -\langle \mathrm{Tr}W^{-1}(x,y)W^{-1}(y,x)\rangle \\ & +2\langle \mathrm{Tr}W^{-1}(y,y)\mathrm{Tr}W^{-1}(x,x)\rangle \ . \qquad (3) \end{aligned}$$

In Eq.(3), "Tr" represents summation over color and Dirac spinor indices. W^{-1}'s are u and d quark propagators, U is the link variable of gluon, and S_g is the pure gauge action of gluons. We assume that the u and d quark propagators are equivalent because u and d quark masses are almost the same. From Eq.(3), we can see that σ propagator is composed of the connected diagram (the first term of Eq.(3)) and the disconnected one (the second term of Eq.(3)). The quantum number of the σ meson ($I=0$ and $J^P=0^+$) is the same as that of the vacuum, and the vacuum expectation value of the σ operator, $\langle\sigma(x)\rangle$, does not vanish. Therefore, the contribution of $\langle\sigma(x)\rangle$ should be subtracted from the σ operator. Thus,

$$\begin{aligned} G(y,x) \ = \ & -\langle \mathrm{Tr}W^{-1}(x,y)W^{-1}(y,x)\rangle \\ & +2\langle \mathrm{Tr}W^{-1}(y,y)\mathrm{Tr}W^{-1}(x,x)\rangle \\ & -2\langle \mathrm{Tr}W^{-1}(y,y)\rangle\langle \mathrm{Tr}W^{-1}(x,x)\rangle (4) \end{aligned}$$

is adopted for the σ propagator. The third term in Eq.(4) corresponds to the subtraction of the vacuum contribution. Our results show that the values of the second and the third terms in Eq.(4) are in the same order. Therefore, in order to obtain the signal correctly as the difference between these terms, the high precision numerical simulations and careful analyses are required.

3. Results of numerical simulations

We calculate the σ propagator in the full QCD by using the Hybrid Monte Carlo (HMC) algorithm, in order to take the dynamical quarks which are important to estimate the disconnected diagrams of the σ propagator. We use the Z_2 noise method to calculate the disconnected diagrams (the second and third terms in Eq.(4)). The 500 or 1000 random Z_2 numbers are generated. The two-flavor Wilson fermion system is simulated on the $8^3\times 16$ lattice.

After 500 trajectories are updated in the quenched QCD, we start to update the configuration in the full QCD by using HMC algorithm. More than 500 trajectories by HMC are spent for thermalization. The σ propagators are calculated on a configuration in every 10 trajectories.

Based on ref.[11], we set $\beta=4.8$ ($a=0.197(2)$ fm, $\kappa_c=0.19286(14)$). Figures 1, 2 and 3 correspond to the three values of the hopping parameter, $\kappa=0.1846$(fig. 1), 0.1874 (fig. 2) and 0.1891(fig. 3), respectively. The numerical results are summarized in Table 1.

4. Concluding Remarks

We have reported our preliminary results on the σ meson propagator based on a full QCD lattice calculation with dynamical fermions. The simulations with larger hopping parameters give us clearer signals. Though our simulation is still far from the chiral limit and no decay channel is yet open, our preliminary results obtained with

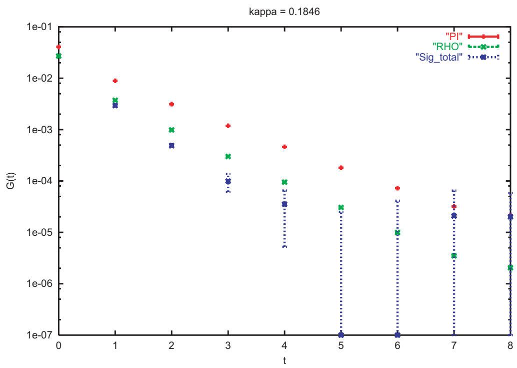

Figure 1. Propagators of π, ρ and σ for κ = 0.1846 with periodic boundary condition.

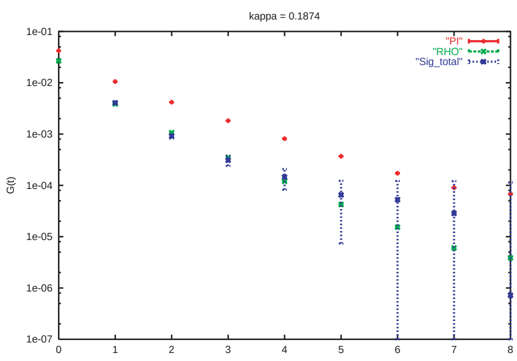

Figure 2. Propagators of π, ρ and σ for κ = 0.1874 with periodic boundary condition.

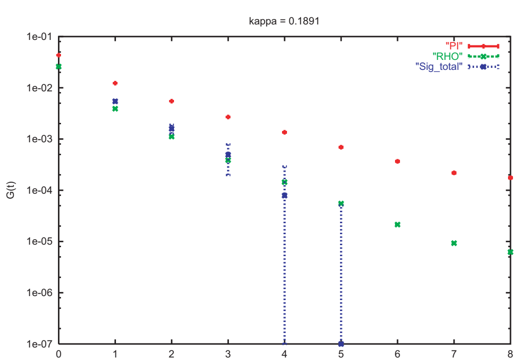

Figure 3. Propagators of π, ρ and σ for κ = 0.1891 with periodic boundary condition.

Table 1
Summary of the results

κ	0.1846	0.1874	0.1891
statistics $^{1)}$	1380	900	180
$m(\pi)/m(\rho)$ $^{2)}$	0.8291(12)	0.7715(17)	0.7026(32)
$m(\pi)/m(\rho)$ $^{3)}$	0.815	0.747	0.68
$m(\sigma)/m(\rho)^{3)}$	1.2	0.96	1

$^{1)}$number of configulations, $^{2)}$CPPACS,
$^{3)}$our result

still a relatively low statics indicate the existence of the light σ meson with the mass in almost the same order as that of the ρ meson. See Table 1.

Acknowledgment This work is supported by Grant-in-Aide for Scientific Research by Monbu-Kagaku-sho (No. 11440080, No. 12554008, No. 12640263 and No. 14540263) and ERI of Tokuyama Univ. Simulations were performed on SR8000 at IMC, Hiroshima Univ., SX5 at RCNP, Osaka Univ., SR8000 at KEK.

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