Sigma Meson Study from Lattice QCD

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December 17, 2016

Introduction

1. Lowest resonance in the spectrum of QCD which is crucial to understand chiral symmetry breaking.
2. The direct determination of σ resonance parameters from QCD is difficult because it is a nonperturbative problem.
3. Its decay width is comparable to its mass. The identification of mass and width has wide variety.
4. Sigma meson may be the mixture of $q\bar{q}$ mesons, molecules, and tetraquarks.

Our goal is to study sigma meson resonance using meson-meson scattering technique from Lattice QCD.

Recent Study from LQCD

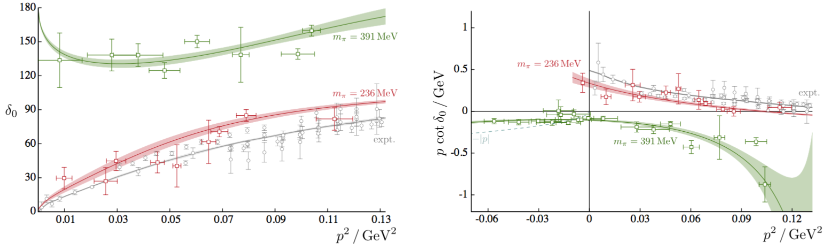


Figure: Sigma meson study from Hadron Spectrum Collaboration[2] at 391 MeV and 236 MeV pion masses.

We plan to study the sigma meson on 315 MeV and 220 MeV pion mass ensembles.

[2] Isoscalar scattering and the meson resonance from QCD, Raul A. Briceno et al

Symmetries and Interpolating Fields

We construct interpolating fields in $I = 0, J = 0$ channel in A_1^+ irrep.

$$q\bar{q} : \frac{1}{\sqrt{2}}\bar{u}(t)\Gamma(\mathbf{P})u(t) + \frac{1}{\sqrt{2}}\bar{d}(t)\Gamma(\mathbf{P})d(t); \tag{1}$$

$$\pi\pi(\mathbf{P}, t) = \frac{1}{\sqrt{3}}[\pi^-(p_1, t)\pi^+(p_2, t) + \pi^+(p_1, t)\pi^-(p_2, t) - \pi^0(p_1, t)\pi^0(p_2, t)] \tag{2}$$

$$T : [ud]^a(\mathbf{0}, t)[\bar{u}\bar{d}]^a(\mathbf{p}, t). \tag{3}$$

$$[ud]^a(\mathbf{0}, t) = \frac{1}{2}\epsilon^{abc}[u^{Tb}(t)C\gamma_5 e^{i\mathbf{0}} d^c(t) - d^{Tb}(t)C\gamma_5 e^{i\mathbf{0}} u^c(t)] \tag{4}$$

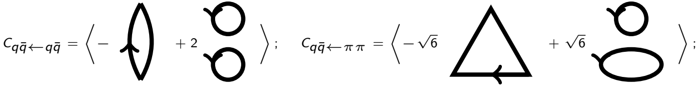
$$[\bar{u}\bar{d}]^a(\mathbf{p}, t) = \frac{1}{2}\epsilon^{abc}[\bar{u}^b(t)C\gamma_5 e^{i\mathbf{p}} \bar{d}^{Tc}(t) - \bar{d}^b(t)C\gamma_5 e^{i\mathbf{p}} \bar{u}^{Tc}(t)] \tag{5}$$

$\mathcal{O}$	$1-2$	3	4	5
	$q\bar{q}_{1-2}$	$\pi\pi_{000}$	$\pi\pi_{100}$	T_{000}

Variational Basis

To see the energy states in elastic region, we use variational method.

$$
C = \begin{pmatrix}
C_{q\bar{q}_{1-2} \leftarrow q\bar{q}_{1-2}} & C_{q\bar{q}_{1-2} \leftarrow \pi\pi_{000}} & C_{q\bar{q}_{1-2} \leftarrow \pi\pi_{100}} & C_{q\bar{q}_{1-2} \leftarrow T_{000}} \\
C_{q\bar{q}_{1-2} \leftarrow \pi\pi_{000}} & C_{\pi\pi_{000} \leftarrow \pi\pi_{000}} & C_{\pi\pi_{000} \leftarrow \pi\pi_{100}} & C_{\pi\pi_{000} \leftarrow T_{000}} \\
C_{q\bar{q}_{1-2} \leftarrow \pi\pi_{100}} & C_{\pi\pi_{000} \leftarrow \pi\pi_{100}} & C_{\pi\pi_{100} \leftarrow \pi\pi_{100}} & C_{\pi\pi_{100} \leftarrow T_{000}} \\
C_{q\bar{q}_{1-2} \leftarrow T_{000}} & C_{\pi\pi_{000} \leftarrow T_{000}} & C_{\pi\pi_{100} \leftarrow T_{000}} & C_{T_{000} \leftarrow T_{000}}
\end{pmatrix} \tag{6}
$$



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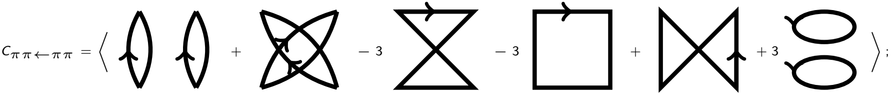

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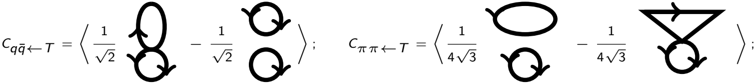

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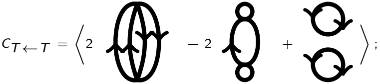

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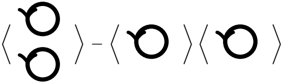
Subtract vacuum contribution:



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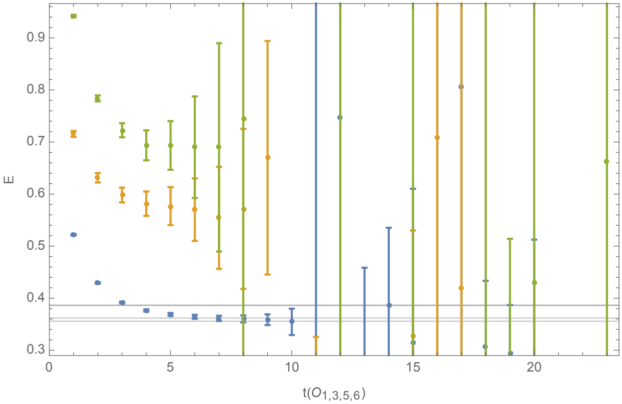
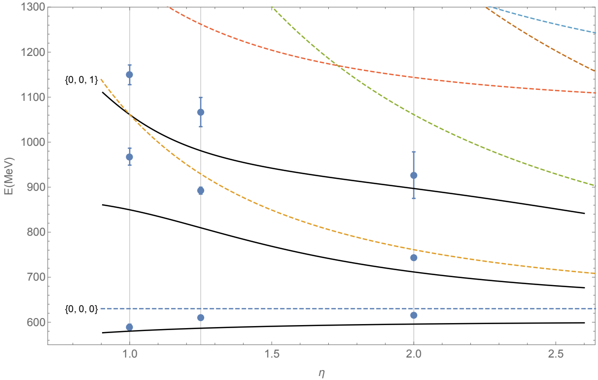
Energy Spectrum

We compute the effective mass for the lowest three energy states at 315 MeV $24^3 48$ ensemble in rest frame.



Preliminary Result

We extract the lowest three energy states for ensembles with different elongation factor η.



Future Plan

1. Cross check the preliminary result.
2. Extend the study to boost frame (total momentum $P \neq 0$).
3. Compute the phase shift and scattering length using Luscher formula.