Approximation of PDEs solutions by means of Machine Learning Models

Master Thesis Internship Training Proposal

Location : CEA DES/ISAS/DM2S, CEA Saclay 91191 Gif-sur-Yvette CEDEX

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A large number of applications in the CEA requires the modelling of physical phenomena, which leads to the construction of Partial Differential Equations (PDEs) that must be solved. Classical methods exist to numerically approximate solutions of these equations, such as finite elements or finite volumes, with some known limitations (dependence to a mesh, high computation ressources). Over the past decade, Machine Learning (ML) methods have brought new tools to solve problems in several fields, with interesting performances in many applications. A specific branch is emerging to propose models to solve PDEs, like [4, 2, 3]. Some of them are based on Neural Networks that embed physical constraints, leading to the concept of Physics Informed Neural Networks (PINNs).

In this Master Thesis training subject we propose to test some of these ML methods for approximating PDEs solutions. The student will be asked to conduct a non-exhaustive state of the art of recent methods for approximating the solution of PDEs thanks to Machine Learning.

For application, we propose to consider several classical types of PDEs like:

- the (non-homogeneous) Poisson equation

$$\partial_x^2 u + \partial_y^2 u = f(u, x, y), \tag{1}$$

- the Burgers equation (with a source term)

$$\partial_t u + \partial_x \left(\frac{u^2}{2} \right) = S(u), \tag{2}$$

- the linear first order wave equation system with a source term

$$\begin{cases} \partial_t u + \partial_x \pi = f(u), & \text{(3a)} \\ \partial_t \pi + a^2 \partial_x u = 0, & \text{(3b)} \end{cases}$$

The first objective is to build approximations of the solution of these equations thanks to different machine learning methods among those provided [4, 2, 3]. One of the goal of the present study is to investigate the performance of these methods in terms of computing cost, their stability and their accuracy. For example, we would like to assess the ability of the method to capture stationary solutions of the equation that balance fluxes terms and (possibly stiff) source terms.

According to the progression of the project, other problems in higher dimensions can be considered, such as the plane channel problem in fluids mechanics, associated to the Navier-Stokes equations.

The candidate methods under study will be implemented in Python using TensorFlow and standard tools like Jupyter and git.

References

[1] Tensorflow. https://www.tensorflow.org/.

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[3] Ravi G. Patel, Indu Manickam, Nathaniel A. Trask, Mitchell A. Wood, Myoungkyu Lee, Ignacio Tomas, and Eric C. Cyr. Thermodynamically consistent physics-informed neural networks for hyperbolic systems. *Journal of Computational Physics*, 449:110754, January 2022.

[4] M. Raissi, P. Perdikaris, and G.E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, February 2019.