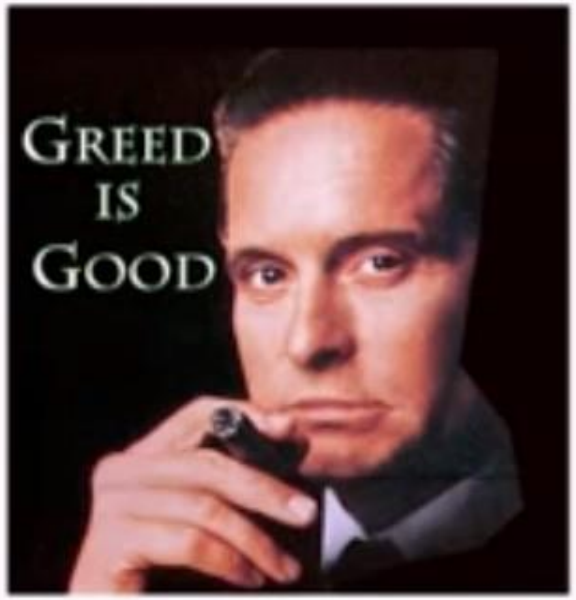
DATA SIENCE OBTIMISATION PROBLEMME

Win+w

The Pros and Cons of Greedy

- Easy to implement
- Computationally efficient



- But does not always yield the best solution
  - Don't even know how good the approximation is

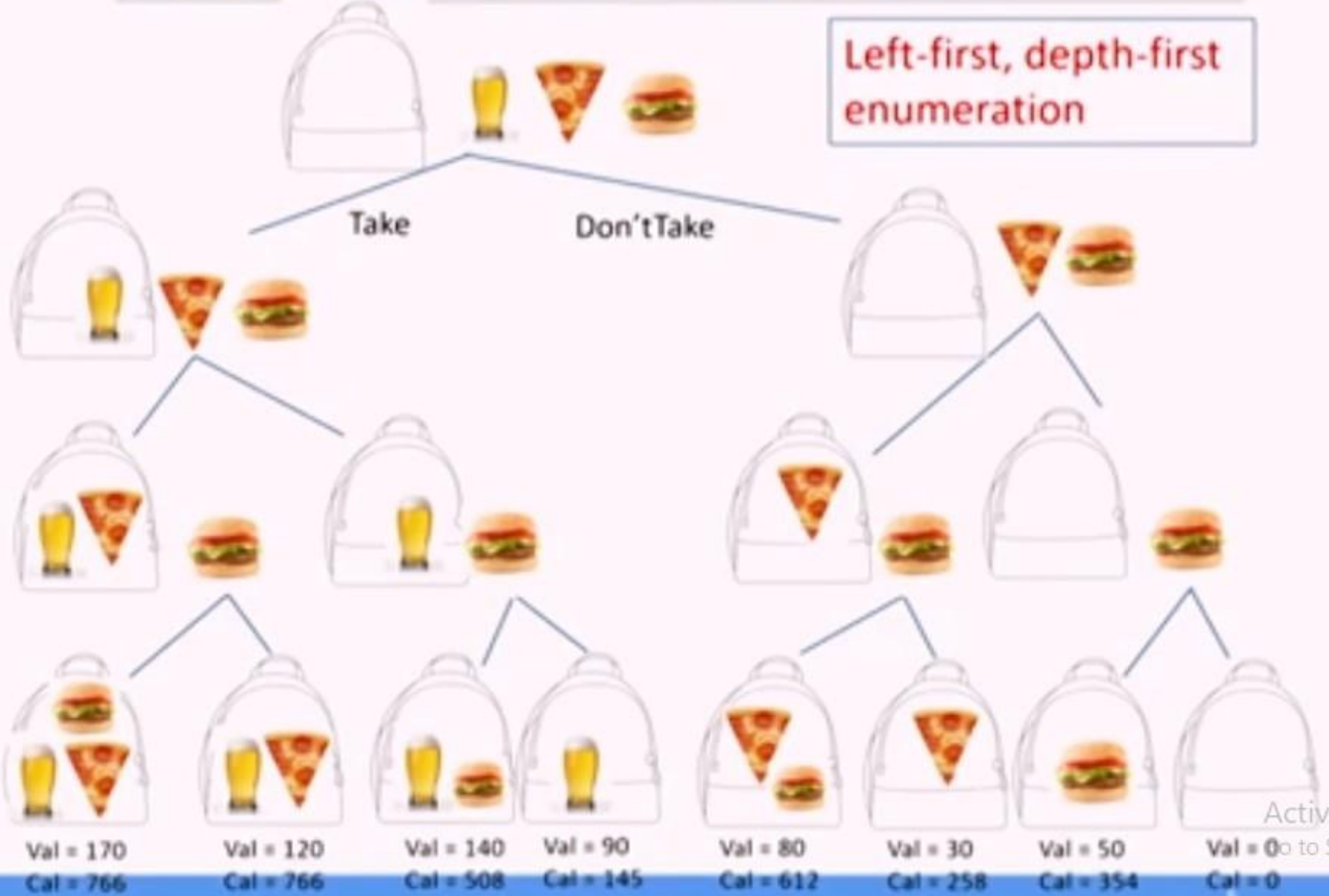
Question 1

Brute Force Algorithm

- 1. Enumerate all possible combinations of items.
- 2. Remove all of the combinations whose total units exceeds the allowed weight.
- 3. From the remaining combinations choose any one whose value is the largest.

A Search Tree Enumerates Possibilities

Left-first, depth-first enumeration



Computational Complexity

- Time based on number of nodes generated
- Number of levels is number of items to choose from
- Number of nodes at level *i* is 2^i
- So, if there are *n* items the number of nodes is
  - $\sum_{i=0}^{i=n} 2^i$
  - I.e., $O(2^{n+1})$
- An obvious optimization: don't explore parts of tree that violate constraint (e.g., too many calories)
  - Doesn't change complexity

Header for Decision Tree Implementation

```
def maxVal(toConsider, avail):
    """Assumes toConsider a list of items,
               avail a weight
       Returns a tuple of the total value of a
           solution to 0/1 knapsack problem and
           the items of that solution"""
```

`toConsider`. Those items that nodes higher up in the tree (corresponding to earlier calls in the recursive call stack) have not yet considered

`avail`. The amount of space still available

Body of maxVal (without comments)

```
if toConsider == [] or avail == 0:
        result = (0, ())
    elif toConsider[0].getUnits() > avail:
        result = maxVal(toConsider[1:], avail)
    else:
        nextItem = toConsider[0]
        withVal, withToTake = maxVal(toConsider[1:],
                                     avail - nextItem.getUnits())
        withVal += nextItem.getValue()
        withoutVal, withoutToTake = maxVal(toConsider[1:], avail)
    if withVal > withoutVal:
            result = (withVal, withToTake + (nextItem,))
        else:
            result = (withoutVal, withoutToTake)
    return result
```

Body of maxVal (without comments)

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if toConsider == [] or avail == 0:
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        withVal += nextItem.getValue()
        withoutVal, withoutToTake = maxVal(toConsider[1:], avail)
    if withVal > withoutVal:
            result = (withVal, withToTake + (nextItem,))
        else:
            result = (withoutVal, withoutToTake)
    return result
```

Does not actually build search tree

Local variable `result` records best solution found so far

Search Tree Worked Great

- Gave us a better answer
- Finished quickly
- But 2^8 is not a large number
  - We should look at what happens when we have a more extensive menu to choose from

Code to Try Larger Examples

```
import random

def buildLargeMenu(numItems, maxVal, maxCost):
    items = []
    for i in range(numItems):
        items.append(Food(str(i),
                          random.randint(1, maxVal),
                          random.randint(1, maxCost)))
    return items


for numItems in (5,10,15,20,25,30,35,40,45,50,55,60):
    items = buildLargeMenu(numItems, 90, 250)
    testMaxVal(items, 750, False)
```

Dynamic Programming?

Sometimes a name is just a name

"The 1950s were not good years for mathematical research... I felt I had to do something to shield Wilson and the Air Force from the fact that I was really doing mathematics... What title, what name, could I choose? ... It's impossible to use the word dynamic in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible. Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to. So I used it as an umbrella for my activities.

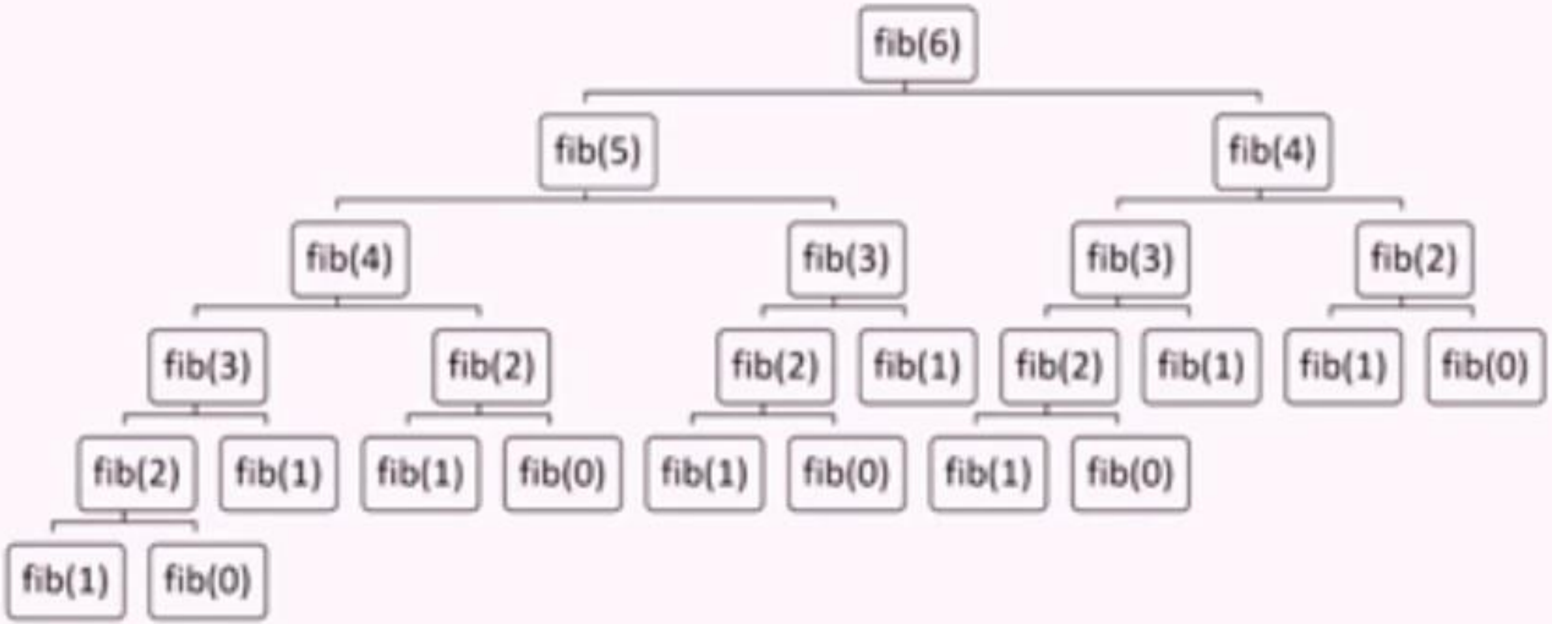
-- Richard Bellman

Recursive Implementation of Fibonnaci

```
def fib(n):
    if n == 0 or n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

fib(120) = 8,670,007,398,507,948,658,051,921

Call Tree for Recursive Fibonnaci(6) = 13



Clearly a Bad Idea to Repeat Work

- Trade a time for space
- Create a table to record what we've done
  - Before computing fib(x), check if value of fib(x) already stored in the table
    - If so, look it up
    - If not, compute it and then add it to table
  - Called memoization

Using a Memo to Compute Fibonnaci

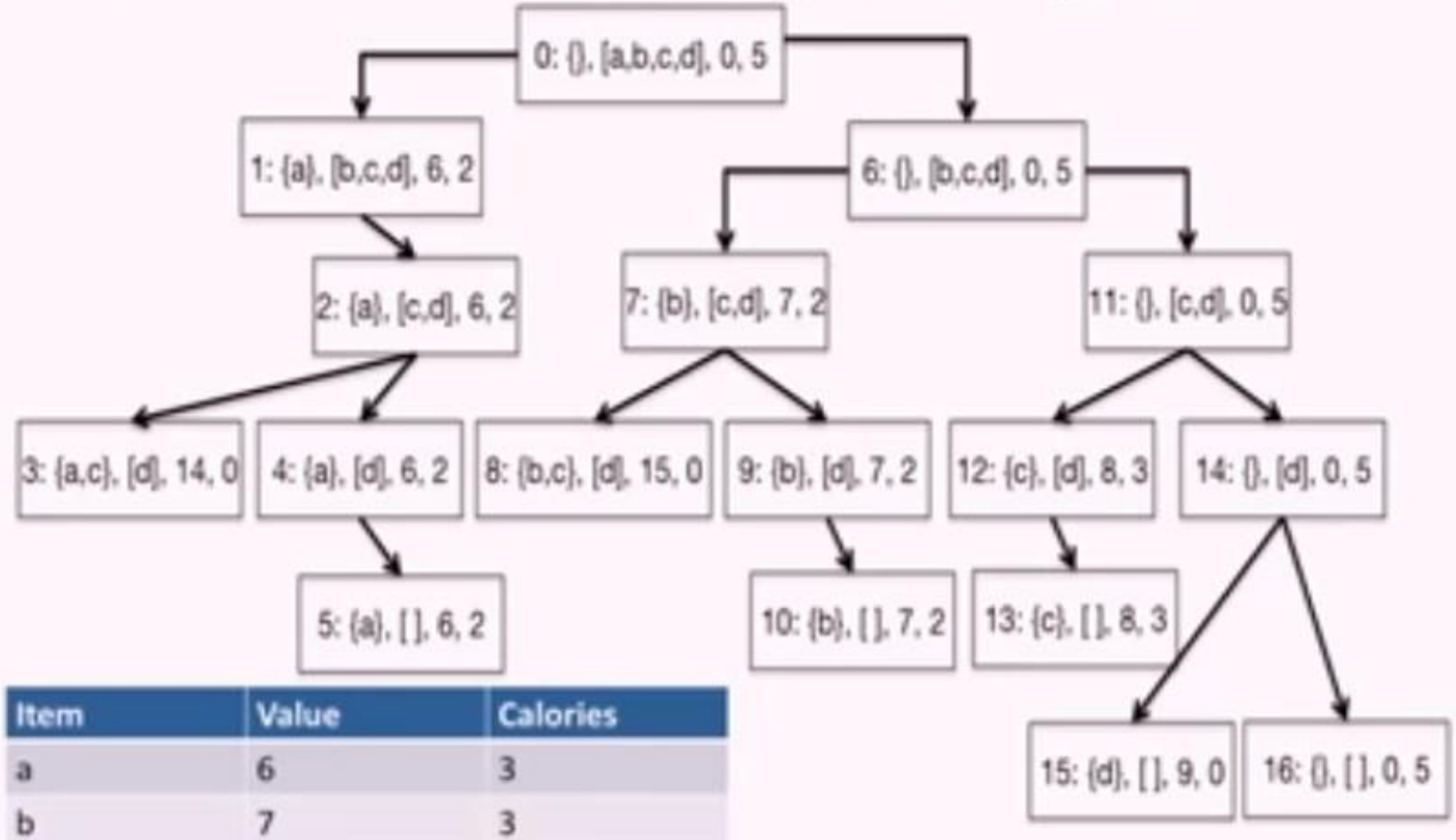
```
def fastFib(n, memo = {}):
    """Assumes n is an int >= 0, memo used only by
         recursive calls
       Returns Fibonacci of n"""
    if n == 0 or n == 1:
        return 1
    try:
        return memo[n]
    except KeyError:
        result = fastFib(n-1, memo) +\
                 fastFib(n-2, memo)
        memo[n] = result
        return result
```

When Does It Work?

- Optimal substructure: a globally optimal solution can be found by combining optimal solutions to local subproblems
  - For $x > 1$, $fib(x) = fib(x - 1) + fib(x - 2)$
- Overlapping subproblems: finding an optimal solution involves solving the same problem multiple times
  - Compute fib(x) or many times

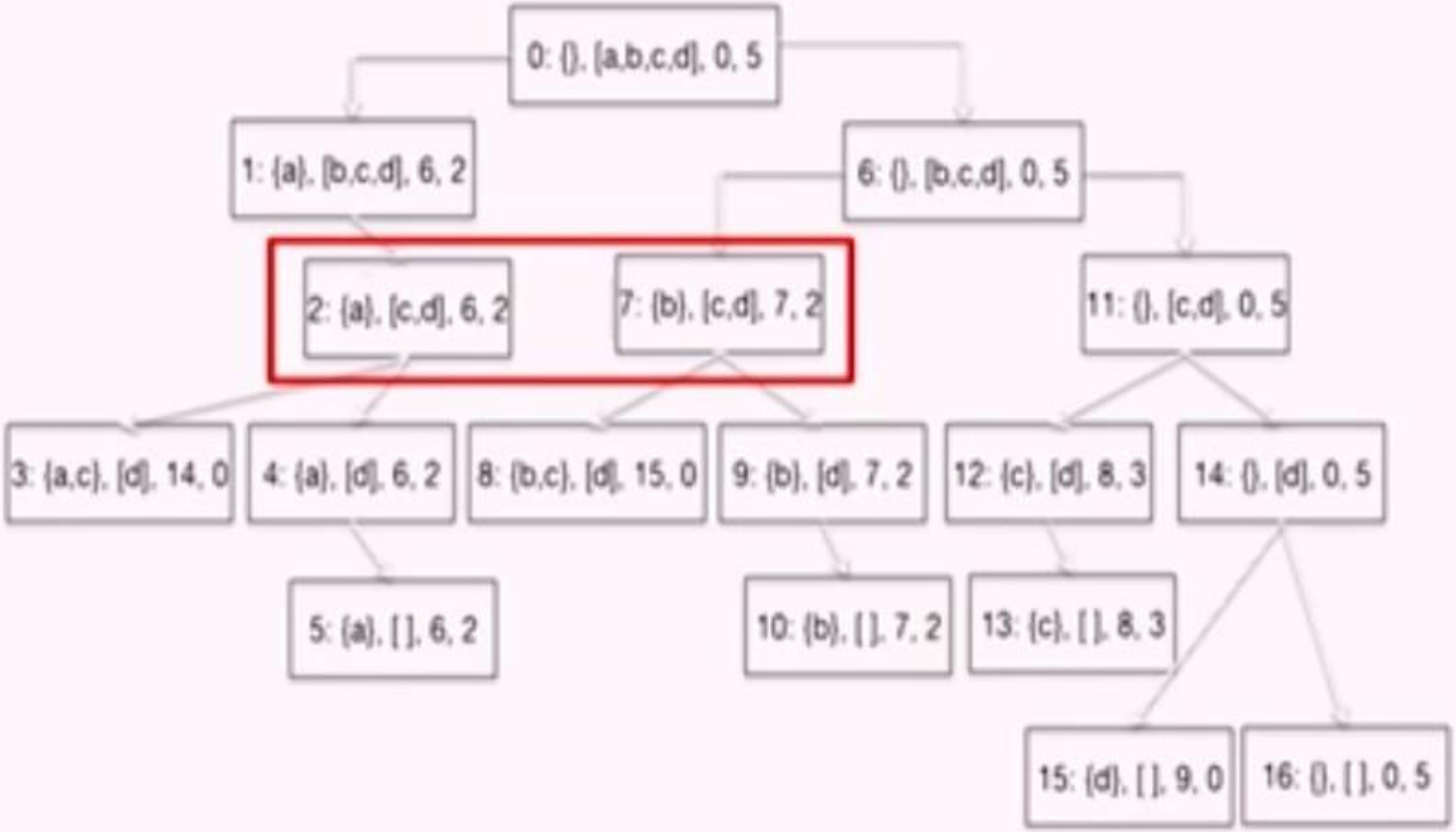
Search Tree

- Each node = <taken, left, value, remaining calories>



Item	Value	Calories
a	6	3
b	7	3
c	8	2
d	9	5

Overlapping Subproblems



Modify maxVal to Use a Memo

- Add memo as a third argument
  - `def fastMaxVal(toConsider, avail, memo = {}):`
- Key of memo is a tuple
  - (items left to be considered, available weight)
  - Items left to be considered represented by `len(toConsider)`
- First thing body of function does is check whether the optimal choice of items given the the available weight is already in the memo
- Last thing body of function does is update the memo

Performance

len(items)	2**len(items)	Number of calls
2	4	7
4	16	25
8	256	427
16	65,536	5,191
32	4,294,967,296	22,701
64	18,446,744,073,709,551,616	42,569
128	Big	83,319
256	Really Big	176,614
512	Ridiculously big	351,230
1024	Absolutely huge	703,802

How Can This Be?

- Problem is exponential
- Have we overturned the laws of the universe?
- Is dynamic programming a miracle?
- No, but computational complexity can be subtle
- Running time of `fastMaxVal` is governed by number of distinct pairs, <toConsider, avail>
  - Number of possible values of `toConsider` bounded by `len(items)`
  - Possible values of `avail` a bit harder to characterize
    - Bounded by number of distinct sums of weights
  - Covered in more detail in assigned reading

Summary of Lectures 1-2

- Many problems of practical importance can be formulated as optimization problems
- Greedy algorithms often provide adequate (though not necessarily optimal) solutions
- Finding an optimal solution is usually exponentially hard
- But dynamic programming often yields good performance for a subclass of optimization problems—those with optimal substructure and overlapping subproblems
  - Solution always correct
  - Fast under the right circumstances

The "Roll-over" Optimization Problem

Score = ((60 – (a+b+c+d+e))*F + a*ps1 + b*ps2 + c*ps3 + d*ps4 + e*ps5

Objective:

- Given values for F, ps1, ps2, ps3, ps4, ps5
- Find values for a, b, c, d, e that maximize score

Constraints:

- a, b, c, d, e are each 10 or 0
- a + b + c + d + e ≥ 20