

# Playing with the E&M Toy Model: A Charged Spinning Sphere

## 1 Comparing the E&M Analogy with Kerr

I argued in [insert section number here] that E&M is a strong analog to gravity. That was a generic field theoretic argument; let's take advantage of this analogy to use electromagnetism to get some concrete predictions about gravity. Taking the analogy as valid, we can construct a toy model to compare with Kerr.

To review, mapping gravity to electromagnetism, we can say:

$$m \rightarrow q \text{ (mass becomes charge)} \quad (1)$$

$$G \rightarrow \frac{1}{4\pi\epsilon_0} \text{ (constants switch)} \quad (2)$$

$$+ \rightarrow - \text{ (gravity is attractive; electricity, repulsive)} \quad (3)$$

Then, making special relativistic considerations, we can derive necessary magnetic fields as corrections to both electricity and gravity.

So, we're studying orbits in the Kerr spacetime, which is the consequence of a spinning mass. Accounting for the above mapping then, we should expect the analogous E&M system to be represented by Fig.1: A charged spinning sphere. Specifically, we want to study the orbit of a charged particle near the sphere. More to the point of this thesis, we seek how the orbits of the particles change when the spin of the central sphere changes.

## 2 A Description of the Relevant Magnetic Vector Potential

All quantities refer to Fig.1. Our system is a solid sphere of radius  $R$  with constant surface charge density  $\sigma$ , rotating about the  $z$  axis with some angular frequency  $\omega$ .

The magnetic vector potential at an arbitrary point  $(r, \theta, \phi)$  outside the sphere is given by  $\mathbf{A} = \frac{\mu_0 R^4 \sigma}{3} \frac{\sin \theta}{r^2} \hat{\phi}$ . The derivation for this is reproduced in Appendix [insert number]. For our purposes, it is helpful to reparameterize the vector potential with the angular

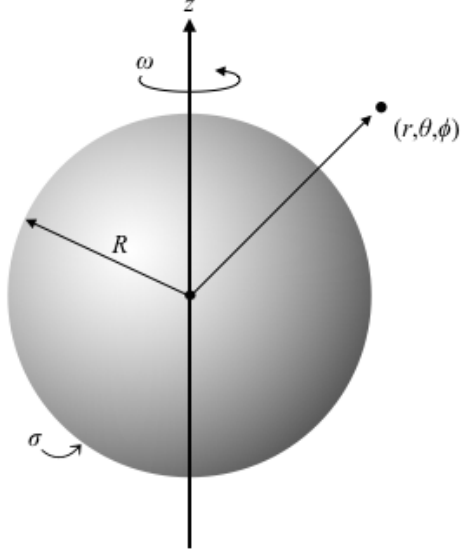


Figure 1

momentum,  $l$  and total charge  $Q$  instead<sup>1</sup>:

$$\mathbf{A} = \frac{1}{2} \frac{\mu_0 Q l \sin \theta}{4\pi r^2} \hat{\phi} \quad (4)$$

The electric potential is unchanged from a static sphere:  $V = \frac{Q}{4\pi\epsilon_0 r}$ .

Then, to probe the trajectories of a particle of charge  $q$  near the spinning sphere, we need to look at the Lagrangian

$$L = \frac{1}{2} m \left( \dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2 \right) - \frac{qQ}{4\pi\epsilon_0 r} + \frac{\mu_0 q Q l \sin^2 \theta}{8\pi r} \dot{\theta}. \quad (5)$$

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<sup>1</sup>This change is made using the angular momentum per unit mass,  $I\omega/M$ , with the moment of inertia of a sphere,  $I = 2MR^2/5$ .