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Chiral Symmetry and Scalar Meson in Hadron and Nuclear Physics [1]

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Abstract

After giving a short introduction to the Nambu-Jona-Lasinio model with an anomaly term, we show the importance of the scalar-scalar correlation in the low-energy hadron dynamics, which correlation may be summarized by a scalar-isoscalar meson, the sigma meson. The discussion is based on the chiral quark model with the sigma-meson degrees of freedom. Possible experiments are proposed to produce the elusive meson in a nucleus and detect it. In relation to a precursory soft mode for the chiral transition, the reason is clarified why the dynamic properties of the superconductor may be described by the diffusive time-dependent Ginzburg-Landau (TDGL) equation. We indicate the chiral symmetry plays a significant role also in nuclei; one may say that the stability of nuclei is due to the chiral symmetry of QCD.

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1 Introduction

The salient features of low-energy hadron physics or QCD may be summarized as follows; (i) *Confinement of the colored quarks and gluons*, (ii) *the dynamical breaking of chiral symmetry* (DBCS), (iii) $U_A(1)$ *anomaly*, (iv) *Explicit* $SU_V(3)$ *breaking*, (v) *Success of the constituent quark model*, (vi) *OZI rule and its violation in mesons and baryons*. Hatsuda and the present author [1] showed that a semi-phenomenological but unified description of the above facts is possible, emphasizing a dominant role of chiral symmetry in low-energy hadron dynamics. The description is based on the $SU(3)$ Nambu-Jona-Laisnio (NJL) model[2]. The model embodies three basic ingredients of QCD, i.e., the dynamical breaking of chiral symmetry (DBCS), $U_A(1)$ anomaly and the explicit symmetry breaking due to the current quark masses. Various empirical aspects of QCD were shown to be realized through interplay among the three ingredients. It was emphasized that the constituent quark model and the chiral symmetry is reconciled in a chiral quark model. It was also shown that the chiral quark model can account for most of the empirical facts on baryons as well as the low-lying mesons. Furthermore, the NJL model allows us to study the change of hadron properties in hot/dense medium in a self-consistent way.

In this report, we shall first give a short introduction of the NJL model with the anomaly term in section 2, which is the basis for the proceeding discussions. Then we pick up and rearrange some topics dealt in the above review article to clarify significance of the sigma meson in the low-energy hadron dynamics. The sigma meson is a scalar and iso-scalar meson and the chiral partner of the pion; see section 3. We show that a chiral quark model incorporating the collectiveness in the scalar channel summarized by the sigma meson nicely accounts for some properties of lowlying baryons, such as π-N Σ term[3]. We also mention that the convergence radius of the chiral perturbation theory is related with the sigma meson mass. In section 4, possible experiments are proposed to detect the elusive meson in the laboratory.

Next, we move to the problem of the fluctuation effects in hot QCD. This is also a topics dealt in Ref. [1], where it is shown that a precursory soft mode exist in the high temperature phase for the chiral transition in QCD. We shall make a comment on the nature of the corresponding mode in the weak-coupling supercondoctor as described by the BCS model[4]: It will be clarified why the dynamic properties of the superconductor may be usually described by the diffusive time-dependent Ginzburg-Landau (TDGL) model; see section 5.

Finally, we indicate that the chiral symmetry plays a significant role in nuclei as well as in one hadron systems. Notice that nuclei are only stable bound systems in the hadron world. One may say that this stability is due to the chiral symmetry of QCD; see section 6.

2 Brief summary of the $SU(3)$ NJL model with an anoamly term

Our model Lagrangian is the generalized Nambu-Jona-Lasinio (NJL) model with the anomaly term[5, 6, 7];

$$\mathcal{L} = \bar{q}i\gamma\cdot\partial q + \sum_{a=0}^{8}\frac{g_S}{2}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2] - \bar{q}\mathbf{m}q + g_D[\det\bar{q}_i(1-\gamma_5)q_j + h.c.],$$

$$\equiv \quad \mathcal{L}_0 + \mathcal{L}_{SB} + \mathcal{L}_S + \mathcal{L}_D \tag{2.1}$$

where the quark field q_i has three colors ($N_c = 3$) and three flavors ($N_f = 3$), λ^a (a=0$\sim$8) are the Gell-Mann matrices with λ_0=$\sqrt{\frac{2}{3}}\mathbf{1}$. Here $\mathcal{L}_0 + \mathcal{L}_S \equiv \mathcal{L}_{NJL}$ is the $U(3)$ generalization of the NJL model and has manifest flavor-$U_L(3)\otimes U_R(3)$ invariance. $\mathcal{L}_{SB}$ is the explicit $SU_V(3)$ breaking part with m_i (i=u, d, s) being the current quark mass. Finally, $\mathcal{L}_D$ in (2.1) denotes a term which has $SU_L(3) \otimes SU_R(3)$ invariance but breaks the $U_A(1)$ symmetry[8, 9]; this term is a reflection of the axial anomaly in QCD. While $\mathcal{L}_S$ does not cause the flavor mixing, the anomaly term does; with the dynamical breaking of chiral symmetry, it induces effective 4-fermion vertices such as $\langle\bar{d}d\rangle(\bar{u}u)(\bar{s}s)$ and $-\langle\bar{d}d\rangle(\bar{u}i\gamma_5 u)(\bar{s}i\gamma_5 s)$, where the former (latter) gives rise to a flavor mixing in the scalar (pseudo-scalar) channels.

For notational convenience, we here introduce the bosonic variables $\Phi_{ij} = \bar{q}_j(1-\gamma_5)q_i$: Note that $\bar{q}(1-\gamma_5)\lambda_a q = \mathrm{Tr}[\lambda_a\Phi]$. The fact that $\mathcal{L}_D$ represents the $U_A(1)$ anomaly can be seen in the anomalous divergence of the flavor singlet axial current $J_5^\mu = \bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d + \bar{s}\gamma^\mu\gamma_5 s$,

$$\partial_\mu J_5^\mu = -4N_f g_D \mathrm{Im}(\det\Phi) + 2i\bar{q}m\gamma_5 q, \tag{2.2}$$

which is to be compared with the usual anomaly equation written in terms of the topological charge density of the gluon field, $\partial_\mu J_5^\mu = 2N_f g^2/32\pi^2 F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + 2i\bar{q}m\gamma_5 q$. Thus one may say that the determinantal 6-fermion operator $-2g_D\mathrm{Im}(\det\Phi)$ simulates the effect of the gluon operator $\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ [10] in the quark sector. For further comments on the model lagrangian and the details of the following discussions, see the review paper [1] or the original papers cited in the beginning of this section.

In the self-consistent mean field theory, the Lagrangian can be rewritten as follows;

$$\mathcal{L} = \mathcal{L}_{MFA} + \mathcal{L}_{res}, \tag{2.3}$$

where

$$\begin{aligned}
\mathcal{L}_{MFA} &= \bar{q}(i\gamma\cdot\partial - M)q - g_S\mathrm{Tr}(\phi^\dagger\phi) - 2g_D(\det\phi + \mathrm{c.c.}), \\
\mathcal{L}_{res} &= g_S : \mathrm{Tr}(\Phi^\dagger\Phi) : \\
&\quad + g_D : [\mathrm{Tr}(\phi\Phi^2) - \mathrm{Tr}(\phi\Phi)\mathrm{Tr}\Phi - \frac{1}{2}\mathrm{Tr}\Phi^2\mathrm{Tr}\phi + \frac{1}{2}\mathrm{Tr}\phi(\mathrm{Tr}\Phi)^2 + \mathrm{h.c.}] : \\
&\quad + g_D : (\det\Phi + \mathrm{h.c.}) : .
\end{aligned} \tag{2.4}$$

Here the Fock terms are omitted, the normal ordering is taken with respect to the Fock vacuum of $\mathcal{L}_{MFA}$ and ϕ is a diagonal 3×3 matrix defined in terms of the quark condensates as

$$\phi = \langle\Phi\rangle_0 \equiv \mathrm{diag.}(\langle\bar{u}u\rangle, \langle\bar{d}d\rangle, \langle\bar{s}s\rangle) \tag{2.5}$$

The "constituent quark mass matrix" $M = \mathrm{diag.}(M_u, M_d, M_s)$ is given in terms of the condensates

$$M_u = m_u - 2g_s\langle\bar{u}u\rangle - 2g_D\langle\bar{d}d\rangle\langle\bar{s}s\rangle, \tag{2.6}$$

M_d and M_s with the subscripts u, d and s being replaced in a cyclic way. The quark condensates are in turn given with M_i;

$$\langle\bar{u}u\rangle = -iN_c\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\frac{1}{\gamma\cdot p - M} = -\frac{N_c}{\pi^2}\int_0^\Lambda p^2dp\frac{M_i}{\sqrt{M_i^2+p^2}}. \tag{2.7}$$

Here the three momentum cut-off Λ is introduced, and $\langle \bar{d}d \rangle$ ($\langle \bar{s}s \rangle$) is obtained by the replacement $M_u \rightarrow M_d$ ($M_u \rightarrow M_s$).

The q-$\bar{q}$ collective excitations above the condensed vacuum are formed due to the residual interaction $\mathcal{L}_{res}$ in (2.4). The 4-fermion part of $\mathcal{L}_{res}$ can be decomposed into physical channels;

$$\begin{aligned} \mathcal{L}_{res}^{(4)} &= \frac{1}{2}[(G_\delta \sum_a^{1,2} + G_{\kappa^\pm} \sum_a^{4,5} + G_{\kappa^0} \sum_a^{6,7}) : S_a^2 : + \sum_{a,b}^{0,3,8} : S_a G_{ab}^S S_b :] \\ &+ \frac{1}{2}[(G_\pi \sum_a^{1,2} + G_{K^\pm} \sum_a^{4,5} + G_{K^0} \sum_a^{6,7}) : P_a^2 : + \sum_{a,b}^{0,3,8} : P_a G_{ab}^P P_b :] \quad , \end{aligned} \tag{2.8}$$

where we have introduced the composite operators $S_a = \bar{q}\lambda_a q$ and $P_a = \bar{q}i\gamma_5\lambda_a q$. The coupling constants (G's) in $\mathcal{L}_{res}^{(4)}$ are summarized in Table 2.1 of Ref.[1]. The suffix of the coupling constants shows the relevant channel where the residual interaction is active. We note that not only $\mathcal{L}_S$ but also $\mathcal{L}_D$ contribute to the residual 4-fermion interaction.

In Table 1, we summarize basic physical quantities calculated in the NJL model together with the corresponding empirical values. The numerical values are obtained with the following parameter set; $\Lambda = 631.4$MeV, $g_S\Lambda^2 = 3.67$, $g_D\Lambda^5 = -9.29$, $m_s = 135.7$MeV, where we have used a three-momentum cutoff scheme. [2]

The predicted constituent quark masses

$$M_u = M_d = 335\text{MeV} \quad \text{and} \quad M_s = 527\text{MeV}, \tag{2.9}$$

are consistent with the phenomenological masses extracted from the baryon magnetic moments [12].

The non-perturbative part of the condensate in Table 1 is defined by subtracting the perturbative contribution from the full condensate [13]

$$\langle \bar{q}q \rangle^{NP} \equiv \langle : \bar{q}q : \rangle = \langle \bar{q}q \rangle - \langle \bar{q}q \rangle^{pert.} = -iN_c \mathrm{tr} \int \frac{d^4p}{(2\pi)^4}[\frac{1}{\gamma\cdot p - M} - \frac{1}{\gamma \cdot p - m}] \tag{2.10}$$

The absolute value of the condensate and the $SU_V(3)$ breaking ratio $\langle \bar{s}s \rangle^{NP}/\langle \bar{u}u \rangle^{NP}$ in the NJL model agree well with those deduced from the QCD sum rules [14]. It should be emphasized that the quark condensates to be deduced from the QCD sum rules are the non-perturbative part as defined above, while those calculated in the lattice simulations are total ones; the total condensate of the s quark is larger in the absolute value than that of the non-strange quarks in accordance with the lattice result[15].

[2] The corresponding invariant cutoff for the non-strange (strange) quarks reads $\Lambda_4 = 2\sqrt{M^2 + \Lambda^2} = 1430\,(1640)$ MeV with the constituent quark mass $M = 335\,(527)$MeV.

	Theory	Empirical values
M_u (M_s)	335 (527)	336 (540) MeV
$\langle \bar{u}u \rangle^{NP}$	$-(245)^3$	$-(225 \pm 25)^3$ MeV^3
$\langle \bar{s}s \rangle^{NP} / \langle \bar{u}u \rangle^{NP}$	0.78	0.8 ± 0.1
m_π (m_K)	138* (496*)	138 (496) MeV
m_η ($m_{\eta'}$)	487 (958*)	549 (958) MeV
m_σ ($m_{\sigma'}$)	668 (1348)	~ 700 (~ 1400) MeV
$\Gamma_{\sigma \to 2\pi}$	~ 900	$\sim$Re m_σ
f_π (f_K)	93.0* (97.7)	93 (113) MeV
f_η ($f_{\eta'}$)	94.3 (90.8)	93$\pm$9 (83$\pm$7) MeV
θ_η (φ_σ)	$-21°$ ($-6.8°$)	$\sim -20°$ ($-$)
$G_{\pi q}$ (G_{Kq})	3.5 (3.6)	~ 3.5 ($-$)
$G_{\pi N}$ ($G_{\sigma N}$)	12.7 (7 $-$ 10)	13.4 ($\sim$10.0)
$\Sigma_{\pi N}$	49 $\pm$7	45 ± 10 MeV

Table 1: Comparison of the theoretical estimates and the experimental/empirical values of the basic physical quantities. * indicates the quantity used as input.

Comments on other quantities listed in the table are given in [1]. As for the sigma meson, we shall say a lot in the following section. Table 1 tells us that the $SU(3)$ NJL model reproduces the fundamental physical quantities in the accuracy of O(10%-15%).

3 Role of the sigma meson in the QCD phenomenology

3.1 The Sigma meson

The sigma meson is the chiral partner of the pion for the $SU_L(2) \otimes SU_R(2)$ chiral symmetry: In the $(1/2, 1/2)$-representation, the sigma field σ constitutes the quartet together with the three pions. This is well represented in the linear sigma model. The order parameter of the chiral transition of QCD is the scalar quark condensate $\langle \bar{q}q \rangle \sim \sigma$, and the vacuum is determined as the sate where the effective potential $\mathcal{V}(\sigma)$ takes the minimum. Let us denote the minimum point by σ_0. Then the particle representing the quantum fluctuation $\tilde{\sigma} \sim \langle : (\bar{q}q)^2 : \rangle$ is the sigma meson. ($\sigma = \sigma_0 + \tilde{\sigma}$). In this sense, the sigma meson is analogous to the Higgs particle in the Weinberg-Salam theory, where the Higgs field is the order parameter, and the quantum fluctuation of the field around the minimum point of the Higgs potential or the effective potential is the Higgs particle in the present world.

In the NJL model, the chiral symmetry is realized linearly like the linear sigma models, hence the appearance of the sigma meson is inevitable: The sigma meson mass m_σ is predicted to be twice of the constituent quark mass in the chiral limit [2, 16], hence $m_\sigma \sim 700$ MeV as in the ladder QCD[17].

There exists, however, a controversy on the identification of the nonet scalar mesons in the particle zoo[18], and some people are skeptical even about the existence of the sigma meson with a rather low mass, say about 600 to 800 MeV [3]. The origin of the such skepticism may be related to the facts that the decay of the sigma meson to two

[3] Our view about the identification of the scalar mesons is given in chapter 3 of ref.[1].

pions gives rise to a huge width $\Gamma = 400 \sim 1000$ MeV of the sigma meson, and that a possible coupling with glue balls with $J^{PC} = 0^{++}$ make the situation obscure.

Nevertheless some studies [20, 21] on the phase shift analysis of the pi-pi scattering in the scalar channel which claims a pole of the scattering matrix in the complex energy plane with the real part $\mathrm{Re}m_\sigma \simeq 500$ MeV and the imaginary part $\mathrm{Im}m_\sigma \simeq 500$MeV. Fig. 1 shows how the phase shift is reproduced with such a broad resonance [21]: Actually the analysis shows the existence three resonances, i.e., $f_0(500)$, $f_0(975)$ and $f_0(1400)$ in the terminology in the PDG[18]. There are two sets of masses (M) and widths (Γ) of resonances obtained to fit the data. One set reads (in MeV); $(M,\ \Gamma) = (506 \pm 10,\ 494 \pm 5), (973 \pm 2,\ 29 \pm 2),\ (1430 \pm 5,\ 145 \pm 25)$ for $f_0(500)$, $f_0(975)$ and $f_0(1400)$, respectively. Another set of masses and widths is not so different from the above values, and predicts a f_0 meson, i.e., the sigma meson with the mass 505$\pm$10 MeV and the width 497$\pm$5 MeV.

There is also a preliminary experimental result at KEK[22], which seems to show a bump around 600 MeV with a width $\sim$ 400 MeV in the reaction $\pi^- \mathrm{p} \rightarrow \mathrm{n}\pi^0\pi^0$. The 2 π^0 are detected by 4 γ's. This is a clever experiment in the sense that by confining to the $2\pi^0$ channel, one can reject the iso-vector channel where we would have a huge yield from the rho meson [4]

Fig.1

If such a scalar meson with a low mass is identified, many experimental facts which have been mysterious can be nicely accounted for in a simple way: The correlation in the scalar channel as summarized by such a scalar meson can account for the $\Delta I = 1/2$ rule for the decay process $\mathrm{K}^0 \rightarrow \pi^+\pi^-$ or $\pi^0\pi^0$ [23]; see Fig.2. In the meson-theoretical model for the nuclear force, a scalar meson exchange with the mass range 500$\sim$ 700 MeV is indispensable to fully account for the state-independent attraction in the intermediate range[24]. These facts indicate that the scalar-scalar correlation is important in the hadron dynamics. This is in a sense natural because the dynamics which is responsible for the correlations in the scalar channel is nothing but the one which drives the chiral symmetry breaking.

Fig.2

In this section, we show that the correlations in the scalar channel as possibly described as the sigma meson are also essential in reproducing some interesting observables such as the π-N sigma term $\Sigma_{\pi N}$[3];

$$\Sigma_{\pi N} = \hat{m}\langle \bar{u}u + \bar{d}d \rangle. \tag{3.1}$$

The empirical value of $\Sigma_{\pi N}$ is reported to be 45 ± 10 MeV[25]. The chiral perturbation theory fails in reproducing the empirical value unless a huge strangeness content of the proton is assumed[3]. Our discussion will be based on a chiral quark model[26]: We

[4]Experimental status about the scalar particles is summarized in an Appendix of ref.[19] as well as in ref.[1]. See these references for other experiments about the sigma meson.

shall show that the chiral quark model based on the NJL model can account for the π-N Σ term (and other related quantities in the scalar channel) by taking into account the correlations in the scalar channel as summarized by the sigma meson[27, 7, 28]; see also [29].

We remark also the convergence radius of the chiral perturbation theory [30, 31] is linked with the mass of the scalar meson. The following subsections are a rewrite of a part of chapter 3 and 4 of ref.[1] with an emphasis put on the role of the sigma meson. See ref.[1] and references cited therein for more details.

3.2 π-N Σ term in a chiral quark model

The basic quantities underlying the following discussion are the quark contents of hadrons $\langle h|\bar{q}_i q_i|h\rangle \equiv \langle \bar{q}_i q_i\rangle_h$ $(i = u, d, s, ...)$. Actually, it is more adequate to call them the scalar charge of the hadron. They are interesting in the relation with the problems of the π-N Σ term $\Sigma_{\pi N}$, the degree of the Okubo-Zweig-Iizuka (OZI) rule[32], the anomalous charm production in high-energy hadron-hadron collisions and so on.[1] In this report, we confine ourselves on the problem of the quark contents of low-lying baryons in the scalar channel, i.e., $\langle \bar{q}_i q_i\rangle_B$.

Feyman-Hellman theorem tells us that once the baryon mass M_B is known as a function of the current quark masses m_i, the quark content of the baryon is calculated as,

$$\langle \bar{q}_i q_i\rangle_B = \frac{\partial M_B}{\partial m_i}. \tag{3.2}$$

Then the problem is to know how M_B is dependent on m_i $(i = u, d, s, ...)$. In this respect, the chiral quark model is useful, where the notion of the chiral symmetry and the constituent quark model are reconciled: In this model, the dependence of M_B on m_i is known through the constituent quark mass M_i which is identified with the sum of the current quark mass m_i and the mass M_i^D generated by the dynamical breaking of chiral symmetry(DBCS); $M_i = M_i(m_u, m_d, m_s, ...)$. On the other hand, M_B is given in terms of M_i's in a constituent quark model and hence the the current quark masses m_i's.

Our theory may be described by the following effective Lagrangian

$$\mathcal{L}^{\rm eff} = \mathcal{L}_{\chi {\rm SB}} + \mathcal{L}_{``OGE''} + \mathcal{L}_{\rm conf}, \tag{3.3}$$

where $\mathcal{L}_{\chi {\rm SB}}$ is the interaction responsible for DBCS and is identified with $\mathcal{L}_{NJL}$. $\mathcal{L}_{\chi {\rm SB}}$ is first switched on and then by adopting a successful constituent quark model, the spin-spin interaction between the constituent quarks $\mathcal{L}_{``OGE''}$ and the confining force $\mathcal{L}_{\rm conf}$ are switched on. Such a two-stage approach was first discussed by Goldman and Haymaker[33]. When the NJL model is adopted to describe DBCS, the constituent quark masses M_u, M_d, M_s are given as the solution of the gap equation Eq.(2.6). As a successful constituent quark model, we adopt the Isgur-Karl model [11]; the baryon mass formula then reads as follows,

$$M_B = M_0 + \sum_i^{u,d,s}[M_i + \frac{a}{2M_i}] + b\sum_{i<j}\frac{\sigma_i\cdot\sigma_j}{M_i M_j}. \tag{3.4}$$

Here M_0 represents the contributions of the confinement potential and the short-range color-electric interaction, which are independent of the constituent masses. $a/2M_i$ denotes the kinetic energy of the confined quarks. The spin-spin term with the coefficient

b is the color-magnetic interaction responsible for the octet-decuplet mass splitting.[5] Instead of taking the detailed form of the quark wave functions inside the baryons, we take the known masses of proton(938), $\Delta(1232)$ and $\Omega(1672)$ to extract the parameters M_0, a and b. The result is $a = (175.2\mathrm{MeV})^2$, $M_0 = -56.4\mathrm{MeV}$, $b = (176.4\mathrm{MeV})^3$, which gives the baryon masses in a excellent agreement (in MeV unit);

$$\begin{aligned} \Lambda(1115) = 1114, \quad \Sigma(1193) = 1186, \quad \Sigma^*(1385) = 1372, \\ \Xi(1320) = 1332, \quad \Xi^*(1507) = 1519. \end{aligned} \tag{3.5}$$

Now, taking the derivatives of the mass formula Eq.(3. 4), one immediately finds that the quark content $\langle \bar{q}_i q_i \rangle_B$ or the scalar charge is given in terms of the scalar charge of the constituent quark,

$$Q_{ji} \equiv \frac{\partial M_j}{\partial m_i} = \langle \bar{q}_i q_i \rangle_{q_j}, \tag{3.6}$$

with $i, j = u, d, s, ...$ Therefore let us first calculate and analyze Q_{ji}.

Q_{ji} can be obtained by differentiating the gap equation Eq.(2. 6) with the quark condensate Eq.(2.7). Due to the dynamical chiral symmetry breaking, $M_{u,d,s}$ are composed of two pieces, i.e., the current quark mass and the dynamical mass

$$M_{u,d,s} = m_{u,d,s} + M^D_{u,d,s} \quad . \tag{3.7}$$

We stress that $M^D_{u,d,s}$ has a purely non-perturbative origin and is a non-linear function of $m_{u,d,s}$.

Using the NJL model, one can go further and identify the effect of $M^D_{u,d,s}$ in a concrete form: Noting that

$$\frac{d\langle \bar{q}_i q_i \rangle}{dM_i} = \Pi^S_i(q^2 = 0), \tag{3.8}$$

where $\Pi^S_i(q^2)$ is the zero-th order polarization in the scalar channel due to the i-quark (i=u, d, s), one has

$$\mathbf{Q} = [\mathbf{1} + \mathbf{V}^\sigma \cdot \mathbf{\Pi}^S(0)]^{-1}, \tag{3.9}$$

where

$$\mathbf{V}^\sigma = 2 \cdot \begin{pmatrix} g_s & g_D\langle \bar{s}s \rangle & g_D\langle \bar{d}d \rangle \\ g_D\langle \bar{s}s \rangle & g_s & g_D\langle \bar{u}u \rangle \\ g_D\langle \bar{d}d \rangle & g_D\langle \bar{u}u \rangle & g_s \end{pmatrix} \tag{3.10}$$

is the interaction Lagrangian in the scalar channel in the flavor basis;

$$\mathcal{L}^\sigma_{res} = \sum_{i,j=u,d,s} : \bar{q}_i q_i V^\sigma_{ij} \bar{q}_j q_j : . \tag{3.11}$$

The off-diagonal terms of Q_{ij} are proportional to g_D, which shows that the flavor mixing is generated by the anomaly term. To clarify the structure of $\mathbf{Q}$, we note that the propagator of the scalar mesons in this model is written as follows;

$$\mathbf{D}_\sigma(q^2) = -[\mathbf{1} + \mathbf{V}_\sigma \cdot \mathbf{\Pi}^S(q^2)]^{-1} \cdot \mathbf{V}_\sigma, \tag{3.12}$$

[5] The spin-spin interaction may include a contribution from the instanton-induced quark-quark interaction [34]. We do not identify the origin of the spin-spin interaction here.

accordingly,

$$\mathbf{Q} = -\mathbf{D}_\sigma(0) \cdot \mathbf{V}_\sigma^{-1}. \tag{3.13}$$

We remark that when the interaction is absent $\mathbf{Q} = \mathbf{1} \equiv \mathbf{Q}^{(0)}$.

As we have already noticed, it is more adequate to call $\mathbf{Q}$ the scalar charge (matrix) of the constituent quarks. Effective charges are usually enhanced (suppressed) due to collective excitations generated by the attractive (repulsive) forces. In our case, we should have an enhancement because the interaction is attractive. The physical origin of the enhancement may be described as follows; the external probe can interact not only with the bare particle but also with the q-$\bar{q}$ cloud surrounding the bare particle. In our model, the cloud is nicely summarized by the scalar meson in the ring approximation. Such an enhancement (suppression) of a charge is well known in the many-body theory, especially in nuclear physics[35]: The effective electric charges of a nucleon in heavy nuclei is enhanced due to the existence of the quadrupole giant resonances, while the effective axial charge g_A is suppressed due to the existence of the Gamow-Teller giant resonance. The sigma meson plays an analogous role to the giant resonances in nuclei.

It is also to be noted that the anomalous quark contents (the degree of the violation of the OZI rule) are related with the mixing property of the scalar mesons.

The typical value for Q_{ji} in the parameter set given in reads

$$\mathbf{Q} = \begin{pmatrix} 1+1.14 & 0.45 & 0.15 \\ 0.45 & 1+1.14 & 0.15 \\ 0.28 & 0.28 & 1+0.42 \end{pmatrix}, \tag{3.14}$$

which shows the following: (i) The nonperturbative correlation in this channel significantly increases the effective charge. (ii) The violation of the OZI rule is not negligibly small in this channel, as is seen from the off-diagonal elements. (iii) The s-quark mixing in the u quark is small in comparison with the d-quark mixing in u (Q_{us}/Q_{ud}=0.15/0.45=1/3); this is due to the mass difference of M_s and M_d.

Before using the full mass formula Eq. (3.3), let us see the result in the simple additive quark ansatz, i.e.,

$$M_P \simeq 2M_u + M_d, \tag{3.15}$$

for the proton, for example. In this approximation, we have[29]

$$\Sigma_{\pi N} \;\; = \;\; 3\hat{m}(Q_{uu} + Q_{ud}) = 43 \text{ MeV}, \tag{3.16}$$

where the $SU(2)$ isospin symmetry is assumed. If there were no interaction between current quarks, only the kinematical piece remains and gives $\Sigma_{\pi N} \simeq 3\hat{m} \simeq (15-24)$ MeV provided that $\hat{m} \simeq (5-8)$ MeV. Thus one sees that the nonperturbative effect of the interaction responsible for the breaking of the chiral symmetry and the generation the dynamical mass is necessary to reproduce the empirical value $\Sigma_{\pi N} = (45 \pm 10)$ MeV. We should also stress again that the nonperturbative effect is intimately related to the existence of the collective excitation in this channel, i.e., the sigma meson.

The strangeness content of the proton is written in the same approximation as

$$\langle \bar{s}s \rangle_p = 3\frac{\partial M_u}{\partial m_s} = 3\frac{\partial M_u^D}{\partial m_s}, \tag{3.17}$$

which shows that the OZI-violating matrix element has the nonperturbative origin. Numerically, the ratio $y \equiv 2\langle \bar{s}s\rangle_N / \langle \bar{u}u + \bar{d}d\rangle_N$ is as small as 0.12. Thus we have seen that the large empirical value of $\Sigma_{\pi N}$ can be obtained without a large strangeness content of the nucleon.[29]

How about the effect of the residual spin-spin interaction and the confinement included in the mass formula Eq.(3.3).[27] Using the mass formula instead of the additive ansatz, we have for the proton, for example,

$$\langle \bar{u}u\rangle_P = 4.97\ (4.73), \quad \langle \bar{d}d\rangle_P = 4.00\ (3.03), \langle \bar{s}s\rangle_P = .53\ (0.46), \tag{3.18}$$

hence

$$\Sigma_{\pi N} = 49\text{MeV}. \tag{3.19}$$

Here the numbers in the paranthesis are the results in the additive ansatz. One sees that the residual interactions enhance the $\Sigma_{\pi N}$ and the empirical value is reproduced.

Here we should mention the cutoff-scheme dependence on $\Sigma_{\pi N}$ [7]. The canonical (three momentum) cutoff is the most reasonable scheme from the physical point of view [1] and Eq.(3.15) is in fact calculated in this scheme. In the other schemes [36], smaller $\Sigma_{\pi N}$ are obtained. This is due to the unphysical quenching of $\langle \bar{q}q\rangle$ (or M) for large m, and such a quenching gives small Q_{ij} and thus small $\Sigma_{\pi N}$.

3.3 The convergence radius of the chiral perturbation theory and the mass of the sigma meson

In the chiral perturbation theory[37], observables are expanded with respect to the current quark masses. The general structure of such an expansion is known to be

$$\mathcal{O}(m) = \mathcal{O}(0) + \sum_{l=1}^{\infty} a_l\ m^l + b_1\, m^{1/2} + b_2\, m \ln m + \cdots \ . \tag{3.20}$$

Here the first term denotes the value at the chiral limit, the second term is analytic terms and the last terms are non-analytic ones. The non-analyticity is due to the masslessness of the NG-boson in the chiral limit, while the coefficients of the analytic terms (a_i) depend on the details of the QCD dynamics. Carruthers and Haymaker (CH) examined the convergence property of this analytic series [30], using the $SU(3)$ linear σ-model. They found that the convergence radius of the analytic series is so small that one cannot reach the physical region of the current quark masses for the strangeness sector. Hatsuda [31] examined the same problem using the NJL model and reached a similar conclusion with CH.

In this subsection, we shall indicate that the convergence radius is intimately related with the mass of the sigma meson in the chiral limit. The following is mostly based on section 3.5 of ref. [31, 1].

For simplicity, let us take the effective potential of the linear σ model in the tree level with an explicit symmetry breaking,

$$V(\sigma) \ = \ V_0(\sigma) - f_\pi m_{NG}^2 \sigma \ , \qquad V_0(\sigma) = \frac{m_\sigma^2}{8 f_\pi^2}(\sigma^2 - f_\pi^2)^2 \ . \tag{3.21}$$

m_σ denotes the second derivative of $V_0(\sigma)$ with respect to σ and is identified as the mass of the scalar meson in the chiral limit. m_σ sets the characteristic scale of the system. m_{NG} is the mass of the NG boson which vanishes in the chiral limit. The vacuum is determined as the point where the effective potential takes the minimum; $dV(\sigma)/d\sigma = 0$, or equivalently $dV_0/d\sigma = f_\pi m_{NG}^2$, which is reduced to

$$F(x) \equiv x(x^2-1) = 2(\frac{m_\pi}{m_\sigma})^2 \equiv a \quad \text{with} \quad x = \sigma/f_\pi. \tag{3.22}$$

$V_0(\sigma)$ and $F(x)$ are shown in Fig.3.6 of ref. [1]. These equations actually determine the extrema of the effective potential. $F(x)$ represents how the effective potential is steep at x. One sees that a is a natural dimensionless expansion parameter. Notice that a is the ratio of pion mass squared to the sigma meson mass squared. In the chiral limit ($a = 0$), Eq.(3.22) has three solutions $x = 0$ (the symmetric phase) and $x = \pm 1$ (the dynamically broken phases). For $a \neq 0$, the attempt to calculate $x(a)$ by the perturbation around $x(a = 0) = 1$ fails at the point where $dx(a)/da = \infty$. This condition gives the convergence radius

$$\mid a \mid < a_{cr} = 2/3\sqrt{3}. \tag{3.23}$$

Actually, the existence of a_{cr} is related to the behavior of $x(a)$ for $a < 0$: Note that the convergence radius of the Taylor series $x(a) = \sum_n c_n a^n$ is determined by the nearest singularity of $x(a)$ in the complex a-plain. For a smaller than $-2/3\sqrt{3}$, there are no solutions which continuously connected to the solution $x = 1$ in the chiral limit. We remark that the critical value a_{cr} is the maximum value of the steepness of the effective potential in the chiral limit. In the infinite steepness limit, the sigma meson mass goes infinity and the linear sigma model is reduced to a non-linear sigma model. Therefore, it is natural that the convergence radius in the physical unit is related to the sigma meson mass, as we will see now.

Let's translate a_{cr} into the critical value of m_{NG}: Owing to (3.23), m_{NG}^2 should satisfy the following relation

$$m_{NG}^2 < \frac{m_\sigma^2}{3\sqrt{3}}. \tag{3.24}$$

In the NJL model, $m_\sigma \simeq 2M_u \simeq 700$ MeV, as noted previously. Weinberg [38] suggests Re $m_\sigma \simeq$ Re $m_\rho \simeq 770$ MeV on the basis of the mended symmetry. If the existence of the sigma meson with such a rather small mass is confirmed, we have $\sim (340\text{MeV})^2$ for the right hand side of Eq. (3.24). It means that $m_\pi^2 = (140\text{MeV})^2$ lies within the convergence radius, while $m_K^2 = (495\text{MeV})^2$ does not, implying that the expansion of observables in the strange sector with respect to m_K is doubtful. Conversely, for m_K to lie within the convergence radius, m_σ in the chiral limit have to be as large as 1.1 GeV.

In summary, we have seen that the convergence radius is related to how steep the effective potential is in the chiral limit. Since the steepness can be translated to the sigma meson mass, the convergence radius may be related to the properties of the sigma meson. The NJL model and the mended symmetry as well as the linear sigma model suggest that the strangeness sector may be dangerous to apply the chiral perturbation theory. Nevertheless, the final answer will be given by model-independent analyses of the effective potential or the nature of the chiral symmetry breaking in QCD. With such analyses, one could also have a good insight into the sigma meson.

4 Producing and detecting the sigma meson in experiment

We have seen that the correlations which may be summarized by the unstable and hence elusive sigma meson play significant roles in the hadron phenomenology at low energies. Therefore one may wonder whether there is any chance to observe the sigma meson clearly. What does come when the environment is changed by rising temperature and/or density? As was first shown by Hatsuda and the present author[16, 39], the sigma meson decreases the mass in association with the chiral restoration in the hot and/or dense medium, and the width of the meson is also expected to decrease because the pion hardly changes the mass as long as the system is in the Nambu-Goldstone phase. Thus one can expect a chance to see the sigma meson as a sharp resonance at high temperature. Such a behavior of the meson may be detected by observing two pion with the invariant mass around several hundred MeV in relativistic heavy ion collisions. As was indicated by Weldon[40], when the charge pions have finite chemical potentials, the process $\sigma \to \gamma \to$2 leptons can be used to detect the sigma meson.

It is worth mentioning that the simulations of the lattice QCD [41] show the decrease of the screening mass $m^{\sigma}_{\rm scr}$ of the sigma meson. Here a screening mass is defined through the space correlation of the relevant operator rather than the time correlation as a dynamical (real) mass. The relation between the screening mass and the dynamical mass as discussed by Hatsuda and Kunihiro is not clearly understood yet. Nevertheless it is known [42] that the NJL model gives the similar behavior for the screening masses in the scalar channels with the dynamical ones. It means that the lattice result on the screening masses may suggest that the dynamical masses also behave in a way as predicted in [39]. [6]

We remark that the logic of this type of physics is not new for nuclear physicists. Studying a possible change of collective modes being associated with a change of the ground state of nuclei is in fact what they have usually been doing: When nuclei are being deformed from a spherical shape, the vibrational modes will soften, i.e., the energy of the 2^+ phonon is decreased. If a nucleus is near the superconducting phase, the pairing vibrational mode is also softened.[35] For more than a decade, people have been eager to search a precursory collective phenomena for the pion condensation in nuclei.[45] These are all the same physics in the logic. In fact the analogy of phase transitions in many-body systems with the chiral transition in QCD is emphasized in [46].

Here we propose several experiments for examining possible restoration of chiral symmetry in nuclei and the possible existence of the sigma meson. One uses pions, another protons and light nuclei and the other electrons. To detect the sigma, one may use 4 γ's and/or two leptons. The latter process is possible because a scalar particle can be converted to a vector particle because of the scalar-vector mixing in the system with a finite baryonic density. This mixing is well known in the Walecka model[44]. Microscopically, the process is described by $\sigma \to {\rm N\bar{N}} \to \gamma$. Here σ may be replaced by any scalar particle, and γ any vector particle with the same quantum numbers other than spin and parity.

1. A (π, 4γ N) A$'$

[6] It is interesting that the decrease of the scalar meson in the medium is also shown [43] with hadron models like the σ-ω model[44], although the role of the chiral symmetry is obscure in such a phenomenological model.

This reaction process is depicted in Fig. 3. In this reaction, the charged pion ($\pi^{\pm}$) is absorbed by a nucleon in the nucleus, then the nucleon emits the sigma meson, which decays into two pions. To make a veto for the two pions from the rho meson, the produced pions should be neutral ones which decay to four γ 's.

Fig.3

2. A (P, 4γ N) A′

This type of the reaction is depicted in Fig. 4. The incident proton, deuteron or ^{3}He ... collides with a nucleon in the nucleus, then the incident particle will emit the sigma meson, which decays into two pions. One may detect 4 γ 's from 2 π^0. The collision with a nucleon may occur after the emission of the sigma meson; the collision process is needed for the energy-momentum matching.

Fig.4

In the detection, one may use the two leptons from the process $\sigma \rightarrow \mathrm{N\bar{N}} \rightarrow \gamma$ mentioned above: See Fig. 5. This detection may gives a clean data, but the yield might be small.

Fig.5

3. A (e$^-$, 4γ e$^-$) A′

The final example uses the electron beam; See Fig. 6. The γ ray emitted from the electron is converted to the omega meson in accord with the vector meson dominance principle. The omega meson may decay into the sigma meson in the baryonic medium via the process $\omega \rightarrow \mathrm{N}\ \bar{\mathrm{N}} \rightarrow \sigma$. The sigma will decay into two pions. One may detect the 4γ's from the 2 neutral pions.

Fig.6

5 Fluctuation effects in hot quark matter and superconductor

A decade ago, in the paper [47] entitled "Fluctuation effects in Hot Quark Matter; Precursors of Chiral Transition at Finite Temperature", Hatsuda and the present author showed that there should exist colorless hadronic modes even in the deconfined and chirally symmetric phase at high temperature, if the chiral transition is not of so strong first order. They are actually the fluctuations of the order parameter $\langle\langle(\bar{q}q)^2\rangle\rangle$ and $\langle\langle(\bar{q}i\gamma_5\tau q)^2\rangle\rangle$. The excitation energies and the widths of the modes decrease as the system approaches the critical point from the above. Thus the modes are a kind of soft modes.

Such long-range fluctuations near the critical point are actually well-known phenomena in many body systems. Apart from the modes in nuclei mentioned in the preceding section, some examples are the soft phonons in (anti-)ferroelectrics, the paramagnon in ferromagnets, the paring vibration in nuclei [1].

In subsequent papers [48], the present author extended the discussion to the vector channel: The quark number susceptibility is related to the fluctuation in the vector channel, and the recent lattice simulations [49] can be interpreted as showing that the interaction in the vector channels is weak at high temperatures.

Now, it is well known that the NJL model was invented on the basis of an analogy between the superconductor and the chirally broken phase in the vacuum[2]. Therefore it is interesting that there seems no propagating soft modes in the material which undergoes the phase transition into the superconducting phase. Actually, dynamic critical phenomena in superconductors can be well described by the time-dependent Ginzburg-Landau equation which is a diffusion equation[50]. It has been recently shown [51] that this is due to the fact that the number of the effective spatial dimensions of the superconductor in the BCS model is one: In fact, the excitation energy for the precursory mode in the BCS model is given by [52]

$$1 + g_{BCS}\int_{-\omega_D}^{\omega_D} d\epsilon \frac{\tanh \beta\epsilon/2}{\epsilon - \omega/2} = 0, \tag{5.1}$$

where g_{BCS} is a constant proportional to the coupling constant and ω_D is the Debye frequency, while the corresponding equation in the NJL model is given by

$$1 + g_{NJL}\int_{0}^{\Lambda} k^2 dk \frac{\tanh \beta k/2}{k - \omega/2} = 0, \tag{5.2}$$

with g_{NJL} being a constant proportional to the coupling constant. These equations show that the BCS model describe the system essentially a one-dimensional system because the cutoff ω_D is small; this means that the frequency region where the electrons feel the attraction is narrow. Thus the precursory mode in the superconductor does not become a propagating one, and the equation describing the low-frequency phenomena is well given by the diffusive TDGL equation[50]. This is in contrast to the chiral transition as described by the NJL model, where the precursory mode is propagating[47]; for the details, see ref.[51]

6 The saturation property of nuclei and chiral symmetry

We have shown and stressed that most of the low-energy phenomena in QCD are largely determined by chiral symmetry and its dynamical breaking rather than the confinement[1]; an interplay between the explicit breaking of chiral symmetry due to the current quark masses and the $U_A(1)$ anomaly adds some interesting variations to the low-energy hadron dynamics. Then how about nuclei, stable many-hadron systems. Actually, they are only the bound systems of hadrons, apart from the possible H-dibaryon conjectured by Jaffe[53]. Here, we would like to indicate that the deepest reason of the stability of nuclei is the chiral symmetry and its dynamical breaking in QCD.

The stability of the nuclei can be summarized by the saturation properties of the binding energy and density[54]: All nuclei except for some smallest nuclei have the almost the same binding energy E_B per nucleon and the central density ρ irrespective of the nucleon number. The corresponding values of the nuclear matter are

$$E_B \simeq -15 \ \ \text{MeV}, \qquad \rho = .17 \ \ \text{fm}^3 \equiv \rho_0 \ \ (k_F = 1.36\text{fm}^{-1}). \tag{6.1}$$

One may notice that
(i) E_B is small compared with the rest mass ~ 1 GeV; nuclei are loosely bound system.
(ii) the nuclear density ρ_0 is remarkably low; it means that the inter-nucleon distance is as distant as 1.8 fm, which is much larger than the radius (~ 0.4 fm) of the repulsive core of the nuclear force.

Why? The answer was given by the nuclear matter theory based on the reaction matrix (G-matrix) initiated by Brueckner and developed in the 60's.[54]

The resultant nuclear density is so low that the nuclear matter may be treated as a good approximation by the superposition of the two-nucleon interactions, i.e., the independent-pair approximation. The effective two-nucleon potential (G-matrix) is then constructed by a ladder approximation which takes into account the Pauli principle in the intermediate states. The reaction matrix is expanded by partial waves with specific relative angular momenta. The main attraction comes from the S-waves, $^1S_0$ and $^3S_1$; the former is primarily due to the iso-singlet scalar meson in the one-boson-exchange (OBE) model, while the latter due to the pion. The former contribution in the absolute value only increases with the density. The $^3S_1$-wave contribution $V_{eff}^{^3S_1}(\rho)$ actually has a coupling to the $^3D_1$ wave and shows a peculiar density dependence as shown in Fig.7; $V_{eff}^{^3S_1}(\rho)$ in the absolute value takes the minimum about the density $\sim \rho_0$, and then the attraction becomes smaller. This is the primary reason of the saturation properties of the nuclear matter[54]. The peculiar density dependence is due to the coupling between the $^3S_1$ and the $^3D_1$ waves by the tensor force of the one-pion-exchange potential (OPEP); the Pauli principle in the intermediate states in the coupling diagrams gives the peculiar bending behavior.

Now the tensor force of OPEP arises because the pion is pseudo-scalar. The density of the bending point is so low because the pion is light. Needless to say, the pion is pseudo-scalar and light because it is the NG-boson of the $SU(2) \otimes SU(2)$ chiral symmetry. Thus one may say the saturation properties of the nuclear matter and hence the stable existence of nuclei are due to the chiral symmetry and its dynamical breaking. A few remars are in order: As for the determination of the range of the OPEP or the tensor force of the nuclear force, the subtle cancellation between the OPEP and the potential due to the ρ meson exchange in the tensor coupling is important. It is also to be remarked that the actual value of the pion mass is determined by the small current quark masses of u and d quarks. Therefore if they were much larger or smaller, nuclei might have not existed or the universe would be much different from the one where we exist.

Fig.7

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Figure Captions

Fig.1 The data and the calculated phase shift of pion-pion scattering in $I = J = 0$ channel.[21]

Fig.2 The diagram which enhances the process with $\Delta I = 1/2$.

Fig.3 The diagram describing the pion-production of the sigma meson. The sigma meson may be detected in a bump of the invariant mass of the 4 γ.

Fig. 4 The diagram describing the nucleon- or light nucleus-production of the sigma meson.

Fig.5 The diagram describing the conversion of the sigma meson to the gamma ray.

Fig. 6 The diagram describing the electro-production of the sigma meson.

Fig. 7 Contribution to the binding energy of nuclear matter of the ^1S and ^3S state as a function the Fermi momentum k_F. The calculation is by D. W. L. Sprung. Taken from a review by Bethe [54].