Study Report: Narrowing

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Abstract

Narrowing...

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In this section, we discuss the use of narrowing techniques on solving equations over an equational theory E. By solving an equation we mean finding a substitution σ such that $s\sigma =_E t\sigma$. These solutions can be found by unification *modulo E* (if such unification algorithm exists for this theory), but here we are interested in applications of rewriting theory to solving such equations.

We start the presentation by the so-called syntactic unification, i.e. E-unification with empty E. The transformation rules given below is due to Martelli and Montanari [citation], the idea is to transform sets of equations to other sets of equations until a termination state is reached; that is, a solution state or a failure one. We also extend this same set of rules to solving E-equations.

Definition 1. Let Σ be a signature. An equational goal is a finite set of Σ-equations.

Table 1 Martelli-Montanari rules

(1) Trivial

$$\{x \stackrel{?}{=} x\} \cup G \implies G$$

Delete trivial equations.

(2) Decompose

$$\{f(s_1, \cdots, s_n) \stackrel{?}{=} f(t_1, \ldots, t_n)\} \cup G \implies \{s_1 \stackrel{?}{=} t_1, \ldots, s_n \stackrel{?}{=} t_n\} \cup G$$

(3) Symbol Clash

$$\{f(s_1, \ldots, s_n) \stackrel{?}{=} g(t_1, \ldots, t_n)\} \cup G \implies \perp \text{ if } f \neq g$$

(4) Orient

$$\{t \stackrel{?}{=} x\} \cup G \implies \{x \stackrel{?}{=} t\} \cup G \text{ if } t \notin V$$

(5) Occurs Check

$$\{x \stackrel{?}{=} t\} \cup G \implies \perp \text{ se } x \in \texttt{vars}(t) \text{ and } x \neq t$$

(6) Variable Elimination

$$\{x \stackrel{?}{=} t\} \cup G \implies G\{x \mapsto t\} \text{ if } x \notin \texttt{vars}(t)$$

The application of the above rules non-deterministically transforms goals into goals:

$$G_0 \implies \cdots \implies G_n$$

Each application of a rule will then called a *elementary derivation step*. As for the case of (Variable Elimination), we may get some substitution on the way, we make them explicit by writing:

$$G_0 \implies G_1 \implies_{\sigma_1} G_2 \implies_{\sigma_2} \cdots \implies_{\sigma_i} \cdots \implies_{\sigma_{n-1}} G_n$$

The computed solution of the derivation chain is then the composition of such substitutions in their order of appearance.

Definition 2. A *successful* derivation chain is a finite sequence of equational goals $G_0, G_1, \dots, G_n$ such that the last goal is empty. We also say a derivation chain has *failed* if it end is the fail symbol $\perp$.

Example 1. 1. We want to determine an mgu of the terms $f(g(x), h(x, u))$ and $f(z, h(f(y, y), z))$. That is, solving the equational goal:

$$\begin{aligned}
&\{f(g(x), h(x, u)) = f(z, h(f(y, y), z))\} \\
&\qquad \overset{(2)}{\Longrightarrow} \{g(x) = z, h(x, u) = h(f(y, y), z)\} \\
&\qquad \overset{(4)}{\Longrightarrow} \{z = g(x), h(x, u) = h(f(y, y), z)\} \\
&\qquad \overset{(6)}{\Longrightarrow}_{[z/g(x)]} \{h(x, u) = h(f(y, y), g(x))\} \\
&\qquad \overset{(2)}{\Longrightarrow} \{x = f(y, y), u = g(x)\} \\
&\qquad \overset{(6)}{\Longrightarrow}_{[x/f(y,y)]} \{u = g(f(y, y))\} \\
&\qquad \overset{(6)}{\Longrightarrow}_{[u/f(y,y)]} \emptyset
\end{aligned}$$

We get as the computed solution to the problem the composition $[z/g(x)][x/f(y, y)][u/g(f(y, y))]$.

2. A failing unification derivation:

$$\begin{aligned}
&\{h(x, y, x) = h(y, g(x), x)\} \\
&\qquad \overset{(2)}{\Longrightarrow} \{x = y, x = g(x), x = x\} \\
&\qquad \overset{(5)}{\Longrightarrow} \perp
\end{aligned}$$

It can be proved that this set of rules derive a correct and terminating procedure for syntactic unification. (add citation here).

The situation changes if we whant to solve equations for a non-empty equation theory E.

References