ρ resonance in $\pi\pi$ channel with LapH smearing

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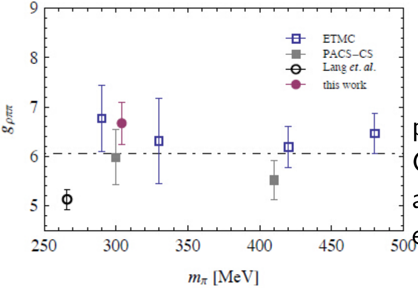
What is interesting?

1. Lüscher pointed out that the discrete spectrum E_n in finite volume $\rightarrow$ phase shift $\delta$of continuum scattering amplitude in the elastic region[1].
2. Focus on vector meson $\rho(770)$ in the $I^G(J^{PC}) = 1^+(1^{--})$ for two pions with back to back $\pi \rightarrow \leftarrow \pi$ channel.
3. Compute energy spectrum for ρ and $\pi\pi$ operators
4. Extract phase shift and compute ρ resonance parameters $g_{\rho\pi\pi}$ and decay width Γ_ρ



previous study of $g_{\rho\pi\pi}$
C.B.Lang et al use Laph smearing and boost frame method in $16^3 32$ ensemble.

Lüscher's formula for phase shift

Table : Resolution of $2J+1$ spherical harmonics into the irreducible representations of O_h and D_{4h}

J	O_h	D_{4h}
0	A_1^+	A_1^+
1	F_1^-	$A_2^- \oplus E^-$
2	$E^+ \oplus F_2^+$	$A_1^+ \oplus B_1^+ \oplus B_2^+ \oplus E^+$
3	$A_2^- \oplus F_1^- \oplus F_2^-$	$A_2^- \oplus B_1^- \oplus B_2^- \oplus 2E^-$
4	$A_1^+ \oplus E^+ \oplus F_1^+ \oplus F_2^+$	$2A_1^+ \oplus A_2^+ \oplus B_1^+ \oplus B_2^+ \oplus 2E^+$

Assume that contribution from angular momenta $l \leq 3$ is negligible. Phase shift $\delta_{l=1}$ is decomposed into A_2^- and E^- representations in D_{4h} group.

From energy spectrum to phase shift

Phase shift for $l = 1$

$$A_2^- : \cot\delta_1(k) \quad = \quad \mathcal{W}_{00} + \frac{2}{\sqrt{5}}\mathcal{W}_{20} \tag{1}$$

$$E^- : \cot\delta_1(k) \quad = \quad \mathcal{W}_{00} - \frac{1}{\sqrt{5}}\mathcal{W}_{20} \tag{2}$$

$$\mathcal{W}_{lm}(1, q^2, \eta) = \frac{\mathcal{Z}_{lm}(1, q^2, \eta)}{\pi^{\frac{3}{2}}\eta q^{l+1}}; \quad q = \frac{kL}{2\pi}; \tag{3}$$

Zeta function

$$Z_{lm}(s; q^2) = \sum_{\mathbf{n}\in L^3} \mathcal{Y}_{lm}(\mathbf{n})(\mathbf{n}^2 - q^2)^{-s} \tag{4}$$

Total energy

$$W = 2\sqrt{m_\pi^2 + k^2}; \quad k = \sqrt{\left(\frac{W}{2}\right)^2 - m_\pi^2} \tag{5}$$

Computing energy spectrum on the lattice

Variational analysis
correlation matrix in the operators basis

$$C(t)_{ij} =< \mathcal{O}_i(t)\mathcal{O}_j^{\dagger}(0) >; i,j = 1,2,..., \text{number of operators} \tag{6}$$

The eigenvalue of the correlation matrix are

$$\lambda^{(n)}(t,t_0) \propto e^{-E_n t}(1+\mathcal{O}(e^{-\Delta E_n t})), n = 1,2,..., \text{number of operators} \tag{7}$$

where $\Delta E_n = E_{\text{Number of operators}+1} - E_n$

Previous work[2]

One local operator ρ and one scattering operator $\pi\pi$ with stochastic evaluation for all-to all propagator $C_{\pi\pi\leftarrow\pi\pi}$ and $C_{\rho\leftarrow\pi\pi}$.

$$\rho = \sum_x \frac{1}{\sqrt{2}}\{\bar{u}(x)\gamma_3 u(x) - \bar{d}(x)\gamma_3 d(x)\} \tag{8}$$

$$\pi\pi = \frac{1}{\sqrt{2}}\{\pi^+(\mathbf{p})\pi^-(-\mathbf{p}) - \pi^-(\mathbf{p})\pi^+(-\mathbf{p})\} \tag{9}$$

Three ensembles $\eta 24\times 24\times 24\times 48$ ($2424^2 48$, $3024^2 48$, $4824^2 48$) with $m_\pi = 304\,\mathrm{MeV}$, $a = 0.1255\,\mathrm{fm}$.

Table : First two energy levels in A_2^- sector

η	1.0	1.25	2.0
aE_0	0.516(10)	0.511(4)	0.442(4)
aE_1	0.660(6)	0.606(10)	0.549(12)
am_π	0.1925(7)	0.1944(6)	0.1946(8)

Recent work: Larger operator basis with LapH smearing[3]

The 3-dimensional gauge-covariant Laplacian matrix

$$\tilde{\Delta}^{ab}(x,y;U)=\sum_{k=1}^{3}\left\{\tilde{U}_k^{ab}(x)\delta(y,x+\hat{k})+\tilde{U}_k^{ba}(y)^{*}\delta(y,x-\hat{k})-2\delta(x,y)\delta^{ab}\right\} \tag{10}$$

Definition of quark smearing operator to certain energy cutoff Λ (only smear spatial and color components)

$$S_{\Lambda}(t)=\sum_{\lambda(t)}^{\Lambda}|\lambda(t)\rangle\langle\lambda(t)|\,; \quad \tilde{u}(t)=S(t)u(t)=\sum_{\lambda_t}|\lambda_t\rangle\langle\lambda_t|\,u(t). \tag{11}$$

Light quark propagator $M^{-1}(t_f,t_i)$ has $(N_s^3 N_t N_c N_d)^2$ entries was projected to momentum space $\tilde{M}(t_f,t_i)$ with $(N_\lambda N_t N_d)^2$ entries.

LapH smearing

Test on $16^3 32$ ensemble (150 configs)

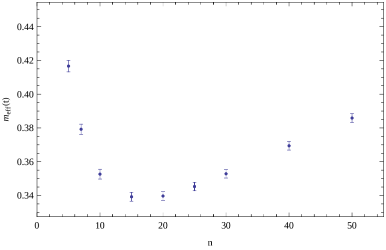


Figure : effective mass vs energy cutoff N_{vec}

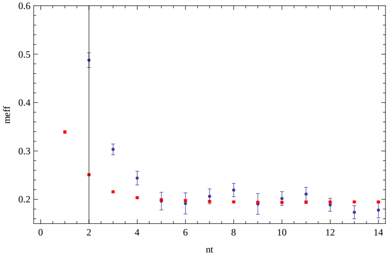


Figure : pion effective mass with and without LapH smearing

Recent work: Larger operator basis with LapH smearing

Four local operator ρ and two scattering operators $\pi\pi$.

$$\rho^J(t_f) = \bar{u}(t_f)\Gamma_{t_f} A_{t_f}(\mathbf{p}) d(t_f); \quad \rho^{J\dagger}(t_i) = \bar{d}(t_i)\Gamma^\dagger_{t_i} A^\dagger_{t_i}(\mathbf{p}) u(t_i) \tag{12}$$

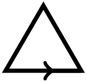
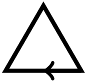
J	Γ_{t_f}	A_{t_f}	$\Gamma^\dagger_{t_i}$	$A^\dagger_{t_i}$
1	γ_i	$e^{i\mathbf{p}}$	$-\gamma_i$	$e^{-i\mathbf{p}}$
2	$\gamma_4\gamma_i$	$e^{i\mathbf{p}}$	$\gamma_4\gamma_i$	$e^{-i\mathbf{p}}$
3	γ_i	$\nabla_j e^{i\mathbf{p}} \nabla_j$	γ_i	$\nabla^\dagger_j e^{-i\mathbf{p}} \nabla^\dagger_j$
4	$\frac{1}{2}$	$\{e^{i\mathbf{p}}, \nabla_i\}$	$-\frac{1}{2}$	$\{e^{-i\mathbf{p}}, \nabla_i\}$

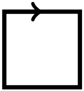
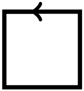
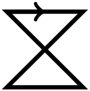
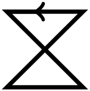
$$\pi\pi(\mathbf{p}) = \frac{1}{\sqrt{2}}\{\pi^+(\mathbf{p})\pi^-(-\mathbf{p}) - \pi^-(\mathbf{p})\pi^+(-\mathbf{p})\}; \quad \mathbf{p} = (1,0,0)\text{and}(1,1,0) \tag{13}$$

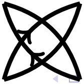
Compute correlation function

$\pi\pi \leftarrow \pi\pi$	$\rho \leftarrow \pi\pi$	$\rho \leftarrow \rho$
$(\mathbf{p_1}, t_f)(\mathbf{p_2}, t_f)$	$(\mathbf{p_1} + \mathbf{p_2}, t_f)$	$(\mathbf{P}, t_f)$
$\gamma_5 \quad \gamma_5$	Γ_J	Γ_J
$\gamma_5 \quad \gamma_5$	$\gamma_5 \quad \gamma_5$	Γ'_J
$(-\mathbf{p_1}, t_i)(-\mathbf{p_2}, t_i)$	$(-\mathbf{p_1}, t_i)(-\mathbf{p_2}, t_i)$	$(-\mathbf{P}, t_i)$

$C_{\rho\leftarrow\rho} = \Big\langle$  $\Big\rangle_U$ (14)

$C_{\rho\leftarrow\pi\pi} = \Big\langle$  $-$  $\Big\rangle_U$ (15)

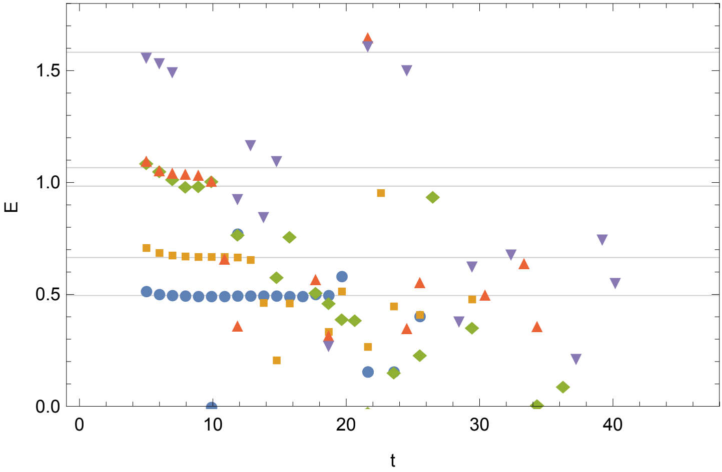
$C_{\pi\pi\leftarrow\pi\pi} = -\Big\langle$  $+$  $-$  $-$ 

$+$  $-$   $\Big\rangle_U$ (16)

5×5 correlation matrix

$$C = \begin{pmatrix} C_{\rho^J \leftarrow \rho^{J'}} & C_{\rho^J \leftarrow \pi\pi(100)} \\ C_{\pi\pi(100) \leftarrow \rho^{J'}} & C_{\pi\pi(100) \leftarrow \pi\pi(100)} \end{pmatrix} \quad (17)$$

extract 5 energy states ($\mathbf{P} = 0$)



Result compared to chiral perturbation theory

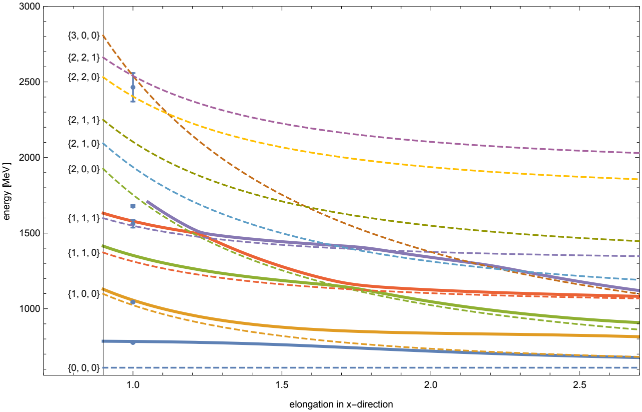


Figure : Simulation on $24^3 48$ $m_\pi = 304\,\mathrm{MeV}$ensemble with chiral perturbation theory. Dashed line indicate the free energy of two pion with certain back to back momentum

6×6 correlation matrix

$$C = \begin{pmatrix} C_{\rho^J \leftarrow \rho^{J'}} & C_{\rho^J \leftarrow \pi\pi(100)} & C_{\rho^J \leftarrow \pi\pi(110)} \\ C_{\pi\pi(100) \leftarrow \rho^{J'}} & C_{\pi\pi(100) \leftarrow \pi\pi(100)} & C_{\pi\pi(100) \leftarrow \pi\pi(110)} \\ C_{\pi\pi(110) \leftarrow \rho^{J}} & C_{\pi\pi(110) \leftarrow \pi\pi(100)} & C_{\pi\pi(110) \leftarrow \pi\pi(110)} \end{pmatrix} \tag{18}$$

extract 6 energy states($\mathbf{P} = 0$)

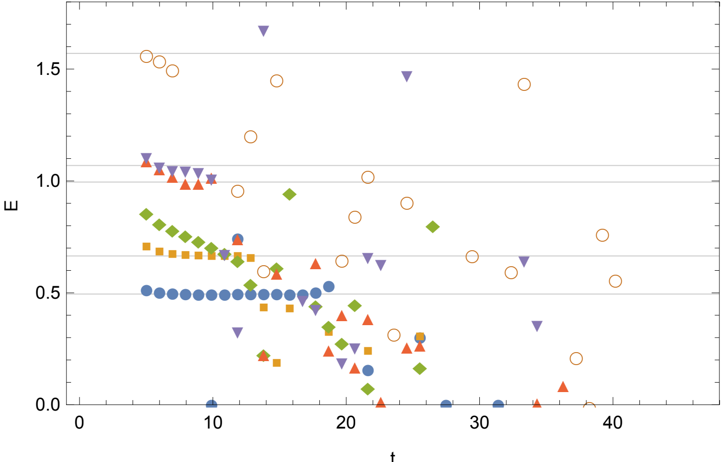
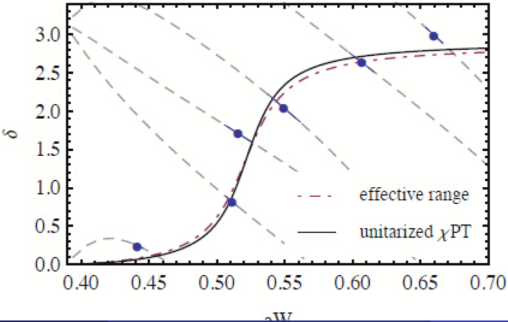


Table : First two energy levels on $\{24^3, 30^3, 48^3\}48$ ensemble(300 configs) in A_2^- sector

η	1.0	1.25	2.0	1.0($LapH$)
aE_0	0.516(10)	0.511(4)	0.442(4)	0.4947(9)
aE_1	0.660(6)	0.606(10)	0.549(12)	0.6648(11)

Figure : phase shift for $\{24^3, 30^3, 48^3\}48$(old result)



Coming soon

1. continue the same analysis ($\mathbf{P} = 0$) on $\{30^3, 48^3\}48$ ensembles (300 configs) and compute the resonance parameter $g_{\rho\pi\pi}$ and decay width Γ_ρ
2. start $\mathbf{P} = (1, 0, 0)$ and $(1, 1, 0)$ on $24^3 48$ ensemble(300 configs) and compute the resonance parameter $g_{\rho\pi\pi}$ and decay width Γ_ρ

n_0	t_0	fitrange	$\tilde{E}_n$	E_n(MeV)	$\chi^2/p.o.f$	$conflv\%$
1	3	$7-18$	0.4947(9)	777.8(1.4)	0.86	56.3
2	3	$7-14$	0.6648(11)	1405(2)	0.87	51.0
3	3	???	???	???	???	???
4	3	$6-11$	0.995(16)	1560(30)	0.67	61.1
5	3	$4-9$	1.069(5)	1680(8)	1.61	16.7
6	3	$4-7$	1.57(6)	2470(90)	0.81	44.2

- M. Luscher, *Nucl.Phys. B* **354** (1991) 531–578.
- C. Pelissier and A. Alexandru, *Phys.Rev. D* **87** (2013) 014503, [arXiv:1211.0092].
- C. Morningstar, John Bulava, Justin Foley, Keisuku J.Juge, David Lenkner, Mike Peardon, Chik Him Wong, *Phys.Rev. D* **83**, 114505 (2011), [arXiv:1104.3870].