Depth Dependence of Submerged Gaseous Expulsion

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A relationship between the pressure of a container of gas and the frequency and amplitude of a sound produced by this container is derived. This relationship is then used to determine the underwater depth at which a human cannot fart due to the ambient pressure. Both analytical and numerical investigations are performed, as well as a careful discussion of several practical considerations.

I. INTRODUCTION

In this paper, we seek to understand the extent of a person's ability to fart. Specifically: How deep can a person submerge themselves before the ambient pressure of the water is too great to be overcome by gluteal action alone?

There is a surprisingly short history in the literature of other research considering this problem. So short, in fact, that we are confident we are the first.

II. THEORY

This problem divides naturally into two parts: First, we determine how much pressure is required to create a fart, then we find the depth at which that pressure is insufficient to overcome the environment, leaving any flatulence confined until it finds itself in more supportive conditions.

II.1. The pressure required to fart

Lacking any sort of probe into an individual's relevant canals, it is difficult to determine the interior pressure that might produce a fart. To circumvent this problem, we can turn to indirect measurement, and rely on the frequency of resultant longitudinal waves of a fart to determine the rest of its properties.

Therefore, we seek an expression that describes how the power of a sound wave depends on that wave's frequency and amplitude. This will allow us to determine lower-bound energies of a fart, from which we can determine the pressure. We review some of the derivation here as we find it helpful in interpreting the ultimate result.

The power, P of any wave is its time-averaged energy per time. Further, we can assume that the displacement $A(x,t)$ of a particle in a single-frequency wave is a sinusoid:

$$A(x,t) = A \sin\left[2\pi\left(\nu t - \frac{x}{\lambda}\right)\right], \tag{1}$$

for amplitude A, frequency ν, and wavelength λ. Taking the time derivative of (1) returns something we can use to calculate the kinetic energy via $K = 1/2mv^2$. Making the replacement $m \to \rho_a ac\bar{T}$ (for ambient density ρ_a, instantaneous cross-sectional area a, wave speed c, and period $\bar{T}$), to work with quantities which are more relevant for wave dynamics, and taking the integral over one period, we arrive at

$$P = 2\pi^2\left(\rho_a A^2\right)\left(\nu^2 ac\right), \tag{2}$$

for the power of a sound wave. Implicit in this derivation, we assumed that the energy of a sound wave is evenly split between kinetic and potential, so $E = 2K$.

Finally, pressure of a gas in a container is the energy per volume V of that container. So, if the fart lasts T seconds, the total pressure, p, which produces a sound with power P is

$$p = 2\pi^2 \frac{\left(\rho_a A^2\right)\left(\nu^2 ac\right)T}{V}. \tag{3}$$

II.2. Overcoming underwater pressure

Pressure in a noncompressible fluid goes as

$$P = \rho_f g h, \tag{4}$$

for density of the fluid ρ_f, depth h and gravitational acceleration g. With this, we arrive at the so-called "Deich-Schnaubelt relation", which says a person cannot fart below a depth of

$$h = 2\pi^2 \frac{\left(\rho_a A^2\right)\left(\nu^2 ac\right)T}{Vg\rho_f}. \tag{5}$$

It's important to note that the quantities ν, A, ρ_a, c are the values at sea level, and are only used to determine the energy, which we assume is invariant to depth. They will all change in a different ambient environment.

III. DATA AND RESULTS

Table I gives the fiducial values of the quantities in (5). The values describing the fart sound wave were derived from a series experiments carried out at Montana

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TABLE I. Fiducial values of (5).

Symbol	Value	Meaning
ρ_a	1.225 kg/m^3	density of air at sea level
A	3×10^{-3} m $\pm 5 \times 10^{-4}$	amplitude of fart
ν	10^3 Hz $\pm 3 \times 10^2$	frequency of fart
a	.1 m	resonant aperture
c	331.2 m/s	speed of sound at sea level
T	2 s ± 1	duration of fart
V	5×10^{-1}m^3 ± 1	volume of resonant chamber
ρ_f	10^3 kg/m^3	density of water

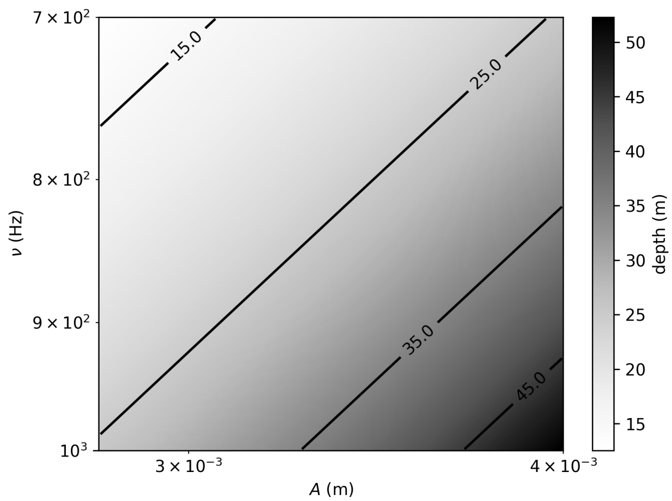

FIG. 1. Depth as a function of sea-level fart amplitude and frequency. 15-, 25-, 35- and 45-meter contours are highlighted.

State. The biological values are order of magnitude estimates, owing to the authors' unwillingness to perform the observations themselves.

Using these values, we arrive at a lower-bound depth of $h \simeq 30$ m $\pm$ 12 below which it is impossible to fart.

It is instructive to study an amplitude-frequency diagram (Fig. 1). The more powerful farts are represented in the bottom-right corner. The corresponding heights scale quadratically, due to the dependence of h on the squares of amplitude and frequency.

IV. PRACTICAL CONSIDERATIONS

Over the course of the current work, we have spent little energy considering in-situ conditions which would bear an effect on the calculations. Nearby wildlife, for instance, would drastically reduce the depth at which one would *want* to fart: Sharks, after all, are famous for their senses of smell.

One interesting phenomenon is that higher-pitched farts are viable for much greater depths (Fig. 2). This means that the deeper you go, the safer you are from sharks, whose auditory range is only sensitive to approximately 800 Hz [1].

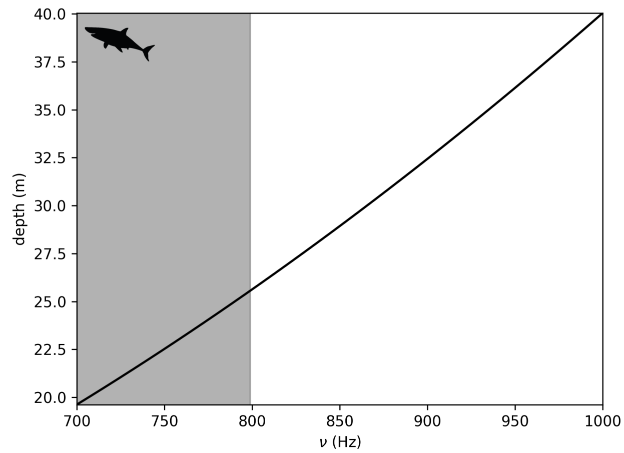

FIG. 2. The achievable depth of a range of sea-level frequencies (black line). The shaded region corresponds to those depths and frequencies where sharks are the most capable.

In addition, we have not considered any effect that something like a wetsuit would have on the situation. It is conceivable that a wetsuit would raise the maximum depth, but it's not clear by how much.

V. DISCUSSION

We have derived a relationship between underwater depth and sea-level fart energies. We used this Deich-Schnaubelt equation, along with measurements of dubious accuracy to lower-bound the maximum depth at which the ambient underwater pressure would overcome the pressure the fart was being expelled at, showing that any inability to fart underwater would not likely be achieved until under 30 m.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES

[1] Molly Edmonds, *How Stuff Works*, https://animals.howstuffworks.com/fish/sharks/shark-senses2.htm, accessed August 28, 2018.