KSP Remote Tech Calculations

Introduction

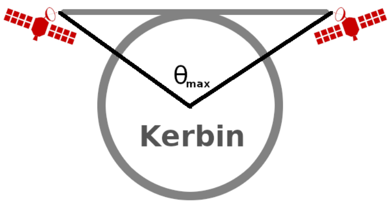
The RemoteTech mod of KSP requires the design and implementation of constellations of relay satellites which are used by remotely-operated satellites and probes. Such constellations usually consist of a number of equally-spaced satellites in circular orbits. The mathematics of those constellations is presented here, with a particular focus on the Kerbin system.

This document lives at https://github.com/deirdreobyrne/KSP/blob/master/RemoteTechMath.pdf, and is discussed on the Kerbal Space Program forums[1]. I would eventually like to close some holes in this analysis, so this document should be considered to be a living document.

PLEASE NOTE THAT THIS VERSION OF THIS DOCUMENT HAS NOT YET BEEN INDEPENDENTLY CHECKED. If you like math, please check my work, and post your findings on the forum.

Step 1 – choose your antenna

Each antenna has a range associated with it. In order to keep a line-of-sight between antennae in a circular orbit, there needs to be a minimum number of antennae in the constellation.



The angle θ_{max} represents the maximum angle at the centre of Kerbin that the satellite constellation can support, which in turn gives the minimum number of satellites in such a constellation. Simple trigonometry gives us the forumula

$$\theta_{max} = 2 \times \tan^{-1}\left(\frac{d}{2r_k}\right), \; n = \frac{360\,°}{\theta_{max}}$$

where d = maximum antenna range, r_k = radius of Kerbin, and n = minimum number of satellites in the constellation (which will, of course, need to be rounded up to an integer).

Almost all of the antennae in the game have a minimum constellation size of 3 when used in Kerbin orbit. The Communotron 16-S needs a minimum of 4, and the weak Reflectron DP-10 needs 8.

Antenna	**Range**	θ_{max}	n
Reflectron DP-10	500km	*45.2°*	*8*
Communotron 16-S	1.5Mm	*102.7°*	*4*
Communotron 16	2.5Mm	*128.7°*	*3*

Step 2 – choose your constellation size

Now that we have our minimum constellation size, we can choose the actual number of satellites we will use in that constellation. This gives the angle θ of our constellation, through the formula $\theta = 360\,°/n$, where n is the number of satellites we've chosen. We can now calculate the range of possible orbits for our constellation.

The restriction on our minimum orbit size is that all satellites need line-of-sight to the next satellite. This leads to the formula

1 https://forum.kerbalspaceprogram.com/index.php?/topic/181918-remotetech-math/

$$SMA_{min}=\frac{r}{\cos(\theta/2)}$$ where r = radius of Kerbin and SMA_{min} = minimum semi-major axis.

Note that SMA_{min} depends only on the radius of Kerbin and the number of satellites in the constellation (assuming you've used the formula in step 1 to ensure that there are sufficient satellites).

The restriction on our maximum orbit size is that the range of our antenna must be such that it can reach its two nearest neighbours. This leads to

$$SMA_{max}=\frac{d}{2\times\sin(\theta/2)}$$ where SMA_{max} = maximum semi-major axis and d = antenna range

From these values, the minimum and maximum orbit altitudes, and the minimum and maxium orbit periods, can be calculated.

Antenna	**Range**	**n**	θ	SMA_{min}	SMA_{max}
Communotron 16-S	1.5Mm	4	*90°*	*848,528m*	*1,060,660m*
Communotron 16-S	1.5Mm	5	*72°*	*741,641m*	*1,275,976m*
Communotron 16	2.5Mm	3	*120°*	*1,200,000m*	*1,443,376m*
Communotron 16	2.5Mm	4	*90°*	*848,528m*	*1,767,767m*

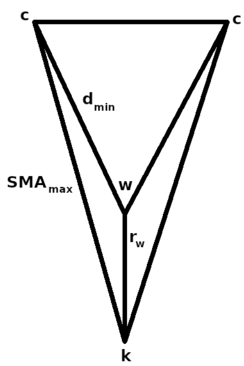
Remember that the radius of Kerbin (600,000m) should be subtracted from the SMA values to get the corresponding orbital heights.

Step 3 – ensure your working satellites can use the constellation

The antenna on your constellation satellites are only half the story. That constellation is used by working satellites, which have their own orbits and their own antennae with their own range.

The restriction on the semi-major axis of the constellation is that a new constellation satellite must come into range of the working satellite the instant the working satellite moves out of range of its current constellation satellite.

We will define d_{min} as the range of the antenna (working or constellation) with the smallest range, and r_w as the distance of the working satellite above the centre of Kerbin. And we will assume the working satellites are co-planar with the constellation, and are in circular orbits.



In this diagram, c is the location of the constellation satellites, w is the location of the working satellite, and k is the centre of Kerbin. Note that the angle *wkc* is $\theta/2$. To solve the triangle for SMA_{max}, we need to first invoke the sine rule to find the angle *kcw.*

$$kcw=\sin^{-1}\left(\frac{r_w\times\sin(\theta/2)}{d_{min}}\right)$$

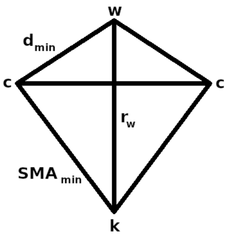
Now we can calculate the angle *cwk*

$$cwk=180°-(\theta/2)-kcw$$

Finally, we re-invoke the sine rule to get SMA_{max}.

$$SMA_{max}=\frac{d_{min}\times\sin(cwk)}{\sin(\theta/2)}$$

Calculating SMA_{min} is very similar



Again, *wkc* is *θ/2*, and again the sine rule gives us the angle *kcw*. However, since *kcw* is an acute angle, we need to use the other solution to the inverse sine function

$$kcw = 180° - \sin^{-1}\left(\frac{r_w \times \sin(\theta/2)}{d_{min}}\right)$$

The rest of the calculation proceeds as above.

				SMA_{max}			SMA_{min}		
d_{min}	**r_w**	**n**	***θ***	***kcw***	***cwk***	***SMA***	***kcw***	***cwk***	***SMA***
2.5Mm	**2.8Mm**	**3**	*120°*	*75.9°*	*44.1°*	*2,008,276m*	*104.1°*	*15.9°*	*791,724m*

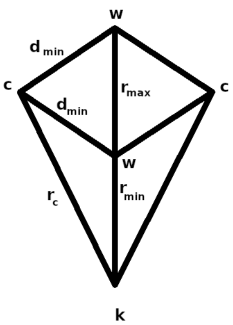
We now have two values for each of SMA_{max} and SMA_{min} – one from the requirement that the constellation satellites can communicate with each other, and the other from the requirement that the working satellites can communicate with the constellation satellites. Our actual minimum semi-major axis is the larger of the two values of SMA_{min}, and our actual maximum semi-major axis the the smaller of the two values of SMA_{max}. From these values, we can calculate the minimum and maximum orbital period of the constellation satellites.

One issue with the communication satellites is that they need to be able to power themselves while in eclipse. If they utilise solar power for energy, then they need to have enough battery capacity to survive the worst eclipse they can come across. We will look at the relevant calculations later. For now, note that the lower the orbit the shorter the amount of time the satellite will spend in eclipse.

Step 4 – choose a constellation semi-major axis

Our constellation will be able to provide constant coverage for a range of orbits, in that there is a range of distances from the centre of Kerbin in which a working satellite is constantly within reach of at least one constellation satellite.

Again, we will define d_{min} as the range of the antenna (working or constellation) with the smallest range, and r_c as the chosen semi-major axis of the constellation satellites. Define r_{max} and r_{min} to be the maximum and minimum distance from the centre of Kerbin the working satellites can have while still maintaining constant contact with the constellation.



The angle *ckw* is *θ/2*, and again we apply the sine rule, this time to find the two angles *kwc*

$$kwc = \sin^{-1}\left(\frac{r_c \times \sin(\theta/2)}{d_{min}}\right),\ kwc = 180° - \sin^{-1}\left(\frac{r_c \times \sin(\theta/2)}{d_{min}}\right)$$

The obtuse angle *kwc* will give us r_{min} at the end of the calculation, and the acute angle will give us r_{max}. The two angles *kcw* are found by the rule that the sum of the angles of a triangle is equal to 180°

$$kcw = 180° - kwc - (\theta/2)$$

Finally we can apply the sine rule again to get r_{min} and r_{max} $r_{min/max} = \frac{d_{min} \times \sin(kcw)}{\sin(\theta/2)}$

d_{min}	r_c	n	θ	r_{max}			r_{min}		
				kwc	*kcw*	r_{max}	*kwc*	*kcw*	r_{min}
1.5Mm	**1.7Mm**	**3**	*120°*	*79.0°*	*41.0°*	*1,137,228m*	*101.0°*	*19.0°*	*562,772m*

Note that in this example, r_{min} is less than the radius of Kerbin, which means there is coverage down to the surface of Kerbin. However that coverage will only cover the lower latitudes. An analysis of this is provided in appendix A.

Step 5 – dealing with eclipses

Once every orbit, the constellation satellites will be eclipsed by Kerbin. If you are unlucky, they will enter an eclipse of the Mun just before or after being eclipsed by Kerbin. And if you are really unlucky, they will be eclipsed by Minmus just before or after that!

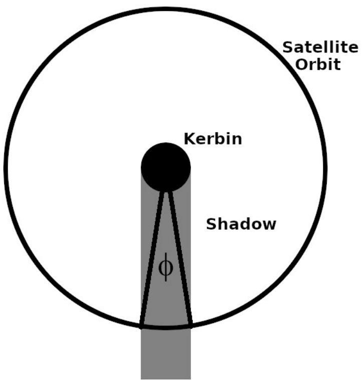
Once every orbit, they will also be eclipsed by the Mun, and probably by Minmus. The worst times for those eclipses to happen are when the satellite is above Kerbin's terminator, as at that time the speed of the satellite relative to the shadow of the Mun and/or Minmus is at its slowest. For the Kerbin system, these potential eclipses are usually the longest.

But that's just the Kerbin system. A satellite in a constellation high enough above Duna, for instance, might experience quite an eclipse from Ike, as Ike is quite large and quite close to Duna. I'm not going to cover such eclipses (yet).

We make the assumptions that the constellation satellites are relatively close to the parent body, and the moons are relatively far away and relatively slow-moving.

Scenario 1 – Kerbin, Mun and Minmus eclipse in quick succession

In general, we need to calculate the amount of time it takes the satellite to cross a shadow cast by a celestial body of given radius. We will make the assumption that the body casting the shadow is effectively stationary during the eclipse, and that Kerbol is a point source of light at an infinite distance.



The angle ϕ is given by

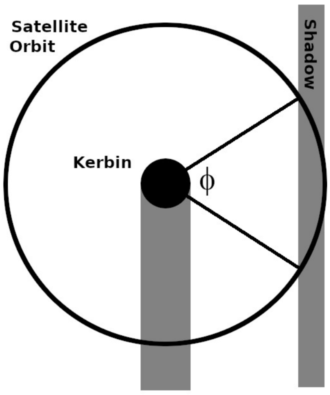
$$\phi = 2\times\sin^{-1}\left(\frac{r}{r_c}\right)$$

where r is the radius of the body casting the shadow, and r_c is the semi-major axis of the satellite orbit. Converting that into an eclipse duration t is simply a matter of scaling to the orbital period of the satellite P

$$t=\frac{P\times\phi}{360°}$$

Values of t should be calculated for eclipses by Kerbin, the Mun, and Minmus, and added together to find an estimate of the worst eclipse the satellite could see passing behind Kerbin.

Scenario 2 – Mun and Minmus eclipse at the worst possible time



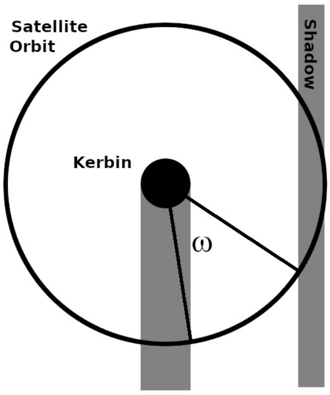
The angle ϕ is given by

$$\phi = 2 \times \cos^{-1}\left(1 - \frac{2 \times r_s}{r_c}\right)$$

where r_c is the semi-major axis of the satellite orbit, and r_s is the radius of the body which is generating the shadow. ϕ can then be converted into an eclipse duration as above.

The value of r_s should be the sum of the radii of all eclipsing bodies (i.e. the radius of the Mun plus that of Minmus).

In scenario 2, there is usually time between the Kerbin eclipse and the Mun+Minmus eclipse for batteries to recharge themselves via solar panels.



The angle ω is given by

$$\omega = \cos^{-1}\left(\frac{r_k}{r_c}\right) - \cos^{-1}\left(1 - \frac{2 \times r_s}{r_c}\right)$$

where r_k is the radius of Kerbin (600,000m), and r_s and r_c are as above.

Step 6 – positioning the satellites

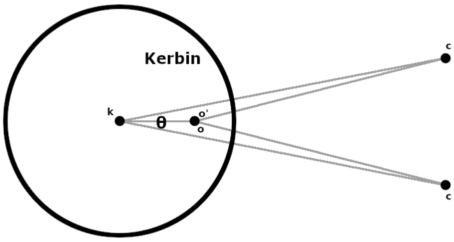
A method for ensuring the satellites in a constellation are in the correct relative positions is to target the last satellite in the constellation, use maneuver nodes, and tweak until the intercept distance is correct for the constellation.

For our constellation, the correct intercept distance is $2 \times r_c \times \sin(\theta/2)$

Alternatively, the constellation satellites can be launched from a mother ship, which is in a suitable resonant orbit. There are two such orbits that are usually used – one with an apoapsis of r_c, and a period of *(n-1)/n* times the period of the constellation satellites (e.g. 2/3 the period of a 3-constellation satellite). A constellation satellite is released at every apoapsis of the mother ship.. The other orbit has a periapsis of r_c, and a period of *(n+1)/n*, where the satellites are released at periapsis. Both orbits require the constellation satellites to go through a relatively low Δv, with the second periapsis-based orbit having a slightly lower Δv requirement.

Appendix A – Kerbin surface coverage

The constellation satellites can find themselves in orbits where they provide coverage for some of Kerbin's surface for certain working antennas. There are two limiting cases we need to consider. The first is that the range of the antennas is such that there is coverage to Kerbin's horizon, and the second is that the range falls short.

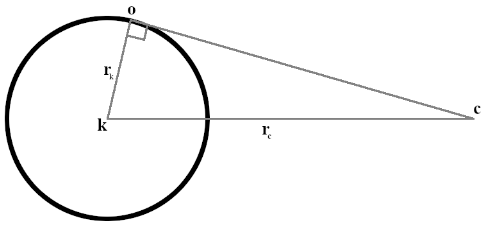


In this diagram, we see Kerbin from above. *k* is the centre of Kerbin, *o* is the observer on the surface of Kerbin, *o'* is the point directly below the observer, and *c* are the constellation satellites. The latitude of the observer is φ.

We first look at the triangle *koo'*, where we see

$$ko' = r_k \times \cos\phi \qquad oo' = r_k \times \sin\phi$$

There are two scenarios we need to consider. The first is that the range of the antennas are such that the constellation satellites have the range to communicate to the horizon of Kerbin.



In this diagram we are looking at an angle perpendicular to the triangle *koc*. We can see that $oc = \sqrt{r_c^2 - r_k^2}$

Now we can consider the triangle *oo'c*. From pythagoras,

$$o'c^2 = oc^2 - oo'^2 = r_c^2 - r_k^2 - r_k^2 \sin^2(\phi)$$

And now we can consider the triangle *ko'c*. From the cosine rule

$$r_c^2 - r_k^2 - r_k^2 \sin^2(\phi) = r_k^2 \cos^2(\phi) + r_c^2 - 2 \times r_k \times r_c \times \cos(\phi) \times \cos(\theta/2)$$

$$2 \times r_k \times r_c \times \cos(\phi) \times \cos(\theta/2) = 2 \times r_k^2$$

$$\phi = \cos^{-1}\left(\frac{r_k}{r_c \times \cos(\theta/2)}\right)$$

For example, when *rc* = 1,300,000m and *θ* = 120°, we get *φ* = 76.7°.

Now we need to consider the situation where the antenna ranges are such that communication to the horizon is not possible. Considering the triangle *oo'c*, we have $oc = d_{min}$, and applying pythagoras, we get $o'c^2 = d_{min}^2 - r_k^2 \sin^2(\phi)$

We can now apply the cosine rule to *ko'c*

$$d_{min}^2 - r_k^2 \sin^2(\phi) = r_k^2 \cos^2(\phi) + r_c^2 - 2 \times r_k \times r_c \times \cos(\phi) \times \cos(\theta/2)$$

$$2 \times r_k \times r_c \times \cos(\phi) \times \cos(\theta/2) = r_c^2 + r_k^2 - d_{min}^2$$

$$\phi = \cos^{-1}\left(\frac{r_c^2 + r_k^2 - d_{min}^2}{2 \times r_k \times r_c \times \cos(\theta/2)}\right)$$

For example, when r_c = 1,600,000m, d_{min} = 1,500,000m and *θ* = 120°, we get *φ* = 79.2°.

Appendix B – a worked example

Number of satellites in the constellation	3
Constellation antenna	Communotron 32
Constellation SMA	1,376,574.6m
Constellation altitude	776,574.6m
Constellation period	1h30m00.0s
Distance between satellites	2,384,297.2m
Maximum Kerbin+Mun+Minmus eclipse	18m21.8s
Worst Mun+Minmus eclipse	25m45.6s

Scenario 2 recharge time	3m09.6s
When paired against the Communotron 16-S in co-planar circular orbit	
Minimum altitude of working satellite	Kerbin to latitude ±77.4°
Maximum altitude of working satellite	998,661.8m
When paired against the Communotron 16 in co-planar circular orbit	
Minimum altitude of working satellite	Kerbin to latitude ±77.4°
Maximum altitude of working satellite	2,285,736.2m
When paired against the Communotron 32 in co-planar circular orbit	
Minimum altitude of working satellite	Kerbin to latitude ±77.4°
Maximum altitude of working satellite	4,944,086.1m