# Tarea 7

Sistema

$$\ddot{y} + 5\dot{y} + 6y = 0.5u(t)$$

Condiciones iniciales

$y(0) = 1$, $\dot{y}(0) = -1$

$$L\{\ddot{y}\} = s^2 Y(s) - s\,y(0) - \dot{y}(0)$$

$$= \boxed{s^2 Y(s) - s + 1}$$

$$L\{5\dot{y}\} = 5(sY(s) - y(0))$$

$$= \boxed{5sY(s) - 5}$$

$$L\{6y\} = \boxed{6Y(s)}$$

**1.** $u(t) = 2\sin(5t)$

$$L\{0.5u(t)\} = L\{\sin(5t)\} = \boxed{\frac{5}{s^2+5}}$$

Por lo anterior se obtiene:

$$s^2 Y(s) + 5sY(s) + 6Y(s) = \frac{5}{s^2+5^2} + s + 4$$

Despejando $Y(s)$:

$$Y(s) = \underbrace{\frac{5}{(s+2)(s+3)(s^2+5^2)}}_{A} + \underbrace{\frac{s}{(s+2)(s+3)}}_{B} + \underbrace{\frac{4}{(s+2)(s+3)}}_{C}$$

Utilizando matlab para calcular las fracciones parciales se obtiene:

Ⓐ:

$$\left(\frac{5}{29}\right)\frac{1}{(s+2)} - \frac{1}{986}\left(25\left(\frac{s}{s^2+5^2}\right)+95\left(\frac{1}{s^2+5^2}\right)\right) - \left(\frac{5}{34}\right)\frac{1}{(s+3)}$$

Ⓑ: $3\left(\frac{1}{s+3}\right) - 2\left(\frac{1}{s+2}\right)$

Ⓒ: $4\left(\left(\frac{1}{s+2}\right)\right) - (4)\frac{1}{(s+3)}$

Ahora aplicando para Ⓐ, Ⓑ y Ⓒ la transformada inversa de Laplace:

Ⓐ: $\left(\frac{5}{29}\right) \cdot e^{-2t} - \frac{25}{986} \cdot \cos(5t) - \frac{95}{986}\sin(t) - \frac{5}{34} \cdot e^{-3t}$.

Ⓑ: $3 \cdot e^{-3t} - 2 \cdot e^{-2t}$

Ⓒ: $4e^{-2t} - 4 \cdot e^{-3t}$

La respuesta $y(t)$ ante la entrada $u(t) = 2\sin(5t)$ es:

$$\boxed{y(t) = \frac{63}{29} \cdot e^{-2t} - \frac{39}{34} e^{-3t} - \frac{25}{986}\cos(5t) - \frac{95}{986} \cdot \sin(t).}$$

2. $u(t) = 4e^{-2t}$

$$\mathcal{L}\{0.5\,u(t)\} = \mathcal{L}\{2e^{-2t}\} = 2.\left(\frac{1}{s+2}\right)$$

obteniendo:

$$s^2Y(s) + 5sY(s) + 6Y(s) = \frac{2}{(s+2)} + s + 4$$

Despejando $Y(s)$:

$$Y(s) = \underbrace{\frac{2}{(s+2)^2(s+3)}}_{A} + \underbrace{\frac{s}{(s+2)(s+3)}}_{B} + \underbrace{\frac{4}{(s+2)(s+3)}}_{C}$$

Utilizando matlab para calcular las fracciones parciales solo de A, ya que B y C ya se calcularon en el numeral anterior.

$$A \quad \frac{-2}{s+2} + \frac{2}{(s+2)^2} + \frac{2}{(s+3)}$$

Ahora aplicando para A la transformada inversa de Laplace

$$A: \quad -2.e^{-2t} + e^{-2t}\cdot(2t) + 2e^{-3t}$$

La respuesta $y(t)$ ante la entrada $u(t) = 4e^{-2t}$ es:

$$y(t) = -2e^{-2t} + 2t.e^{-2t} + 2e^{-3t} + 3e^{-3t} - 2e^{-2t} + 4e^{-2t} - 4e^{-3t}$$

$$y(t) = (\cancel{-2} + 2t \cancel{-2} \cancel{+4}).e^{-2t} + (2+3-4)e^{-3t}$$

$$\boxed{y(t) = 2t.e^{-2t} + e^{-3t}}$$

3. $u(t) = 6e^{-3t} \cdot \cos(2t)$

$$\mathcal{L}\{0.5\,u(t)\} = \mathcal{L}\{3e^{-3t}\cos(2t)\} = \frac{3(s+3)}{(s+3)^2+4}.$$

Por lo anterior se obtiene

$$s^2Y(s) + 5sY(s) + 6Y(s) = \frac{3(s+3)}{(s+3)^2+4} + s + 4.$$

Despejando $Y(s)$:

$$Y(s) = \frac{3\cancel{(s+3)}}{(s+2)\cancel{(s+3)}((s^2+5^{?2})+4)} + \frac{s}{(s+2)(s+3)} + \frac{4}{(s+2)(s+3)}$$

$$Y(s) = \underbrace{\frac{3}{(s+2)(s^2+5^{?2})+4)}}_{A} + \underbrace{\frac{s}{(s+2)(s+3)}}_{B} + \underbrace{\frac{4}{(s+2)(s+3)}}_{C}$$

Utilizando matlab para calcular las fracciones parciales de A

A: $\frac{3}{5(s+2)} + \frac{-3s-12}{5(s^2+6s+13)}$

Ahora aplicando para A la transformada inversa de laplace

A $\frac{3}{5}e^{-2t} - \frac{3}{5} \cdot e^{-3t}\cos(2t) - \frac{3}{10}e^{-3t}\sin(2t)$

La respuesta $y(t)$ ante la entrada $u(t)=6e^{-3t}\cos(2t)$ es

$$y(t)=\frac{3}{5}e^{-2t}-\frac{3}{5}e^{-3t}\cos(2t)-\frac{3}{10}e^{-3t}\sin(2t)+3e^{-3t}-2e^{-2t}+4e^{-2t}-4e^{-3t}$$

$$y(t)=\left(\frac{3}{5}-2+4\right)e^{-2t}+(3-4)e^{-3t}-\frac{3}{5}e^{-3t}\cos(2t)-\frac{3}{10}e^{-3t}\sin(2t)$$

$$\boxed{y(t)=\frac{13}{5}e^{-2t}-e^{-3t}-\frac{3}{5}e^{-3t}\cos(2t)-\frac{3}{10}e^{-3t}\sin(2t)}$$

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4. Funciones de transferencia

Power amplifier: $G_1(s)=\dfrac{25}{0.003s+1}$

Solenoid: $G_2(s)=\dfrac{1.6}{0.002s+1}$

Spool Value: $G_3(s)=\dfrac{1}{0.035s^2+7s+1800}$

Teniendo en cuenta que $G(s)=G_1(s)\,G_2(s)\cdot G_3(s)$, se obtiene:

$$G(s)=\frac{40}{(0.003s+1)(0.002s+1)(0.035s^2+7s+1800)}$$

$$G(s)=\frac{40}{0.003\left(s+\frac{1000}{3}\right)0.002(s+500)\,0.035\left(s^2+200s+\frac{360000}{7}\right)}$$

$$G(s)=\frac{40}{2.1\times10^{-7}}\left(\frac{1}{\left(s+\frac{1000}{3}\right)(s+500)\left(s^2+200+\frac{360000}{7}\right)}\right)$$

Dado que $Z(s) = G(s) \cdot E_{in}(s)$.

$$e_{in}(t) = 2\sin t \rightarrow \boxed{E_{in}(s) = \frac{2}{s^2+1}}$$

Multiplicando $G(s)$ y $E_{in}(s)$ se obtiene:

$$Z(s) = \frac{80}{2.1\times10^{-7}}\left(\frac{1}{(s+1000/3)(s+500)(s^2+200s+360000/7)(s^2+1)}\right)$$

Encontrado los fracciones parciales en matlab y aplicando la transformada inversa de laplace se obtiene la respuesta $z(t)$ del sistema

$$\boxed{z(t) = \frac{80}{2.1\times10^{-7}}\left(\frac{834941623424\,e^{-\frac{1000}{3}t}}{148239062572126301642 7125} - \frac{2^{35}\cdot 3e^{-500t}}{125\cdot13\cdot19\cdot53^2\cdot89\cdot7607\cdot1473399833} - \frac{5181040386678002732374470871552\cos(t)}{4996019204886013522578893976005690739\,5761}\right)}$$

5. $e_{in}(t) = 4e^{-t}\cos 2t \rightarrow E_{in}(s) = 4\frac{(s+1)}{(s+1)^2+4}$

Multiplicando $G(s)$ y $E_{in}(s)$ se obtiene:

$$Z(s) = \frac{160}{2.1\times10^{-7}}\left(\frac{s+1}{(s+1000/3)(s+500)(s^2+200s+360000/7)((s+1)^2+4)}\right)$$

Encontrando las fracciones parciales en matlab y aplicando la transformada inversa de laplace se obtiene la respuesta $z(t)$ del sistema:

(Nota: la respuesta era demasiado larga asi que adjunto un pantallazo de matlab)

$$\frac{3918973587573028600627 2\,\mathrm{e}^{-500\,t}}{8616859128411867384106 25} - \frac{2114125250578724587180032\,\mathrm{e}^{-\frac{1000\,t}{3}}}{1473549727151086429738562 5}$$

$$+\frac{7127574191829879689158917574406044914193665973532474874632601 6\,\mathrm{e}^{-100\,t}\left(\cos(\sigma_1)-\frac{41890035012192455560851180284297897574 4\,\sqrt{2}\,\sqrt{5693899500982857}\,\sin(\sigma_1)}{31332039659153225065829766081285748882704182 11}\right)}{85653537687489126286922630574878943183469945559757843735352690 75}$$

$$+\frac{4400884698984967957463499630513163975065 6\,\mathrm{e}^{-t}\left(\cos(2\,t)+\frac{2487752734587638972\,\sin(2\,t)}{1400903460894131761 63}\right)}{4907873495280002463734175521503271154838 25}$$

where

$$\sigma_1 = \frac{\sqrt{2}\,\sqrt{5693899500982857}\,t}{524288}$$